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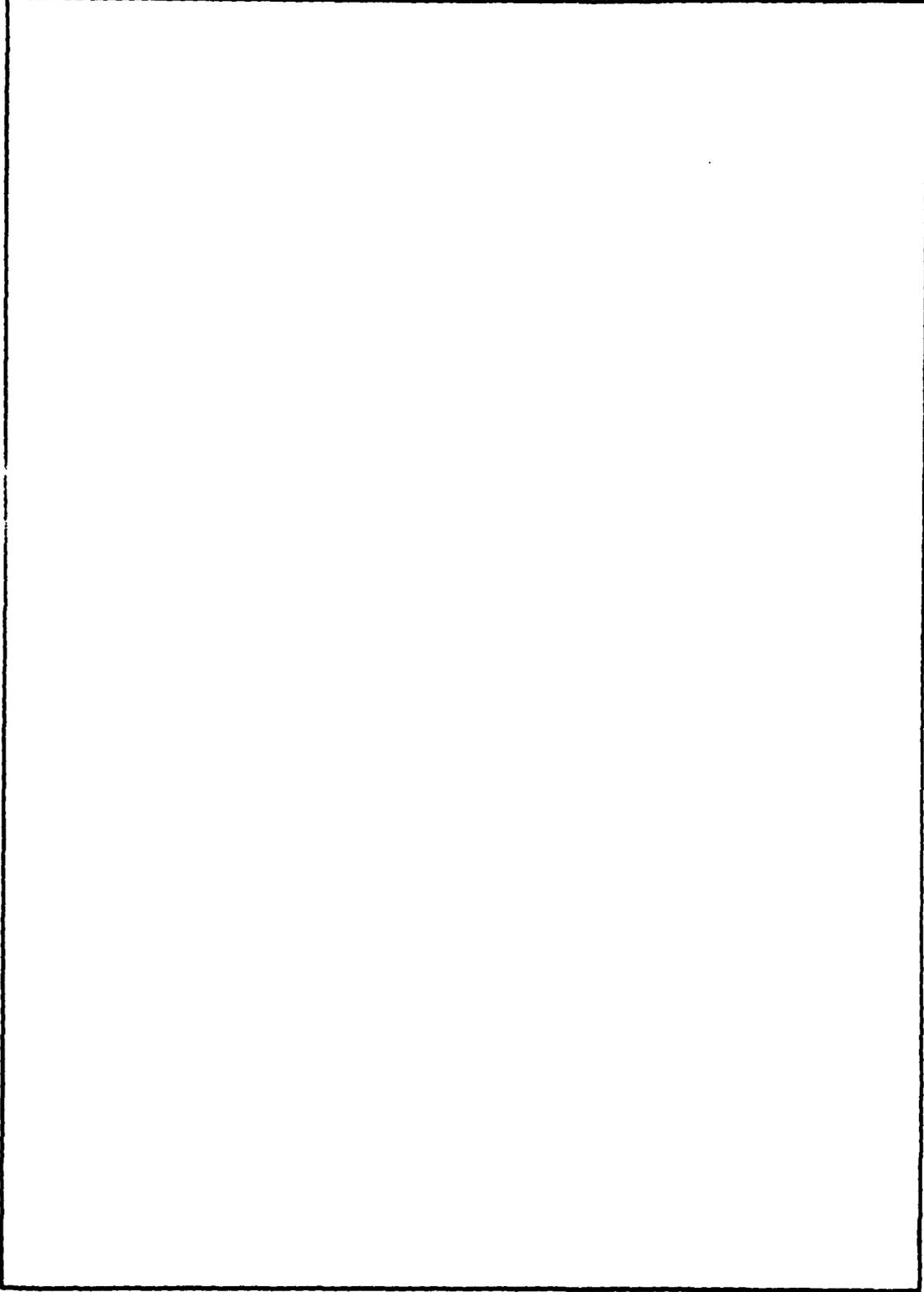
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## ABSTRACT

The NASTRAN structural analysis program has been modified so that its TRAPAX and TRIAAX finite elements may handle piezoelectric material properties. These elements are solid, axisymmetric ring elements capable of handling non-axisymmetric loads. The natural frequencies of a piezoelectric hollow cylinder and a piezoelectric disk with electroded surfaces were computed with this modified version of NASTRAN and were found to be in excellent agreement with experimental and other numerical results.

## ADMINISTRATIVE INFORMATION

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## INTRODUCTION

The analysis of sonar transducers requires accounting for the effects of their piezoelectric materials. The theory for these materials couples the structural displacements to the electric potentials.<sup>1,2\*</sup> This theory has been incorporated into the TRAPAX and TRIAAX finite elements of the NASTRAN structural analysis computer program.<sup>3</sup> These elements, trapezoidal and triangular in cross-section respectively, are solid, axisymmetric rings whose degrees-of-freedom are expanded into Fourier series, thus allowing nonaxisymmetric loads.

The complete details of the TRAPAX and TRIAAX finite elements and of finite element piezoelectric theory are well-documented<sup>1,2,3</sup> and will not be repeated here. However, some of the ideas will be briefly discussed.

The constitutive relations of the material may be written as

$$\begin{Bmatrix} \{\sigma\} \\ \{D\} \end{Bmatrix} = \begin{bmatrix} [c^E] & [e] \\ [e]^T & -[\epsilon^S] \end{bmatrix} \begin{Bmatrix} \{\epsilon\} \\ \{E\} \end{Bmatrix} \quad (1)$$

---

\*A complete listing of references is given on page 25.

where  $\{\sigma\}$  = stress components =  $[\sigma_{rr}, \sigma_{zz}, \sigma_{\theta\theta}, \sigma_{rz}, \sigma_{r\theta}, \sigma_{z\theta}]^T$

$\{D\}$  = components of electric flux density =  $[D_{rr}, D_{zz}, D_{\theta\theta}]^T$

