Fluorics 41: Single-Sided Control Port Characteristics of Laminar Proportional Amplifiers for Arbitrary Input Loading

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This report presents a technique for analytically predicting single-sided control port characteristics of a laminar proportional amplifier (LPA) with arbitrary input circuits at the opposite control using calculable differential characteristics. Specific cases dealt with include a constant pressure at the opposite control, a constant flow at the opposite control, and a series and shunt flow resistance at the opposite control.
Item 20 (cont'd)

Experiments using amplifiers at various aspect ratios (height-to-width ratio of supply nozzle) and supply pressures, which imply a large range of LPA resistance, show a maximum error of 11 percent between actual and computed single-sided control port flow resistance.
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16. Control port 1 (C1) pressure-flow characteristic for series and shunt resistance at control port 2, with \( P_s = 0.46 \) kPa

17. Control port 1 (C1) pressure-flow characteristic for series and shunt resistance at control port 2, with \( P_s = 0.15 \) kPa
1. INTRODUCTION

Over the past five years, considerable effort has been expended to analytically determine
the differential input or control (pressure-flow) characteristics of laminar proportional amplifiers
(LPA’s).\textsuperscript{1,2} With increasing emphasis on sensing/control systems, however, it has become im-
portant to analytically predict single-sided input characteristics for various input circuits at the
opposite control. Unfortunately, little or no work has been done along these lines and thus
information is unavailable in technical literature. The ability to predict single-sided input char-
acteristics would greatly facilitate the design of many fluidic systems such as fluidic temperature
sensors, strain gages, speed sensors, and all devices that operate on the back-pressure principle.

Figure 1 depicts the current state-of-the-art LPA (Harry Diamond Laboratories—HDL—
Model 3.1.1.8), which will be the only component considered. With reference to the interaction
region (dotted line), it is apparent that if a pressure is applied at control port 1 with control port
2 open to ambient ($P_{c2} = 0$), a deflection of the supply jet would result. Consider now the case
where the same pressure is applied at control port 1 while control port 2 is held at constant
pressure other than zero ($P_{c2} = \text{constant}$). The amount of jet deflection is now reduced, yielding
a new pressure-flow ($P-Q$) relationship for control port 1. This effect is not limited only to
constant pressures at the opposite control, but has been observed for constant flows and arbitrary
input circuits at port 2 as well.

Analytical expressions are derived for the pressure-flow relationships at control port 1 for
three cases: (1) constant pressure on control port 2, (2) constant flow on control port 2, and (3)
a series/shunt resistance (as being a representation of some general load) at control port 2. These
expressions are derived by use of calculable differential pressure-flow relationships. Experimental
verification for various aspect ratios (height-to-width ratio of supply nozzle, denoted by $\sigma$) and
supply pressures is presented, followed by the summary and conclusions.

\textsuperscript{1} F. M. Manion and T. M. Drzewiecki, Analytical Design of Laminar Proportional Amplifiers, Proceedings of The
\textsuperscript{2} T. M. Drzewiecki, Fluidics 38. A Computer-Aided Design Analysis for the Static and Dynamic Port Characteristics
Figure 1. HDL Model 3.1.1.8 LPA.
2. ANALYSIS

Recently, a computer algorithm has been developed\(^2\) that can predict differential control port characteristics of LPA’s. The analysis presented here is an extension of this previous work in that computed differential characteristics are used as a basis for the derivation of single-sided characteristics. Only the deflected-jet and centered-jet characteristics need be known. These are shown in figure 2.

