ADA082510

TECHNICAL REPORT DRCPM-MP-79-1

LEVEL II

ESTIMATING THE SYSTEM RELIABILITY LOWER CONFIDENCE LIMIT FROM DATA DERIVED FROM SYSTEM AND SUBSYSTEM TEST RESULTS

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September 1979

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER DRCPM-MP-79-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Estimating the System Reliability Lower Confidence Limit from Data Derived from System and Subsystem Test Results.		5. TYPE OF REPORT & PERIOD COVERED Technical Rep[redacted]t,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Julian L. Cothran		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Commander US Army Missile Command ATTN: DRCPM-MPT Redstone Arsenal, Alabama 35809		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Commander US Army Missile Command ATTN: DRSMI-TI Redstone Arsenal, Alabama 35809		12. REPORT DATE Sep[redacted]79
		13. NUMBER OF PAGES 89
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Missile weapon reliability
Reliability scoring inputs
Weapon System Effectiveness Industry Advisory Committee (WSEIAC)
System effectiveness model
Confidence Limit
Forward Edge of the Battle Area (FEBA)
Camp Meidell Inequality
Maximum Likelihood Estimates (MLE)
Confidence Level

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Presently a problem exists in being able to present a confidence level on missile weapon reliability. This paper suggests a method for solving the problem. This method provides the means for estimating the system reliability lower confidence limit with specified confidence level from reliability phase scoring data derived from system and subsystem test results; and yields a quick and good approximation to a more rigorous and lengthy analysis. The method does not require any assumptions concerning the form of the reliability phase distributions nor their test sample size. (continued)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) cont.

This paper formulates and demonstrates the suggested method and then compares it with several others. The method is then applied to a realistic missile weapon system in its development life cycle phase. A comprehensive explanation of the testing philosophy and reliability phase scoring criteria and methodology is given.

ACKNOWLEDGEMENTS

I wish to thank Mr. Murray D. Hillman for suggesting the topic and providing his helpful comments during the preparation of the paper. I also wish to thank Mr. Chester L. Hopkins for his assistance with the computer program used to provide a comparison of applicable methods and Ms. Patricia Hughes and Mrs. Linda Maples for typing the paper.

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I. INTRODUCTION

An engineer would be rather foolish and risk his reputation if he assigned a definite value to the system reliability of a missile weapon system based on a few firings. He will estimate the system reliability, though, and ascertain with a certain degree of confidence that his estimate is within a certain interval. The degree of confidence expresses his state of mind and reflects the extent of misjudgement he wishes to avoid; thus it is a reflection of his conservatism. If a number is assigned to this confidence, it becomes a confidence level and specifies the percentage of the statements that he expects will be correct. At this stage of the analysis, the engineer will determine his trade-off point between the desired confidence level and the quantity of testing (in this case, the number of missile launchings) based on available time and money. Thus, he can establish a quantitative interval on his estimates which reflects his professional confidence and represents his monetary limitations. He has also helped to alleviate an inherent flaw in higher management decision making, that of misinterpretation of data due to a lack of statistical background, which contributes to possible gross errors in judgement since intuition cannot be used as a safeguard. He has also presented the decision maker with a knowledge of the risk involved and of the limitations of his information.

Thus, for any one-shot weapon system or explosive device where the test data are limited, the best that the engineer can say of the true system reliability, or some other form of output measurement, is that it lies within some confidence limits with a certain degree of confidence. These limits with the associated degree of confidence have to be clearly defined, based on the many variables and limitations of the unit under test, and the test itself, in order to provide the most informative statements based on the test results. When working with confidence measurements, two basic principles are common: (1) for a given quantity of test data, the higher the confidence level, the larger the confidence interval and vice versa, and (2) the greater the quantity of test data the narrower the confidence interval for any specified confidence level.

These concepts provide the underlying framework for determining a value for the output of a system and quantifying the worth of that value. The ultimate output of any system is the performance of some intended function, usually referred to as the mission with respect to weapon systems. The term used to describe the overall capability of a system to accomplish its mission is system effectiveness. [80, p.1]*

In 1965 a special committee known as the Weapon System Effectiveness Industry Advisory Committee (WSEIAC), chartered through Department of Defense Directive, published their

*Numbers in brackets throughout the paper indicate references.

reports. These reports addressed the problem of developing a standard technique to apprise management of current and predicted weapon system effectiveness during all phases of the weapon system life cycle. WSEIAC states that effectiveness is a function of system availability, dependability, and capability. [29, p. 42], [2] The result of the expression of these parameters in the system effectiveness model is used to provide decision information to high level management with a required high degree of confidence in the model, thus providing a measure of a system worth.

It is sometimes necessary to modify the basic guidance provided by WSEIAC in order to model a particular system since effectiveness is influenced by: (1) the way the equipment is designed and built, (2) used and maintained, (3) rules of engagement, (4) fiscal control and (5) many other administrative policy decisions. With this in mind, a general system effectiveness definition for a one-shot device such as a missile is: "System effectiveness is the probability that the system (missile) will operate successfully (kill the target) when called upon to do so under specified conditions." [80, p. 5] This definition is also structured to give an effectiveness expression for the respective system. As an example, the system effectiveness expression for a Forward Edge of the Battle Area (FEBA), man-portable, shoulder fired, air defense missile system might be:

$$E_s = R_s \times R_w \times L_m \times P_{det}$$

E_s = System Effectiveness

R_s = Preuse Reliability = $R_{stockpile} \times R_{field\ environment}$

R_w = Weapon Reliability = $R_{prefire} \times R_{fire} \times R_{warhead\ detonation} (R_{fuze} \times R_{warhead})$

L_m = Missile Lethality

P_{det} = Probability of Detection, Evaluation and Transfer.

The majority of terms in this expression are determined from actual tests on components of the system or the entire system itself. The system level tests are structured such that results are applicable to the specific phases (terms) in the reliability expressions of the system effectiveness model. For convenience and ease of derivation, the phases are assumed independent and test results are usually expressed as binomial results or success ratios taken from attribute data. By definition, the point estimates of each reliability phase, obtained from reliability scoring of test data, are combined as a series product to produce weapon and preuse reliability at a required confidence level. At this point, an inherent problem arises in generating the necessary reliability values at a predesignated required confidence.

One of the acceptable ways of determining system reliability with confidence is by combining the point estimates of the respective system reliability scoring phases by a single-thread approach. This approach, which successively discards failures based on equal sample tests, enables a confidence level to be placed on the binomial system reliability estimate. An example of this approach for a three phase reliability expression is:

$$\frac{18}{20} \times \frac{15}{18} \times \frac{12}{15} = \frac{12}{20}$$

This approach does not represent the real world conditions due to time, money, and realistic test programs which yield unequal sample sizes for each phase. Test programs are structured such that each respective type of test is independent of another type and each type corresponds to a particular reliability scoring phase or results of a combination of phases. These tests and/or reliability phases represent respective distributions which are generally unknown as well as describing a system with an unknown distribution.

Presently a problem exists in being able to place a confidence level on missile weapon reliability. This paper suggests a method for solving the problem. This method provides the means for estimating the system reliability lower confidence limit with specified confidence level from reliability phase scoring data derived from system and subsystem test results and yields a quick and good approximation to a more rigorous and lengthy analysis. The method does not require any assumptions concerning the form of the reliability phase distributions nor their test sample size.

A. METHOD

Since most component and system data are made up of attributes in the form of observable two state test results based on a go/no-go criterion, data are collected in the form of binomial success ratios. These ratios are characterized as success probabilities, $\hat{p}$'s, on the components of the system or $\hat{R}$'s when concerning the system, where $\hat{p}$ is an estimate obtained as a probability from $\hat{p} = \frac{(n-d)}{n}$ when n components are tested and d of them fail, so that n-d complete the test successfully, likewise concerning system test results. Thus it is known, as in the common coin tossing example, that this probability estimate has a binomial distribution about the true p or R.

With the necessary attribute test data collected, the reliability of the system is represented as a mathematical function of random variables of the form $\hat{R}$ ($\hat{p}_1$, $\hat{p}_2$, . . . $\hat{p}_n$) which is representative of a system built up from various series and parallel arrangements of components. In this case the reliability system estimate, $\hat{R}$, may be taken as an unbiased estimate of the parameter $R(p_1, p_2, \ldots, p_n)$, that is, the system reliability. If, as in the case of interest here, the reliability of the system is a mathematical function of system reliability phase scoring estimates, it would have the form $\hat{R}(\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_n)$ and would be taken as the unbiased estimate of the system reliability $R(r_1, r_2, \ldots, r_n)$. At this point, the question arises as to whether the results of any test or phase are dependent upon the results of other tests or phases. The answer is intuitively yes, and should, if possible, be taken into account in any reliability prediction and the determination of confidence limits. However, in this paper, it will be assumed that all test results are independent, thus uncorrelated, in order to simplify the discussion and keep the algebraic computations to a minimum.

Since one of the prime objectives of this paper is to present a relatively simple method of determining an estimate for the lower confidence limit on system reliability, the method for generating a one-sided tolerance, or control, limit in construction of statistical quality control charts was adopted. This limit is given by the quantity $\hat{R}\text{-}K\sigma_{\hat{R}}$ with the property that the probability is γ that at least a proportion $(1\text{-}\beta)$ of the distribution will be contained within the interval $\hat{R}\text{-}K\sigma_{\hat{R}}$ and plus one. The quantity K is called a tolerance factor and is a function of γ and β.

Normally the assumption of normality of system performance, based on the central limit theorem, is applied, but this is applicable only when system performance is the sum of the effects of many component or subsystem test results, or reliability scoring phases, with no single one having a dominant variance. Since sample sizes are usually not large enough for the central limit theorem to apply, the indiscriminate assumptions of normality could lead to erroneous conclusions. Based on this, the estimate of system variance will be derived from the generation of system moments which neither requires assumptions concerning the form of the scoring phase distributions nor test sample sizes associated with each phase. It also allows an analysis to be made of the importance of each phase variable through the examination of the magnitude of its partial derivative. This method is commonly referred to as statistical error propagation or the delta method and is developed through the concept of expected values with a Taylor series expansion about the point at which each of the component variables takes on its expected value, or first moment.

Since independence has been assumed (it need not be) and a distribution-free method for obtaining system parameters has been adopted, the Camp-Meidell Inequality for determining the tolerance factor, K, will be used. This is a distribution-free and fairly conservative method, but a necessary one since neither the phase nor system distributions are known. (This method is a modified, more accurate version of method number 6 described in the literature review which follows.)

The remainder of Chapter I gives the literature review. In Chapter 2 the proposed method is formulated, explained, demonstrated, and compared. Chapter 3 applies the proposed method to a realistic missile weapon system in its development life cycle phase. It presents a comprehensive explanation of the testing philosophy and reliability phase scoring criteria and methodology involved in the system development. Chapter 4 summarizes the results and offers recommendations.

B. LITERATURE REVIEW

A comprehensive literature search and investigation of the problem was made. This section reviews the investigation and presents some method descriptions and comments on selected methods which are deemed sufficient in order to cover the broad spectrum of solutions that exist in the literature, which might be applicable. Through this investigation, it is evident that direct, exacting analytic methods for the problem solution are not feasible due to the complexity of the system (the mathematical model which defines its reliability), the nonavailability of a computer facility with "canned" programs for the solution of the problem, and not enough time available for a rigorous solution in support of management decisions.

A review of the existing literature follows:

1. A method for determining confidence interval estimation of the reliability of multi-component systems using component test data was developed by Johnson [34] using the exact multivariate binomial distribution. Basic to this method is the dependence of the reliability of the system on the reliability of its components which is determined by the structure of the system such that the reliability of the system can be expressed as a known function of the failure rates of the components. The presentation assumes an ordering of the N points of the sample space of the vector $d = (d_1, d_2, \ldots, d_c)$, where d_i is the number of components in the sample of the i^{th} type of component that failed when tested. Superscripts designate the ordering of the sample point. Thus with the above assumptions, the construction of a one-side confidence interval for system reliability is the solution of the following nonlinear mathematical programming problem:

Minimize the objective function

$$R = R\ (p_1,\ p_2,\ \ldots,\ p_c)$$

subject to the constraints that

$$\sum_{r=1}^{k} \prod_{i=1}^{c} \binom{n_i}{d_i^r} p_i^{\,d_i^r} (1-p_i)^{n_i - d_i^r} \geq 1-\gamma$$

$$0 \geq p_i \geq 1$$

where: $R(p)$ = the reliability function for the particular system under consideration

n_i = the number of items tested of the i^{th} type of component in the system

d_i = the number of failures observed in the i^{th} type component

k = the order index of d

γ = the confidence coefficient (level)

c = the number of different types of components.

This method requires a high capacity, high speed digital computer and reprogramming for each specific problem based on the system structure. It should also be emphasized that component (subsystem) failure rates will most often be unknown and will have to be estimated from test results.

2. The linearization method of obtaining confidence intervals for system reliability based on the asymptotic normality of the maximum likelihood estimates (MLE) was presented by De Cicco [19] and further enunciated by Rosenblatt [61] and Johnson [34]. This method develops as follows:

If the system reliability function is written as

$$R = R\ (p_1,\ p_2,\ \ldots,\ p_c)$$

the MLE of R, $\hat{R}$, can be obtained by replacing the p's (p_i) by their (its) MLE $\hat{p}_i = d_i/n_i$, i = 1, 2, . . ., c, [36, p.199] where d_i is the number of i^{th} type components that failed during tests and n_i is the number of items tested of the i^{th} type of component. Here $\hat{p}_i$ (the MLE of the failure rate of the i^{th} type component) has an asymptotic normal distribution with mean p_i and variance $p_i(1 - p_i)/n_i$. [24, p.133] It also follows that R is asymptotically normal with mean R and variance [57, p.207]

$$\sigma^2 = \sum_{i=1}^{c} \left(\frac{\partial R}{\partial p_i}\right)^2 \text{var}\ (\hat{p}_i).$$

If $\hat{p}_i$ is substituted for the unknown parameter p_i in the equation for σ^2, approximate confidence intervals can be obtained such that

$$p\{\hat{R} - Z_{\frac{1+\gamma}{2}}\hat{\sigma} \le R \le \hat{R} + Z_{\frac{1+\gamma}{2}}\hat{\sigma}\} \simeq \gamma.$$

Thus an approximate 100γ per cent one-sided lower confidence limit for R can be constructed from the above.

In using this method it should be recognized that it might produce confidence intervals that do not contain R thus the true confidence coefficient might be considerably smaller than intended. It should also be recognized that computations for determining the MLE are more difficult than for determining moments and that the MLE sometimes yields biased estimates.

3. Shooman [66] addresses the question of approximations and bounds on system reliability when considering a system of n components (subsystem) with information on each component reliability but little information on their interconnection. The essential factors for the approaches discussed are: the component hazards, the system structural model, and the dependence of the components. For a r-out-of-n structure, expressions for the exact reliability were derived using the binomial distribution for independent and identical components or by a structural-model-approach if the components were dependent. For this same structure, the Poisson approximation of the binomial covering the ends of the reliability range was given. This derivation used the average probabilities of success and failure and the normal approximation was presented to cover the middle. Bounds on series structures with many elements were de ved using the failure law of complex equipment. This was also presented as a pessimistic estimate if parallel paths were present in the system.

An alternative to the other approaches was the development of bounds on system reliability through the application of cut-set and tie-set analysis when a more complex problem was encountered in which the form of the reliability function became very difficult to write down. This analysis was developed from the properties of the reliability graph for any system not containing dependent failures.

For the application of the above approaches, it is expected that the bounds will be quite loose and that care and judgement be applied in their utilization.

A brief discussion of interval estimates (confidence interval) was presented in which a component hazard rate was assumed and the maximum-likelihood-estimate (MLE) was computed for system mean, variance, and reliability function. These values were then used to form a confidence interval based on the assumed estimated distribution.

Another method was to calculate the variance of the MLE of the system and apply the Tchebycheff or Gauss inequality to predict the probability that a random variable (system reliability function) of unspecified distribution lies in an interval.

4. Another approach for determining system reliability confidence intervals with specified confidence level from component data was presented by Stolp and Welch [75]. They proposed an extension of the Neyman-Pearson definition of a confidence interval for one random variable to a confidence interval for a function of several random variables.