$\{\epsilon\}$  = mechanical strain components

$\{E\}$  = electric field components

$[c^E]$  = elastic stiffness tensor evaluated at constant electric field

$[e]$  = piezoelectric tensor

$[\epsilon^S]$  = dielectric tensor evaluated at constant mechanical strain

The displacement vector of a point within an element is taken to be

$$\{\bar{u}\} = \begin{Bmatrix} u \\ v \\ w \\ \phi \end{Bmatrix} \quad (2)$$

where  $u$ ,  $v$ , and  $w$  are the ring displacements in the radial, tangential, and axial directions, respectively, and  $\phi$  is the electric potential. The latter degree-of-freedom is taken to be the fourth degree-of-freedom at each ring. Each of these quantities is expanded into a Fourier series with respect to the azimuth position  $\theta$ . The Fourier series for the electric potential  $\phi$  has the same form as the Fourier series for radial displacement  $u$ , as given in the NASTRAN Theoretical Manual.<sup>3</sup>

The "stiffness" matrix for the  $N^{\text{th}}$  harmonic is

$$[K^{(N)}] = \pi \int_j \int_z [B^{(N)}]^T \begin{bmatrix} [c^E] & [e] \\ [e]^T & -[\epsilon^S] \end{bmatrix} [B^{(N)}] r dr dz \quad (3)$$

where  $[B^{(N)}]$  is the matrix of "strain"- "displacement" coefficients for the  $N^{\text{th}}$  harmonic.

Equations (2) and (3) indicate that the matrix equation to be solved for static analysis may be partitioned as follows:

$$\begin{bmatrix} [K_{\delta\delta}] & [K_{\delta\phi}] \\ [K_{\phi\delta}] & [K_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{\delta\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{F_{\delta}\} \\ \{F_{\phi}\} \end{Bmatrix} \quad (4)$$

where  $\{\delta\} = [u_1, v_1, w_1, \dots, u_n, v_n, w_n]^T$

$\{\phi\} = [\phi_1, \dots, \phi_n]^T$

$\{F_\delta\}$  = vector of structural forces

and  $\{F_\phi\}$  = vector of electrical charges

Note, however, that the program assumes that the electric potential  $\phi_i$  is the fourth degree-of-freedom of grid point  $i$ .

Both lumped and consistent mass matrices are available and are of standard structural form, i.e., the mass matrix does not couple the structural and electrical unknowns. The structural damping matrix also is of standard structural form. Both point charges and surface charges are also available.

#### NASTRAN MODIFICATIONS

Numerous modifications were required to incorporate the piezoelectric material capability into NASTRAN. These modifications included new bulk data cards, new outlooks on existing data cards, and subroutine changes, some of which were the result of existing errors. All modifications were made to NASTRAN Level 17.5.

#### DATA CARDS

Four new bulk data cards have been introduced into NASTRAN, and two existing bulk data cards can be viewed in a different light to accommodate piezoelectric materials. Also, the NASTRAN card contains a new parameter.

#### New Cards

The four new bulk data cards added to the program describe piezoelectric material properties. These properties are frequently described by the following matrices:

$$[S^E] = \begin{bmatrix} S_{11}^E & S_{12}^E & S_{13}^E & 0 & 0 & 0 \\ S_{12}^E & S_{11}^E & S_{13}^E & 0 & 0 & 0 \\ S_{13}^E & S_{13}^E & S_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66}^E \end{bmatrix} \quad (5)$$

where  $S_{66}^E = 2(S_{11}^E - S_{12}^E)$

$$[d] = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

$$[\epsilon^S] = \begin{bmatrix} \epsilon_{11}^S & 0 & 0 \\ 0 & \epsilon_{11}^S & 0 \\ 0 & 0 & \epsilon_{33}^S \end{bmatrix} \quad (7)$$

The matrices in Equation (1) are computed as follows:

$$[c^E] = [S^E]^{-1} \quad (8)$$

$$[e] = [d][c^E] \quad (9)$$

and  $[\epsilon^S]$  is given by Equation (7).

Two of the new data cards, MATPZ1 and MATPZ2, describe the piezoelectric material properties in two different ways. MATPZ1 is used to specify the parameters in Equations (5)-(7). MATPZ2 is more general and allows the user to enter the full matrices  $[c^E]$ ,  $[e]$ , and  $[\epsilon^S]$ . The

only assumption concerning these matrices is that  $[c^E]$  and  $[\epsilon^S]$  are symmetric. Figures 1 and 2 describe MATPZ1 and MATPZ2.

CAUTION: Piezoelectric electric material properties are usually specified with respect to a standard set of material axes 1, 2, 3. For axisymmetric solids, direction 1 coincides with the z-axis and direction 2 coincides with the  $\theta$ -axis. Polarization direction 3 may vary in the R-Z plane and, for radial polarization, coincides with the R-axis. When a user specifies properties on a MATPZ1 card, the transformation from the 1, 2, 3 directions to the R, Z,  $\theta$  directions is performed by NASTRAN. However, such a transformation is not performed by NASTRAN when the MATPZ2 card is used. Also, the ordering of the components of the stress and strain vectors is somewhat different for conventional piezoelectric specifications and for NASTRAN. The difference is that the ordering of the Z- $\theta$  and R-Z shears is interchanged. Once again, NASTRAN performs the transformation for MATPZ1, but not for MATPZ2.

The other new data cards, MTTPZ1 and MTTPZ2, allow the values on MATPZ1 and MATPZ2 to be temperature-dependent. (However, the TRAPAX and TRIAAX elements have not yet been modified to handle the combination of thermal loads and piezoelectric materials.) Figures 3 and 4 describe MTTPZ1 and MTTPZ2.

#### Existing Cards

Point and surface charges may be specified. These charges are analogous to structural point loads and pressures, respectively, and are entered into  $\{F_\phi\}$  in Equation (4). Since the electric potential is associated with degree-of-freedom 4, point charges may be applied to specific harmonics with MOMAX bulk data cards or may be specified by MOMENT, MOMENT1, or MOMENT2 cards applied to POINTAX points. In the latter case, the direction of the "moment" must be about the radial direction, i.e., degree-of-freedom 4. The MKS unit of the point charge is coulombs.

The PRESAX bulk data card is used to specify surface charges. However, in order to distinguish between surface charges and structural pressure loads within the same problem, the first-specified ring

Input Data Card MATPZ1 Material Property Definition

Description: Defines the material properties for linear, temperature-independent, piezoelectric materials.