![Figure 2. LPA differential input characteristics.](image)

The centered-jet characteristic corresponds to the control port pressure-flow relationship while the supply jet is maintained at its center position. The deflected-jet characteristic, on the other hand, corresponds to the control port pressure-flow relationship as the jet is being deflected with a “push-pull” differential input signal. This, in turn, states that the bias or average control pressure, \(P_b\), where

\[ P_b = \frac{P_{c1} + P_{c2}}{2}, \]

is constant for a given deflected-jet characteristic. Thus, an infinite number of deflected-jet characteristics exist—one for every value of bias pressure, \(P_b\). Fortunately, however, the inverse slope, \(R_d\), of the deflected-jet characteristic is nearly constant with bias pressure, \(P_b\), for \(P_b\) not close to zero. The following analysis assumes that

(a) Centered-jet and deflected-jet characteristics are linear.

(b) The inverse slope of the deflected-jet characteristics is constant with bias pressure.

(c) The derived single-sided characteristic will also be linear.

2.1 Constant Pressure on Opposite Control

The physical configuration for this case is shown in figure 3. Here, the pressure-flow \((P-Q)\) relationship at control port \(C_1\) is desired while control port \(C_2\) is held at constant pressure, \(P^*\).

With the assumption of a linear pressure-flow relationship at \(C_1\), only the inverse slope and

pressure axis intersection need be determined. An expression for the inverse slope can be derived easily after two \( P-Q \) coordinates have been defined.

For convenience, assume that the jet is initially centered. This corresponds to point 1 in figure 4. Coordinates for point 1 are

\[
P_{c11} = P^* 
\]

and

\[
Q_{c11} = \frac{P^* - P_{\text{offset}}}{R_{c1}}
\]

for

\[
P_b = P^* .
\]
Now, by increasing the bias pressure to $2P^*$ while holding $P_{c2}$ equal to $P^*$, $P_{e12}$ is found by use of the bias pressure equation

$$P_b = 2P^* = \frac{P_{c12} + P^*}{2},$$

or

$$P_{e12} = 3P^*$$

From above, then, $\Delta P_{e1} = P_{e12} - P_{e11} = 2P^*$. The quantity $\Delta Q_{e1} = Q_{e12} - Q_{e11}$ can now be formulated after several intermediate steps. Since $Q_{e11}$ is already known, only $Q_{e12}$ need be found. Again, referring to figure 4, it is seen that

$$Q_{c12} = \Delta Q_c - Q_{e22}$$

where

$$\Delta Q_c = \frac{2P^*}{R_d}$$

and

$$Q_{e22} = Q_5 - Q_4$$

where

$$Q_5 = \frac{P^*}{R_d}$$

and

$$Q_4 = \frac{2P^* - P_{\text{offset}}}{R_{ej}}$$

The final expression for $\Delta Q_{e1}$ becomes

$$\Delta Q_{e1} = \frac{P^*(R_{c1} + R_d)}{R_{c1}R_d}$$

which yields

$$R_{c1} = \frac{\Delta P_{e1}}{\Delta Q_{e1}} = \frac{2R_{c1}R_d}{R_{c1} + R_d}. \quad (2)$$

The intersection of the $C1$ curve with the pressure axis can now easily be found to be

$$P_{c1} \bigg|_{Q_{c1}=0} = \frac{P^*(R_{ej} - R_{c1}) + R_{c1}P_{\text{offset}}}{R_{ej}}$$

thus completing the entire expression,

$$P_{c1} = R_{c1}Q_{c1} + \frac{P^*(R_{ej} - R_{c1}) + R_{c1}P_{\text{offset}}}{R_{ej}}. \quad (3)$$
2.2. Constant Flow at Control Port C2

Consider now the case where a constant amount of flow is maintained at C2. This occurs in reality when a very high resistance from a constant pressure is applied. Variations in the control resistance do not materially affect the total resistance; hence, flow is approximately constant. Again, by construction and assumption of linearity, the inverse slope, $R_{cl}$, of the $P-Q$ characteristic can be determined.