This technique involves: (a) a point estimator of the total weapon reliability as a function of the subsystem point estimators, (b) the derivation of the sampling distribution of the total weapon statistic, (c) a specified overall level of confidence for the weapon and the constraint that this overall confidence be shared equally among the subsystems, and (d) the assumption that all subtier levels of the weapon system and the complete system itself conform to the binomial distribution. The authors determined the unique number of tests to be run with zero failures in order to satisfy the requirements and extended their approach and philosophy to the case of one or more failures. The mathematics of their technique are based on the relationship

$$R_i^{N_i} = 1-C_i \quad \text{(for the zero failure case)}$$

which says that reliability and lack of confidence behave in the same manner. Therefore, for a series configuration, $\Pi R_i = R_s$, then $\Pi(1-C_i) = 1-C_s$,

and $$P(f \leq k) + P(f > k) = 1 \quad (f = \text{failure}),$$

which says: $P(R \geq R_o) + P(R < R_o) = 1$ (R = reliability).

Therefore, $P(R \geq R_o) = C$ (confidence)

and $P(R \leq R_o) = 1C$ (lack of confidence).

where: R_i = the estimated reliability of the i^{th} subsystem

N_i = the number of tests required of the i^{th} subsystem with zero failures for a specified C_i, confidence

$1-C_i$ = the lack of confidence of the estimated reliability for the i^{th} subsystem

C_i = the probability of (k) or fewer failures in the i^{th} subsystem:

$$P\left[x \leq (k)\right] = \sum_{x=0}^{k} \binom{N}{x} \hat{R}^{x} (1-\hat{R})^{N-x} .$$

Using the series law of reliability (product rule), the following reasoning was used to combine the confidence levels:

if $R_i^{N_i} = 1-C_i, \quad i = 1,2, \ldots, n$

and $R_s^{N_s} = 1-C_s$

where R_s is the reliability of the system and C_s is the confidence level of the estimated system reliability such that

$$R_s^{N_n} = R_1^{N_1} R_2^{N_2} \ldots R_i^{N_i} \ldots R_n^{N_n}$$

then

$$1-C_s = (1-C_1)(1-C_2)\ldots(1-C_i)\ldots(1-C_n).$$

Upon imposing a uniform subsystem confidence constraint they get

$$C_s = 1-(1-C_i)^n$$

or

$$C_i = 1-(1-C_s)^{1/n} \quad .$$

It should be noted that if the number of subsystems (components) increases, for a given weapon confidence level, their confidence levels decrease, or stated another way, as the number of subsystems is increased, the subsystem confidence level decreases thereby effectively increasing the subsystem reliability value. For example, 20 subsystems would share a 90 per cent weapon confidence at the 11 per cent confidence level and 25 subsystems would share at approximately 8 per cent.

5. A method for determining the lower reliability limit of the system with respective system confidence level was proposed by Chance [14]. This method is based on the joint probability of occurrence for the system, paced by the system reliability estimate, and based on the enumeration of all the ways the subsystem probabilities can occur excluding all the enumerations which exceed the lower system reliability limits. It is dependent on the following assumptions:

- The true reliability of the total population is based on the reliability of the sample except for the zero failure case.

- The lower subsystem reliability limit is dependent on the confidence level, sample size and reliability estimator. Since this is the lower subsystem reliability limit, then the product of these lower limits gives the lower system reliability limit.

- The system confidence level is determined from the allowable subsystem probability of occurrence. In some instances, a subsystem probability of occurrence outside of the confidence interval is used in calculating the system confidence level due to the reliability product rule.

The mechanics of this method are:

First, compute the point reliability estimate for each subsystem based on the number of test runs and the respective number of successes using

$$R(t_i) = \frac{n(t_i)}{N_i} \quad \text{or rather} \quad \frac{x_i}{N_i} = R_{E_i}$$

where $n(t) = x$ = number of successes in N trials or tests and R_E is the reliability estimator. Since the reliability point estimator obtained from the sample test results of the zero failure case cannot be used as the unbiased reliability estimator since it is 1.0, he used the reliability estimator

$$R_{E_i}^{N_i} = \gamma_i \text{ or } R_{E_i} = (1-\beta_i)^{1/N_i}. \qquad \text{(See Method No. 4)}$$

With the respective reliability estimator for each subsystem, he calculated the probability of occurrence for each subsystem from the binomial formula

$$G_i = p(x_i = D_i) = \binom{N_i}{x_i} R_{E_i}^{x_i} (1-R_{E_i})^{N_i - x_i}$$

where $D_i = N_i - A_i$ such that A_i is the number of failures in N_i tests all of the i^{th} enumeration.

Calculations were then made in which the probability of occurrence (G_i) for each subsystem was accumulated until just before it was greater than the confidence value B_i. Then using the corresponding point estimate reliability as the lower subsystem reliability limit, he multiplied each lower subsystem reliability limit to calculate the lower system reliability limit such that

$$R_{LS} \leq (R_{L1})(R_{L2}) \ldots (R_{LNS}) .$$

Note that only those reliability enumerations which did not exceed the lower system reliability limits were considered. He then summed all of the acceptable enumerations, governed by the above equation, to give the system confidence level

$$G_s = \sum_{i=1}^{N_s} (G_i) .$$

It should be observed that as the number of subsystems, tests, and failures increase, the trail of all the possible combinations of probabilities of occurrence, which are governed by the pacing equation

$$R_{LS} \leq (R_{L1})(R_{L2}) \ldots (R_{LNS}).$$

will become a monstrous problem and demand a great deal of computer time.

6. The next method is attributable to several authors who have independently arrived at or proposed, similar methods of solution. An explanation of this method will mainly be taken from the paper by Dutcit [20] and one by DeCicco [18]. Two other references which mention the use of this solution are Lloyd and Lipow [41] and Shooman [66].

This technique is general in the sense that it is not limited to only a series configured system but can have a series-parallel structured system in any configuration. The underlying conditions of this approach are: (a) the components, subsystems, and system are described by mathematical functions composed of binomial parameters, (b) the "error propagation" formula is used to define the expected values as well as the variance of a function, and (c) the Tchebycheff inequality is used in order to yield a one-sided confidence limit.

From the derivation and results of the "error propagation" formula, a function of n variables $f(x_1, x_2, \ldots, x_n)$ which is expanded into a Taylor series about a particular point, say $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$, gives

$$f(x_1, x_2, \ldots, x_n) = f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) +$$

$$(x_1 - \hat{x}_1) \left.\frac{\partial f}{\partial x_i}\right|_{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n} + \ldots +$$

$$(x_n - \hat{x}_n) \left.\frac{\partial f}{\partial x_n}\right|_{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n} + \text{higher order terms.}$$

If the higher order terms are neglected and the assumption is made that the random variables are independent with means at $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$, the expected value of the function is given by

$$E\{f(x_1, x_2, \ldots, x_n)\} \simeq f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$$

and the approximate variance given by

$$\mathrm{VAR}\{f(x_1, x_2, \ldots, x_n)\} \simeq \mathrm{VAR}\,(x_1) \left\{\left.\frac{\partial f}{\partial x_1}\right|_{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n}\right\}^2$$

$$+ \ldots + \mathrm{VAR}(x_n)\left\{\left.\frac{\partial f}{\partial x_n}\right|_{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n}\right\}^2. \qquad [8, p. 62]$$

The equation for the reliability of a component is

$$\hat{R}_i = \frac{x_i}{M_i}$$

where:

x_i = the number of successful functionings of the i^{th} item (component) under test

M_i = the total number of tests of the i^{th} component

$\hat{R}_i$ = a best estimate of a binomial parameter,
which is a proportion describing a population where the proportion R_i of the individuals has a certain characteristic at a 50 per cent confidence level. The equation for the reliability of a system (series, parallel, r-out-of-n, a combination, etc.) is written in the usual way but with the component estimates, as described above, such that $\hat{R}=f(\hat{R}_i)$. The total variance of the system reliability is then calculated from this equation.

After determining the necessary, respective parameters, the authors determined the confidence interval which was developed from Tchebycheff's inequality through the use of distribution free methods. [8, p. 34]

These results were then utilized in the general equation for computing the lower confidence limit as

$$\text{Per cent Confidence Level } R \geq \hat{R} - A\sigma_{\hat{R}}$$

where "A" is not dependent on the distribution of R and is derived from the inequality.

When using and developing this method, the equations (mathematics) can become cumbersome to work with, thus all authors referenced have deleted the necessary derivations which explain and justify their results.

7. Lloyd and Lipow [41] have covered the basics as discussed in Shooman [66], reference method 3, and have presented rather thorough, very excellent coverage of Reliability Estimation and confidence limits in chapters 7 and 8 (with appendices) as well as sections 9.2.2 and 9.2.3. An attempt to discuss their literature will not be given here, but attention is called to their work and the derivations of exact system confidence limits which are presented.

Due to the mathematics and involved nature of the methods, they are not readily adaptable to the aim of this paper. For a brief but thorough discussion of their work (methods). see Rosenblatt [61] and Schick [64].

8. A very excellent paper confined to the problem of estimating a probability (reliability) associated with a system via using data obtained from tests of subsystems or components was presented by Rosenblatt [61]. This paper is especially singled out as suggested reading if the subject of confidence limits for the reliability of complex systems and associated confidence level is of interest. The problem of confidence limits for system reliability is formulated in a general manner but with the necessary and appropriate mathematical exercise. The literature on the subject. up to the publication date of the paper, is more than adequately covered as well as explanations, problem definitions, and comparisons of "exact" methods with alternative approximate methods.

9. One of the best known methods for combining the component reliability estimates to determine approximate confidence limits for the system reliability was developed by Madansky [42]. A synopsis of the Madansky method can be found in the papers by Shick [64], Rosenblatt [61], Johnson [34], and others. It is based on the observed failures of the individual components and assumes that the failures are independent and that for each component they are binomially distributed with the actual component reliability being unknown. He uses a likelihood ratio test based on the fact that minus two time the logarithm of the likelihood ratio is distributed asymptotically as chi-square with one degree of freedom, which is discussed by Wilks [82]. Madansky then derives the confidence limits based on a general method for testing hypothesis.

This method uses the likelihood ratio test for testing the null hpyothesis

$$H_0: f(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n) = \hat{R}_s$$

against the alternate hypothesis

$$H_1: f(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n) < \hat{R}_s$$

where $f(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n)$ is the system reliability structure function with n subsystems or components and $\hat{R}_i = x_i/n_i$ is an estimate of the i^{th} subsystem reliability, such that x_i successes were observed in n_i trials.

$\hat{R}_s$ is an estimate of the system reliability.

The likelihood ratio statistic is then given by

$$\rho = L/L_{max}$$

where

$$L = \max \prod_{i=1}^{n} b(x_i, n_i, \hat{R}_i)$$

$$f = \hat{R}_s$$

and

$$L_{max} = \max \prod_{i=1}^{n} b(x_1, n_i, \hat{R}_i)$$

$$0 \leq \hat{R}_i \leq 1$$

such that $b(x_i, n_i, \hat{R}_i)$ is the binomial. (A brief review of maximum-likelihood estimators can be obtained by referring to Shooman [66, p. 87]).

Thus the confidence interval for system reliability is found as the set

$$\{\hat{R}_s | -2 \ln\rho \leq \chi^2_{1-\gamma,1}\}$$

which is the set of $\hat{R}_i$'s that were not rejected as null hypotheses by the likelihood ratio test when $\chi^2_{1-\gamma,1} = \chi^2_{\alpha,1}$ which is the upper 100α percentile of the chi-square distribution with one degree of freedom.

It should be noted that this approximation is inapplicable when there are no observed failures and that it requires a computer program for solution. It should be used for moderate reliabilities.

10. A Bayesian procedure was originated by Mardo, Cole, Seibel, and Stephenson reference [74] which yields good results. The procedure assumes the system prior reliability is

uniformly distributed and the subsystem prior reliabilities are equally beta distributed. The latter distribution is a natural or cognate prior for binomial data, i.e., the posterior from Bayes Theorem is also beta and has the following mean and variance:

$$f_{BETA}(R|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} R^{a-1} (1-R)^{b-1}$$

$$E(R) = \frac{a}{a+b}$$

$$Var(R) = \frac{ab}{(a+b)^2(a+b+1)}$$

Using the following formulas for the mean and variance of the product of independent random variables, the mean and variance of the subsystem priors are determined:

$$E(xy) = E(x)\ E(y)$$

$$Var(xy) = [Var(x) + E^2(x)][Var(y) + E^2(y)] - E^2(x)\ E^2(y)$$

The beta parameters, a and b, are then calculated from the above mean and variance by the method of moments.

The subsystem data are combined with the subsystem priors in accordance with Bayes' Theorem to obtain the subsystem posteriors:

$$a_i'' = a' + S_i$$

$$b_i'' = b' + (N_i - S_i)$$

where N_i and S_i are the sample size and number of successes in the i^{th} subsystem. The means and variances of the subsystem posterior reliability distributions are then calculated using the formulas for these parameters from the beta distribution.

The system posterior mean and variance is determined from the subsystem means and variances assuming the reliability product formula for a series system and using the above formulas for the mean and variance of a product of independent random variables. The beta parameters of the system are then calculated algebraically from the mean and variance using the method of moments as before.

The lower Bayesian confidence limit for system reliability (R_L) is then calculated by integrating the posterior reliability density function between this limit and one, and setting the integral equal to the confidence.

Whether or not one agrees with the Bayesian approach or the assumption of a uniform prior for the system reliability, the method appears to yield good results according to a Monte Carlo simulation study reported in the referenced literature.

11. Confidence intervals for a system of binomial parameters have been addressed by several authors. A few of the most prominent ones are Clopper and Pearson [15], Buehler [10], Brownlee [9], and Mood and Graybill [48]. A brief explanation of Buehler's method is presented here.

In Buehler's article, he uses the Poisson approximation to the binomial distribution for obtaining one-sided confidence limits for the product of two binomial parameters, such that

$$\binom{n_i}{d_i} q_i^{d_i}(1-q_i)^{n_i-d_i} \text{ is replaced by } \frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}$$

where n_i is the test sample size of the i^{th} component, d_i is the number of failures observed in that sample and q_i is the failure rate of the i^{th} component so that $\lambda_i = n_i q_i$.

If $\bar{q}(d_1)$ and $\bar{q}(d_2)$ are the upper one-sided confidence limits, with $\sqrt{\gamma}$ confidence coefficient (level), for the failure rates of two respective components of the system, whose failure rates are q_1 and q_2, then

$$\Pr\{q_1 \le \bar{q}(d_1)\} \ge \sqrt{\gamma}$$

and

$$\Pr\{q_2 \le \bar{q}(d_2)\} \ge \sqrt{\gamma}$$

thus

$$\Pr\{q_1 q_2 \le \bar{q}(d_1)\bar{q}(d_2)\} \ge \Pr\{q_1 \le \bar{q}(d_1)\}\Pr\{q_2 \le \bar{q}(d_2)\} \ge \gamma$$

This method is specialized for small probabilities of failure and for moderate sample sizes which exceed 40. The rationale for the lower limit is developed the same way as for the upper.

12. A practical way of solving the situation in which the system reliability function is a product of dissimilar functions is by the use of a Monte Carlo simulation. This method has been proposed by several authors, a few are Orkand [54], Levy [37], Levy and Moore [38], and Mann [43].

This method is developed as follows for estimating the system reliability value of a multi-variable system function.

- Obtain a value for each variable in the system function by randomly sampling its respective probability density.

- Substitute this value into the reliability system function to obtain a sample reliability value, system reliability estimate, for a specified mission time.

- Repeat the above steps until a sufficient number of system reliability point estimates have been obtained.

- Evaluate the point estimates of system reliability in order to determine the confidence limits for the desired confidence level. This step is accomplished by ordering the system point estimates in increasing magnitude, thus building a step wise cumulative distribution of the system reliability with each step being of height 1/n, where n is the number of system reliability point estimates obtained. System confidence limits are then obtained by choosing the value corresponding to the desired limit percentage point.

The utility of this method is limited by its dependence on a computer, the requirement that each respective parameter probability density function be known as well as how well it describes the variations of the parameter, and how many sample point estimates are used to describe the system.

