AFAPL-TR-78-6
PART IV

LEVEL III

2

A074827
A III

SC

ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE

PART IV CYLINDRICAL ROLLER BEARINGS

SHAKER RESEARCH CORP.
BALLSTON LAKE, NEW YORK 12019

DECEMBER 1979

TECHNICAL REPORT AFAPL-TR-78-6, PART IV
INTERIM REPORT FOR PERIOD APRIL 1978 – AUGUST 1979

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

AIR FORCE AERO PROPULSION LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any was supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

John B. Schrand

JOHN B. SCHRAND
Project Engineer

Howard F. Jones

HOWARD F. JONES
Chief, Lubrication Branch

FOR THE COMMANDER

Robert D. Sherrill

ROBERT D. SHERRILL
Acting Chief, Fuels and Lubrication Division

If your address has changed, if you wish to be removed from our mailing list, or if the addressee is no longer employed by your organization, please notify AFWAL/POSL, WPAFB OH 45433 to help us maintain a current mailing list.

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

AIR FORCE/56780/11 March 1980 — 260

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFAPL-TR-78-6 [redacted] PT-4
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE. Part IV. Cylindrical Roller Bearings.
5. TYPE OF REPORT & PERIOD COVERED: Technical - Interim, April 1978 - August 1979
6. PERFORMING ORG. REPORT NUMBER: SRC-79-TR-45
7. AUTHOR(s): A. B. Jones, J. M. McGrew, Jr
8. CONTRACT OR GRANT NUMBER(s): F33615-76-C-2038
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Shaker Research Corporation, Northway 10 Executive Park, Ballston Lake, N.Y. 12019
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 3048-06-85
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Aero Propulsion Laboratory/SFL, Air Force Systems Command, Wright-Patterson AFB, Ohio 45433
12. REPORT DATE: Dec[redacted]79
13. NUMBER OF PAGES: 74
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 73
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Tapered Roller Bearings
Tapered Roller Bearing Stiffness
Roller Bearings
Roller Bearing Stiffness
Turbine Bearings
Rotordynamics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is an update of the original Part IV of the Rotor-Bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X). A computer program is given for preparation of cylindrical roller bearing stiffness data input for rotordynamic response programs. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of cylindrical roller bearing stiffness. The resulting program is reasonably small and easy to use.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

The work reported herein was performed during the period 15 April 1978 to 15 June 1979, under the direction of John B. Schrand (AFWAL/POSL) and Dr James F. Dill (AFWAL/POSL), Project Engineers. The report was released by the authors in August 1979.

Accession For	
NTIS GRA&I	☒
DDC TAB	☐
Unannounced	☐
Justification______	
By______	
Distribution/	
Availability Codes	
Dist	Avail and/or special
A	

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

NOMENCLATURE

Symbol		Units
b_x	Semi-width of contact ellipse at X	in.
B	Corner break	in.
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	$\frac{\text{lb-sec}}{\text{in}}$
$\underline{\underline{B}}_N$	Damping matrix $\begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & (\underline{\underline{B}}_N)_{\text{angular}} & \end{bmatrix}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{\underline{B}}_N)_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{\underline{B}})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb-sec}}{\text{radian}}$
C_i	A constant, $C_i = \begin{cases} 1 & \text{for } i = 1 \\ -1 & \text{for } i = 2 \end{cases}$	-
d	Roller diameter	in.
E	Pitch diameter	in.
E_E	Modulus of elasticity for roller body	lbs/in^2
E_R	Modulus of elasticity for race body	lbs/in^2

F_c	Roller centrifugal force	lbs.
F_i	External applied force, i = x, y, z	lbs.
F'_i	Reaction force, positive in direction opposite to displacements, i = x, y, z	lbs.
$\underline{F}$	Force Matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
G	Distance along roller element from extreme end of effective length to point where crown drop is measured	in.
H	Roller crown radius minus the rise of the arc at midpoint of effective length	in.
K	Roller-race stiffness	lbs/in
K_{ij}	Stiffness component, change of force in i direction due to displacement in j direction; i = x, y, z; j = x, y, z	lbs/in
$\underline{\underline{K}}_N$	Stiffness matrix $\begin{bmatrix} (\underline{\underline{K}}_N)_{lineal} & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & (\underline{\underline{K}}_N)_{angular} & \\ 0 & 0 & & \end{bmatrix}$	lbs/in
$(\underline{\underline{K}})_{lineal}$	Stiffness matrix due to lateral displacements $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{lineal}$	lbs/in
$(\underline{\underline{K}})_{angular}$	Stiffness matrix due to angular rotations $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{angular}$	$\frac{in\text{-}lb}{rad}$

ℓ_e	Effective length of roller load carrying surface	in.
ℓ_F	Length of flat portion of roller measured along roller element	in.
ℓ_T	Total length of roller measured parallel to roller axis	in.
m	Mass of roller	lbs.
M_i	External applied moment, i = x, y, z	lbs-in
M_i'	Reaction moment, i = x, y, z	lbs-in
M_1	Outer race/roller contact moment	lbs-in
M_2	Inner race/roller contact moment	lbs-in
n	Number of rollers	
N_1	Outer ring rotational speed	rpm
N_2	Inner ring rotational speed	rpm
P_x	Contact unit loading	lbs/in
P_x'	Current estimate of contact unit loading	lbs/in
P_D	Diametral clearance	in.
P_{1_q}	Outer contact load on qth roller	lbs.
P_{2_q}	Inner contact load on qth roller	lbs.
q	Roller position index	-
R_c	Roller crown radius	in.
$\underset{\sim}{W}_N$	Column Matrix $= \begin{bmatrix} \delta_x \\ \delta_y \\ \Theta_x \\ \Theta_y \end{bmatrix}$	

x, y, z	Bearing coordinate system	in.
X, Y	Roller coordinate system	in.
x_o	Static component of displacement	in.
x'	Dynamic component of displacement	in.
X_{A_i}	Extremity of contact pattern measured parallel to roller axis to the left of the midpoint of the effective length	in.
X_{B_i}	Extremity of contact pattern measured parallel to roller axis to the right of the midpoint of the effective length	in.
X_A^*	Maximum permissible distance of contact pattern extremity from midpoint of effective length measured along race to the left	in.
X_B^*	Maximum permissible distance of contact pattern extremity from midpoint of effective length measured along race to the right	in.
$\underline{\underline{Z}}_N$	Impedance matrix = $\underline{\underline{K}}_N + i\,\nu \underline{\underline{B}}_N$	

Other notations as defined in text.

GREEK SYMBOLS

γ	d/E	-
δ	Displacement	in.
δ_x	Linear displacement in x direction	in.
δ_y	Linear displacement in y direction	in.
δ_z	Linear displacement in z direction	in.
Δ	Approach of inner race to outer race at azimuth ϕ	in.
Δ_x	Approach of roller to race at X	in.
Δ_{1q}	Approach of roller to outer race (cup) at qth roller	in.
Δ_{2q}	Approach of roller to inner race (cup) at qth roller	in.
ε_i	Residues of simultaneous equations	-
η_E	Roller elastic constant = $\frac{4(1 - {}^2_E)}{E_E}$	$\frac{in^2}{lb}$
η_R	Race elastic constant = $\frac{4(1 - {}^2_R)}{E_R}$	$\frac{in^2}{lb}$
Θ	Misalignment of inner race with respect to outer race	radians, $^\circ$
Θ_x	Angular rotation about x axis	radians, $^\circ$
Θ_y	Angular rotation about y axis	radians, $^\circ$
Θ_z	Angular rotation about z axis	radians, $^\circ$
ν	Frequency of rotation	rad/sec
ν_E	Poisson's ratio for roller	
ν_R	Poisson's ratio for race	

ρ	Material density	lbs/in^3
ϕ	Circumferential roller position	radians, °
Ω_E	Orbital velocity of roller	rad/sec
Ω_1	Angular velocity of outer ring	rad/sec
Ω_2	Angular velocity of inner ring	rad/sec
∇	Crown drop	in.
∇'	Crown drop at distance G from roller extremity	in.