The supply jet is initially assumed to be in its centered position, which corresponds to point 1 in figure 5. Pressure-flow coordinates are

$$P_{c11} = Q^* R_{cj} + P_{offset}$$

and

$$Q_{c11} = Q^* .$$

Now, the bias pressure is increased to $P_3$ so that the corresponding flow rate is $2Q^*$ while a constant flow rate of $Q^*$ is still maintained at control port 2. This defines point 2 such that

$$P_{c12} = P_3 + Q^* R_d$$

where

$$P_3 = P_{c11} + Q^* R_{cj} \text{ and } Q_{c12} = 3Q^* .$$

The inverse slope, $R_{c1}$, of the control port 1 characteristic can now be formed.

$$R_{c1} = \frac{\Delta P_{c1}}{\Delta Q_{c1}} = \frac{P_3 + Q^* R_d - Q^* R_{cj} - P_{offset}}{2Q^*}$$

or

$$R_{c1} = \frac{R_{cj} + R_d}{2} . \quad (3)$$

---

![Figure 5. Construction of single-sided characteristic.](image-url)
Pressure-flow coordinates $P_{c11}$ and $Q_{c11}$ are now used to determine the intersection with the pressure axis,

$$b = P_{c11} - Q_{c11}R_{c1}$$

or

$$b = Q^*R_{c1} + P_{offset} - Q^*R_{c1},$$

thus completing the final expression,

$$P_{c1} = Q_{c1}R_{c1} + Q^*(R_{cj} - R_{c1}) + P_{offset}. \quad (4)$$

2.3. Series and Shunt Resistance at Control Port C2

Consider the configuration shown in figure 6.

![LPA Physical Configuration](image)

Figure 6. LPA physical configuration for series and shunt resistance at control port C2.

In order to derive an expression for the pressure-flow relationship of control port 1, a load line for control port 2 is first constructed (fig. 7). From continuity it is seen that

$$Q_2 = Q_b + Q_{c2},$$

where

$$Q_2 = \frac{P_1 - P_{c2}}{R_1} \quad \text{and} \quad Q_b = \frac{P_{c2}}{R_b}$$

or

$$Q_{c2} = \frac{P_1 - P_{c2}}{R_1} - \frac{P_{c2}}{R_b}, \quad (5)$$

thus defining the $P-Q$ relationship for control port 2. Since point 1 lies on the jet-centered curve, $P_{c1} = P_{c2}$ and $Q_{c1} = Q_{c2}$, equating the flows yields

$$\frac{P_{c1} - P_{offset}}{R_{cj}} = \frac{P_1 - P_{c2}}{R_1} - \frac{P_{c2}}{R_b}.$$
However, \( P_{c1} = P_{c2} \); hence,
\[
\frac{P_{c1} - P_{\text{offset}}}{R_{cj}} = \frac{P_1 - P_{c1}}{R_1} - \frac{P_{c1}}{R_b}
\]
or
\[
P_{c11} = \frac{P_1}{R_1 \left( \frac{1}{R_{cj}} + \frac{1}{R_1} + \frac{1}{R_b} \right)} + \frac{P_{\text{offset}}}{R_{cj} \left( \frac{1}{R_{cj}} + \frac{1}{R_1} + \frac{1}{R_b} \right)}
\]
and
\[
Q_{c11} = \frac{P_{c11} - P_{\text{offset}}}{R_{cj}}
\]
thus defining point 1. Once again the bias pressure is increased and the jet is deflected so that \( P_{c22} \) is the intersection of the deflected jet characteristic, the load line for \( C2 \), and the pressure axis (this can be done for convenience without any loss of generality). \( P_{c12} \) can now be solved for with the use of the constant bias conditions on the deflected-jet curve,
\[
P_3 = \frac{P_{c22} + P_{c12}}{2}
\]
or
\[
P_{c12} = 2P_3 - P_{c22} \tag{6}
\]
\( P_{c22} \) is obtained from equation 5 with \( Q_{c2} = 0 \),
\[
P_{c22} = \frac{P_1 R_b}{R_b + R_1} \tag{7}
\]
$P_3$ is obtained by solving the centered-jet and deflected-jet characteristics simultaneously.