13. The Bayesian approach for determining system reliability with confidence has received increased interest in recent years and has created a controversy between classical and Bayesian statisticians. Quite a few authors have begun to address this method and the controversy that has evolved. Comparisons and/or comments on the Bayesian versus the classical are addressed by Easterling [21], Crelin [17], Canavos [13], Schafer [63], Bonis [7], and others. Application of the Bayesian approach is given by Schafer [63] and his other

articles as discussed in this reference; Wolf [83]; Springer and Thompson [69, 70]; Zimmer, Breipohl, and Prairie [84]; Hahn and Shapiro [30]; Fertig [25]; and others.

This method is based on Bayes' Theorem which follows directly from the concept of conditional probability and provides a mechanism for combining the initial or prior probability concerning the occurrence of some event with related experimental data to obtain a revised or posterior probability. This theorem stated as:

$$\Pr(A_i|B) = \Pr(A_i) \frac{\Pr(B|A_i)}{\sum_{i=1}^{n} \Pr(B|A_i)\Pr(A_i)} \quad \text{for } i = 1, 2, \ldots, n$$

has a two term right-hand side: $\Pr(A_i)$, the prior probability, and

$$\frac{\Pr(B|A_i)}{\sum_{i=1}^{n} \Pr(B|A_i)\Pr(A_i)},$$

the factor by which the prior probability is revised on the basis of the experimental data. [30, p. 21]

Although the use of this method for determining system reliability from component or phase test data seems to be promising, since it is neither limited to specific distributions nor test sample size, it has not been accepted for use in reliability assessment of weapon systems due to:

- The difficulty in determining the prior distribution.
- A lack of feel for the sensitivity of a Bayes point estimate to the prior distribution.
- An inherent dislike for mixing intuitive information with real data, since it deteriorates the objectivity of the analysis.
- The method is not really understood.

14. In concluding the review of the literature on the subject, several additional references and methods will be mentioned but no attempt will be made to give an explanation or comment. This portion will only serve to alert the reader to what exists on the subject and where it might be found. The author hopes that the above explanations and the rather cursive along-the-scale view of the methods that were presented has given the reader an introduction to the variety of methods that might be applied toward the solution of the problem. The author also hopes that the reader has been left with an understanding of the applicability of the methods such that he can decipher which ones seem reasonable and which ones seem to be in "left-field."

Articles addressing the reliability of multi-component complex systems have been written by Abraham [1]; Messinger and Shooman [46]; Birnbaum, Esary, and Saunders [6]; Tomsky, Chow, and Schiller [77]; Irwin [33]; Barlow and Proschan [4]; Pieruschka [56]; and Takenaga [76]. Comparisons of various methods have been made by Myhre and Sanders [50], and Aggarwal, Misra, and Gupta [3]. The explanation of confidence intervals and limits with respect to component and system reliability is given by: Neyman [51]; Ireson [32]; Bazovsky [5]; Roberts [60]; Mann [44]; Goodman and Madansky [27]; Soanes [72]; and Proschan [57]. Articles on computer programs for obtaining confidence intervals have been written by Engelman, Roach, and Schick [23], and Springer and Thompson [71]. Specific subjects of Pr $(\underline{Y} < \overline{X})$ has been addressed by Moore and Taylor [49]; systems with few failures by Saunders [62]; dependent failures by Shooman [67]; bounds and propagation of uncertainties in system reliability by Shaw and Shooman [65]; reliability of series systems from component test data by Connor and Wells [16], and Mann and Grubbs [45]; and systems analysis by Whitehouse [81].

2. AN ESTIMATE OF THE LOWER CONFIDENCE LIMIT ON SYSTEM RELIABILITY

This chapter uses some of the previously presented concepts and definitions to create a confidence limit at a specified level on missile weapon reliability without having to resort to some rigorous and lengthy solution. It establishes an estimate of the lowest value necessary in order to meet confidence specifications, thus establishing a lower bound on the required reliability interval.

The suggested method adopts the technique for generating a one-sided tolerance, or control limit in construction of statistical quality control charts. This limit is given by the quantity $\hat{R} - K\hat{\sigma}_R$ with the property that the probability is γ that at least a proportion $(1 - \beta)$ of the distribution will be contained within the interval $\hat{R} - K\hat{\sigma}_R$ and plus one; where $\hat{R}$ is the point estimate of weapon reliability, K is a tolerance factor, $\sigma_{\hat{R}}$ is the standard deviation of the reliability model, γ is the confidence level, and β is the level of significance. This control equation is applied in the following manner:

- Construct a weapon reliability model from which the point estimate of weapon reliability, $\hat{R}$, is obtained. Component estimates in the model are obtained from observations (statistical data) taken during subsystem and system level tests.

- Determine the variance and thus, the standard deviation of the model. This is best accomplished by use of the generation of system moments, sometimes referred to as the "error propogation method" which does not require assumptions concerning the form ot the reliability phase distributions nor their test sample size. This is primary, since the reliability phase distributions are unknown and the phase test sample sizes are usually different.

- Determine the tolerance factor, K. This is done by use of the Camp-Meidell inequality which is a distribution-free and fairly conservative method and one which applies to any set of numbers whether the numbers are viewed as constituting a sample or a population. The most general and easily applied use of this inequality is restricted to certain circumstances. These circumstances are:

  — that the distribution be unimodal

  — that the mode approximately coincide with the arithmetic mean

  — that the distribution be smooth.

The unimodal case has been used in this paper since the majority of the distributions that would describe the results of test data on missile systems come very close to meeting the prescribed conditions. Camp [12] presents the cases for $c = 0$, 1, and 2, where c is a measure of skewness such that the inequality is applied to functions with more than one mode. He emphasizes the case for $c = 0$ (one mode) and points out that in a badly skewed function, the origin might be chosen at the mode instead of the mean. Under this circumstance, the probability and moments could be defined with respect to the new origin, thus giving the unimodal case.

- Substitute the values obtained for the above parameters into the control equation, $\hat{R}$-$K\sigma_{\hat{R}}$, to yield the lower limit on weapon reliability at the designed confidence level.

A. CONSTRUCTION OF THE RELIABILITY MODEL AND ITS VARIANCE

Considering a series configured system as one which creates a single path from cause to effect, the requirement that all elements work successfully for system success is imposed. Thus, the event representing system success is the intersection of the x's where x_i is the event signifying the success of the i^{th} element and $i = 1, \ldots, m$ such that the system is made up of m elements. In other words, the probability of system success is the reliability of the system given by:

$$R = P_s = P(x_1 x_2 \ldots x_m)$$

and if the events are independent

$$R = \prod_{i=1}^{m} P(x_i). \qquad (1)$$

For a parallel configured system, the probability of success is given by the probability of the union of the m successful events, as:

$$R = P_s = P(x_1 + x_2 + \ldots + x_m)$$

and if the events are independent

$$R = 1 - \prod_{i=1}^{m} P(\bar{x}_i). \qquad (2)$$

where $\bar{x}_i$ represents the failure of the i^{th} element.

Considering a general series — parallel configured system, the reliability equation would be:

$$R = \{1-(1-x_1)^a\}\{1-(1-x_2)^b\}\ldots\{1-(1-x_i)^t\}\ldots\{1-(1-x_m)^k\}$$

or

$$R = \prod_{i=1}^{m} \{1-(1-x_i)^t\} \qquad (3)$$

where t elements are in parallel in set i in which all x's have the same probability and there are m sets in the system. This is illustrated in *Figure 1*. (The r-out-of-n case is presented in Chapter 3.)

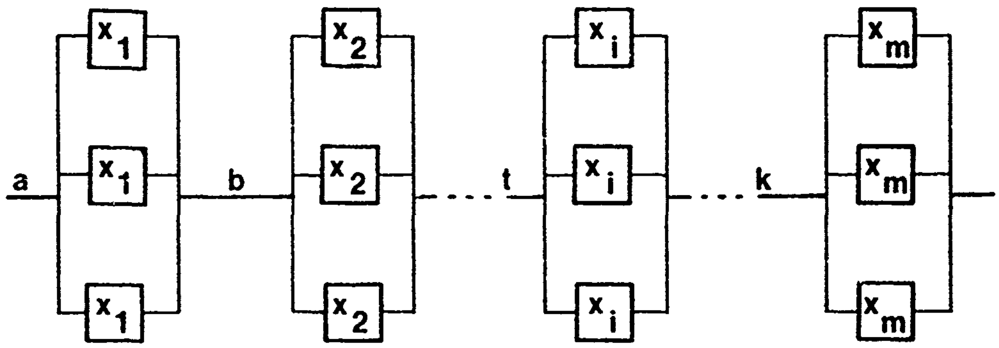


Figure 1. Series - parallel configured reliability model.

After determining the system reliability structure model, which would be one or a combination of the above configurations, it is necessary to determine its standard deviation. This value is required as a component in the control equation $\hat{R} - K\sigma_{\hat{R}}$. In order to determine this value, the following equation is used from Hahn and Shapiro [30, pp. 231, 255] and Tukey [78, pp. V1-V12].

$$\mathrm{Var}(R) = \sum_{i=1}^{m} \left(\frac{\partial R}{\partial x_i}\right)^2 \mathrm{Var}(x_i) + \sum_{i=1}^{m} \left(\frac{\partial R}{\partial x_i}\right)\left(\frac{\partial^2 R}{\partial x_i^2}\right) E\{x_i - E(x_i)\}^3 \quad (4)$$

Since all values were determined from tests on samples, either directly or indirectly, they are estimates of the respective event they represent.

Since the emphasis has already been placed on attribute values representing successful or failed events, due to test structuring, the discussion will be restricted to the use of the binomial distribution. (It should be noted that the x_i's $(i = 1 \ldots m)$ can generally be described by any distribution and not just the binomial.) Therefore, applying equation (4) to determine the variance of a series configured system of independent sets it becomes:

$$\mathrm{Var}(\hat{R}) = \sum_{i=1}^{m} (\hat{p}_1 \cdot \hat{p}_2 \cdots \hat{p}_{i-1} \cdot \hat{p}_{i+1} \cdots \hat{p}_m)^2 \frac{p_i q_i}{n_i} \quad (5)$$

where n_i is the recorded number of binomial tests on the i^{th} set, p_i is the associated binomial parameter for the set of which $\hat{p}_i$ denotes the observed proportion of successes in the n_i trials (the attribute test data $\hat{p}_i = S_i/n_i$), q_i is the binomial parameter associated with the failures of the i^{th} set ($q_i = 1 - p_i$), and there are m sets in the system.

Similarly, the variance of a parallel configured system of independent elements can be written as:

$$Var(\hat{R}) = \left[t(1-\hat{p})^{t-1}\right]^2 \frac{pq}{n} \tag{6}$$

where there are t identical elements in parallel comprising a set of which p is the binomial parameter for the set, and n is the recorded number of binomial tests performed on the set. The second term on the right hand side of equation (4) contributes only slightly in this case, therefore, it was dropped. The slight contribution that it would have made would tend to decrease the value given by the first term on the right hand side of equation (4). Thus, in this case, equation (6) represents a maximum value for system variance.

Using the same rationale for designating the parameters for the system variance as used in the series and parallel model cases and per the illustration, *Figure 1*, the variance for a series-parallel configured model is:

$$Var(\hat{R}) = \sum_{i=1}^{m} \{ \left[1-(1-\hat{p}_1)^a\right] \ldots \left[1-(1-\hat{p}_{i-1})^s\right]\left[1-(1-\hat{p}_{i+1})^u\right]$$

$$\ldots \left[1-(1-\hat{p}_m)^k\right]\left[t(1-\hat{p}_i)^{t-1}\right] \}^2 \frac{p_i q_i}{n_i} \tag{7}$$

B. DETERMINATION OF THE TOLERANCE FACTOR, K, AND LOWER SYSTEM CONFIDENCE LIMIT ON RELIABILITY

The Camp-Meidell inequality will be used to determine the tolerance factor, K, which is another term in the control equation $\hat{R} - K\sigma_{\hat{R}}$.

This inequality is given by:

$$P_{K\sigma} \geq 1 - \frac{1}{2.25K^2}$$

where $P_{K\sigma}$ is the probability of confidence within the interval $\pm K\sigma$ [68, p. 97],[55],[12]. If it is desirable to have 90 percent confidence that the true system reliability will be greater than or equal to a lower confidence limit given by $\hat{R}-K\sigma_{\hat{R}}$, the value for K is determined with respect to the specified confidence. The systems reliability estimate and standard deviation are found by use of equations 1, 2 3, 5, 6, and 7, as applicable.

K is determined for a 90 percent confidence by:

$$.90 \geq 1 - \frac{1}{2.25K^2}$$

such that

$$K \leq 2.108$$

and for a desirable 70 percent confidence, $K \leqslant 1.217$. Thus, in the first instance there is 90 percent confidence that the true system reliability is greater than or equal to $\hat{R} - 2.108\sigma_{\hat{R}}$.

C. NUMERICAL EXAMPLE AND COMPARISON OF METHOD

Consider a series system reliability model made up of 5 subsystems or reliability scoring phases, each of which has a true fraction defective of .01 (q = .01) and each of which was subjected to 50 binomial tests. It is required to have 90 percent confidence in system reliability and is desirable to determine the lower confidence limit on the system reliability.

From equation 1, the reliability of the system is $\hat{R} = .95099$. This value is taken as an estimate or as the true system reliability depending on whether or not the assumption is made that the true reliability of the subsystems or phases is as designated. In order to determine the system standard deviation, equation 5 is used which gives $\sigma_{\hat{R}}^2 = .000914$ ($\sigma_{\hat{R}} = .03022$) and the tolerance factor is $K \leq 2.108$. By combining these values into the control equation, the result is:

90 percent Lower Confidence Limit on System R = .951-2.11(.03) = .888.

A comparison of 5 methods, each of which evaluates the above example, has been made. The values for all but the first method (Method A) were generated from a computer program provided by Mr. Dick Nutt and Mr. Chester Hopkins [53].

The purpose of the program was to verify the correctness of the candidate methods through simulation in order to provide a method(s) for solution of the problem. The problem is the inability to place a confidence level on missile weapon reliability by use of a procedure that is, or will be, agreed to Army-wide.

The program is written for a serial system reliability model of which the binomial is the parent population for all testing and scoring phases. The three candidate methods used in the program were considered to be the most appropriate ones for application to missile weapon systems and present testing philosophy. The fourth method incorporated into the program provides a form of check since it is purely binomial.

The methods compared are briefly described below with the comparison results provided in *Table 1*.

Method A is the proposed method presented in this paper.

Method B was provided by Mr. Ray Heathcock, MICOM, Product Assurance Directorate. It is a modified version of referenced method 7 in the literature search and is basically explained in Lloyd and Lipow [41, p. 227]. This method is applicable to the binomial as the parent population and for an independent serial system reliability model, providing its highest efficiency when sample test sizes are equal. The point estimate for the model is given as

$$\hat{R} = \prod_{i=1}^{k} \frac{N_i - f_i}{N_i}$$

where N_i is the number of tests and f_i the number of failures in the i^{th} phase, respectively. The quantity $N_{AVN}(1 - \hat{R})$, where N_{AVN} is the average test sample size for the system, is then considered to represent the number of system failures, F, in N_{AVN} tests on the system. Using these values, an estimate of the lower confidence limit on system reliability is found from the binomial tables.

Method C is a generalized version of referenced method 4 in the literature search. It was provided by Mr. Chester Hopkins and is applicable to any combination of test sample sizes, number of failures, or distributions.

Method D is the Bayesian procedure which was explained as method 10 in the literature search. It is applicable to the binomial and uses the incomplete beta functions as the cognate prior.

Method E provides a lower bound on the reliability of the system from pure binomial computations.

TABLE 1. POINT ESTIMATES GIVEN BY DIFFERENT METHODS

(90% Confidence (γ=0.90), Assumed System Reliability ≈ .95099)

Method	Point Estimate of the Lower Confidence Limit on System Reliability
A	.888
B	.888
C	.759
D	.892
E	.904

From a review of *Table 1*, it is obvious that the proposed method (Method A) provides as good an estimate of the lower system reliability limit with a specified confidence as any of the other candidate methods. A review of the methods compared indicates that the proposed method is as adaptable as any of the other methods, and requires only hand calculations.