SUBSCRIPTS

Symbol	Description
b	Refers to bearing
E	Refers to roller
F	Refers to flat
i	Index, i = 1, 2, 3 or i = x, y, z
i,j	Refers to index of stiffness matrix; i.e., force in i direction due to displacement in j direction
p	Refers to pedestal
q	Refers to roller circumferential position
R	Refers to race
T	Refers to total
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I

INTRODUCTION

The original Rotor-Bearing Dynamics Design Technology Series AFAPL-TR-65-45 (Parts I through X) included a volume, Part IV(1), which presented design data for typical deep-groove and angular contact ball bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of rolling element bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (2) considers both elastohydrodynamic and cage effects for ball bearings. The more recent computer based design guide prepared by Crecelius and Pirvics (3) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

---

(1) Lewis, P. and Malanoski, S.B., "Rotor Bearing Dynamics Design Technology. Part IV: Ball Bearing Design Data Technical Report," AFAPL-TR-65-45, Part IV. Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio.

(2) Mauriello, J.A., LaGasse, and Jones, A.B., "Rolling Element Bearing Retainer Analysis," DAAJ02-69-C-0080, TR105.7.10, USAAMRDL-TR-72-45.

(3) Crecelius, W.T. and Pirvics, J., "Computer Program Operation Manual on "SHABERTH" a Computer Program for the Analysis of the Steady State and Transient Thermal Performance of Shaft Bearing Systems," AFAPL-TR-76-90, Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio, October 1976.

Thus, very sophisticated analytical tools are available for the design and application of rolling element bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition, both computer programs are very large and require an extensive computer facility for use.

Part II(4) of the revised series provided an update of the original Part IV (1). Those aspects of the original Part IV(1) which treated general design aspects of ball bearings, load capacity, speed limitations, etc. were deleted since their coverage is superficial compared to the more sophisticated computer tools now available (2,3). Only those parts directly connected with preparation of input for the rotordynamic response programs (Part I(5) of the revised series) were retained. The stiffness data included in the original Part IV were also updated. A later volume (Part III(6) of the revised series), enlarged the treatment to include the tapered roller bearing.

The present volume (Part IV of the revised series) extends the treatment of rolling element bearings to the cylindrical roller bearing. The complete stiffness matrix is calculated including centrifugal effects. Considerations

---

(4) Jones, A.B., and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide--Part II: Ball Bearings, "AFAPL-TR-78-6, Part II, February 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

(5) Pan, C.H.T., Wu, E.R., and Krauter, A.I., "Rotor Bearing Dynamics Technology Design Guide: Part 1, Flexible Rotor Dynamics," AFAPL-TR-78-6, Part I, June 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

(6) Jones, A.B., and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide - Part III: Tapered Roller Bearings," AFAPL-TR-78-6, Part III, February 1979, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of cylindrical roller bearing stiffness. The resulting program (Appendix) is reasonably small and easy to use.

SECTION II

ANALYSIS

2.1 General Bearing Model and Coordinate System

Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- The restraining force/moment may not be temporally in phase with the displacement/inclination.
- The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described in Reference 5 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):

$$\underline{R}_N = - \underline{\underline{Z}}_N \cdot \underline{W}_N \qquad (1)$$

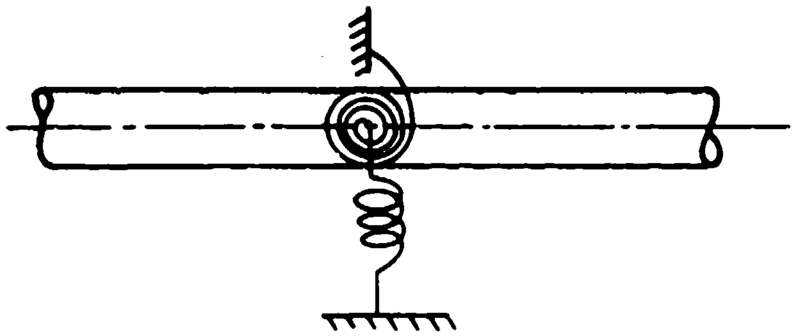

Figure 1a   Bearing Stiffness Model

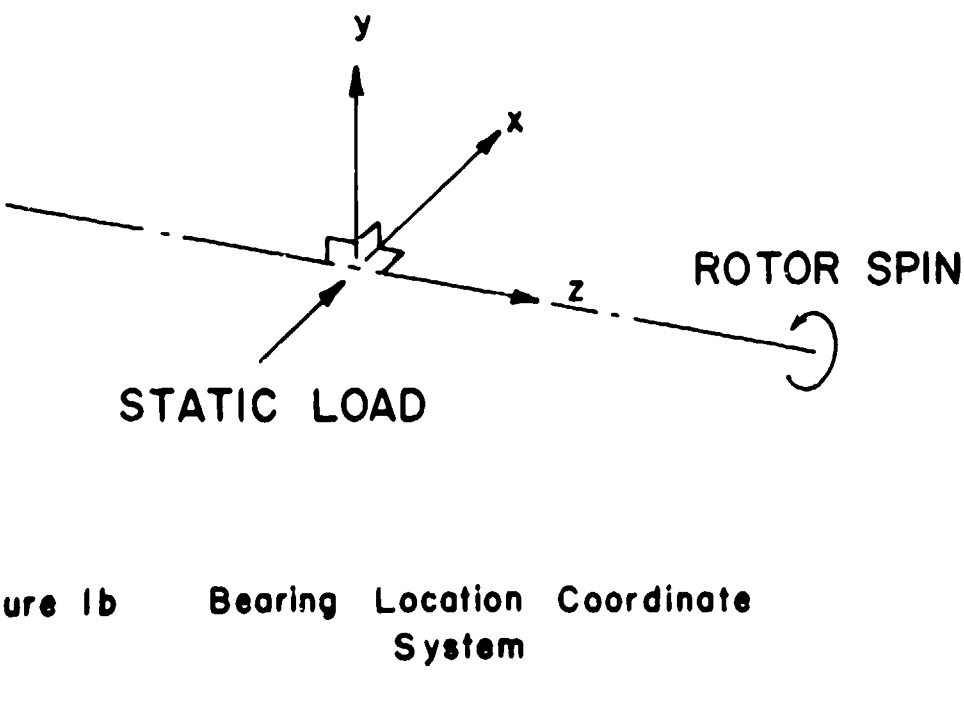

Figure 1b   Bearing Location Coordinate System

where $\underline{W}_N$ is a column vector containing elements which are the two lateral displacements (δ_x, δ_y) and the two lateral inclinations (Θ_x, Θ_y) of the rotor axis at the bearing station N.

Employing a right-handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right-handed triad (x, y, z). (δ_x, δ_y) are respectively lateral lineal displacement components of the rotor axis along the(x, y) directions. (Θ_x, Θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that Θ_y is a rotation about the y-axis, while Θ_x is a rotation about the negative x-axis.