Format:

1	2	3	4	5	6	7	8	9	10
MATPZ1	MID	$S_{11}^E$	$S_{33}^E$	$S_{44}^E$	$S_{12}^E$	$S_{13}^E$	$d_{31}$	$d_{33}$	+i
+A	$d_{15}$	$\epsilon_{11}^S/\epsilon_0$	$\epsilon_{33}^S/\epsilon_0$	RHO	A	TREF	GE		

<u>Field</u>	<u>Contents</u>
MID	Material identification number (Integer > 0)
$S_{11}^E - d_{15}$	Piezoelectric constants multiplied by $10^{12}$ (Real)
$\epsilon_{11}^S/\epsilon_0, \epsilon_{33}^S/\epsilon_0$	Piezoelectric constants, where $\epsilon_0$ is taken to be $8.854 \times 10^{-12}$ farad/meter (Real)
RHO	Mass density (Real)
A	Thermal expansion coefficient (Real)
TREF	Thermal expansion reference temperature (Real)
GE	Structural element damping coefficient (Real)

Remarks:

1. MID must be unique with respect to all other material cards.
2. MATPZ1 materials may be made temperature-dependent by use of the MTPZ1 card.
3. MATPZ1 may be referenced only by PTRAPAX and PTRIAAX cards.
4. Matrix  $[S^E]$  must be nonsingular.

Figure 1 - MATPZ1 Bulk Data Card

Input Data Card MATPZ2 Material Property Definition

Description: Defines the material properties for linear, temperature-independent, piezoelectric materials.

Format:

1	2	3	4	5	6	7	8	9	10
MATPZ2	MID	$c_{11}^E$	$c_{12}^E$	$c_{13}^E$	$c_{14}^E$	$c_{15}^E$	$c_{16}^E$	$c_{22}^E$	-A
+A	$c_{23}^E$	$c_{24}^E$	$c_{25}^E$	$c_{26}^E$	$c_{33}^E$	$c_{34}^E$	$c_{35}^E$	$c_{36}^E$	+B
+B	$c_{44}^E$	$c_{45}^E$	$c_{46}^E$	$c_{55}^E$	$c_{56}^E$	$c_{66}^E$	$e_{11}$	$e_{12}$	+C
+C	$e_{13}$	$e_{14}$	$e_{15}$	$e_{16}$	$e_{21}$	$e_{22}$	$e_{23}$	$e_{24}$	+D
+D	$e_{25}$	$e_{26}$	$e_{31}$	$e_{32}$	$e_{33}$	$e_{34}$	$e_{35}$	$e_{36}$	+E
+E	$\epsilon_{11}^S$	$\epsilon_{12}^S$	$\epsilon_{13}^S$	$\epsilon_{22}^S$	$\epsilon_{23}^S$	$\epsilon_{33}^S$	RHO	AX	+F
+F	AY	AZ	TREF	GE					

<u>Field</u>	<u>Contents</u>
MID	Material identification number (Integer > 0)
$c_{11}^E - \epsilon_{33}^S$	Piezoelectric constants (Real)
RHO	Mass density (Real)
AX,AY,AZ	Thermal expansion coefficients (Real)
TREF	Thermal expansion reference temperature (Real)
GE	Structural element damping coefficient (Real)

Remarks:

1. MID must be unique with respect to all other material cards.
2. MATPZ2 materials may be made temperature-dependent by use of the MATPZ2 card.
3. MATPZ2 may be referenced only by PTRAPAX and PTRIAAX cards.
4. See CAUTION on page 5.

Figure 2 - MATPZ2 Bulk Data Card

Input Data Card MTTPZ1 Material Temperature Dependence

Description: Specifies table references for those piezoelectric material properties on a MATPZ1 card which are temperature-dependent.

Format:

1	2	3	4	5	6	7	8	9	10
MTTPZ1	MID	R1	R2	R3	R4	R5	R6	R7	+A
+A	R8	R9	R10	R11	R12	R13	R14		

Field

Contents

MID

Material property identification number which matches the identification number on some basic MATPZ1 card (Integer > 0)

Ri

References to table identification numbers for the corresponding fields on the MATPZ1 card (Integer  $\geq 0$  or blank)

Remarks:

1. Blank or zero entries mean no table dependence of the referenced quantity on the basic MATPZ1 card, and the quantity remains constant.
2. TABLEM1, TABLEM2, TABLEM3, and TABLEM4 type tables may be used.
3. Material properties given on the basic MATPZ1 card are initial values. If two or more quantities are to retain a fixed relationship, then two or more tables must be input to define the relationship.

Figure 3 - MTTPZ1 Bulk Data Card

Input Data Card MTPZ2 Material Temperature Dependence

Description: Specifies table references for those piezoelectric material properties on a MATPZ2 card which are temperature-dependent.

Format:

1	2	3	4	5	6	7	8	9	10
MTPZ2	MID	R1	R2	R3	R4	R5	R6	R7	+A
				(etc.)					
+F	R48	R49	R50	R51					

Field

Contents

MID

Material property identification number which matches the identification number on some basic MATPZ2 card (Integer > 0)

Ri

References to table identification numbers for the corresponding fields on the MATPZ2 card (Integer ≥ 0 or blank)

Remarks:

1. Blank or zero entries mean no table dependence of the referenced quantity on the basic MATPZ2 card, and the quantity remains constant.
2. TABLEM1, TABLEM2, TABLEM3, and TABLEM4 type tables may be used.
3. Material properties given on the basic MATPZ2 card are initial values. If two or more quantities are to retain a fixed relationship, then two or more tables must be input to define the relationship.

Figure 4 - MTPZ2 Bulk Data Card

identification number on the PRESAX card (field 4) must be made negative if a surface charge is desired. Modifications have been made to NASTRAN to allow a negative ring identification number only when the parameter SYSTEM(78) is set to 1 on the NASTRAN card.