\[
\frac{P_3 - P_{\text{offset}}}{R_{cj}} = \frac{P_3}{R_d} - \frac{P_1 R_b}{(R_b + R_1) R_d}
\]

or

\[
P_3 = \frac{P_{\text{offset}} R_d}{R_{cj}} - \frac{P_1 R_b}{R_{cj} \left( \frac{R_d}{R_{cj}} - 1 \right)} - \frac{P_1 R_b}{(R_b + R_1) \left( \frac{R_d}{R_{cj}} - 1 \right)}
\]

(8)

With equation 6, $P_{c12}$ is found to be

\[
P_{c12} = \frac{P_1 R_b \left( 1 + \frac{R_d}{R_{cj}} \right)}{(R_b + R_1) \left( 1 - \frac{R_d}{R_{cj}} \right)} - \frac{2 R_d P_{\text{offset}}}{R_{cj} \left( 1 - \frac{R_d}{R_{cj}} \right)}
\]

(9)

$Q_{c12}$ can now be obtained by the use of the equation of the deflected-jet curve,

\[
P_{c12} = R_d Q_{c12} + P_{c22}
\]

From this,

\[
Q_{c12} = \frac{P_{c12} - P_{c22}}{R_d} = \frac{(2P_3 - P_{c22}) - P_{c22}}{R_d} = \frac{2(P_3 - P_{c22})}{R_d}
\]

or

\[
Q_{c12} = \frac{2P_1 R_b}{R_{cj} (R_b + R_1) \left( 1 - \frac{R_d}{R_{cj}} \right)} - \frac{2P_{\text{offset}}}{R_{cj} \left( 1 - \frac{R_d}{R_{cj}} \right)}
\]

(10)

With two $P-Q$ coordinates known, $R_{c1}$, the inverse slope of the $C1$ characteristic, is formed.

\[
R_{c1} = \frac{\Delta P_{c1}}{\Delta Q_{c1}} = \frac{P_{c12} - P_{c11}}{Q_{c12} - Q_{c11}}
\]

(11)

where

\[
P_{c12} = \frac{P_1 R_b \left( 1 + \frac{R_d}{R_{cj}} \right)}{(R_b + R_1) \left( 1 - \frac{R_d}{R_{cj}} \right)} - \frac{2 R_d P_{\text{offset}}}{R_{cj} \left( 1 - \frac{R_d}{R_{cj}} \right)}
\]

(11a)

\[
P_{c11} = \frac{P_1}{R_1 \left( \frac{1}{R_{cj}} + \frac{1}{R_b} + \frac{1}{R_1} \right)} + \frac{P_{\text{offset}}}{R_{cj} \left( \frac{1}{R_{cj}} + \frac{1}{R_1} + \frac{1}{R_b} \right)}
\]

(11b)

\[
Q_{c12} = \frac{2P_1 R_b}{R_{cj} (R + R_1) \left( 1 - \frac{R_d}{R_{cj}} \right)} - \frac{2P_{\text{offset}}}{R_{cj} \left( 1 - \frac{R_d}{R_{cj}} \right)}
\]

(11c)
The pressure axis intercept is now determined by substitution of the expressions for \( P_{c11} \) and \( Q_{c11} \) into the equation

\[
Q_{c11} = \frac{P_1}{R_{cl}R_1\left(\frac{1}{R_{cl}} + \frac{1}{R_1} + \frac{1}{R_b}\right)} + \left[1 - R_{cl}\left(\frac{1}{R_{cl}} + \frac{1}{R_1} + \frac{1}{R_b}\right)\right] P_{\text{offset}} \tag{11d}
\]

Thus, the complete equation becomes

\[
b = P_{c11} - R_cQ_{c11} \tag{12}
\]

Thus, the complete equation becomes

\[
P_{c1} = Q_{c1}R_{c1} + (P_{c11} - R_cQ_{c11}) \tag{13}
\]

with the corresponding quantities defined in equations 11a to d.