$\underline{\underline{Z}}_N$ is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{\underline{Z}}_N = \underline{\underline{K}}_N + i\nu\underline{\underline{B}}_N \tag{2}$$

where $\underline{\underline{K}}_N$ is the stiffness matrix and $\underline{\underline{B}}_N$ is the damping matrix. ν is the frequency of vibration. Most commonly, lateral lineal and angular displacements do not interact with each other so that the non-vanishing portions of $\underline{\underline{K}}_N$ and $\underline{\underline{B}}_N$ are separate 2 x 2 matrices. That is

$$\underline{\underline{K}}_N = \begin{bmatrix} (\underline{\underline{K}}_N)_{\text{lineal}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & (\underline{\underline{K}}_N)_{\text{angular}} \end{bmatrix} \tag{3}$$

$$\underline{\underline{B}}_N = \begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & (\underline{\underline{B}}_N)_{\text{angular}} & \end{bmatrix} \tag{4}$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{\underline{K}})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \tag{5}$$

$$(\underline{\underline{B}})_{\text{lineal}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}} \tag{6}$$

$$(\underline{\underline{K}})_{\text{angular}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}} \tag{7}$$

$$(\underline{\underline{B}})_{\text{angular}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}} \tag{8}$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{\underline{Z}}_N = (\underline{\underline{Z}}_b^{-1} + \underline{\underline{Z}}_p^{-1}) \tag{9}$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{\underline{Z}}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as (δ_x, δ_y, δ_z, Θ_x, Θ_y) in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (Θ_x, Θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = \underline{\underline{Z}} \cdot \underline{x} \tag{10}$$

$\underline{F}$ is a column vector comprising the five reaction components (F_x, F_y, F_z, M_x, M_y), while $\underline{x}$ is the displacement vector (δ_x, δ_y, δ_z, Θ_x, Θ_y). $\underline{\underline{Z}}$ is a (5 x 5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_o + \underline{x}'; \quad \underline{F} = \underline{F}_o + \underline{F}' \tag{11}$$

($\underline{x}'$, $\underline{F}'$) are respectively presumed to be infinitesimal in comparison with ($\underline{x}_o$, $\underline{F}_o$). Accordingly, Z_{ij} are regarded as dependent on $\underline{x}_o$ but not on $\underline{x}'$. To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

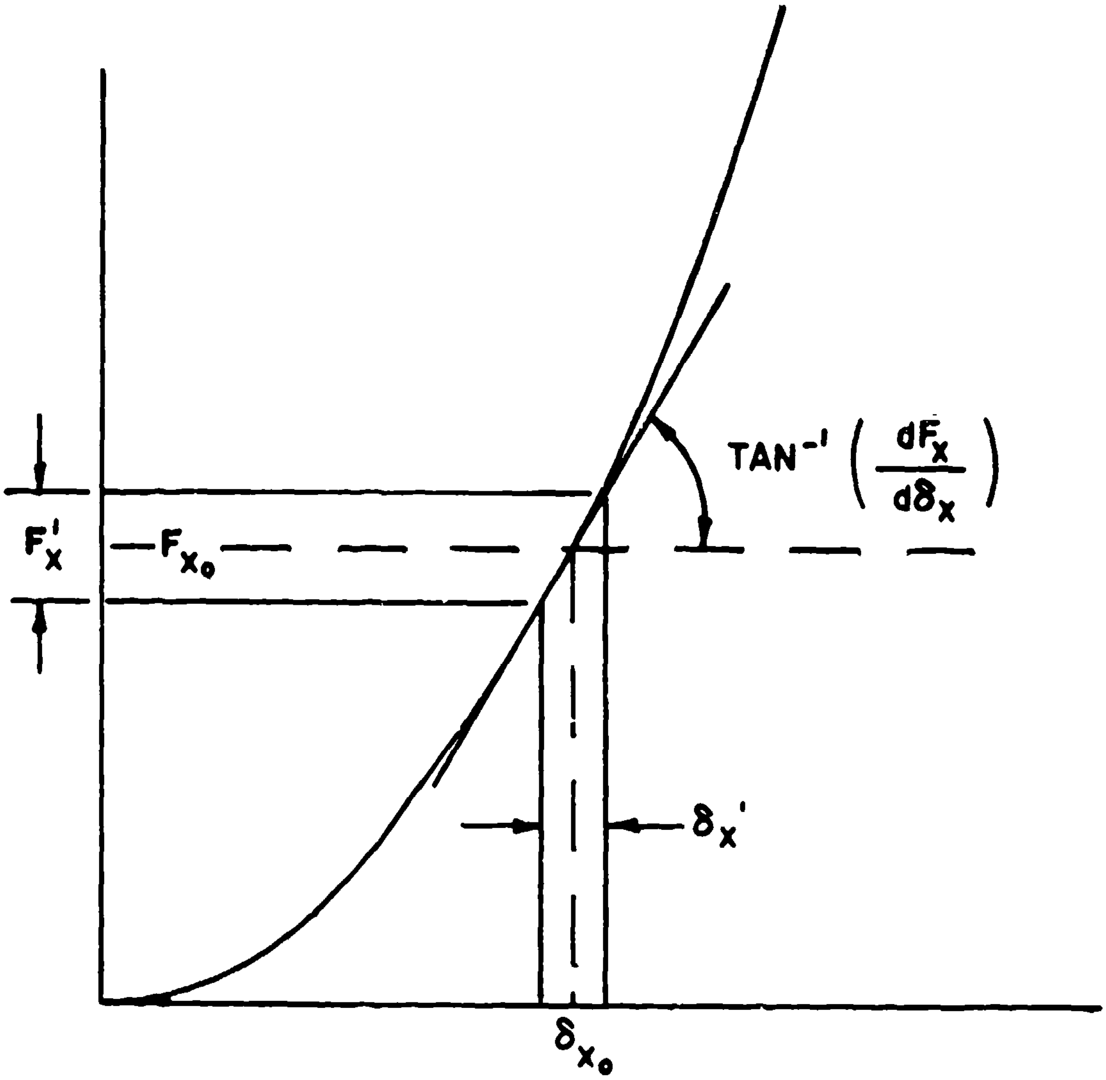

Figure 2   Linearization of Cylindrical Roller Bearing Stiffness

As illustrated, the load-displacement relationship is a 10/9 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_o}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta_x} \delta'_x \tag{12}$$

where δ'_x is the incremental displacement. $\partial F_x/\partial \delta_x$ will depend on the amplitude of δ_{x_o}.

The question of history dependence is resolved by regarding $\underline{x}'$ as periodic motions at any frequency ν of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν.

To avoid notational clumsiness, the primes will be dropped from ($\underline{F}'$, x') which are understood to be dynamic perturbation quantitites unless the subscript "o" is used to designate the static equilibrium condition.

2.3 Cylindrical Roller Bearing Characterization

In many ways the cylindrical roller bearing is much simpler to model from a rotor dynamic point of view than a fluid film bearing. In general, the following two simplifications can be made:

- The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 illustrates a typical radial roller bearing having cylindrical rollers. In the bearing illustrated the rollers are guided by closely fitting flanges on the outer ring. Other constructions are common and all carry load in the manner to be described.

The bearing is referred to an orthogonal xyz coordinate system with the bearing axis directed along z. The bearing reacts to the linear displacements δ_x and δ_y and to the rotations or misalignments Θ_x and Θ_y. All are shown in their positive senses in Figure 4.

Figure 5 illustrates the roller index q.