The NASTRAN card allows the user to override various NASTRAN system parameters. SYSTEM(78), set aside for specifying piezoelectric materials, has a default value of zero, which implies that no piezoelectric materials are allowed. If SYSTEM(78) = 1, piezoelectric materials are allowed and coupling occurs between the structural and electric degrees-of-freedom. If SYSTEM(78) = 2, piezoelectric materials are allowed, but no coupling occurs.

Setting SYSTEM(78) to its proper value is important for a number of reasons:

1. If SYSTEM(78) = 0, no piezoelectric materials are expected, and MATPZ1 and MATPZ2 cards will not be searched.
2. If SYSTEM(78)  $\neq$  1, a negative ring identification number is not allowed on the PRESAX card.
3. If SYSTEM(78)  $\neq$  1, NASTRAN will automatically constrain degree-of-freedom 4 (the electric potential) at each ring for the zero harmonic in the AXISYMMETRIC = COSINE case.
4. If SYSTEM(78) = 2, some time will be saved in generating the "stiffness" matrix over the time for the SYSTEM(78) = 1 case.
5. If SYSTEM(78)  $\neq$  1, degrees-of-freedom 4, 5, and 6 must be removed from the problem via the SPCAX or RINGAX cards. If SYSTEM(78) = 1, only degrees of freedom 5 and 6 must be removed.

#### SUBROUTINES

Numerous modifications have been made to NASTRAN Level 17.5 to allow piezoelectric materials, point and surface charges, and complex stress and complex force output for TRAPAX and TRIAAX elements, and to correct existing errors related to these elements. The modifications for each subroutine are described in this section.

#### DELKLS

The modification to this subroutine allows for finer finite element meshes with the TRIAAX and TRAPAX elements by tightening a check on the difference between radial coordinates of the grid points of an element.

#### EMA

If a structural damping coefficient is present, only the uncoupled structural terms of the stiffness matrix are multiplied by it. All the piezoelectric terms of the stiffness matrix are set to 0.

#### GPTABD

An array COMP3 was added to describe the manner in which the real and imaginary parts of complex stresses and forces are related to each other for output for the TRIAAX and TRAPAX elements. Also, the E array for these elements was updated to point to the new information in COMP3.

#### GP3A

Provision was made to allow a negative ring identification number on the PRESAX card to indicate a surface charge rather than a structural pressure.

#### IFP

Since the MATPZ2 and MTPZ2 bulk data cards are fixed ended and contain more than 50 items each, dimensions for arrays I, M, MF, and M1 in /IFPDTA/ were increased to 100. Also, standard modifications required to access new bulk data cards were made.

#### IFPDCO

As in IFP, dimensions for /IFPDTA/ were increased.

#### IFP3

Modifications to IFP3 were required to allow negative ring identification numbers on PRESAX cards for surface charge density loading.

#### IFP3B

In standard structural axisymmetric problems, IFP3B constrains degrees-of-freedom 2, 4, and 6 in the zeroth harmonic for the COSINE case. Since voltage is degree-of-freedom 4, IFP3B was modified to constrain only degrees-of-freedom 2 and 6 in piezoelectric problems.

IFS1P

Dimensions in /IFPDTA/ were increased, and checks were added for the data on new cards MATPZ1, MATPZ2, MTPZ1, and MTPZ2.

IFS2P

Dimensions in /IPDTA/ were increased as in IFP.

IFS3P

Dimensions in /IFPDTA/ were increased as in IFP, and changes were made to allow the PRESAX card to have a negative ring identification number in piezoelectric problems.

IFS4P, IFS5P, IFXOBD

Dimensions in /IFPDTA/ were increased as in IFP.

IFXOBD

In IFX1BD, all available spaces for bulk data cards were used by the standard program. In order to increase the number of spaces available, space had to be increased in the IB and IC arrays in IFXOBD, which contains bit specifications for the bulk data, parameter, and case control cards in IFX1BD. Associated bit specifications also had to be entered.

IFX1BD - IFX7BD

Standard additions were made for new bulk data cards MATPZ1, MATPZ2, MTPZ1, and MTPZ2.

OFFZZZZ (OFF)

Two errors were corrected. Output for real, sort 2, transient analyses of axisymmetric problems is now printed rather than punched. Also, if the problem is a real, sort 2, statics analysis, the entire label of the heading is deleted rather than only part of it. Modifications allow for the output from axisymmetric problems of new displacement vector headings for real, sort 2, transient problems; for real, sort 2, static problems; and for complex, sort 2, frequency response problems. Special packing formats added to OFF5BD are also checked.

OFF1A

This subroutine contains FORMAT statements for output headings. Two existing FORMAT statements were modified to allow the output of charges and flux densities. Thirteen FORMAT statements were added to provide a

complete set of output headings. These FORMAT statements include headings for stresses, forces, loads, and displacements for complex output, sort 1 and sort 2, and real, sort 2, transient and static output.

OFPIBD

This BLOCK DATA subprogram is pointed to from one of 10 tables, depending on the type of stresses or forces being processed for output. It points into OFP5BD to construct the correct FORMAT statement. Modifications and additions allow for the addition of charges and flux densities to the output for any type of real or complex problem containing TRAPAX and TRIAAX elements.

OFP5BD

Pieces of FORMAT statements from this block data were modified or added to allow the output of charges and flux densities.

OF2PBD, OF3PBD, OF3SBD, OF4PBD, OF5PBD,  
OF6PBD, OF7PBD, OF7SBD, OF8PBD

These BLOCK DATA subprograms are indirectly pointed to from OFPZZZZ and, in turn, point to OFPIBD and OFP1 or OFP1A. They modify or provide the capability for output of the following types of problems: complex stresses, sort 1; real stresses, sort 2 (not statics); real stresses, sort 2 (statics); complex stresses, sort 2; real forces, sort 1; complex forces, sort 1; real forces, sort 2 (not statics); real forces, sort 2 (statics); and complex forces, sort 2.

PREMATZ (PREMAT)

This subroutine now allows piezoelectric material properties to be read from the MATPZ1, MATPZ2, MTTPZ1, and MTTPZ2 cards already described. If the piezoelectric material properties are specified on a MATPZ1 card, they are transformed to the form found on a MATPZ2 card (Equations 8 and 9). All the piezoelectric material constants are stored in PZOUT(1-51), which is part of named common MATPZ.