3. APPLICATION TO SNIPER WEAPON SYSTEM

This chapter applies the proposed method to the reliability assessment in the development program of the SNIPER Weapon System. This is a fictitious system but is presented with enough detail and realism in order to convey the involvement of testing and reliability scoring as well as the assessment effort. The explanation is given at the milestone decision point of the Defense System Acquisition Review Council III (DSARC III) between the Development and Production Phases of the System Life Cycle. A favorable DSARC III decision authorizes production and standard type classification.

General information is presented on the deployment mission and system description. An overall description of the system development and operational test programs, along with the testing philosophy, is presented in order to enhance a thorough understanding of the reliability phase scoring methodology, scoring criteria, and phase definition. The application of the proposed method is presented for the weapon and system reliability assessment of the results of the Prototype Qualification Test-Government (PQT-G) Test Program.

Planning and structuring of the development test programs, along with reliability scoring of the test results, were accomplished through the "jury of opinion" concept. This concept is based on the belief that a group review is less likely to overlook errors of judgement than is an individual. It provides a control on the process through its members (reviewers) concentrating on identifying and reviewing the respective reliability estimates rather than concentrating on their detailed calculations. The "jury of opinion" concept is accomplished through a SNIPER Test Integration Working Group (STIWG), comprised of the following primary members: (1) SNIPER Project Manager's Office, system developed; (2) System Prime Contractor; (3) US Army Test and Evaluation Command (TECOM); (4) US Army Materiel Systems Analysis Activity (AMSAA), major systems evaluator; (5) US Army Training and Doctrine Command (TRADOC), user; (6) US Army Operational Test and Evaluation Agency (OTEA); (7) US Army Logistics Evaluation Agency (LEA); and, (8) Electronics Research and Development Command (ERADCOM), Office of Missile Electronic Warfare (OMEW). The reliability scoring committee is a subgroup of the STIWG and is comprised of the majority of the STIWG primary members.

A. SYSTEM DESCRIPTION

SNIPER is a member of the family of Short Range Air Defense (SHORAD) weapons protecting the Field Army units. The system will normally be employed to provide low-altitude air defense for battalions, squadrons, and company-size units operating near the forward edge of the battle area (FEBA). The system may also be employed to provide air defense for surface-to-surface and air defense missile sites and small vital areas when no other ground-based air defense means are available. The weapon will also be used in the early phases of airmobile and airborne operations.

The SNIPER Weapon System mainly consists of the Weapon Round and Battery Coolant Unit (BCU). The SNIPER Weapon Round consists of a guided missile in a launch tube assembly (the Missile Round) mated to a separable gripstock. The guided missile consists of a guidance section, warhead or telemetry section, the flight and launch motors, and tail assembly. The separable gripstock unit is easily attached to a missile round to provide an operational weapon round and is easily detached following missile launch for immediate use with another missile round. The weapon round, as defined, does include an installed BCU which is a one-time use, throw-away item. It provides both the electrical power and detector coolant to the weapon prior to launch.

B. DEVELOPMENT AND OPERATIONAL TEST PROGRAM

Full Scale Development Testing of the SNIPER Air Defense Weapon System stressed the coordination of test planning, combining of test objectives and cross-utilization of test resources by all concerned test activities to provide the most cost and time effective program. This was implemented through the STIWG.

The overall SNIPER Development Test Program activities were Engineering Design Test-Contractor (EDT-C), Engineering Design Test-Government (EDT-G), Prototype Qualification Test-Contractor (PQT-C), Prototype Qualification Test-Government (PQT-G), Operational Test II (OT II), and Production Prototype Test (PPT).

These tests were designed to minimize the risk of a major discrepancy occurring during later test and evaluation, to provide answers to critical questions and issues, to support major decision points and key milestones, and to provide a smooth transition from developement to production. To obtain these objectives, a test philosophy was formulated and applied which proposed: (1) progressive difficulty in flight tests, (2) validation of the computer simulation by flight test data including minimum critical boundaries and low probability of hit trajectories, (3) the use of simulation to define missile dynamic boundaries, and (4) the use of full scale tactical targets only against trajectories demanding maximum target performance such as high speed and difficult maneuver.

The objectives, under the prescribed test philosophy, were accomplished by: (1) a controlled progressive build-up from piece part and component testing through launch testing and flight testing against increasingly more difficult targets in EDT-C and EDT-G, (2) a system level demonstration in a field environment in the Contractor Demonstration (CD) portion of EDT-C, (3) a final system level demonstration in PQT-G, and (4) a preproduction evaluation during PPT.

C. RELIABILITY PHASE SCORING CRITERIA FOR SYSTEM EFFECTIVENESS

The SNIPER scoring criteria for system effectiveness were developed in order to document the firing results in an orderly manner, and to permit assessment of the system for conformity to the requirements document. The scoring criteria contain the method for the scoring of SNIPER flight tests from the time the weapons are accepted by the government through the completion of the mission or flight test.

Procedures were defined for scoring reliability and system effectiveness. The reliability scoring provides a performance measure of the ability of the system to operate without failure. System effectiveness is defined as the probability that the system will operate successfully (kill the target) when called upon to do so under specified conditions. The system characteristics assessed by reliability contribute to the system effectiveness.

These procedures permit assessment of the weapon system from stockpile through flight test. Each test was scored as to whether a valid test of the system occurred or the test was invalidated for some specific reason. The valid tests were further scored as to whether the test was successful or a failure occurred. The successful tests were categorized in terms of significant results and the failures were identified as completely as possible in order that corrective actions could be taken where practical. This scoring technique provided a method for predicting future effectiveness and reliability values by recognizing the effect of corrective action of certain failures that occurred in tests.

The system effectiveness (E_s) for a SNIPER weapon against each type of threat aircraft was defined in the requirements document by effectiveness limits. The E_s is an average value determined by testing or combination of testing and simulation against a sampling of targets presented within the system performance boundaries. The following definitions and equations apply:

$$E_s = R_s \times \underbrace{R_{pf} \times R_f \times R_{whd}} \times L_m \times P_{det}$$

or

$$E_s = R_s \times R_{wpn} \times L_m \times P_{det}$$

where:

R_s = Preuse Reliability: The probability that a government accepted weapon and BCU are capable of being successfully activated at the point of initiation of activation of the weapon. This definition excludes damage through mishandling.

R_{pf} = Prefire Reliability. The probability that a weapon and BCU will function without failure from the moment the BCU is activated until the beginning of trigger pull.

R_f = Firing Reliability. The probability that the weapon and BCU (less warhead detonation functions) will perform without any malfunction which would cause engagement failure from the beginning of trigger pull to point of closest approach to the target.

R_{whd} = Warhead Detonation Reliability. The probability that the fuze will perform without any malfunction which would cause mission failure from the point of flight motor ignition until the end of flight, and that the warhead will detonate upon fuze command.

L_m = Missile Lethality. The probability that a missile which functions properly through warhead detonation will kill the target. L_m is a function of intercept geometry, accuracy, and target structural characteristics.

P_{det} = Probability of detection, evaluation and transfer. The probability that a target which presents itself within the performance envelope of the system will be detected, evaluated, and properly engaged by the gunner. For purposes of engineering development, the performance of the gunner will not degrade this probability.

R_{wpn} = Weapon Reliability. The probability that from battery coolant unit insertion to the point of closest missile approach to the target and warhead detonation, component malfunction will not cause engagement failure ($R_{wpn} = R_{pf} \times R_f \times R_{whd}$).

For the purpose of reliability testing to demonstrate that the system met the specification requirements, a failure was defined as any malfunction in the prefire, fire, or warhead phase which caused target engagement failure. During the development program, the assessment of the attained weapon reliability was based on data from the flight test program which was evaluated in accordance with *Figure 2*, with the following general scoring criteria:

1. For instrumented flights, the initial criterion for scoring benign environment, point source missile flight reliability shall be that the missile trajectory intersects a sphere centered at the centroid of the target. Final reliability scoring will be established by analysis of all pertinent flight data. Anomalies exhibited in flight characteristics, telemetered monitor functions, and target excursions at boundary proximities may require a validated computer simulation to determine final reliability scoring.

2. The initial criterion for scoring plume target missile flight reliability shall be that the missile flight trajectory intersect the space envelopes defined in Appendix A. Trajectories which intersect the space envelopes will initially be scored reliable. Trajectories which do not intersect the described space envelopes will initially be scored as unreliable. Final reliability scoring will be established by analysis of all pertinent flight data. Anomalies exhibited in flight characteristics, telemetered monitor functions, target excursions, or boundary proximities may require evaluation by a validated computer simulation to determine final reliability scoring. Missile trajectories which fall outside the envelopes defined in Appendix A, evidence

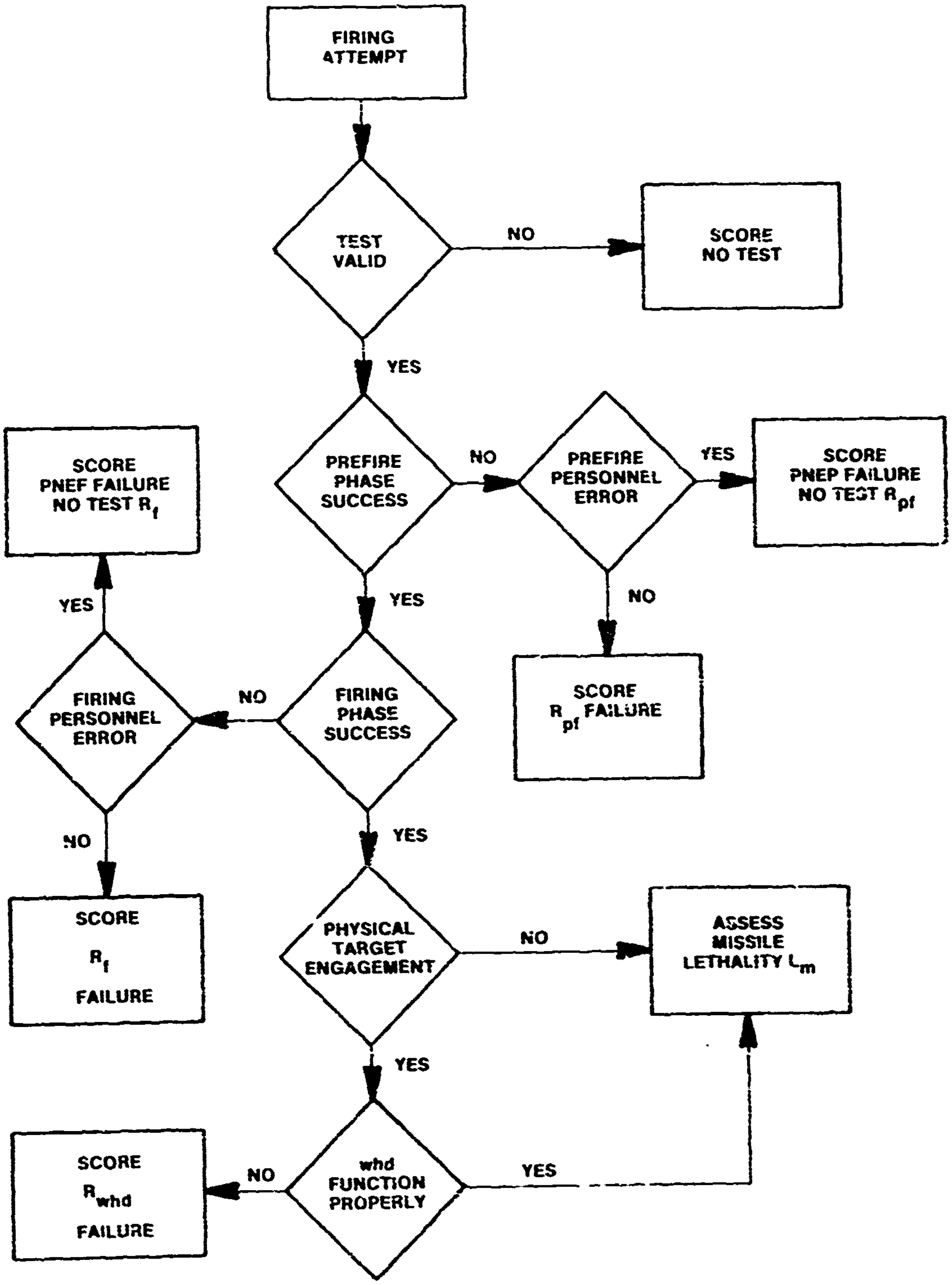


Figure 2. Flow chart.

no flight malfunction, and the simulation reproduces the intercept geometry within the accuracy of the simulation validation plan shall be finally scored reliable. Flight data which establish positive evidence of missile malfunction, independent of flight intercept geometry, shall cause the flight to be scored unreliable.

For the pre-use reliability phase, it was assumed that when the weapon was issued, no gunner maintenance that would preclude the weapon's readiness to fire would occur. The value for this phase will be determined by the Stockpile Surveillance Program during the Use Phase of the System Life Cycle. However, during development testing, R_s was assessed from the result environmental, nonenvironmental, and field handling test results on the missile and weapon round prior to subjecting them to flight tests.

D. RELIABILITY PHASE SCORING METHODOLOGY

The PQT-C and PQT-G test programs consisted of both ground tests and flight tests. The ground tests were composed of environmental tests to specification levels, while the flight tests consisted of flight tests of non-conditioned rounds (rounds which had not been exposed to any form of environmental conditioning or tests) and those rounds that completed the environmental or field testing (conditioned). In order to optimize the generation of engineering data from the environmental tests with minimum expenditures of resources, the weapon and missile rounds were subjected to sequential environmental levels that represented a life time of exposure to that environment. Some rounds were exposed to as many as nine sequential environments before being flight tested (see Appendix B).

Data from the PQT-C and PQT-G test programs were used as the reliability data base for computing Preuse Reliability Values. (Preuse Reliability is defined as the probability that a government accepted weapon and BCU are capable of being successfully activated at the point of initiation of activation of the weapon.) AR 702-3, Army Materiel Reliability, Availability and Maintainability (RAM), paragraph 2-12, Operational Mode Summary and Missile Profile, indicates that "RAM characteristics will be evaluated in accordance with the relative frequency of uses defined in the operational mode summary, rather than overall inclusive potential uses or at the rare extreme uses." In other words, this statement indicates that reliability should be assessed from data representing all environmental levels and operational conditions, not just the extreme cases.

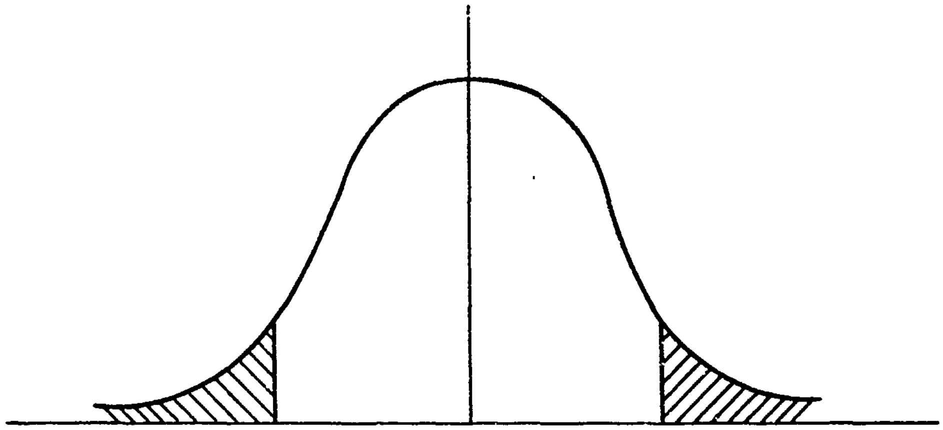
With the above in mind, it was concluded that the SNIPER Preuse Reliability data were comprised of two distinct samples, conditioned and non-conditioned, that must be combined by some realistic means into a reliability estimate. At one extreme are weapons only exposed to the transportation and handling necessary to ship from place of manufacture to White Sands Missile Range (WSMR), where they are assembled and delivered to the flight range; and at the other extreme are rounds preconditioned to life cycle levels of various environments prior to flight testing. The conditioned rounds are subjected to sequential environmental conditions of up to nine environments. Close evaluation indicated that a single data point for each conditioned round was unrealistic, since the data would represent a very small segment of the projected stockpile/field condition. Investigation of the development of Mil-Std-810 life cycle test requirements indicated that each dynamic test represented at least the 95th to 99th worse case condition, while the climatic tests were worse than the 99th percentile. Assuming each environment is meeting the conservative 95th percentile conditions, the following table gives the probability of any one weapon ever being exposed to the specified level of a single or sequence of environments results.