2.3.1 Stiffness

The total characterization of a cylindrical roller bearing's stiffness can be expressed by the matrix.

$$[K] = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \Theta_x} & \frac{\partial F_x}{\partial \Theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \Theta_x} & \frac{\partial F_y}{\partial \Theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \Theta_x} & \frac{\partial F_z}{\partial \Theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \Theta_x} & \frac{\partial M_x}{\partial \Theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \Theta_x} & \frac{\partial M_y}{\partial \Theta_y} \end{bmatrix} \quad (13)$$

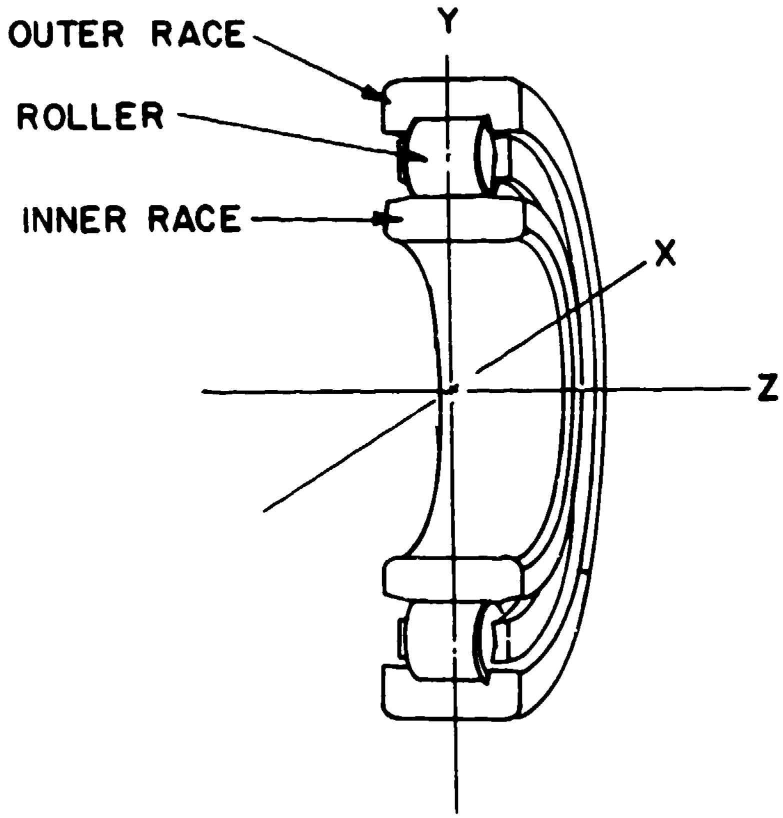

Figure 3. Cylindrical Roller Bearing

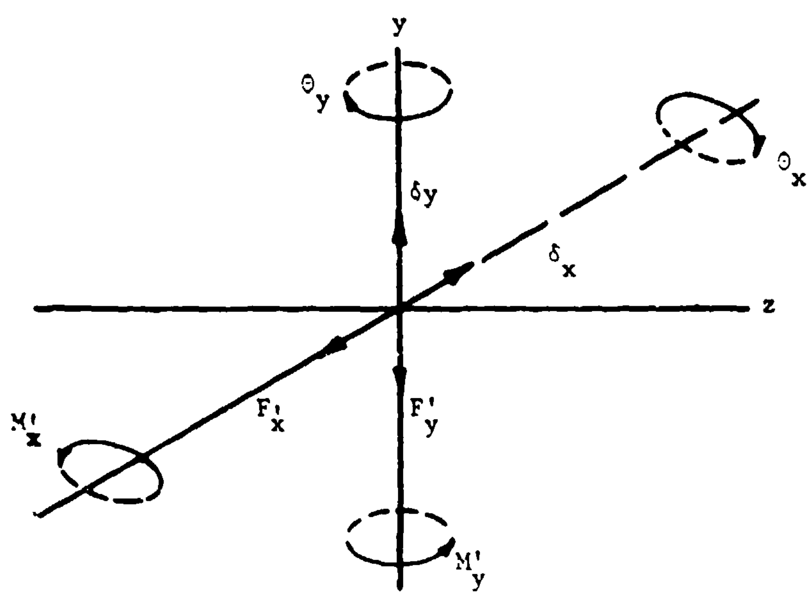

Figure 4. Bearing Coordinate System

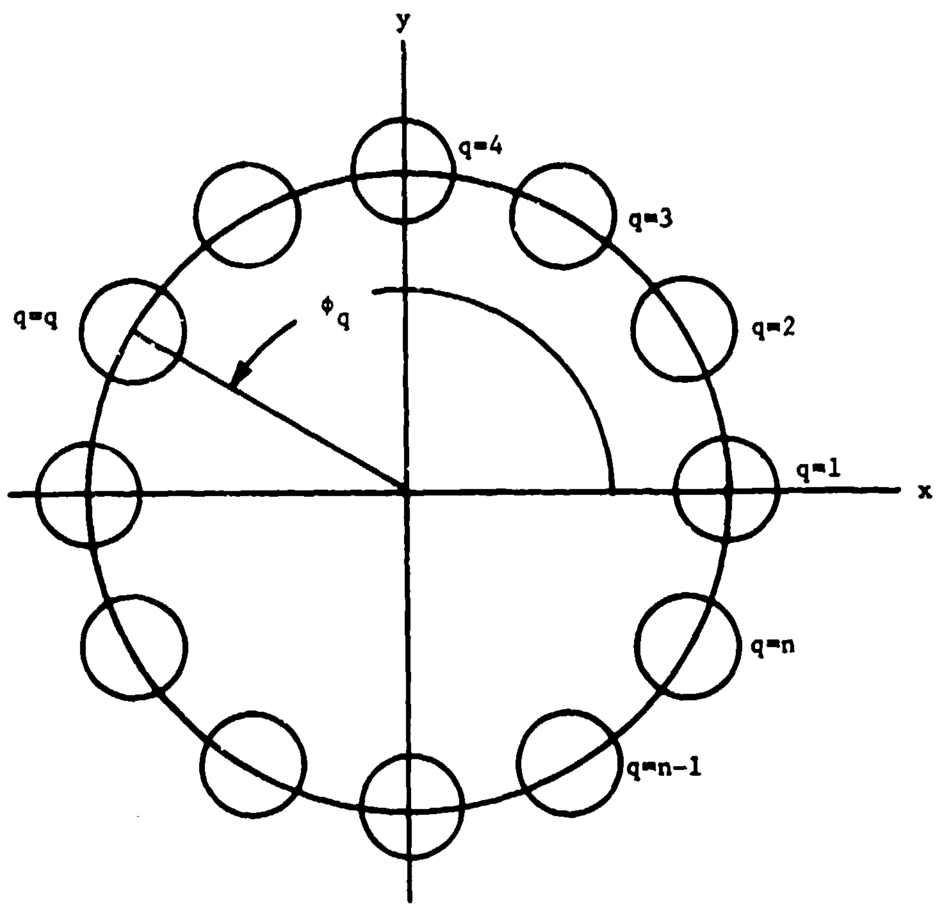

Figure 5. Cylindrical Roller Bearing Index, q

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$(\underline{K})_{\text{lineal}} = \begin{bmatrix} \dfrac{\partial F_x}{\partial x} & \dfrac{\partial F_x}{\partial y} \\ \dfrac{\partial F_y}{\partial x} & \dfrac{\partial F_y}{\partial y} \end{bmatrix} \tag{14}$$

$$(\underline{K})_{\text{angular}} = \begin{bmatrix} \dfrac{\partial M_x}{\partial \Theta_x} & \dfrac{\partial M_x}{\partial \Theta_y} \\ \dfrac{\partial M_y}{\partial \Theta_x} & \dfrac{\partial M_y}{\partial \Theta_y} \end{bmatrix} \tag{15}$$

Note that the axial components of stiffness are zero for a cylindrical roller bearing. However, these terms have been retained on the general cylindrical roller bearing stiffness matrix for consistency with the ball and tapered roller bearing analyses included in earlier volumes of this series (4,6).