3. OBSERVED PERFORMANCE

In order to verify the foregoing analysis, the centered-jet and deflected-jet characteristics were first experimentally determined. Figure 8 shows the technique for obtaining the centered-jet curve.

By applying flow to controls 1 and 2 in a common-mode fashion, the supply jet remains nominally centered. Thus, the requirement that \( P_{c1} = P_{c2} \) and \( Q_{c1} = Q_{c2} = \frac{1}{2}Q_{\text{tot}} \) is satisfied. The deflected-jet characteristic, on the other hand, must satisfy the requirement that

\[
P_b = \frac{P_{c1} + P_{c2}}{2} = \text{constant}
\]

for all points on the curve. With the assumption of linear characteristics, only two points need be obtained to determine the deflection resistance (inverse slope), \( R_d \). Since \( R_d \) is constant with bias pressure, \( P_b \), values for \( P_b \) at which \( R_d \) is determined were arbitrarily selected over a range of 5 to 15 percent of supply pressure. Once a bias pressure is selected, the first point is found on the centered-jet curve where \( P_{c1} = P_{c2} - P_b \). The \( P-Q \) characteristic is now determined for control port 1 with control port 2 open to ambient \( (P_{c2} = 0) \). Again, the intersection of the

![Figure 8. Method for obtaining centered-jet characteristic.](image-url)
deflected-jet curve with the $P_{c2} = 0$ curve must satisfy

$$P_b = \frac{P_{c1} + P_{c2}}{2} = \frac{P_{c1}}{2} \text{ for } P_{c2} = 0.$$

Thus, the second point is quickly determined to be where $P_{c1} = 2P_b$ on the $P_{c2} = 0$ curve. All deflected-jet characteristics presented here were determined in the manner outlined above. It should be noted that both $R_{cl}$ and $R_d$ may be computed$^2$ for a general design.

Experimental data for the three cases shown in section 2 at various aspect ratios (height-to-width ratio of supply nozzle, denoted by $\sigma$) and supply pressures appear in figures 9 through 17. For a constant pressure on control port 2 (fig. 9 to 11) the inverse slope, $R_{cl}$, of the $C1$ characteristics is seen to be in good agreement with predicted performance, exhibiting a maximum deviation from experiment of 11 percent. Good agreement between experimentally determined values of $R_{c1}$ and predicted values can also be seen for a constant flow rate at control port 2 (fig. 12 to 14). Here the error did not exceed 8 percent. Lastly, predicted and experimental values of $R_{c1}$ for a series and shunt resistance at control port 2 agree reasonably well (11-percent maximum error) as can be seen in figures 15 to 17. Actual and computed pressure axis intercepts differed significantly in many cases, however. The reason for this becomes apparent upon inspection of

---

Figure 10. Control port 1 (C1) pressure flow characteristic for $P_{c2} = \text{constant}$, with $P_s = 0.476$ kPa.

the centered-jet characteristics. As mentioned previously, all characteristics were assumed to be straight lines. This assumption is violated for points on the centered-jet curve near zero, due to nonlinearity. Nonetheless, actual values of $P_{\text{offset}}$ (intersections of centered-jet curve and pressure axis) were used in the computation. If a straight-line tangent were drawn on the centered-jet curve, an “apparent” pressure axis intercept could be determined that would provide better agreement. For $P_{c2} = 0$ in figures 9, 10, and 11, the nonlinear nature of the $P_{c2} = 0$ characteristics near zero was responsible for additional error. This effect is particularly pronounced in figure 10, where actual and computed pressure axis intercepts differed by nearly 60 percent. Again, construction of an “apparent” intercept improves accuracy considerably.