TABLE 2. PROBABILITY OF EXPOSURE TO A SEQUENCE OF ENVIRONMENTAL TESTS

Number of Environmental Stresses	Percent Exposed to Combined Environmental Stress
1	5
2	.25
3	.0125
4	.00625
5	.00003125
6	.0000015625
7	.000000078125
8	.00000000390625
9	.0000000001953125

As the above data indicate, five percent of the weapons would be exposed to the environmental level of any one environment, but only one in ten thousand weapons would ever receive the combined stress of three life cycle environments, and, of course, only eight weapons in 10 billion would ever be exposed to the specified level of seven sequential environments. As noted, this degree of sequential testing becomes very unrealistic quite rapidly.

In order to make a viable estimate of Preuse Reliability, the decision was made that while working within the available data base, the data should be combined on as equal a rating as possible between non-conditioned and conditioned rounds. The logic for this is presented by the following illustration,

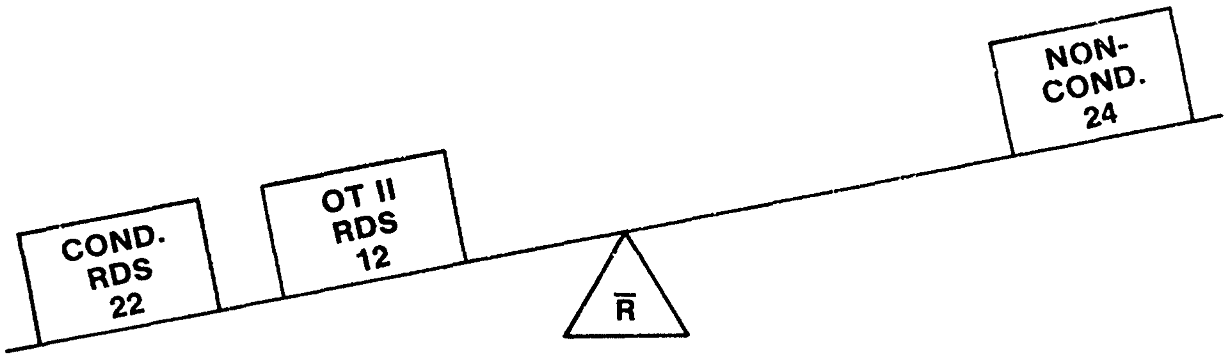


where $\hat{r}$ is the reliability estimate. This assumes that equal sample sizes will be available for the non-conditioned rounds and conditioned rounds (one datum point following each environment).

As noted by review of the reliability data base for PQT-G and OT II (*Figure 3*), the assumption of equal samples for both conditioned and non-conditioned rounds did not hold. Instead of making $\hat{r} = (\hat{r}_1 + \hat{r}_2)/2$, where $\hat{r}_1$ and $\hat{r}_2$ are the means of the 5th and 95th percentile, it was decided to let $\hat{r} = \sum_{i=1}^{n} r_i/n$ and let the reliability estimate become more pessimistic with the heavier weighting of conditioned data. In addition, as review of the data will confirm (see Appendix B), some data points were actually accumulated after multiple environmental exposure instead of after each exposure, which added an additional degree of pessimism.

In the final assessment, preuse reliability data were accumulated from each operational check of the hardware by TECOM that was either the first performed on the hardware or the first performed after the hardware was exposed to a new environment. Each datum point was given an equal weight and Preuse Reliability was calculated as a success ratio (successes/total tests) for the missile round, BCU and separable gripstock.

The SNIPER R_s Model, *Figure 4*, illustrates the relationship of the various weapon system components and their effect on R_s or Preuse Reliability. The model is factored into two distinct parts, stockpile storage environment and field environment. The tests conducted during PQT-C and PQT-G yielded data that were primarily related to the field environment



CONDITIONED RDS.

21-TRANSPORTATION VIBRATION
19-36 INCH DROP
18-21 INCH DROP
0-DUST
9-HUMIDITY
8-SALT FOG
10-MUD
9-ICING
1-FUNGUS
11-HIGH TEMP.
10-LOW TEMP.
8-IMMERSION
9-RAIN
18-LOOSE CARGO
11-FLIGHT

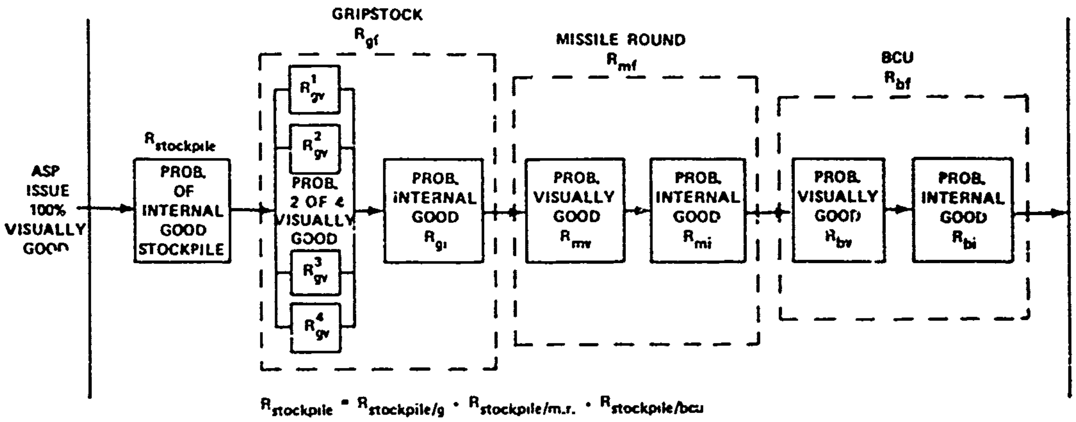
OT II RDS.

8-FIELD TRANSPORTATION AND HANDLING
11-FLIGHT

NON-CONDITIONED RDS.

49-MINIMUM TRANSPORTATION AND HANDLING
24-FLIGHT

Figure 3. PQT-G and OT II reliability sample.



$$R_{stockpile} = R_{stockpile/g} \cdot R_{stockpile/m.r.} \cdot R_{stockpile/bcu}$$

$$R_{gf} = \left\{ R^4_{gv} + 4\, R^3_{gv} (1 - R_{gv}) + 6R^2_{gv} (1 - R^2_{gv}) \right\} \cdot R_{gi}$$

$$R_{mf} = R_{mv} \cdot R_{mi}$$

$$R_{bf} = R_{bv} \cdot R_{bi}$$

$$R_{field\ environ.} = R_{gf} \cdot R_{mf} \cdot R_{bf}$$

$$R_s = R_{stockpile} \cdot R_{field\ environ.}$$

R_s – PREUSE RELIABILITY

$R_{stockpile}$ – PROBABILITY OF NO INTERNAL FAULURE DURING THEATER OR STOCKPILE STORAGE. /g REFERS TO GRIPSTOCK; /m.r. REFERS TO MISSILE ROUND. /bcu REFERS TO BCU.

$R_{field\ environment}$ – PROBABILITY OF NO FAULURE DURING PERIOD BETWEEN ISSUE AT AMMUNITION SUPPLY POINT AND FIRING THE WEAPON.

R_{gf} – GRIPSTOCK FIELD ENVIPONMENT RELIABILITY; R_{gv} REFERS TO "VISUALLY GOOD" GRIPSTOCKS IN FIELD ENVIRONMENT; R_{gi} REFERS TO "INTERNALLY GOOD" GRIPSTOCKS IN FIELD ENVIRONMENT.

R_{mf} – MISSILE ROUND FIELD ENVIRONMENT RELIABILITY

R_{bf} – BCU FIELD ENVIRONMENT RELIABILITY

Rw – WEAPON RELIABILITY

R_{pf} – PREFIRE RELIABILITY; /gpst REFERS TO GRIPSTOCK, /m.r. REFERS TO MISSILE ROUNDS, /bcu REFERS TO BCU.

P_f – FIRING RELIABILITY; /gpst REFERS TO GRIPSTOCK; /m.r. REFERS TO MISSILE ROUND; /bcu REFERS TO BCU.

R_{whd} – WARHEAD DETONATION RELIABILITY WHICH IS PRODUCT OF R_{fuze} (FUSE RELIABILITY) AND $R_{w/h}$ (WARHEAD RELIABILITY).

Figure 4. SNIPER R_S model for E_S (Preuse Phase — Basic Load).

with very little inference to the stockpile storage environment. For purposes of assessment during ED, the stockpile storage factor was assigned a value of 1.0. In calculating the field environmental factor, it is necessary to treat the separable gripstock as a redundant system when considering the mission effects of visually detectable failure modes. The equations given in *Figure 4* provide the relationship of visual and internal (electrical) failure of the gripstock to mission success for a basic load of six missile rounds and four gripstocks. No distinction is made between visual and internal failure of the missile round and BCU.

Appendix B gives a detailed summary of the reliability scoring of the SNIPER PQT-G environmental testing from which the scoring methodology for the calculation of R_s can be followed. It provides the environmental test matrix, the reliability scoring of the test matrix rounds, a description of specific problems found, and specifies the environmental levels used during the test.

The flight test results were scored in accordance with the criteria by evaluating the required input data shown in *Table 3*. The rationale for the assessment of the Preuse Reliability Scoring Phase has been given above. The scoring of Weapon Reliability was subdivided into Prefire, Firing, and Warhead Reliability Scoring Phases and subsequently divided for data gathering and functional purposes as shown in *Figure 5*.

A detailed summary of the reliability scoring of the flight tests has not been provided. The author feels that Appendix B provides sufficient answers to the "How" and "Why" of the reliability scoring when supplemented with the following "When."

1. R_s was scored for the gripstock and missile round after each environment-ground test.

2. R_s for missile round was scored for all missile rounds.

3. R_s for BCU was scored for all BCU's.

4. R_{pf} was scored for the gripstock and missile round after each activation, flight, and ground test.

5. R_f was scored for the gripstock and missile round during the flight test only.

6. R_{fuze} was scored for the fuze during flight test only.

7. $R_{w/h}$ was scored for the warhead (w/h) during flight test only.

TABLE 3. RELIABILITY SCORING INPUTS

ENVIRONMENTAL DATA - BCU, MISSILE ROUND, GRIPSTOCK

TARGET PERFORMANCE/POSITION DATA

GUNNER PERFORMANCE

BCU PERFORMANCE

LAND LINE DATA (PREFIRE - SPIN-UP, COOLDOWN, ACQUISITION SIGNAL)

LAUNCH CAMERA DATA - TIP-OFF, ROLL RATE, GUNNER

MISS DISTANCE - VISUAL, MDI, OPTICS

- RELIABILITY CYLINDER - QUICK LOOK
- SIMULATION DISPERSION

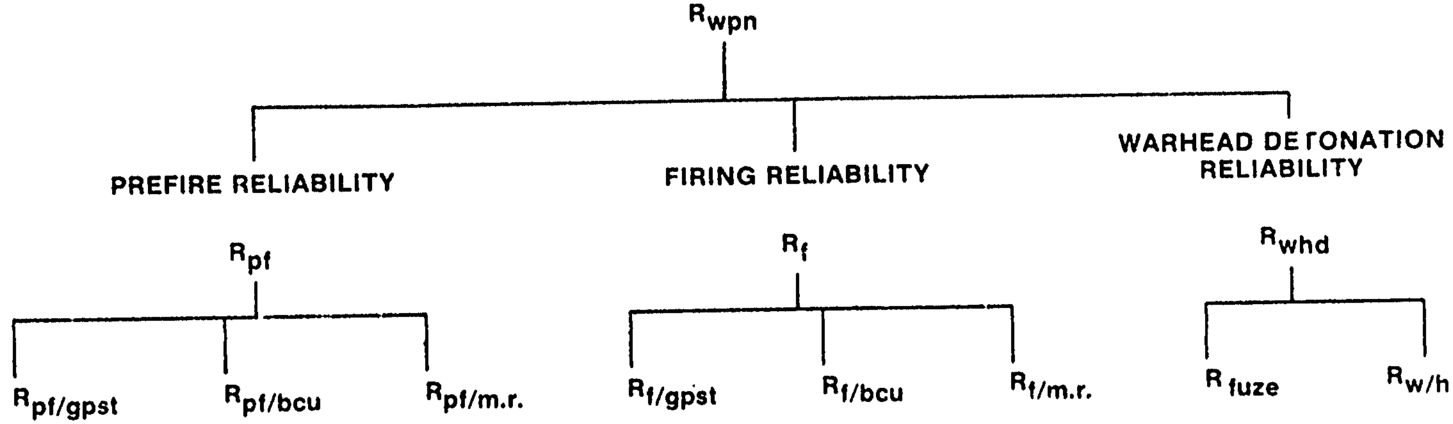
TM RECORDS - GUIDANCE AND FUZE FUNCTIONS

END GAME CAMERAS - TOWER SHOTS

- FUZE DELAY TIME - W/H PENETRATION
- W/H PERFORMANCE

VISUAL OBSERVATION

- W/H PERFORMANCE - HIGH ORDER
- W/H SELF-DESTRUCT



WHERE:

$R_{pf/gpst}$ = PREFIRE RELIABILITY OF THE GRIPSTOCK

$R_{pf/bcu}$ = PREFIRE RELIABILITY OF THE BCU

$R_{pf/m.r.}$ = PREFIRE RELIABILITY OF THE MISSILE RND.

$R_{f/gpst}$ = FIRING RELIABILITY OF THE GRIPSTOCK

$R_{f/bcu}$ = FIRING RELIABILITY OF THE BCU

$R_{f/m.r.}$ = FIRING RELIABILITY OF THE MISSILE RND.

R_{fuze} = RELIABILITY OF THE FUZE

$R_{w/h}$ = RELIABILITY OF THE WARHEAD

Figure 5. Weapon reliability scoring.

E. CONFIDENCE IN WEAPON AND SYSTEM RELIABILITY

Army Regulation 702-3, Army Materiel Reliability, Availability, and Maintainability (RAM) requires that DT II/OT II be designed to evaluate RAM characteristics and assure program continuation if sufficient reliability growth of the system has been achieved. It specifies that sufficient reliability growth will have been achieved if the minimum acceptable value (MAV) is demonstrated at high confidence during the DT/OT preceding the decision to type classify standard. Thus, the regulation requires the evaluation of a lower confidence limit on weapon reliability at a required (system specified) confidence.

The lower confidence limit on weapon reliability will be evaluated for the SNIPER System by applying the proposed method as described in Chapter 2. The weapon reliability scoring breakdown is given in *Figure 5* and the pertinent data from the reliability phase scoring results of PQT-G are provided in *Table 4*. The sample sizes and percent defective for the respective components of each scoring phase provide a feel for the wide range of sample sizes and data fluctuations.

For the assessment of the lower confidence limit on Weapon Reliability, the value for each parameter in the control equation, $\hat{R}_{wpn} - K\sigma_{\hat{R}_{wpn}}$, is determined as follows:

1. The value for $\hat{R}_{wpn}$ is obtained by use of equation 1 from Chapter 2 and is the product of the values for the reliability scoring phases given in *Figure 5*. This product yields $\hat{R}_{wpn} = .8592$.

2. The tolerance factor for both a 90 percent and 70 percent confidence value was generated in Chapter 2 such that $K \leq 2.108$ for the 90 percent case and $K \leq 1.217$ for the 70 percent confidence case.

3. The standard deviation for weapon reliability is determined by application of equation 5 from Chapter 2 and the substitution of the respective data values from *Table 4*. This yields $Var(\hat{R}_{wpn}) = .00149$ and $\sigma_{\hat{R}_{wpn}} = .0386$.