2.3.2 Damping

An extensive search of the literature revealed no experimental damping data for cylindrical roller bearings. As the current state-of-the-art does not permit accurate calculation of cylindrical roller bearing damping, no damping data is included in this report.

2.4 Cylindrical Roller Bearing Under Radial Loading

Solution for the stiffness matrix of a cylindrical roller bearing under radial loading is a tedious problem and requires the use of a digital computer. In this section, the derivation of the solution is described. A computer program for obtaining the solution is included in the Appendix.

2.4.1 Bearing Applied Forces and Moments

In the present problem an external force is applied to the inner ring along x, only, and the inner ring is constrained to movement along x. There may, however, be initial displacements in other modes which are fixed and which result in reactions in other modes as will be shown.

As the result of the four displacements described previously in Figures 3 and 4, there are the reactions F'_x, F'_y, M'_x, and M'_y. F'_x and F'_y are forces. M'_x and M'_y are moments. All are shown in their positive sense in Figure 4. External force F_x is applied at the inner ring center. The senses of the signs are the same as for the reaction F'_x.

2.4.2 Roller Geometry

Figure 6 shows the boundary dimensions of a typical cylindrical roller. Roller mass, moment of inertia, and location of the center of gravity are calculated assuming the roller is a flat-ended, cylinder bounded by d and ℓ_T.

ℓ_e is the effective length of the roller and represents the maximum working length of the roller. The contact pattern between roller and race must lie within the effective length.

ℓ_T is the total length of the roller.

ℓ_F is the length of the flat portion of the roller profile and may be zero for a fully-crowned roller.

R_c is the roller crown radius and is struck from the central plane of the roller. ∇ is the drop of the crown measured at the extremities of the effective length. Thus we can write:

$$\nabla = H - \sqrt{R_c^2 - \left(\frac{\ell_e}{2}\right)^2} \qquad (16)$$

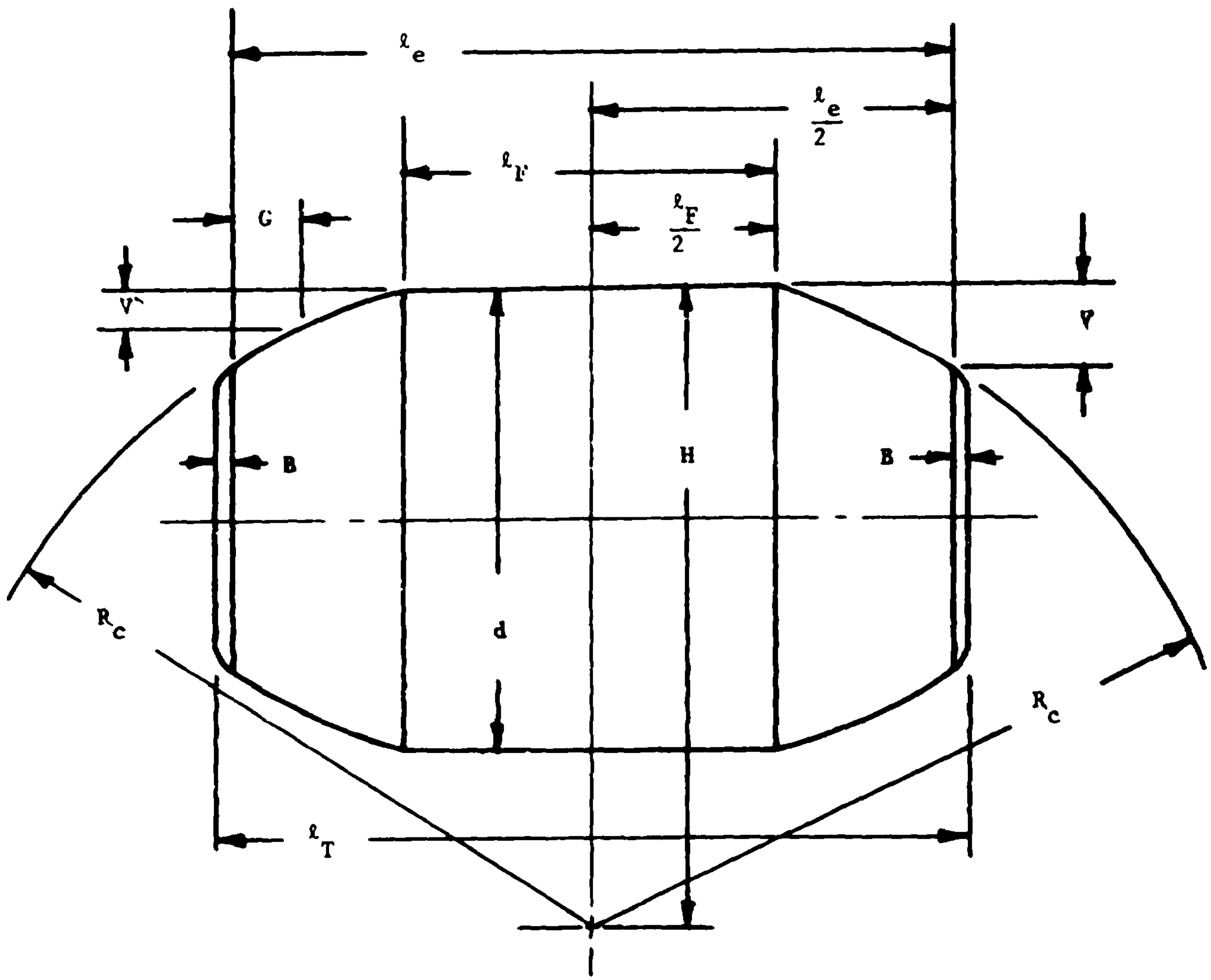

Figure 6. Dimensions of Roller Profile and Crown

where

$$H = \sqrt{R_c^2 - \left(\frac{\ell_F}{2}\right)^2} \qquad (17)$$

The crown drop, ∇, is used in checking the roller profile. If the crown radius is known, the drop at a distance G from the extremity of the effective length is

$$\nabla' = H - \quad R_c^2 - \left(\frac{\ell_e}{2} - G\right)^2 \qquad (18)$$

If the drop ∇' at G is known the crown radius is

$$R_c = \sqrt{\left[\frac{\left(\frac{\ell_e}{2} - G\right)^2 - \left(\frac{\ell_F}{2}\right)^2 - \nabla'^2}{2\nabla'}\right]^2 + \left(\frac{\ell_e}{2} - G\right)^2} \qquad (19)$$

B is the corner break which is the same at both ends of the roll. The exact form of the corner break is unimportant as long as the corner blends smoothly into the roller crown. The effects of the corner breaks are neglected in calculating roller mass. The effect of the crown is also neglected.

The mass of the roller is

$$m = \frac{(\ell_e + 2B)\, d^2 \rho}{491.98} \qquad (20)$$

where ρ is the material density of the roller in lb/in^3.

