"PROBABILITY TRANSFORMS" OF DIGITAL PICTURES

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ABSTRACT

Let \( g \) be any local property (e.g., gray level or gradient magnitude) defined on a digital picture. Let \( p_g(z) \) be the relative frequency with which \( g \) has value \( z \). At each point \((x,y)\) of the picture we can display \( p_g(g(x,y)) \), appropriately scaled; the result is called the \( p_g \)-transform of the picture. Alternatively, we can use joint or conditional frequencies of pairs of local properties to define transforms. This note gives examples of such transforms for various \( g \)'s and \( h \)'s and discusses their possible uses and limitations.

The support of the Defense Advanced Research Projects Agency and the U.S. Army Night Vision Laboratory under Contract DAAG-53-76C-0138 (DARPA Order 3206) is gratefully acknowledged, as is the help of Kathryn Riley in preparing this paper.
1. **Introduction**

Haralick [1] has introduced a "texture transform" which measures the "typicality" of each point's neighborhood in a digital picture. It is defined as follows: For each pair of gray levels \( z, w \), and any given unit displacement \( \Delta = (\Delta x, \Delta y) \), let \( p_\Delta(z,w) \) be the relative frequency with which pairs of points ((\( x, y \)), (\( x+\Delta x, y+\Delta y \))) have the pair of gray levels (\( z, w \)). (Thus \( p_\Delta(z,w) \) is the (\( z, w \)) element in the "cooccurrence matrix" for displacement \( \Delta \).) By definition, \( p_\Delta(z,w) \) is high if gray level \( z \) occurs often adjacent (in direction \( \Delta \)) to gray level \( w \). Let \( \mathcal{N} \) be the set of displacements \( \Delta \) corresponding to the eight neighbors of a point. Then the *texture transform* of the given picture is the array whose value at \( (x,y) \) is

\[
\frac{1}{8} \sum_{\Delta \in \mathcal{N}} p_\Delta(g(x,y) | g(x+\Delta x, y+\Delta y))
\]

Evidently this is high if \( g(x,y) \) is a "typical" neighbor of its neighbors. For example, it should be high in the interiors of large, uniform regions, and low at region borders. The texture transform can be displayed as a picture by appropriately scaling its values.

This note presents a general class of "texture transforms" in which we use frequencies of arbitrary local property values,

*Haralick gives a more general definition in which an arbitrary function of \( P \) can be used; but the simplified definition given above is sufficient for our purposes.*
rather than joint frequencies of pairs of gray levels. We call the result a "probability transform", since its values are relative frequencies, which can be regarded as probability estimates. The general definition, and some examples, are given in Section 2. In Section 3 we briefly consider "second-order probability transforms" whose values are the joint or conditional relative frequencies with which a pair of local properties has a given pair of values.
2. **First-order probability transforms**

Let $g$ be any local property (e.g., gray level, gradient magnitude, etc.) defined on a digital picture. Let $p_g(z)$ be the relative frequency with which $g$ has value $z$. The $p_g$-transform of the picture is simply the array of values $p_g(g(x,y))$. This array can be displayed as a picture after appropriate rescaling.

Figure 1-9 show the $p_g$-transforms of a set of pictures for the following local properties:

a) **gray level**: $g(x,y) = f(x,y)$

b) **local average gray level**: $g(x,y) = \frac{1}{9} \sum_{(x+\Delta x, y+\Delta y) \in N} f(x+\Delta x, y+\Delta y) \equiv \bar{f}(x,y)$
   (the average is taken over the 3-by-3 neighborhood centered at $(x,y)$)

c) **Laplacian magnitude**: $g(x,y) = |f(x,y) - \bar{f}(x,y)|$

d) **gradient magnitude**: $g(x,y) = \max[|f(x,y) - f(x+1,y+1)|, |f(x+1,y) - f(x,y+1)|]$  

The properties themselves are displayed as parts (a-d) of each figure, and the corresponding transforms as parts (e-h).

The general nature of a $p_g$-transform can be predicted from a knowledge of the histogram of $g(x,y)$ values. The following are some simple examples:

1. If the $g(x,y)$ histogram is flat, the relative frequencies are all equal, so that $p_g(g(x,y))$ is a constant.

2. At the other extreme, if the $g$ histogram consists of a few spikes, then $p_g(g(x,y))$ too has only a few distinct values, which depend on the heights of the spikes.
(3) Suppose that the $g$ histogram is monotonic, e.g., the histogram of gradient or Laplacian magnitudes, for most pictures, looks like a negative exponential with values decreasing sharply from a maximum at $z=0$. Then the $p_g$ values are also all distinct, and the $p_g$-transform can be regarded as a nonlinear rescaling (and complementation) of the original $g$ values.

(4) More generally, let the $g$ histogram be unimodal. If the peak is symmetric, then the pairs of equally frequent values map into the same $p_g$ value; thus the $p_g$-transform can be regarded as "folding over" the $g$ values and then nonlinearly rescaling them. If the peak is nonsymmetric, but still monotonic on each side, the rescaling affects the two sides differently. In particular, if there is a flat shoulder on one side of the peak, all $g$ values on that shoulder map into a constant.

(f) Finally, suppose that the $g$ histogram is bimodal. If the peaks are equal and symmetric, we have quadruples of equally frequent values, so that the $p_g$-transform can be regarded as a double folding and rescaling of the $g$ values. If the peaks are unequal, the $p_g$ values into which the small peak maps are a subset of those into which the large peak maps.