PRESAX

A negative ring identification number specified on a PRESAX card indicates a surface charge load rather than a structural pressure load. This subroutine has been modified to identify surface charge loads, and to compute the charge to be applied at the rings specified on the PRESAX card.

### SDR2D

The pointers into the displacement vector file were modified to handle any type of complex problem for the TRIAAX and TRAPAX elements.

### SDR2E

Modifications were made to handle the complex stress and force output from STRAX2 or STPAX2. The imaginary part of the complex stress and force output is stored in variables ISAVES and ISAVEF. The dimension of each of these variables was doubled to handle the additional charges and flux densities. These variables are also part of a new named common ISAVE.

### STPAX1

This subroutine forms the stiffness and stress matrices for TRAPAX elements for stress and force recovery. Two of the new matrices to be formed are  $[K_{\delta\phi}]$  and  $[K_{\phi\phi}]$ , the piezoelectric stiffness matrix partitions of Equation (4).  $[K_{\phi\delta}]$  is not formed because

$$[K_{\phi\delta}] = [K_{\delta\phi}]^T \quad (10)$$

To generate  $[K_{\delta\phi}]$  and  $[K_{\phi\phi}]$  a new 4x4 transformation matrix from field coordinates to grid point degrees of freedom is formed. It is stored in GBP and is similar to GABABQ used to generate  $[K_{\delta\delta}]$ . A piezoelectric materials matrix (Equation (1)) is formed.  $[c^E]$  is stored in EE(1-21),  $[e]$  is stored in EE(37-54), and  $[c^S]$  is stored in EE(55-63).  $[B^{(N)}]$  of Equation (3) is never explicitly formed. Rather,  $[K_{\delta\phi}]$  and  $[K_{\phi\phi}]$  are generated term by term incorporating the matrix triple product and integration over r and z found in Equation (3). The new untransformed partitions of the stiffness matrix are stored in ACURL(145-208). Once these partitions have been transformed, they are stored in AKUPH and AKPH2, corresponding to  $[K_{\delta\phi}]$  and  $[K_{\phi\phi}]$ , respectively. Three new partitions of the master stress matrix must also be generated. The augmented stress matrix is of the form:

$$[S] = \begin{bmatrix} [S_0]_{30 \times 12} & [S_{P1}]_{30 \times 4} \\ [S_{P2}]_{15 \times 12} & [S_{P3}]_{15 \times 4} \end{bmatrix} \quad (11)$$

where  $[S]$  = augmented master stress matrix

$[S_0]$  = original stress matrix<sup>4</sup>

$[S_{p1}], [S_{p2}], [S_{p3}]$  = piezoelectric partitions of the master stress matrix

$[S_{p1}], [S_{p2}],$  and  $[S_{p3}]$  are stored in SELP1, SELP2, and SELP3, respectively. To generate the three new stress matrix partitions, another 3x4 transformation matrix is required. It is stored in WJP and is similar to WJ used to generate  $[S_0]$ . The new stress and stiffness matrix partitions are passed to STPAX2 through a named common SDR2X5. The complete NASTRAN method for formulating the stiffness and stress matrices is described in the NASTRAN Programmers Manual.<sup>4</sup>

STPAX2

With the augmented stress and stiffness matrices from STPAX1, forces, charges, stresses, and flux densities are computed according to Equation (4) and

$$\begin{bmatrix} [S_0] & [S_{p1}] \\ [S_{p2}] & [S_{p3}] \end{bmatrix} \begin{Bmatrix} \{f\} \\ \{q\} \end{Bmatrix} = \begin{Bmatrix} \{T_\sigma\} \\ \{T_\phi\} \end{Bmatrix} \quad (12)$$

where  $\{T_\sigma\}$  = stress (second-order tensor)

and  $\{T_\phi\}$  = flux density (vector)

$\{F_x\}, \{F_y\}, \{T_\sigma\},$  and  $\{T_\phi\}$  are stored in EFORC, ECHRG, ESTRES, and EFLUX, respectively. These results for each harmonic and angle are then accumulated in BLOCK if they are real output or the imaginary part of complex output. If they are the real part of complex output, they are accumulated in CLOCK. However, the results for each harmonic at zero angle are always stored in STRES and FORCE, whether they are real or complex.

STPAX3

Stresses and forces from BLOCK and CLOCK are stored in FORCE and STRESS or SAVEF and SAVES. In real problems, output from BLOCK is transferred to CLOCK and stored in FORCE and STRESS. In complex problems, the imaginary part of the output stored in BLOCK is saved in SAVEF and SAVES.

The real part of complex output from CLOCK is stored in FORCE and STRESS.

#### STRAX1

The modifications are identical to those in STPAX1 except that the TRIAAX element has three grid points rather than four. Also, for the TRAPAX element, stresses are computed at the four grid points and the centroid, rather than just at the centroid as for the TRIAAX element. These considerations change the size of the following matrices to those indicated: transformation matrix GABABP(3x3), ACURL(117),  $[S_0]$ (6x9),  $[S_{p1}]$ (6x3),  $[S_{p2}]$ (3x9),  $[S_{p3}]$ (3x3), and transformation matrix WJP(3x3).

#### STRAX2

The modifications are identical to those in STPAX2, with the exception noted for STRAX1.

#### STRAX3

The modifications are identical to those in STPAX3, with the exception noted for STRAX1.

#### TRAPAX

Modifications to this subroutine are identical to those for STPAX1 for the generation of the stiffness matrix partitions  $[K_{\delta\delta}]$ ,  $[K_{\delta\phi}]$ , and  $[K_{\phi\phi}]$ . In addition, the stiffness matrix returned from this subroutine must be in a usable form to compute grid point displacements. Therefore,  $[K_{\delta\delta}]$ ,  $[K_{\delta\phi}]$ ,  $[K_{\delta\phi}]^T$ , and  $[K_{\phi\phi}]$  must be assembled into a total element stiffness matrix. Since the electric potential  $\phi$  is the fourth degree-of-freedom,  $[K_{\delta\phi}]$ ,  $[K_{\delta\phi}]^T$ , and  $[K_{\phi\phi}]$  must be merged with  $[K_{\delta\delta}]$ . These partitions are inserted in every fourth column and row. No modifications were required in the generation of the mass matrix to account for the piezoelectric capability. However, two existing errors in TRAPAX were corrected. In the first of these, the terms of the lumped mass matrix were not divided by four. Without dividing by four, the total lumped mass was assigned to each grid point rather than only one-fourth of it. In the second, an error in the coding, the consistent mass matrix was not transformed from field coordinates to grid point degrees-of-freedom.