The centrifugal force acting on the roller is

$$F_c = \frac{mE\Omega_E^2}{2} \qquad (21)$$

E is the pitch diameter of the bearing in inches. Ω_E is the orbital velocity of the roller and is

$$\Omega_E = \frac{\Omega_1 (1+\gamma) + \Omega_2\, (1-\gamma)}{2} \qquad (22)$$

where

$$\gamma = \frac{d}{E} \tag{23}$$

Ω_1 and Ω_2 are the angular velocities of outer and inner rings, respectively, in radians/sec.

The subscripts 1 and 2, used in Equation 7 and subsequently, refer to outer and inner contacts respectively.

Figure 7 shows the forces and moments acting on a roller which is in contact with both outer and inner races.

P_1 and P_2 are the contact loads. M_1 and M_2 are the contact moments resulting from non-uniform loading along the roller's length. F_c is the centrifugal force acting at the mass center of the roller.

In the present problem the radial roller bearing is loaded by a single force applied along x and working displacements are limited to a single deflection along x.

Assume, for the moment, that working displacements are possible along x and y and about x and y. Then the approach of inner and outer race for a roller at azimuth ϕ is

$$\Delta = (\delta_x+\delta''_x) \cos\phi + (\delta_y+\delta''_y) \sin\phi - \frac{P_D}{2} \tag{24}$$

where P_D is the diametral clearance.

The relative misalignment of the inner race with respect to the outer is

$$\Theta = (\Theta_x+\Theta''_x) \sin\phi + (\Theta_y+\Theta''_y) \cos\phi \tag{25}$$

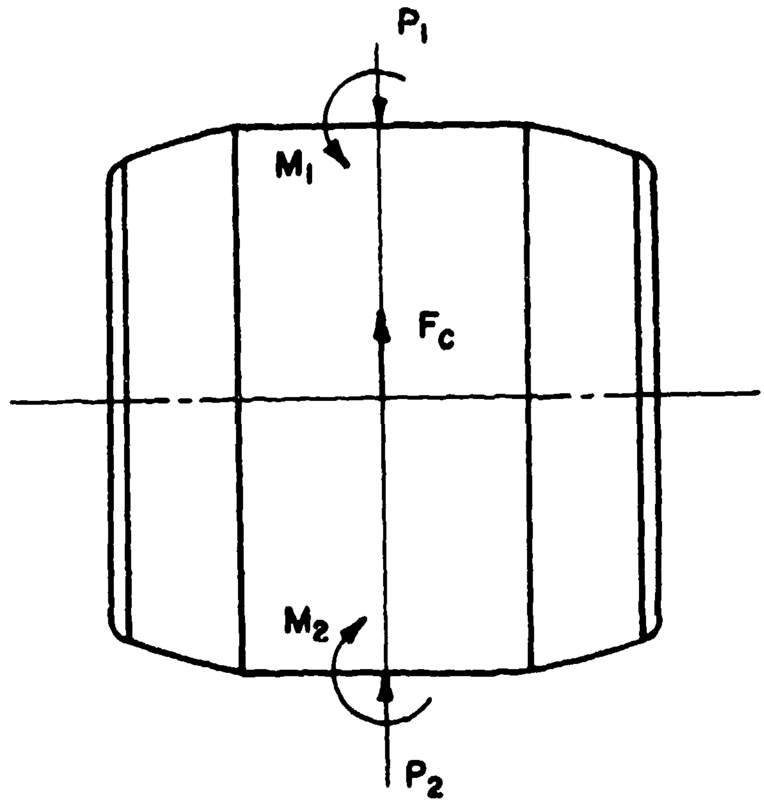

Figure 7. Forces and Moments Acting on Roller

The double primed items in Equations 24 and 25 are initial displacements. If initial displacements exist for a particular mode, working displacements in that mode are prevented.

For the present problem the linear displacement δ_y and the rotations Θ_x and Θ_y are non-existent and the approach and misalignment of the races are, for this case,

$$\Delta = (\delta_x+\delta_x'') \cos\phi + \delta_y'' \sin\phi - \frac{P_D}{2} \tag{26}$$

and

$$\Theta = \Theta_x'' \sin\phi + \Theta_y'' \cos\phi \tag{27}$$

When Θ is positive the left end of the roller, viewed with the axis horizontal, tends to be pinched.

If Δ_1 is the approach of the roller to the outer race, the approach of the roller to the inner race, at the midpoint of the roller is

$$\Delta_2 = \Delta-\Delta_1 \tag{28}$$

and the misalignment at the inner contact is

$$\Theta_2 = \Theta-\Theta_1 \tag{29}$$

Figure 8 illustrates the geometric intersection of a roller and raceway.

The profiles of race and roller bodies are referred to an XY coordinate system. Note that the X axis is positive to the left of the origin.