4. SUMMARY AND CONCLUSIONS

Coupled with an existing computer algorithm for predicting differential control port characteristics, a purely analytical approach was presented for modeling single-sided control port characteristics. Three specific cases were dealt with—constant pressure on control port 2, constant flow on control port 2, and a series/shunt resistance connected to control port 2. Predicted and experimental values of $R_{c1}$ (inverse slope of C1 characteristic) agreed well (11-percent maximum error) for the range of aspect ratios and supply pressures tested. Significant error, however, was encountered between experimental and computed values of the C1 characteristic.
Figure 11. Control port 1 (C1) pressure-flow characteristic for $P_{c2}$ constant, with $P_s = 0.15$ kPa.
Figure 12. Control port 1 (C1) pressure-flow characteristic for $Q_{c2} =$ constant, with $P_s = 0.67$ kPa.

Figure 13. Control port 1 (C1) pressure-flow characteristic for $Q_{c2} =$ constant, with $P_s = 0.476$ kPa.
Figure 14. Control port 1 (C1) pressure-flow characteristic for $Q_{c2}$ = constant, with $P_s = 0.15$ kPa.

![Graph showing control port 1 (C1) pressure-flow characteristic for $Q_{c2}$ = constant, with $P_s = 0.15$ kPa.]

<table>
<thead>
<tr>
<th>Control Pressure, $P_c$ (kPa)</th>
<th>CONTROL PRESSURE, $P_c$ (kPa)</th>
<th>CONTROL PORT 2 FLOW RATE, $Q_{c2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.15</td>
<td>$Q_{c2} = 0.10Q_s$</td>
</tr>
<tr>
<td>0.15</td>
<td>0.15</td>
<td>$Q_{c2} = 0.13Q_s$</td>
</tr>
</tbody>
</table>

Figure 15. Control port 1 (C1) pressure-flow characteristic for series and shunt resistance at control port 2, with $P_s = 0.67$ kPa.

![Graph showing control port 1 (C1) pressure-flow characteristic for series and shunt resistance at control port 2, with $P_s = 0.67$ kPa.]

<table>
<thead>
<tr>
<th>Control Pressure, $P_c$ (kPa)</th>
<th>CONTROL PRESSURE, $P_c$ (kPa)</th>
<th>CONTROL PORT 2 FLOW RATE, $Q_{c2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>$Q_{c2} = 0.10Q_s$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>$Q_{c2} = 0.13Q_s$</td>
</tr>
</tbody>
</table>
pressure axis intercept. This is due, at least in part, to the nonlinear behavior of the centered-jet curve near zero. Better agreement could be obtained through the use of a straight-line or "apparent" pressure axis intercept ($P_{offset}$) for the centered-jet characteristic. In practice, however, the inverse slope, $R_{cl}$, of the $C1$ characteristic is generally of more importance than the pressure axis intercept.

Figure 17. Control port 1 ($C1$) pressure-flow characteristic for series and shunt resistance at control port 2, with $P_s = 0.15$ kPa.
From the results obtained, it would appear that the analytical approach is of a general nature. If a load line can be constructed for control port 2, then the C1 characteristic can be determined. This could be extended to the most general case, where orifices (nonlinear resistances) are present at control port 2, thus allowing purely analytical design for any configuration.

**NOMENCLATURE**

*b*  
pressure axis intercept of C1 characteristic

*C1*  
control port 1

*C2*  
control port 2

*P*  
pressure (kPa)

*Q*  
volumetric flow (m³/s)

*R*  
fluid resistance \( P/Q\) (kPa/m³/s)

*σ*  
aspect ratio (height-to-width ratio of supply nozzle)

**Subscripts**

*b*  
Bias

*cj*  
Centered-jet

*c1*  
Control port 1

*c2*  
Control port 2

*d*  
Deflected-jet

*offset*  
Pressure axis intercept of the centered-jet curve

*tot*  
Total

**Superscripts**

*  
Constant quantity
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