#### TRIAAX

Modifications to this subroutine are identical to those for STRAX1 for the generation of the stiffness matrix partitions  $[K_{\delta\delta}]$ ,  $[K_{\delta\phi}]$ , and  $[K_{\phi\phi}]$ . The formulation of the element stiffness matrix and the mass matrices follows the same procedure as does the TRAPAX subroutine. One existing error was corrected in TRIAAX. The terms of the lumped mass matrix were not divided by three. Without dividing by three, the total lumped mass was assigned to each grid point rather than only one-third of it.

#### TTLPGE

Modifications update the NASTRAN title page to Level 17.5.1 from Level 17.5.0 and to a system generation date of 8/15/79 from 11/30/78.

#### OVERLAY

Some minor additions to the overlay structures of links 8 and 13 are required. Since the materials information specified on MATPZ1 and MATPZ2 cards is returned to the appropriate routines via named common /MATPZ/, this name must be added to the overlay structure of links 8 and 13 and may be included in the same segment as named commons /MATIN/ and /MATOUT/, the standard materials named common. Also, in link 13, new named common /ISAVE/ should be added to the same overlay segment as /SDR2X7/. This new common allows the transfer of information between subroutines SDR2E and STPAX3 or STRAX3.

Finally, the increased lengths of subroutines in link 13 (STPAX1, STPAX2, STPAX3, STRAX1, STRAX2, and STRAX3) necessitate checking open core named commons /SDRA2/ and /SDR2XX/ for proper positioning. This consideration is automatically accounted for in link 8 by EMG.

## NOTES

A few notes concerning piezoelectric problems with NASTRAN will probably be useful.

1. In order to use piezoelectric materials, SYSTEM(78) must be set to 1 or 2 on the NASTRAN card. (The default value is 0.) A value of one indicates electrical-structural coupling, a value of 2 allows the use of piezoelectric materials but does not take into account any electrical effects. The latter case requires that the degrees-of-freedom corresponding to the electric potential be constrained.

2. The electric potential at each ring is considered to be degree-of-freedom 4. Degrees-of-freedom 5 and 6 always have zero stiffness and must be removed from the problem with SPCAX or RINGAX cards. Electroded surfaces (surfaces of constant potential) may be specified with MPCAX cards.

3. Only TRAPAX and TRIAAX elements may reference piezoelectric material cards MATPZ1 and MATPZ2.

4. Standard material cards MAT1 and MAT3 are allowed in problems which also contain piezoelectric materials.

5. The  $S^E$  and  $d$  values on MATPZ1 cards will be multiplied by  $10^{-12}$  by NASTRAN. Also the value of  $\epsilon_0$  is fixed in NASTRAN as  $8.854 \times 10^{-12}$  farad/meter.

6. As may be seen in Equation (3), the lower right-hand portion of the stiffness matrix is negative-definite. This situation does not affect NASTRAN execution except that grid point singularity warning messages are issued for all unconstrained electric potentials.

7. To specify surface charge loads, the first ring identification number on the PRESAX card (field 4) must be negative. This format change will allow NASTRAN to distinguish between electrical charges and structural pressures within a piezoelectric run. This change is allowed only when SYSTEM(78) = 1.

8. Lumped mass and consistent mass are available for TRAPAX and TRIAAX elements. The mass associated with the electric potential degree-of-freedom is zero. Therefore, if a normal modes analysis by GIVENS

method is run, all unconstrained electric potentials must appear on OMIT cards.

9. If a structural damping coefficient is specified on a MATPZ1 or MATPZ2 card in a dynamics problem, the terms of the resulting structural damping matrix corresponding to electric potentials will be zero. The uniform structural damping parameter G in direct frequency response problems should not be used, since its use will result in structural damping terms corresponding to the electric potentials.

10. Prior to the present work, NASTRAN could not handle stresses or forces, whether real or complex, in axisymmetric (AXIC) dynamics problems. NASTRAN can now handle all such uses for the TRAPAX and TRIAAX finite elements.

11. Material properties specified on MATPZ1 cards are transformed by NASTRAN from the standard 1, 2, 3 material directions to the R, Z,  $\theta$  directions. Also, the transformation required due to a switch in the order of the R-Z and Z- $\theta$  shears between conventional specifications and NASTRAN is performed for MATPZ1 properties. Material properties on MATPZ2 cards are used by NASTRAN as they appear on the card. Therefore, any required transformation must be performed by the user.

#### SAMPLE PROBLEMS

Two sample problems were run to check results from TRIAAX and TRAPAX piezoelectric finite elements against experimental and other calculated results.

##### Problem 1

The natural frequencies of a radially polarized PZT-4 hollow cylinder were computed. Figure 5 shows the finite element mesh and boundary conditions of the problem described by Allik.<sup>1</sup> Results from the present work were compared with the results from the mesh in Figure 5.