The equation of the race surface is

$$Y = 0 \tag{30}$$

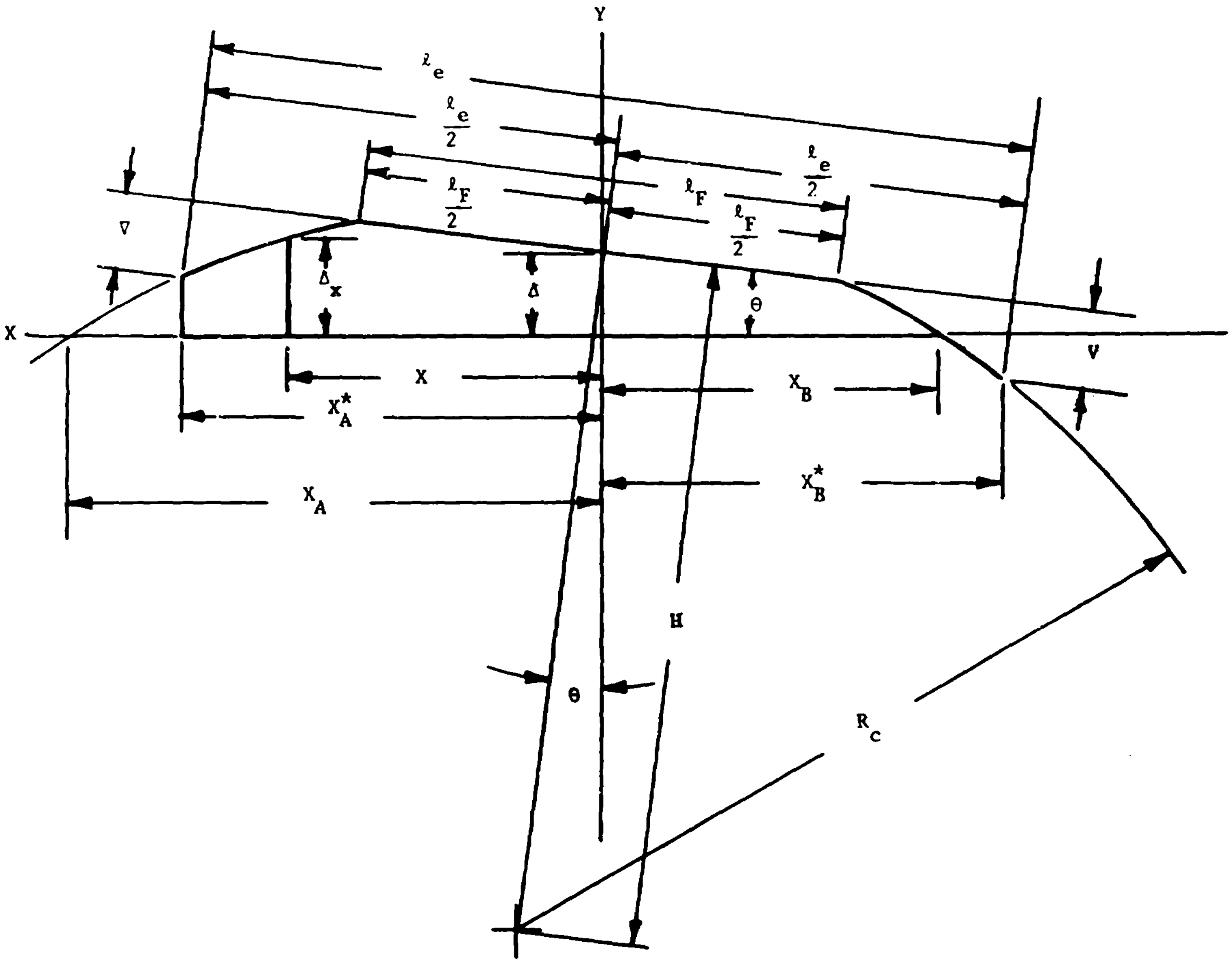

Figure 8. Geometric Intersection of a Roller and Raceway

The equation of the flat portion of the roller is

$$Y = \Delta_i + X\tan\Theta_i \tag{31}$$

The equation of the crowned portion of the roller is

$$(X - H\sin\Theta_i)^2 + (Y + H\cos\Theta_i - \Delta_i)^2 = R_c^2 \tag{32}$$

In the above the subscript i is 1 for an outer contact and 2 for an inner contact.

The intersections of the race and the crowned surface of the roller occur at X_{A_i} and X_{B_i}.

$$x_{A_i} = \sqrt{R_c^2 - (H\cos\Theta_i - \Delta_i)^2} + H\sin\Theta_i \tag{33}$$

$$x_{B_i} = \sqrt{R_c^2 - (H\cos\Theta_i - \Delta_i)^2} + H\sin\Theta_i \tag{34}$$

X_{A_i} and X_{B_i} must be within the projected extremities of the roller crown.

That is

$$X_{A_i} \leqslant X^*_{A_i} \tag{35}$$

$$X_{B_i} \geqslant X^*_{B_i} \tag{36}$$

where

$$X^*_{A_i} = \frac{\ell_e}{2} \cos\Theta_i + \nabla\sin\Theta_i \qquad (37)$$

$$X^*_{B_i} = \frac{\ell_e}{2} \cos\Theta_i + \nabla\sin\Theta_i \qquad (38)$$

If the quantity under the radical in Equations 33 and 34 is zero or negative there is no contact between roll and race.

If $\frac{\ell_e}{2} \cos\Theta_i \geq X_{A_i}$ there is also no contact.

If $X_{A_i} > X^*_{A_i}$, X_{A_i} is set equal to $X^*_{A_i}$.

If $X_{B_i} < X^*_{B_i}$, X_{B_i} is set equal to $X^*_{B_i}$.

If $\frac{\ell_F}{2} \cos\Theta_i > X_{B_i} > -\frac{\ell_F}{2} \cos\Theta_i$ and

$X_{A_i} > \frac{\ell_F}{2} \cos\Theta_i$ the value of X_{B_i} is

$$X_{B_i} = -\frac{\Delta_i}{\tan\Theta_i} \qquad 39)$$

From Figure 7 the conditions for roller equilibrium are

$$-P_1 + P_2 + F_c = 0 \qquad (40)$$

$$M_1 - M_2 = 0 \qquad (41)$$

Equations 40 and 41 are a set of non-linear simultaneous equations in which the variables are Δ_1 and Θ_1 at the outer contact of the particular roller under consideration.

From Figure 8, the intrusion of the roller into the raceway is

$$\Delta_x = \Delta_i + X\tan\Theta_i \qquad |X| \leq \frac{\ell_F}{2}\cos\Theta_i \tag{42}$$

$$\Delta_x = \sqrt{R_c^2 - (X - H\sin\Theta)^2} - H\cos\Theta_i + \Delta_i \qquad |X| > \frac{\ell_F}{2}\cos\Theta_i \tag{43}$$

The derivatives of Δ_x with respect to Θ_i will be required later. They are

$$\frac{d\Delta_x}{d\Theta_i} = \frac{X}{\cos^2\Theta_i} \qquad |X| \leq \frac{\ell_F}{2}\cos\Theta_i \tag{44}$$

$$\frac{d\Delta_x}{d\Theta_i} = \frac{(X - H\sin\Theta_i)\ H\cos\Theta_i}{\sqrt{R_c^2 - (X - H\sin\Theta_i)^2}} + H\sin\Theta_i \qquad |X| > \frac{\ell_F}{2}\cos\Theta_i \tag{45}$$

Lundberg (7) gives the approach Δ_x of two cylindrical bodies pressed together with the uniform loading p_x as

$$\Delta_x = \frac{(\eta_R + \eta E)}{2\pi}\, p_x \left[1.8864 + \ln\left(\frac{X_A - X_B}{2b_x}\right)\right] \tag{46}$$

η_R and η_E are elastic constants for race and roller, or respectively, having the form

$$\eta_R = \frac{(1-\nu_R^{\ 2})}{E_R} \quad \text{or} \quad \eta_E = \frac{(1-\nu_E^{\ 2})}{E_E} \tag{47}$$

where ν is Poisson's ratio and $E_{R,E}$ is the modulus of elasticity.

b_x is the semi-width of the pressure area in the rolling direction

---

7. G. Lundberg: Elastische Beruhrung ziveier Halbraume, VDI Forschung, Sept./Oct., 1939.

$$b_x = \left[\frac{(\eta_R+\eta_E)}{2\pi} p_x\, d\, (1 + C_i\gamma_i)\right]^{1/2} \tag{48}$$

C_i is 1 for an outer contact and -1 for an inner contact. γ is given by Equation 23.

The value of p_x corresponding to Δ_x is required. It cannot be obtained from Equation 46 in closed form. It can be obtained numerically as follows.

Let p_x' be an estimate of p_x. A good starting value is

$$p_x' = \frac{5\text{x}10^7 \Delta_x^{10/9}}{(X_A-X_B)^{1/9}} \tag{49}$$

An improved value of p_x' is

$$\bar{p}_x = p_x' - \frac{(\Delta_x'-\Delta_x)}{\dfrac{d\Delta_x'}{dp_x'}} \tag{50}$$

Δ_x' is the approach of race and roller bodies calculated for the current estimate p_x' using Equation 46.

$\dfrac{d\Delta_x'}{dp_x'}$ is obtained from Equations 46 and 48 using the current estimate p_x' and is

$$\frac{d\Delta_x'}{dp_x'} = \frac{(\eta_R+\eta_E)}{2\pi}\left[1.3864 + \ln\left|\frac{X_A-X_B}{2b_x}\right|\right] \tag{51}$$

Iteration of Equation 50 yields p_x to any desired accuracy.

The contact force P and the moment M are

$$P_i = \int_{X_{B_i}}^{X_{A_i}} p_x dX \tag{52}$$

$$M_i = \int_{X_{B_i}}^{X_{A_i}} p_x dX \tag{53}$$

Equations 40 and 41 may now be solved numerically for Δ_1 and Θ_1.