We can now comment on the examples shown in Figures 1-9 in the light of these general remarks. The histograms for the pictures in Figures 1-9 are shown in Figures 10-18.
a-b,e-f) For the first four pictures, there is a large peak in the histogram of gray levels or local average gray levels, representing background points. In the handwriting and chromosome pictures, the object points are represented by a small, separate peak; in the cloud picture, by a relatively flat shoulder; and in the tank picture, by a shallower-sloping flank. Thus for the tank picture, for example, most of the low-probability points come from the tank rather than from the background, so that the transform yields a roughly complemented picture on a noisier background. For the cloud picture, the cloud points map into a relatively constant value corresponding to the relatively low probability of the flat shoulder. The results for the chromosome picture are more interesting; the cross-sections of the chromosomes evidently contain a flat zone encircling their centers, and points in this zone have relatively high frequencies, so that they are slightly darker in the transform. For the terrain pictures (Figs. 5-9), the transform produces a remarkable enhancement; but the results are sensitive to noise. (compare Fig. 6 with Fig. 5).

c-d, g-h) The histograms of gradient and Laplacian values for all the pictures all decline sharply from a peak at zero, so that these g-transforms of the pictures are essentially rescaled, complemented displays of the gradient or Laplacian magnitude.
In [2] it was shown that probability transforms may be of some value in smoothing a picture. For example, suppose that we average each point with those k of its neighbors that have highest frequencies of occurrence. If a point is near a region border, this will tend to average it with neighbors that are interior to the region, so that the averaging should not blur the border.

The situation is more complicated with regard to using probability transforms for segmentation. Our examples show that small differences in the original may be enhanced on the transform; but the "folding" in the transforms confounds values that were originally easy to distinguish. It appears that the value of probability transforms for segmentation must be assessed on a case by case basis.

The histogram of a g-transform will usually not be very useful as an aid in segmenting the transform. To see why, note that the number of points having value v in the transform must be a multiple of v. In fact, a point has value v in the g-transform iff its g-value in the picture occurred with frequency proportional to v, so that the number of such points is proportional to v. The proportionality factor depends on how many g-values in the picture had that same relative frequency. If values having the same frequency are rare, the proportionality factor will always be the same, and the histogram of the transform will be ramplike.
(It will not be a solid ramp, since not every frequency of occurrence will actually occur; but it will be a set of spikes whose envelope is a ramp.) When two g-values do have the same frequency, the corresponding spike is doubled in height. These remarks were confirmed by the histograms shown in Figures 10-18.
3. **Second-order probability transforms**

Let $g$ and $h$ be any local properties defined on a digital picture, and let $p_{gh}(z|w)$ be the relative frequency with which $g$ has value $z$, given that $h$ has value $w$. The array of values $p_{gh}(g(x,y)|h(x,y))$ will be called the $p_{gh}$ transform of the picture. Similarly, let $p_{gh}(z,w)$ be the relative frequency with which $g$ and $h$ have the pair of values $(z,w)$; the array $p_{gh}(g(x,y),h(x,y))$ will be called the picture's $p_{gh}$-transform.

If we let $h(x,y) = g(x+\Delta x, y+\Delta y)$, we see that $p_{\Delta}$ defined in Section 1 is a special type of $p_{gh}$, and that Haralick's texture transform is just the average of eight such $p_{gh}$-transforms, one for each neighbor.

Figures 19-27 show the $p_{gh}$ and $p_{g|h}$ transforms of the same set of pictures as in Figure 1 for $g =$ gray level (i.e., $g(x,y) = f(x,y)$) and the following $h$'s:

a) Laplacian magnitude
b) Gradient magnitude
c) Average gray level
d) Gray level of the right-hand neighbor

Parts (a-d) show the transforms based on joint frequencies, and parts (e-h) those based on conditional frequencies. The histograms for these pictures are shown in Figures 28-36. Many of the second-order joint transforms resemble the first-order transforms, but the second-order conditional transforms do not; the
latter yield some interesting enhancements (e.g., see the chromosome picture). In most cases, however, the second-order transforms do not appear to be as useful as the first-order ones.

To further illustrate the usefulness of probability transforms in smoothing, Figures 37 and 38 show five iterations of a local weighted averaging scheme, where the weights given to the neighbors of a given point depend on how high their joint or conditional (gray level, gradient magnitude) frequencies are, rather than on how similar their gray levels are. The results are much better than those obtained in [2] based on gray level similarity.
4. Concluding remarks

This note has presented some generalizations of Haralick's "texture transform," and has discussed their possible uses and limitations. Such transforms have potential applications to image enhancement, noise cleaning and segmentation. They certainly provide new ways of looking at a picture which, in some cases, may yield useful insights.
References


Figure Captions

Figures

1-9  First-order probability transforms for a set of pictures
   a) Original
   b) 3x3 local average
   c) Roberts gradient magnitude
   d) Laplacian magnitude
   e-h) Probability transforms of (a-d)

10-18  Histograms of the pictures in Figures 1-9

19-27  Second-order probability transforms for the pictures in Figures 1-9
   a-d) Transforms based on joint frequencies
   e-h) Transforms based on conditional frequencies
   The first property is always the gray level;
   the second is
      a,e) Laplacian magnitude
      b,f) Roberts gradient magnitude
      c,g) 3x3 local average gray level
      d,h) Gray level of right-hand neighbor

28-36  Histograms of the pictures in Figures 19-27

37  Noise cleaning by iterated weighted averaging, where the weight given to each neighbor is proportional to joint (gray level, gradient magnitude) frequency

38  Analogous to Figure 37, using conditional frequency
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**Contract or Grant Number:** DAAG-53-76C-0138

**Prepared for:** U.S. Army Night Vision Laboratory  
Fort Belvoir, VA 22060

**Number of Pages:** 22

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