For the present analysis of the hollow cylinder, four finite element meshes were used. Mesh 1, shown in Figure 6, consists of 56 TRIAAX elements with 45 nodes. One other mesh with TRIAAX elements was

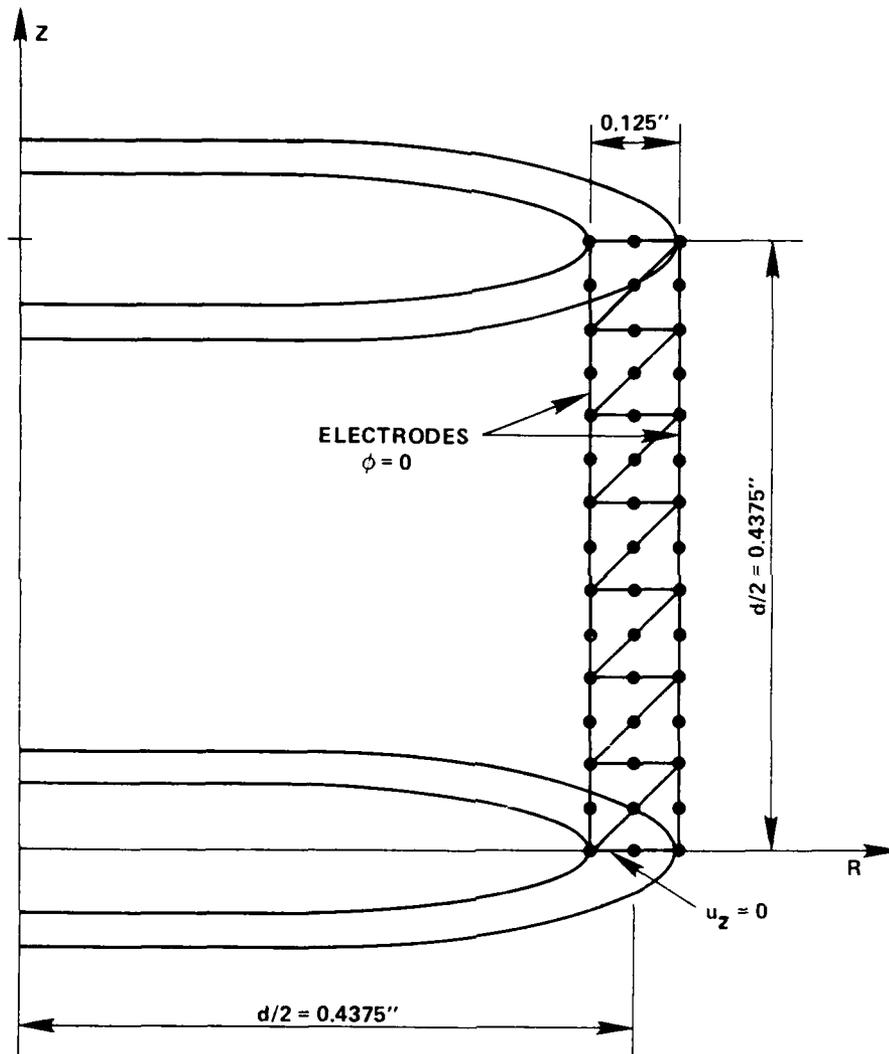


Figure 5 - Finite Element Model of Hollow Cylinder

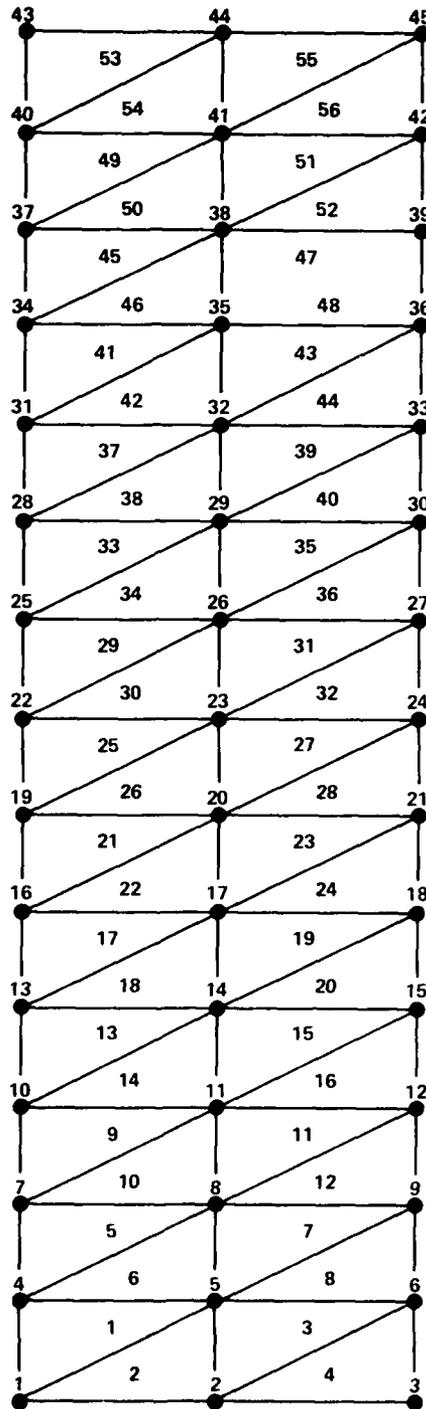


Figure 6 - Finite Element Mesh 1 of Hollow Cylinder

analyzed, quadrupling the number of elements of the first mesh. Mesh 2 consists of 224 TRIAAX elements and 145 nodes. One analysis was done using TRAPAX elements. Mesh 3 consists of 112 TRAPAX elements and 145 nodes.

For each mesh two analyses were done, the first considering the effect of piezoelectric coupling and the second neglecting piezoelectric coupling. The results of all the analyses are presented in Tables 1 and 2. Allik's results were estimated from a graph in that article.<sup>1</sup> Those results were obtained using quadratic isoparametric triangular finite elements and the MARTSAM program.<sup>5</sup>

TABLE 1 - NATURAL FREQUENCIES OF HOLLOW CYLINDER INCLUDING PIEZOELECTRIC COUPLING

Mode	Natural Frequencies (Rad/Sec x 10 <sup>-5</sup> )			
	Mesh from Ref. (1)	Mesh 1	Mesh 2	Mesh 3
1 (R)	2.88	2.91	2.90	2.90
2 (R)	3.25	3.37	3.21	3.23
1 (L)	5.13	5.14	5.12	5.08
3 (R)	7.25	7.90	7.05	7.20
4 (R)	14.0	13.9	13.1	13.5
2 (L)	15.0	14.9	14.5	14.4

(R) denotes radial mode  
(L) denotes longitudinal mode

#### Problem 2

The natural frequencies of an axially polarized PZT-4 piezoelectric disk were computed. This disk is shown in Figure 7. Both the upper and lower electroded surfaces are grounded. The results of the present work were compared to calculated and experimental results of the same disk provided by the Naval Underwater Systems Center (NUSC).