Substituting the above values into the control equation, the following is obtained:

90 percent Lower Confidence Limit on Weapon R = .8592 - 2.108 (.0386)

90 percent Lower Confidence Limit on Weapon R = .778

70 percent Lower Confidence Limit on Weapon R = .8592 - 1.217 (.0386)

70 percent Lower Confidence Limit on Weapon R = .8122

TABLE 4. RELIABILITY PHASE SCORING RESULTS-PQT-G DATA

Phase	Phase Element	Sample Size	Percent Defective ($\hat{q}$)
Preuse (R_s)			
Stockpile	(assessed as 1.0 during ED)		
Field Environment	Gripstock (R_{gf})		
	gv (visual)	71	5.6
	gi (internal)	67	3
	Missile Round (R_{mf})	153	10.4
	BCU (R_{bf})	138	1.4
Weapon Prefire (R_{pf})	Gripstock ($R_{pf/gpst}$)	493	3
	Missile Round ($R_{pf/m.r.}$)	309	2
	BCU ($R_{pf/bcu}$)	107	2
Firing (R_f)	Gripstock ($R_{f/gpst}$)	201	2
	Missile Round ($R_{f/m.r.}$)	30	3
	BCU ($R_{f/bcu}$)	128	.8
Warhead Detonation (R_{whd})	Fuze (R_{fuze})	44	2.2
	Warhead ($R_{w/h}$)	6	0

Although the application of confidence to system reliability is not a requirement, it is easily obtained by applying the proposed method. The system reliability assessment will be obtained for the model $\hat{R} = \hat{R}_{v} \cdot \hat{R}_{wpn}$ by combining the model given in *Figure 4* and the weapon reliability expression given in *Figure 5* for which the expanded equation is given in *Table 5*. The necessary data are provided in *Table 4*.

By substituting the assessed value of each term from *Table 4* into the expanded equation for system reliability given in *Table 5*, the point estimate for system reliability is $\hat{R} = .7358$.

Applying equation 4 from Chapter 2 to the expanded equation for system reliability given in *Table 5* and noting that $E\{x_i - E(x_i)\}^3$ in equation 4 is the third central moment for each respective term in the system reliability expression (*Table 5*), the system variance is calculated as shown below. The calculations are shown for each individual term in the expanded system reliability equation. The values for these individual terms are then summed, per equation 4 from Chapter 2. to yield the system variance. The respective calculations are:

1. The calculations for the contribution to system variance from the first term, $\hat{R}_{gv}$, in the system reliability model are:

$$
\begin{aligned}
\mathrm{Var}\,(\hat{R}(\text{1st term only}, \hat{R}_{gv})) = {} & \{ \left[12\hat{R}_{gv} - 24\hat{R}^2_{gv} + 12\hat{R}^3_{gv} \right] \cdot \hat{R}_{gi} \\
& \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{wpn} \}^2 \; \frac{p_{gv} q_{gv}}{n_{gv}} + \{ \left[12\hat{R}_{gv} \right. \\
& \left. - 24\hat{R}^2_{gv} + 12\hat{R}^3_{gv} \right] \cdot \hat{R}_{gi} \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \\
& \cdot \hat{R}_{wpn} \} \cdot \{ \left[12 - 48\hat{R}_{gv} + 36\hat{R}^2_{gv} \right] \cdot \hat{R}_{gi} \\
& \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{wpn} \} \cdot \{ \hat{R}_{gv} (\hat{R}_{gv} - 1) \\
& \cdot (\hat{R}_{gv} - 2) + 3\hat{R}_{gv} (\hat{R}_{gv} - 1) + \hat{R}_{gv} - 3(\hat{R}^2_{gv}) \\
& \cdot (\hat{R}_{gv} - 1) - 3\hat{R}^2_{gv} + 2\hat{R}^3_{gv} \} \\
= {} & \{ .00068 \cdot .00074 \} + \{ .02615 \cdot (-.90645) \\
& \cdot (-.29942) \}
\end{aligned}
$$

TABLE 5. SNIPER SYSTEM RELIABILITY MODEL

$$\hat{R} = \hat{R}_{s} \cdot \hat{R}_{wpn}$$

$$= \hat{R}_{stockpile} \cdot \hat{R}_{field\ environment} \cdot \hat{R}_{wpn}$$

$$= 1\ (\hat{R}_{field\ environment} \cdot \hat{R}_{wpn})$$

$$= \hat{R}_{field\ environment} \cdot \hat{R}_{pf} \cdot \hat{R}_{f} \cdot \hat{R}_{whd}$$

$$= \hat{R}_{gf} \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{pf} \cdot \hat{R}_{f} \cdot \hat{R}_{whd}$$

$$= \hat{R}_{gf} \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{pf/gpst} \cdot \hat{R}_{pf/bcu} \cdot \hat{R}_{pf/m.r.} \cdot \hat{R}_{g/gpst} \cdot \hat{R}_{f/bcu} \cdot \hat{R}_{f/m.r.}$$

$$\cdot \hat{R}_{fuze} \cdot \hat{R}_{w/h}$$

$$\hat{R} = \left[\{\hat{R}^{4}_{gv} + 4\hat{R}^{3}_{gv}\ (1-\hat{R}_{gv}) + 6\hat{R}^{2}_{gv}\ (1-\hat{R}_{gv})^{2}\} \cdot \hat{R}_{gi} \right] \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{pf/gpst} \cdot \hat{R}_{pf/bcu}$$

$$\cdot \hat{R}_{pf/m.r.} \cdot \hat{R}_{f/gpst} \cdot \hat{R}_{f/bcu} \cdot \hat{R}_{f/m.r.} \cdot \hat{R}_{fuze} \cdot \hat{R}_{w/h}$$

$$= .000001 + .007098.$$

$$Var(\hat{R}(\text{1st term only}, \hat{R}_{gv})) = .00710.$$

2. The calculations for the contribution to system varia ~om the second term, $\hat{R}_{gi}$, in the system reliability model are:

$$Var(\hat{R}(\text{2nd term only}, \hat{R}_{gi})) = \{\left[\hat{R}^4_{gv} + 4\hat{R}^3_{gv}(1-\hat{R}_{gv}) + 6\hat{R}^2_{gv}(1-\hat{R}_{gv})^2\right] \cdot \hat{R}_{mf} \cdot \hat{R}_{bf} \cdot \hat{R}_{wpn}\}^2 \frac{p_{gi}q_{gi}}{n_{gi}}$$

$$= \{[.99933] \cdot .8960 \cdot .9860 \cdot .85916\}^2 \cdot .00043.$$

$$Var(\hat{R}(\text{2nd term only}, \hat{R}_{gi})) = .00025.$$

3. The calculations for the contribution to system variance from the third term, $\hat{R}_{mf}$, in the system reliability model are:

$$Var(\hat{R}(\text{3rd term only}, \hat{R}_{mf})) = \{\left[\{\hat{R}^4_{gv} + 4\hat{R}^3_{gv}(1-\hat{R}_{gv}) + 6\hat{R}^2_{gv} \cdot (1-\hat{R}_{gv})^2\} \cdot \hat{R}_{gi}\right] \cdot \hat{R}_{bf} \cdot \hat{R}_{wpn}\}^2 \frac{p_{mf}\, q_{mf}}{n_{mf}}$$

$$= \{\left[\{.99933\} \cdot .9700\right] \cdot .9860 \cdot .85916\}^2 \cdot .00061.$$

$$Var(\hat{R}(\text{3rd term only}, \hat{R}_{mf})) = .00041.$$

4. The fourth term, $\hat{R}_{bf}$, contribution is:

$$Var(\hat{R}(\text{4th term only}, \hat{R}_{bf})) = \{\left[\{\hat{R}^4_{gv} + 4\hat{R}^3_{gv}(1-\hat{R}_{gv}) + 6\hat{R}^2_{gv}(1-\hat{R}_{gv})^2\} \cdot \hat{R}_{gi}\right] \cdot \hat{R}_{mf} \cdot \hat{R}_{wpn}\}^2 \frac{p_{bf}q_{bf}}{n_{bf}}$$

$$= \{ \left[\{.99933\} \cdot .9700 \right] \cdot .8960 \cdot .85916\}^2$$

$$\cdot \ .0001 \ .$$

$$Var(\hat{R}(4th\ term\ only,\ \hat{R}_{bf})) = .00006.$$

5. The fifth term, $\hat{R}_{pf/gpst}$, contribution is:

$$Var(\hat{R}(5th\ term\ only,\ \hat{R}_{pf/gpst})) = \quad \cdot \hat{R}_{pf/bcu} \cdot \hat{R}_{pf/m.r.} \cdot \hat{R}_f$$

$$\cdot R_{whd}\}^2 \frac{p_{pf/gpst}q_{pf/gpst}}{n_{pf/gpst}}$$

$$= \{.8564 \cdot .9800 \cdot .9800 \cdot .9430$$

$$\cdot .9780\}^2 \cdot .000059.$$

$$Var(\hat{R}(5th\ term\ only,\ \hat{R}_{pf/gpst})) = .00003$$

6. The sixth term, $\hat{R}_{pf/bcu}$, contribution is:

$$Var(\hat{R}(6th\ term\ only,\ \hat{R}_{pf/bcu})) = \{\hat{R}_s \quad \hat{R}_{pf/gpst} \cdot \hat{R}_{pf/m.r.} \cdot \hat{R}_f$$

$$\cdot \hat{R}_{whd}\}^2 \frac{p_{pf/bcu}q_{pf/bcu}}{n_{pf/bcu}}$$

$$= \{.8564 \cdot .9700 \cdot .9800 \cdot .9430$$

$$\cdot .9780\}^2 \cdot .000183.$$

$$Var(\hat{R}(6th\ term\ only,\ \hat{R}_{pf/bcu})) = .00010.$$

7. The seventh term, $\hat{R}_{pf/m.r.}$, contribution is:

$$Var(\hat{R}(7th\ term\ only,\ \hat{R}_{pf/m.r.})) = \{\hat{R}_s \cdot \hat{R}_{pf/gpst} \cdot \hat{R}_{pf/bcu} \cdot \hat{R}_f \cdot \hat{R}_{whd}\}^2 \frac{p_{pf/m.r.}q_{pf/m.r.}}{n_{pf/m.r.}}$$

$$= \{.8564 \cdot .9700 \cdot .9800 \cdot .9430 \cdot .9780\}^2 \cdot .00097.$$

$$Var(\hat{R}(7th\ term\ only,\ \hat{R}_{pf/m.r.})) = .00055.$$

8. The eighth term, $\hat{R}_{f/gpst}$, contribution is:

$$Var(\hat{R}(8th\ term\ only,\ \hat{R}_{f/gpst})) = \{\hat{R}_s \cdot \hat{R}_{pf} \cdot \hat{R}_{f/bcu} \cdot \hat{R}_{f/m.r.} \cdot \hat{R}_{whd}\}^2 \frac{p_{f/gpst}q_{f/gpst}}{n_{f/gpst}}$$

$$= \{.8564 \cdot .93159 \cdot .9920 \cdot .9700 \cdot .9780\}^2 \cdot .000098.$$

$$Var(\hat{R}(8th\ term\ only,\ \hat{R}_{f/gpst})) = .00006.$$

9. The ninth term, $\hat{R}_{f/bcu}$, contribution is:

$$Var(\hat{R}(9th\ term\ only,\ \hat{R}_{f/bcu})) = \{\hat{R}_s \cdot \hat{R}_{pf} \cdot \hat{R}_{f/gpst} \cdot \hat{R}_{f/m.r.} \cdot \hat{R}_{whd}\}^2 \frac{p_{f/bcu}q_{f/bcu}}{n_{f/bcu}}$$

$$= \{.8564 \cdot .93159 \cdot .9800 \cdot .9700 \cdot .9780\}^2 \cdot .000062.$$

$$Var(\hat{R}(9th\ term\ only, \hat{R}_{f/bcu})) = .00003.$$

10. The tenth term, $\hat{R}_{f/m.r.}$, contribution is:

$$Var(\hat{R}(10th\ term\ only, \hat{R}_{f/m.r.})) = \{\hat{R}_s \cdot \hat{R}_{pf} \cdot \hat{R}_{f/gpst} \cdot \hat{R}_{f/bcu} \cdot \hat{R}_{whd}\}^2 \frac{p_{f/m.r.} q_{f/m.r.}}{n_{f/m.r.}}$$

$$= \{.8564 \cdot .93159 \cdot .9800 \cdot .9920 \cdot .9780\}^2 \cdot .00097.$$

$$Var(\hat{R}(10th\ term\ only, \hat{R}_{f/m.r.})) = .00056.$$

11. The eleventh term, $\hat{R}_{fuze}$, contribution is:

$$Var(\hat{R}(11th\ term\ only, \hat{R}_{fuze})) = \{\hat{R}_s \cdot \hat{R}_{pf} \cdot \hat{R}_f \cdot \hat{R}_{w/h}\}^2 \cdot \frac{p_{fuze}\, q_{fuze}}{n_{fuze}}$$

$$= \{.8564 \cdot .93159 \cdot .9430 \cdot 1.0\}^2 \cdot .000489.$$

$$Var(\hat{R}(11th\ term\ only, \hat{R}_{fuze})) = .00028.$$

12. The twelfth term, $\hat{R}_{w/h}$, contribution is:

$$\mathrm{Var}(\hat{R}(\text{12th term only}, \hat{R}_{w/h})) = \{\hat{R}_s \cdot \hat{R}_{pf} \cdot \hat{R}_f \cdot \hat{R}_{fuze}\}^2 \cdot \frac{p_{w/h} q_{w/h}}{n_{w/h}}$$

$$= \{.8564 \cdot .93159 \cdot .9430 \cdot .9780\}^2 \cdot 0 \; .$$

$$\mathrm{Var}(\hat{R}(\text{12th term only}, \hat{R}_{w/h})) = 0.$$

Summing the calculated values (rounded off to the 5th decimal place) from 1 through 12 above, the system variance is:

$$\mathrm{Var}(\hat{R}) = .00710 + .00025 + .00041 + .00006 + .00003 + .00010 + .00055 + .00006 + .00003 + .00056 + .00028 + 0$$

$$\mathrm{Var}(\hat{R}) = .00943.$$

Therefore, $\sigma_{\hat{R}} = .0971$.

Combining the values for all respective terms in the control equation, the lower confidence limit on system reliability is determined as:

$$\hat{R} - K\sigma_{\hat{R}} = .7358 - 2.108\ (.0971) = .531.$$

Therefore, the

90 percent Lower Confidence Limit on System R = .531

and the

70 percent Lower Confidence Limit on System R = .618.

5. CONCLUSIONS AND RECOMMENDATIONS

The aim of this paper has been met since a method has been presented, as described in Chapter 2, which provides a quick and very good means of determining a definite lower confidence limit on system reliability given a designated confidence level.

Only desk calculations are necessary for the application of the method. It can provide a fairly accurate reliability estimate in time to support management decisions when not enough time is available for rigorous, exacting analytic solutions or computer facilities with "canned" applicable programs are not available. It is not limited, in that it applies to practically any reliability system model, phase sample sizes and combinations of different phase distributions. It is especially adaptable and easily used with the common serial reliability model applicable to missile weapon systems and the present testing philosophy and reliability phase scoring methodology.

The recommendation is twofold: (1) That the method be used by MICOM and the respective industrial community as a "quick-look" procedure for generating a lower confidence limit on weapon reliability during the system development phase; and (2) that the method be used during the production phase to provide an "on-the-spot" check of system reliability and the inherent confidence in the production process.

APPENDIX A

SNIPER

DETERMINATION OF FLIGHT RELIABILITY

To determine the flight reliability (R_f) of SNIPER, various limitations for different target conditions were considered. These conditions are:

a. Benign, with or without maneuvers.

b. Afterburner, with or without maneuvers.

c. Afterburner maneuvers, and countermeasures.

Figures A-1, *A-2*, and *A-3* show the space envelopes for the above conditions. The space envelopes are cylinders of lengths and diameters to represent success/failure under different target conditions. Where miss distance instrumentation was used, a first "look" determination of the success/failure was made. If a flight was outside of the specific space envelope, a valid simulation was used to determine the success/failure condition by considering the actual flight of a typical good missile. In the cases of gross failures, ballistic, loss of acquisition, or excessive miss distance no simulation was required. In service practice and gunner training firings where only a miss distance indicator is used, a first "look" determination will be made and then validated by a valid simulation representing range conditions similar to the particular flight.