The mesh for the present analysis of the piezoelectric disk contains 205 nodes and 320 TRIAAX elements. The NUSC model contained 83 nodes and 28 quadratic isoparametric triangular elements and was run with the MARTSAM program. The results of the analyses are presented in Table 3.

TABLE 2 - NATURAL FREQUENCIES OF HOLLOW CYLINDER  
NEGLECTING PIEZOELECTRIC COUPLING

Mode	Natural Frequencies (Rad/Sec x 10 <sup>-5</sup> )			
	Mesh from Ref. (1)	Mesh 1	Mesh 2	Mesh 3
1 (R)	2.88	2.90	2.90	2.90
2 (R)	3.25	3.37	3.21	3.16
1 (L)	5.13	5.09	5.08	5.08
3 (R)	6.50	7.58	6.69	6.34
4 (R)	11.9	13.1	11.9	11.4

(R) denotes radial mode  
(L) denotes longitudinal mode

TABLE 3 - NATURAL FREQUENCIES OF PIEZOELECTRIC DISK

Mode	Natural Frequencies (cps)		
	NUSC Mesh Experimental	NUSC Mesh Calculated	NASTRAN Mesh
1	22042	23298	24323
2	-----	59805	61114
3	-----	103048	104689

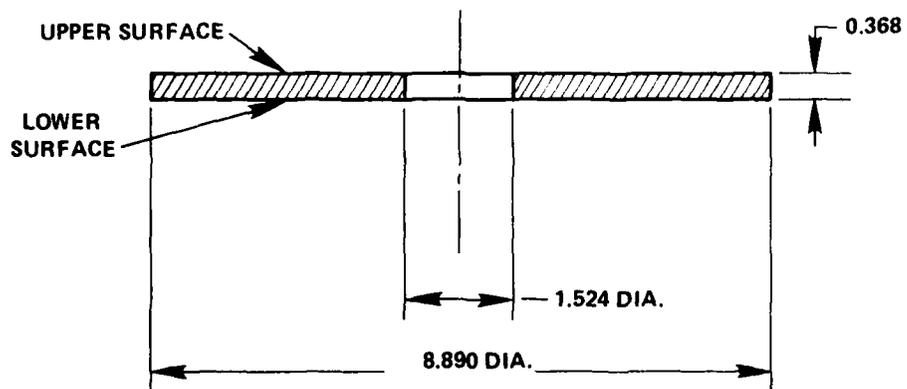
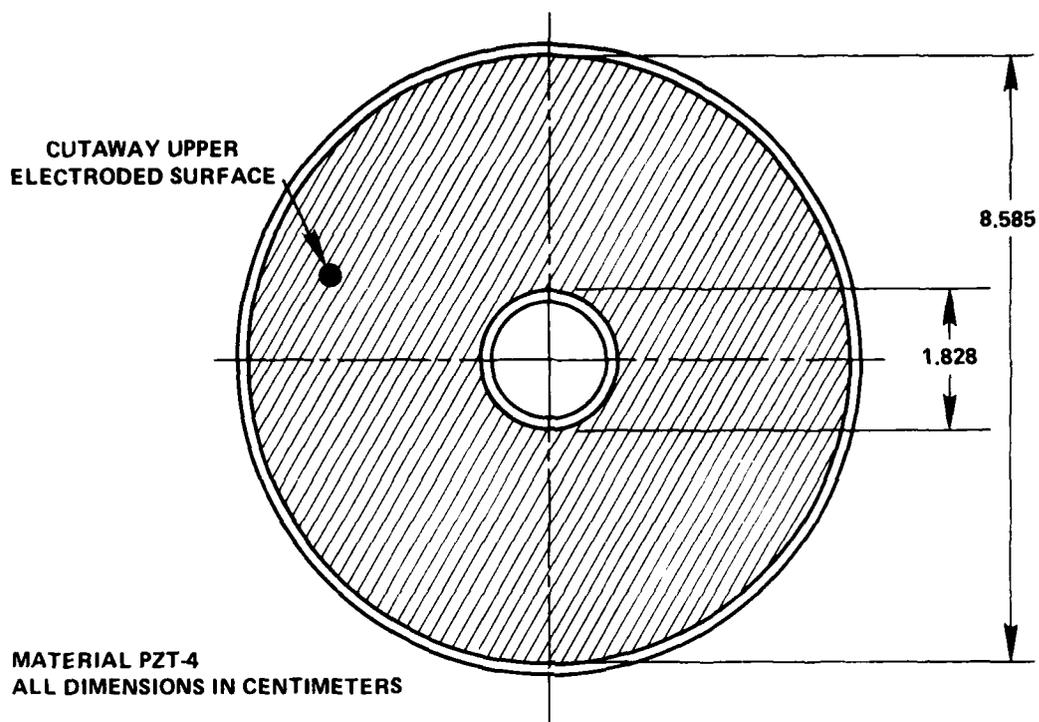


Figure 7 - Piezoelectric Disk

#### REFERENCES

1. Allik, H., "Piezoelectric Finite Element Analysis of Axisymmetric Solids," General Dynamics/Electric Boat Division Report U411-70-047, EBdiv Sonar Development No. 7.32 (April 1970).
2. Allik, H. and T.J.R. Hughes, "Finite Element Method for Piezoelectric Vibration," Int. J. Num. Methods in Engr., Vol. 2, No. 2, pp. 151-157 (April-June 1970).
3. "The NASTRAN Theoretical Manual (Level 17.5)," NASA SP-221(05) (December 1978).
4. "The NASTRAN Programmers' Manual (Level 17.5)," NASA SP-223(05) (December 1978).
5. "MARTSAM 4, A Version of GENSAM, MARTSAM Users' Manual," edited by H. Allik, P. Cacciatore, R. Gauthier, and S. Gordon, General Dynamics/Electric Boat Division Report U440-74-043, Sonar Dev. No. 7.67 (May 1974).

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