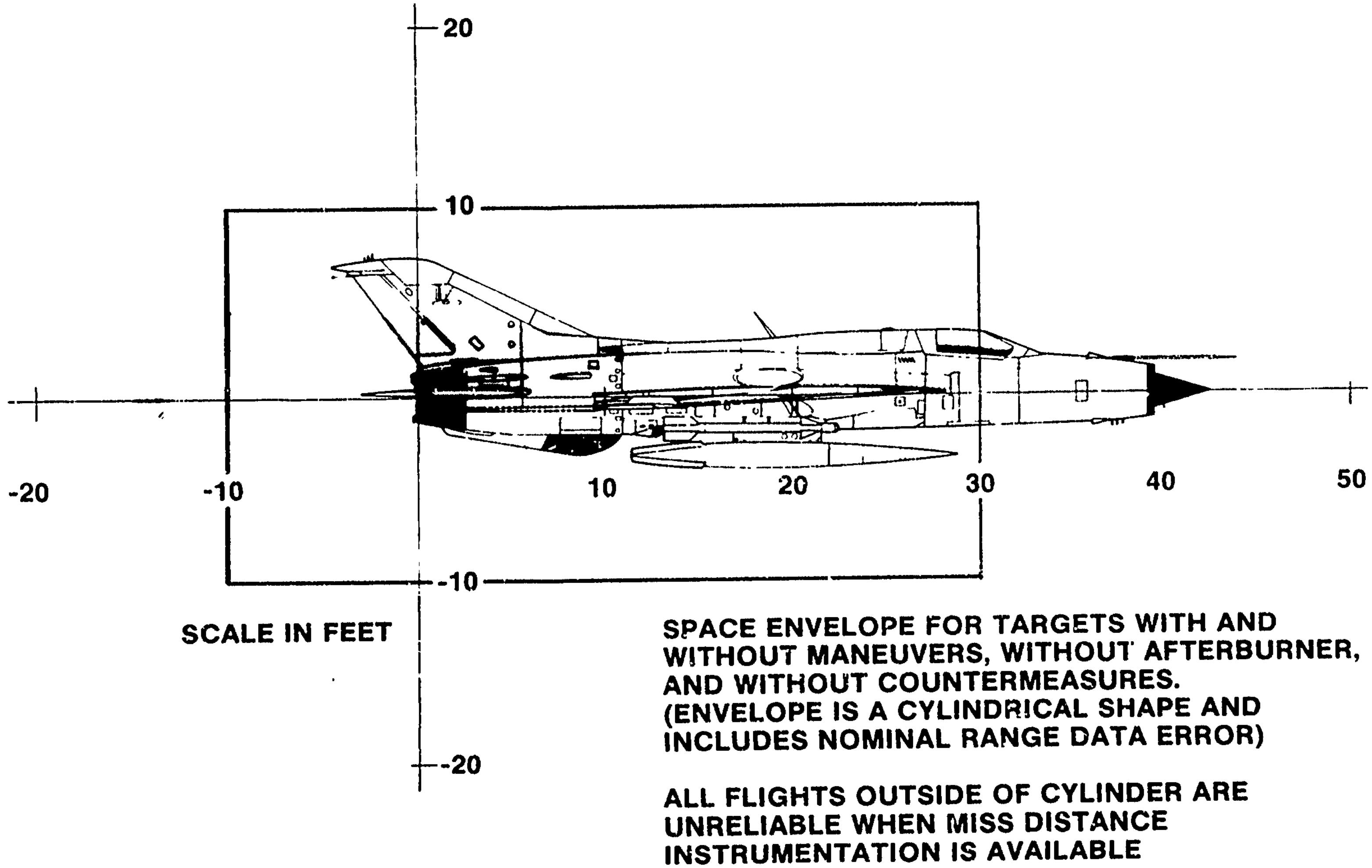


Figure A-1. Reliability envelope.

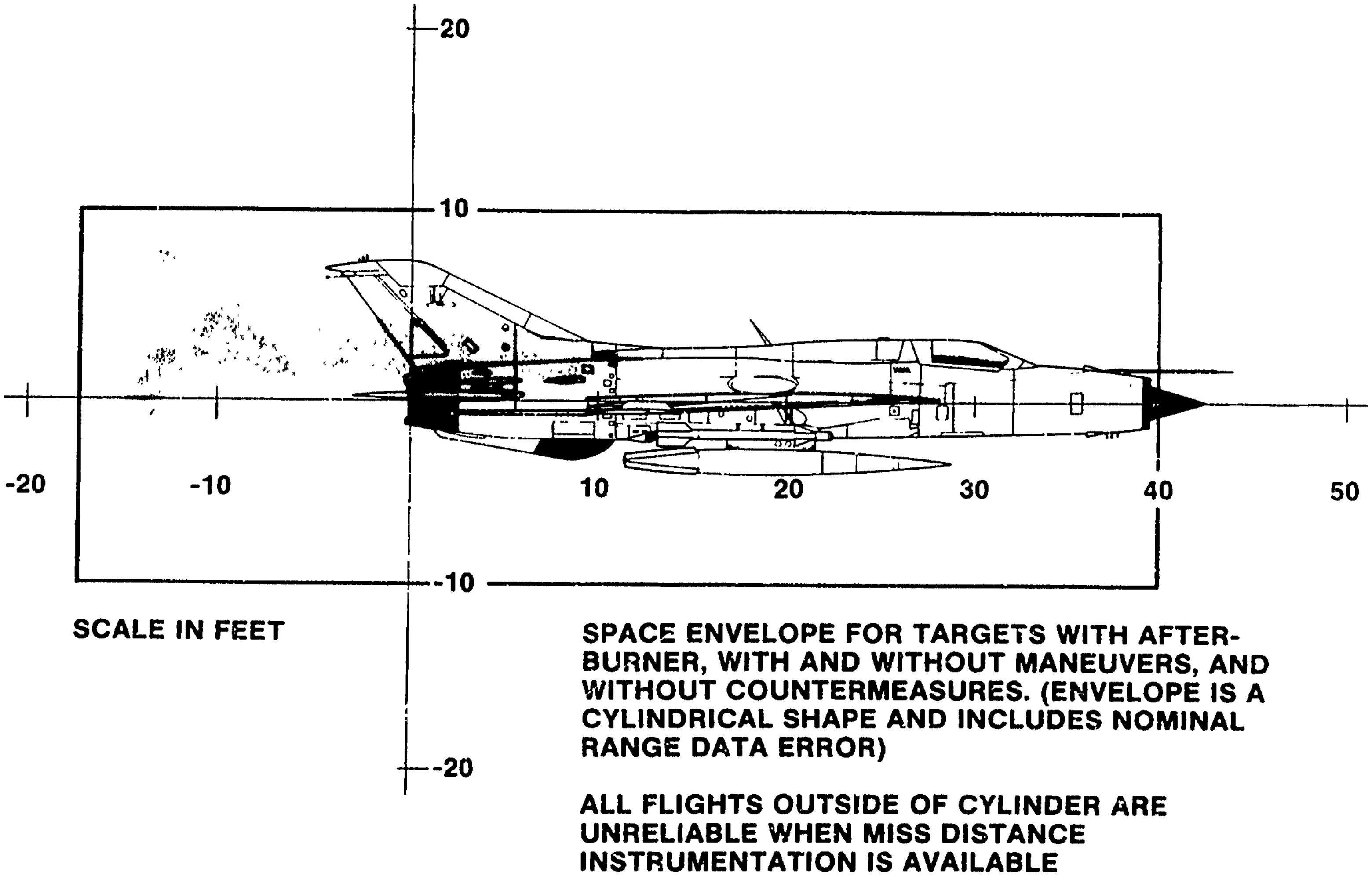


Figure A-2. Reliability envelope.

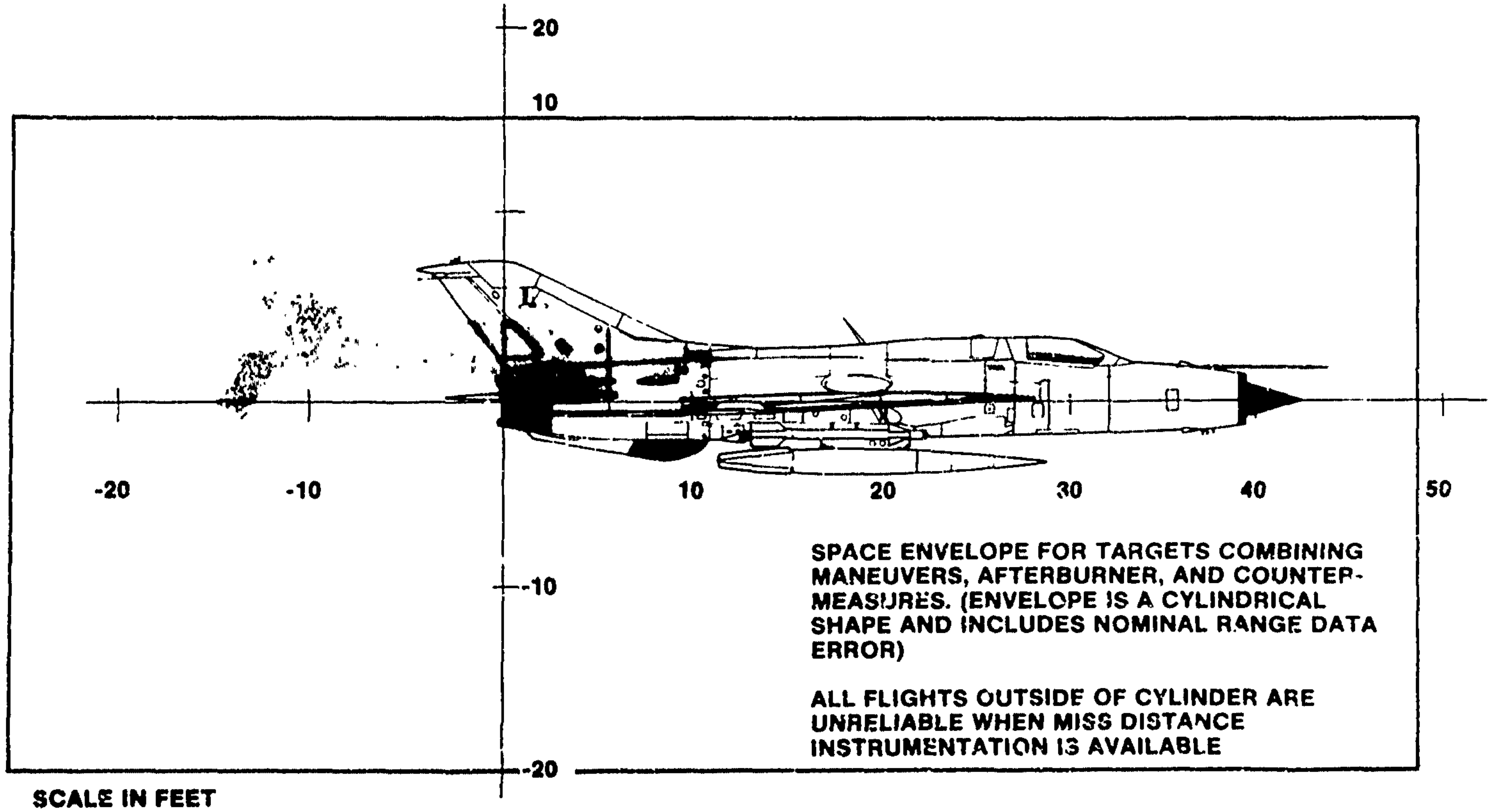


Figure A-3. Reliability envelope.

APFENDIX B

RESULTS OF

SNIPER

PQT-G ENVIRONMENTAL TESTING

This Appendix summarizes the reliability scoring of the SNIPER PQT-G environmental testing. It provides the environmental test matrix, the reliability scoring of the test matrix rounds, a description of specific problems found, and specifies the environmental levels used during the test.

Legend: TC-? — Telemeter round cold condition
TH-? — Telemeter round hot condition
WC-? — Warhead round cold condition
WH-? — Warhead round hot condition
MR — Missile Round
WR — Weapon Round
TLM — Telemeter
WH — Warhead
S — Success
F — Failure
NT — No Test
T.V. — Transportation vibration test
36 in. — 36 inch drop test
L.C. — Loose cargo bounce test
21 in. — 21 inch drop test
Humid — Humidity test
S. Fog — Salt Fog Test
Immer — Immersion Test

WC-1

Missile Round was subjected to x-ray inspection by the contractor and a visual inspection prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo and 21" drop before first operational check, i.e., polecheck. Round was incorrectly dropped on the forward end during the 21" drop test and the IR window cracked; this was not scored as a reliability failure due to the incorrect drop. The window was replaced by the contractor and the round subjected to icing and mud environments. Visual examination and x-ray inspection were performed after each environment with exception of mud where only a visual was conducted. Prior to flight test, the round was subjected to operational test (polecheck). WC-1 successfully completed all environmental testing without failure. Round was fired on 20 November 76 as PQTG-10.

TABLE B-1. PQT-G ENVIRONMENTAL TESTS

ENVIRONMENT: RND	TYPE	CND	TRANS VIB	36 IN. DROP	LOOSE CARGO	21 IN. DROP	RAIN	HUMID	SALT FOG	MUD	IMMER	ICING	FUNGUS
TC-1	MR/TLM	COLD	X	X	X	X				X		X	
TC-2	MR/TLM	COLD	X	X	X	X				X		X	
TC-3	MR/TLM	COLD	X	X	X	X				X		X	
TC-4	WR/TLM	COLD	X	X	X	X				X		X	
TC-5	WR/TLM	COLD	X	X	X	X				X		X	
TC-6	WR/TLM	COLD	X										
WC-1	MR/WH	COLD	X	X	X	X				X		X	
WC-2	MR/WH	COLD	X	X	X	X				X		X	
WC-3	WR/WH	COLD	X	X	X	X				X		X	
WC-4	WR/WH	COLD	X	X	X	X				X		X	
WH-1	MR/WH	HOT	X	X	X	X	X	X	X		X		
WH-2	MR/WH	HOT	X	X									
WH-3	WR/WH	HOT	X	X	X	X	X	X	X	X	X		
WH-4	WR/WH	HOT	X	X	X	X	X	X	X		X		
TH-1	MR/TLM	HOT	X	X	X	X	X	X	X		X		
TH-2	MR/TLM	HOT	X										
TH-3	MR/TLM	HOT	X	X	X	X	X	X	X		X		
TH 4	WR/TLM	HOT	X	X	X	X	X	X	X		X		
TH-5	WR/TLM	HOT	X	X	X	X	X	X	X		X		
TH-6	WR/TLM	HOT	X	X	X	X	X	X	X		X		
TH-7	WR/TLM	HOT	X	X	X	X	X	X					
TCF-1	M/TLM	HOT											X
TCF-2	WR/TLM	HOT											X

TABLE B-2. RELIABILITY ASSESSMENT OF PQT-G ENVIRONMENTAL TESTS

TYPE ROUND	GRIPSTOCK	ENVIRONMENTAL TESTS										REMARKS
	M.R.	T.V.	36 in.	L.C.	21 in.	RAIN	HUMID	S. FOG	MUD	IMMER	ICE	
WH-1*	NA	-	-	-	-	-	-	-		-		INCORRECT DROP
	000133	S	S	S	NT	S	S	S		F		NO TARGET LOCK
WH-2*	NA	-	-									UMBILICAL RETRACT
	000104	S	F									
WH-3*	101	S	S	S	S	S	S	S	S	S		NO GAS FLOW-
	000224	S	S	S	S	S	S	S	S	NT		OVER TEST
WH-4*	190	S	S	S	S	S	S	S		S		GAS LINE BROKEN
	000105	S	S	S	S	S	F	S		S		
WC-1*	NA	-	-	-	-				-		-	INCORRECT DROP
	000222	S	S	S	NT				S		S	
WC-2*	NA	-	-	-	-				-		-	INCORRECT DROP
	000220	S	S	S	NT				S		S	
WC-3*	101	S	S	S	S				S		S	
	000224	S	S	S	S				S		S	
WC-4*	196	S	S	S	S				S		S	
	000103	S	S	S	S				S		S	

*NO DISASSEMBLY

TABLE B-2. (CONTINUED)

TYPE ROUND	GRIPSTOCK	ENVIRONMENTAL TESTS										REMARKS
	M.R.	T.V.	36 in.	L.C.	21 in.	RAIN	HUMID	S. FOG	MUD	IMMER	ICE	
TC-1	NA	-	-	-	-				-		-	
	000187	S	S	S	S				S		S	
TC-2	NA	-	-	-	-				-		-	
	000190	S	S	S	S				S		S	
TC-3	NA	-	-	-	-				-		-	
	000189	S	S	S	S				S		S	
TC-4	189	S	S	S	S				S		S	
	000128	S	S	S	S				S		S	
TC-5	201	S	S	S	S				S		S	
	000140	S	S	S	S				S		S	
TC-6	191	F										IMP GEN FAILED
	000136	F										GYRO BALANCE LOOSE
TCF-2	156	FUNGUS ONLY			F							ELEC FAIL OF GPST
	000191				S							

TABLE B-2. (CONCLUDED)

TYPE ROUND	GRIPSTOCK	ENVIRONMENTAL TESTS										REMARKS
	M.R.	T.V.	36 in.	L.C.	21 in.	RAIN	HUMID	S. FOG	MUD	IMMER	ICE	
TH-1*	NA	-**	-	-	-	-	-	-		-		SIGHT BROKEN
	000192	S	S	S	S	S	F	S		F		GAS LINE BROKEN
TH-2	NA	-										UMBILICAL RETRACT
	000227	F										
TH-3*	NA	-	-	-	-	-	-	-		-		
	000249	S	S	S	S	S	S	S		F		CASTLE NUT
TH-4	177	S	S	S	S	S	S	S		S		
	000203	S	S	S	S	S	S	S		S		SIGHT BROKEN
TH-5	197	F										IMP GEN FAILED
	000199	S	S	S	S	S	S	S		F		NO GAS FLOW
TH-6*	185	S	S**	S	S	S	S	S		S		GAS LINE FAILURE,
	000135	S	F	S	S	S	S	S		F		WATER IN MISSILE
TH-7*	191	NT	NT	NT	NT	NT	NT					FAILED IN TC-6
	000266	S	S	S	S	S	F					CASTLE NUT

*NO DISASSEMBLY
**NO DISASSEMBLY BEYOND THIS TREATMENT

TABLE B-3. PQT-G ENVIRONMENTAL TEST LEVELS

ENVIRONMENT	LEVEL
Transportation Vibration	Logarithmic sweep rate of 15 ± 1 minutes from 5 to 500 to 5 Hz, 1.0 to less than .0058 to 1.0 inch double amplitude respectively, and 2.5 to 3.5 to 2.5g peak respectively. There was no resonant dwell and the duration of curves AX and AY, Method 514.1, Procedure X, MIL-STD-810B (described above) were imposed for 1.0 hours along each major axis. Temperature conditioned, -50 and +160 deg F, respectively.
Loose Cargo Bounce	Package tester was operated in a synchronous mode to produce a 1 inch double amplitude displacement bounce at 284 ± 2 rpm for a total of 1.5 hours. Item rotated each one-half hour period so that it has rested on 3 axis. Temperature conditioned, -50 and +160 deg F, respectively.
Rough Handling in Container Test (36 inch drop)	The test specimen, in its container, was dropped 36 inches on 2 inch thick plywood backed by concrete (or steel) base. Temperature conditioned -50 and +140 deg F, respectively.
Rough Handling Out-of-Container Test (21 inch drop)	The test specimen, out of its container, was dropped 21 inches on concrete. Temperature conditioned, -40 and +140 deg F, respectively.

TABLE B-3. (CONTINUED)

ENVIRONMENT	LEVEL
Rain	Per MIL-STD-810B, Method 506, Procedure I. Specimen was exposed to a simulated rain at a rate of 5 ± 1 inches per hour for 10 minutes. The rainfall rate was then changed to 12 ± 1 inches per hour and held for 5 minutes. The rate was then reduced to 5 ± 1 inches per hour for 15 minutes. (Total time of rain exposure was 30 minutes.) Starting 5 minutes after initiation of rain, the wind source was turned on and adjusted to produce a horizontal wind velocity of 40 miles per hour. The wind source was maintained at this velocity for 15 minutes then turned off. Specimen out of container.
Humidity	Per MIL-STD-810B, Method 507, Procedure I. Specimen was subjected to 10 cycles (not less than 240 hours) consisting of the higher chamber temperature of +160 ± 2 degrees F with a relative humidity of 95 percent to the lower chamber temperature of +82 ± 2 degrees F with a relative humidity of 85 percent.
Salt Fog	Per MIL-STD-810B, Method 509, Procedure I. Specimen was exposed for 48 hours to the following environments: Chamber Temperature: + 95 ± 3 deg F Relative Humidity: 95 to 98% Ph of Salt solution collected: 6.5 to 7.2 The specimen was then stored in an ambient atmosphere for 48 hours for drying.

TABLE B-3. (CONCLUDED)

ENVIRONMENT	LEVEL
Mud	The specimen was submerged to 3/4 of its diameter in mud of semifluid consistency for a period of 5 minutes each for the top, bottom, and right sides.
Icing	For both container and bare specimen. With a test room ambient temperature of + 77 ± 2 deg F, the test items were immersed in water at a temperature of + 52 to + 92 deg F for 5 minutes. The immersion depth was 36 ± 5 inches. The test items were transferred with 1 minute to a -50 deg F test chamber and maintained for no longer than 8 hours.
Fungus	Per MIL-STD-810B, Method 508, except the test time was extended to 90 days with inspections at 30 day intervals. Test performed on 1 bare missile and 1 weapon round.

WC-2

Missile round was subjected to an x-ray inspection by the contractor and a visual inspection prior to initiation of tests. Round was exposed to transportation vibration, 36" drop, loose cargo and 21" drop before initial operational check (polecheck). Visual examination and x-ray inspection were performed after each environment. Round was incorrectly dropped on forward end during 21" drop test, resulting in internal damage to the guidance section. The round was returned to the contractor for repair where the head coil and detector were replaced. This was not scored as a reliability failure because of the incorrect drop. The round was returned to the test program and subsequently exposed to mud and icing tests. The round completed the environmental exposures in an operational condition, (without failure) but was not flight tested because of problems with target availability. The planned flight objective was deferred until the PPT Program.

WC-3

Weapon Round received an x-ray inspection by the contractor and a visual inspection prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop and icing before being subjected to first operational check. Round was then subjected to mud. Visual examination and x-ray inspection were performed after each environmental treatment with exception of mud where only a visual was conducted. An operational test was made after the completion of all environments, prior to flight test. The round exhibited no failure during environmental test sequence and was flight tested as PQTG-1. Round impacted ground due to target presentation being too low; target beyond boundary conditions.

WC-4

Weapon Round received an x-ray inspection by the contractor and a visual inspection prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop and mud before first operational test (polecheck). After this test, round was subjected to icing. Visual examination and x-ray inspection were performed after each environment, with exception of icing, where only a visual was conducted. Operational test was performed on the round after completion of all environmental treatments and prior to flight test. The round had no failures during the complete test sequence and was flight tested as PQTG-2.

WH-1

Missile Round received an x-ray inspection by the contractor and a visual inspection prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo and 21" drop prior to the first operational test. During 21" drop, the round was incorrectly dropped on the forward end and sustained damage to the sight mount and sight. These items were replaced by the contractor and the round returned to test. No reliability failure was charged because the damage occurred as a result of over-test environment. The round was then subjected to rain, humidity, salt fog and immersion with operational tests following each environment. Visual examination followed each environment, with x-ray inspection conducted after each dynamic environment and prior to flight test. After the immersion environment, the round was found to have a gyro caging problem. This hardware was returned to the contractor for analysis.

WH-2

Missile Round received an x-ray inspection by the contractor and a visual inspection prior to initiation of environmental test. Round was exposed to transportation vibration and 36" drop. A visual examination and x-ray inspection was performed after each environment. The first operational test was performed after 36" drop. During the 36" drop, the round slid forward in the launch tube and broke the IR window. Analysis indicated that the lock spring retention force on the umbilical plug was approximately 10 lbs, which is below specification requirements of 15-25 lbs. A deformed spring was the cause. No design change is planned, rather it is felt that increased production control (x-ray inspection) will resolve the problem. This unit was not repaired.

WH-3

Weapon Round received an x-ray inspection by the contractor and a visual examination prior to initiation of environmental tests. Round was exposed to transportation vibration, 36" drop, loose cargo, 21" drop and rain before the first operational test. Then the unit received humidity, salt fog, mud and immersion with operational checks after each environment. Visual examinations were made after each environment and x-ray inspection made after each dynamic environment. The round failed the operational test following the immersion environment due to no gas flow; investigation revealed the unit had been overtested in immersion by being submerged for two hours instead of required five minutes. Analysis of hardware revealed a clogged gas line caused by leaking rubber wiper in gas line insert assembly. The wiper allowed water to enter the line during the immersion test. It is postulated that damage to the wiper was caused by the polecheck adapter since tests have indicated that BCU insertion will not cause problem.

WH-4

Weapon Round received x-ray inspection by the contractor and visual inspection prior to start of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop and rain before the first operational test was made. After these tests the round received salt fog, humidity and immersion with operational test after each environment. Visual examinations were made after each environment and x-ray inspection made after each dynamic environment and prior to flight. The round failed the operational test following humidity because of a broken gas line. Breakage was caused by intergrandular corrosion caused by salt spray, which weakens the tube making it susceptible to mechanical failure. The tube will be painted in production to prevent corrosion. Round was repaired by replacing the exterior gas line and during assembly test there was an out of tolerance condition (low resistance between two pins) found in the fuze. Immersion was run with hardware out of tolerance with no additional problems. Later analysis determined that the lower resistance level would have no effect on flight performance. However, the round was not flight tested because of problems with target availability.

TC-1

Missile Round received x-ray inspection, launcher assembly tests, rate table test, and operational test prior to the initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop, mud and icing environments. Following each environment, except mud, the round was exposed to a visual examination, operational test, disassembly, rate table test, assembly, and operational test. X-Ray inspection was performed after all environments except mud. After the mud environment, the round received a visual examination and operational test. The round passed all environments without failure and was flight tested as PQTG-18.

TC-2

Missile Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to the initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop, mud and icing. Following each environment, except mud, the round was subjected to a visual inspection, operational test, disassembly, rate table test, assembly, and operational test. X-Ray inspection was performed after all environments except mud. After the mud environment, the round received a visual inspection and operational test. The round passed all environments without failure and was flight tested as PQTG-19.

TC-3

Missile Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36″ drop, loose cargo, 21″ drop, mud, icing. Following each environment except mud, the round was exposed to a visual inspection, operational test, disassembly, rate table test, assembly and operational test. During the disassembly/assembly operation following 36″ drop, the water block cable was damaged and replaced; this was not considered an environmental failure. X-ray inspection was performed after all environments except mud. After mud, the round was given a visual inspection and operational test. The round successfully passed all environments without failure and was flight tested as PQTG-23. Although general indication was that the round was good, the flight was "no tested" for reliability due to personnel error/mount problem that induced improper lead during the prefire operation.

TC-4

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36″ drop, loose cargo, 21″ drop, icing and mud. After each environment except icing, the round received a visual inspection, operational test, disassembly, rate table test, assembly and operational test. X-ray inspection was performed after all environments except icing. After icing, the round was given a visual inspection and operational test. The round passed all environmental tests without failure and was flight tested as PQTG-16.

TC-5

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36″ drop, loose cargo, 21″ drop, icing and mud. After each environment except icing the round received a visual inspection. operational test, disassembly, rate table test, assembly and operational test. X-ray inspection was performed after all environments except icing. After icing, the round was given a visual inspection and operational test. The round passed all environmental tets and was successfully flight tested as PQTG-21.

TC-6

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of the environmental tests. Round was subjected to transportation vibration, given a rate table test and then an operational test. A vibration was noticed in the guidance section; analyses later determined that the guidance section had an unbalanced gyro. One of the small balance weights had come off due to inadequate bonding. The gripstock was found to have no impulse generator output. The hardware was not repaired and put back into environmental testing.

TCF-2

Weapon Round received visual inspection, rate table test and operational test prior to being innoculated with the fungus bacteria. The round entered the environment on 6 November 1976 and was removed on 5 January 1977. After completion of the environment, the round was visually examined operationally tested. The gripstock was found to have an electrical malfunction, not related to the environment. The missile round successfully passed the environment and was flight tested as PQTG-25.

TH-1

Missile Round received x-ray inspection, launcher assembly test, rate table test, and operational test prior to initiation of the environmental tests. Round was subjected to transportation vibration, 36″ drop, loose cargo, 21″ drop, rain, salt fog, humidity and immersion. Visual examination was performed after each environment. After transportation vibration, the round was given an operational test, disassembly, rate table test, assembly and operational test. After humidity, cracks were observed in sight frame and aft sight hinge inclosure fixture was broken from sight which caused a boresight problem. After immersion, the round was given x-ray inspection and operational test and found to have a broken gas line. The line was replaced and the round was fired as PQTG-26.

TH-2

Missile Round received x-ray inspection, launcher assembly test, rate table test and an operational test prior to initiation of environmental tests. The round was subjected to transportation vibration and received a visual examination, x-ray inspection and an operational test. It was discovered the round had moved in the launch tube. Damage to the missile round included sheared gas line, water block cable broken, blowout disk ruptured from inside, the IR window broken and the umbilical retracted. Analysis indicated the cause of failure was a deformed lock spring in the missile detent. No design change is planned, but an inspection point will be added during production to prevent recurrence. The hardware was not repaired.

TH-3

Missile Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo. 21" drop, rain, humidity, salt fog and immersion. Visual examination was performed after each environment and x-ray inspection was performed after humidity. At the completion of immersion, water was noted in the launch tube. Examination revealed that the castle nut had cracked. Cause was attributed to impure lexan resulting in reduced physical properties. Increased quality control and the addition of fiber glass in the area of the castle nut is being incorporated into the hardware for the production prototype test. Water was detected in the missile dome; extent of damage was not determined; hardware was not repaired.

TH-4

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop, rain, salt fog, humidity and immersion. Visual examination was conducted after each environment, x-ray inspection, disassembly, rate table test, assembly and operational tests were conducted after transportation vibration, 36" drop, loose cargo, 21" drop and rain, humidity (no x-ray) and immersion. After humidity the sight was found to be broken and the gas line was broken during disassembly operation; line was replaced and round continued. The round was flight tested as PQTG-24.

TH-5

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration, 36" drop, loose cargo, 21" drop, rain, humidity, salt fog and immersion. Visual examination and operating tests were performed after each environment; x-ray inspection, disassembly, rate table test and assembly performed after transportation vibration, 36" drop, loose cargo, 21" drop and humidity (no x-ray). After immersion, the round was found to have a plugged cryostat (no gas flow); round has been returned to the contractor for failure analysis.

TH-6

Weapon Round received x-ray inspection, launcher assembly test, rate table test and operational test prior to initiation of environmental tests. Round was subjected to transportation vibration and 36" drop where it was found to have a broken gas line; line was replaced and unit put back in test. Round then received loose cargo, 21" drop, rain and salt fog humidity and immersion. Visual examination was conducted after each test; x-ray inspection and operational test after transportation vibration, 36" drop and immersion; disassembly, rate table test and assembly after transportation vibration and 36" drop. After humidity the aft sight hinge was found broken. The round would not track at the completion of immersion; IR window broken and water beads on the missile. It is felt that the IR window was cracked during handling prior to immersion, although records do not show it. This round was not repaired.

TH-7

Weapon Round received visual examination and operational test prior to initiation of environmental tests. Gripstock serial number 191 was mistakenly used with this round (it has previously failed during tests of TC-6) and all gripstock data was discounted. The missile round received transportation vibration, 36" drop, loose cargo, 21" drop, rain and humidity. Visual examination was made after each environmental and an operational test after humidity. During humidity, the castle nut separated and allowed moisture in the launch tube. This problem resulted from the poor physical properties of impure lexan. Corrective action is provided by increased quality control and the addition of fiber glass. The round was not repaired.

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