If estimates are made of the variables Δ_1 and Θ_1 Equations 40 and 41 may not be satisfied and there will be the residues ε_1 and ε_2 for Equations 40 and 41 respectively. Differentiating Equations 40 and 41 gives:

$$\frac{d\varepsilon_1}{d\Delta_1} = -\frac{dP_1}{d\Delta_1} + \frac{dP_2}{d\Delta_2}\frac{d\Delta_2}{d\Delta_1} \tag{54}$$

$$\frac{d\varepsilon_1}{d\Theta_1} = -\frac{dP_1}{d\Theta_1} + \frac{dP_2}{d\Theta_2}\frac{d\Theta_2}{d\Theta_1} \tag{55}$$

$$\frac{d\varepsilon_2}{d\Delta_1} = \frac{dM_1}{d\Delta_1} - \frac{dM_2}{d\Delta_2}\frac{d\Delta_2}{d\Delta_1} \tag{56}$$

$$\frac{d\varepsilon_2}{d\Theta_1} = \frac{dM_1}{d\Theta_1} - \frac{dM_2}{d\Theta_2}\frac{d\Theta_2}{d\Theta_1} \tag{57}$$

where

$$\frac{d\Delta_2}{d\Delta_1} = -1 \tag{58}$$

$$\frac{d\Theta_2}{d\Theta_1} = -1 \tag{59}$$

If Δ_1' and Θ_1' are current estimates of the variables, improved values are Δ_1 and Θ_1

$$\Delta_1 = \Delta_1' - \frac{\begin{vmatrix} \varepsilon_1 & \dfrac{d\varepsilon_1}{d\Theta_1} \\[2ex] \varepsilon_2 & \dfrac{d\varepsilon_2}{d\Theta_1} \end{vmatrix}}{\begin{vmatrix} \dfrac{d\varepsilon_1}{d\Delta_1} & \dfrac{d\varepsilon_1}{d\Theta_1} \\[2ex] \dfrac{d\varepsilon_2}{d\Delta_1} & \dfrac{d\varepsilon_2}{d\Theta_1} \end{vmatrix}} \tag{60}$$

$$\Theta_1 = \Theta_1' - \frac{\begin{vmatrix} \dfrac{d\varepsilon_1}{d\Delta_1} & \varepsilon_1 \\[2ex] \dfrac{d\varepsilon_2}{d\Delta_1} & \varepsilon_2 \end{vmatrix}}{\begin{vmatrix} \dfrac{d\varepsilon_1}{d\Delta_1} & \dfrac{d\varepsilon_1}{d\Theta_1} \\[2ex] \dfrac{d\varepsilon_2}{d\Delta_1} & \dfrac{d\varepsilon_1}{d\Theta_1} \end{vmatrix}} \tag{61}$$

The right members of Equations 60 and 61 are evaluated at current estimates.

Iteration of Equations 60 and 61 yields Δ_1 and Θ_1 to any desired accuracy.

The derivatives of P_i and M_i with respect to Δ_i and Θ_i are

$$\frac{dP_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} \, dX \tag{62}$$

$$\frac{dP_i}{d\Theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Theta_i} \, dX \tag{63}$$

$$\frac{dM_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} \, dX \tag{64}$$

$$\frac{dM_i}{d\Theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Theta_i} \, dX \tag{65}$$

The value of $\frac{dp_x}{d\Delta_x}$ is obtained from Equation 51 and the value of $\frac{d\Delta_x}{d\Delta_i}$ is unity.

If Equations 27, 28, 40 and 41 are differentiated with respect to Δ, there results four simultaneous equations which are linear in $\frac{d\Delta_1}{d\Delta}$, $\frac{d\Delta_2}{d\Delta}$, $\frac{d\Theta_1}{d\Delta}$, and $\frac{d\Theta_2}{d\Delta}$ and from which the latter can be obtained. Only $\frac{d\Delta_1}{d\Delta}$ and $\frac{d\Theta_1}{d\Delta}$ are of interest here.

$$- \frac{dP_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta} + \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta} - \frac{dP_1}{d\Theta_1} \frac{d\Theta_1}{d\Delta} + \frac{dP_2}{d\Theta_2} \frac{d\Theta_2}{d\Delta} = 0 \qquad (66)$$

$$\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta} - \frac{dM_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta} + \frac{dM_1}{d\Theta_1} \frac{d\Theta_1}{d\Delta} - \frac{dM_2}{d\Theta_2} \frac{d\Theta_2}{d\Delta} = 0 \qquad (67)$$

$$\frac{d\Delta_1}{d\Delta} + \frac{d\Delta_2}{d\Delta} = 1 \qquad (68)$$

$$\frac{d\Theta_1}{d\Delta} + \frac{d\Theta_2}{d\Delta} = 0 \qquad (69)$$

Equations 66 through 69 are easily solved for $\frac{d\Delta_1}{d\Delta}$, $\frac{d\Theta_1}{d\Delta}$, $\frac{d\Delta_1}{d\Theta}$ and $\frac{d\Theta_1}{d\Theta}$ in a similar manner.

The reactions of the bearing on the shaft are

$$F'_x = \sum_{q=1}^{n} P_{1_q} \cos\phi_q \qquad (70)$$

$$F'_y = \sum_{q=1}^{n} P_{1_q} \sin\phi_q \qquad (71)$$

$$F'_z = 0 \qquad (72)$$

$$M'_x = \sum_{q=1}^{n} M_{1_q} \sin\phi_q \qquad (73)$$

$$M'_y = \sum_{q=1}^{n} M_{1_q} \cos\phi_q \qquad (74)$$

The conditions for equilibrium under the single radial load F_x is

$$F'_x + F_x = 0 \tag{75}$$

The only working response of the bearing is the linear displacement δ_x.

If δ'_x is an estimate of δ_x an improved value of δ_x is

$$\delta_x = \delta'_x - \frac{(F'_x + F_x)}{\dfrac{dF'_x}{d\delta_x}} \tag{76}$$

The right member of Equation 76 is calculated at current values of δ_x.

Iteration of Equation 76 yields δ_x to any desired accuracy.

Although only $\frac{dF'_x}{d\delta_x}$ is required in the foregoing, all the derivatives are required to complete the matrix needed for stiffness calculations. The elements of the complete matrix are

$$\frac{dF'_x}{d(\delta_x, \delta_y, \Theta_x, \Theta_y)} = \sum_{q=1}^{n} \cos\phi_q \frac{dP_{1_q}}{d(\delta_x, \delta_y, \Theta_x, \Theta_y)} \tag{77}$$

$$\frac{dF'_y}{d(\delta_x, \delta_y, \Theta_x, \Theta_y)} = \sum_{q=1}^{n} \sin\phi_q \frac{dP_{1_q}}{d(\delta_x, \delta_y, \Theta_x, \Theta_y)} \tag{78}$$

$$\frac{dF'_z}{d(\delta_x, \delta_y, \Theta_x, \Theta_y)} = 0 \tag{79}$$

$$\frac{dM'_x}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} = \sum_{q=1}^{n} \sin\phi_q \frac{dM_{1_q}}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} \tag{80}$$

$$\frac{dN'_y}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} = \sum_{q=1}^{n} \cos\phi_q \frac{dM_{1_q}}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} \tag{81}$$

$$\frac{d(F'_x,F'_y,F'_z,M'_x,M'_y}{d\delta_z} = 0 \tag{82}$$

where

$$\frac{dP_{1_q}}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} = \left[\frac{dP_{1_q}}{d\Delta_{1_q}} \frac{d\Delta_{1_q}}{d\Delta_q} + \frac{dP_{1_q}}{d\Theta_{1_q}} \frac{d\Theta_{1_q}}{d\Delta_q}\right] \frac{d\Delta_q}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)}$$
$$+ \left[\frac{dP_{1_q}}{d\Delta_{1_q}} \frac{d\Delta_{1_q}}{d\Theta_q} + \frac{dP_{1_q}}{d\Theta_{1_q}} \frac{d\Theta_{1_q}}{d\Theta_q}\right] \frac{d\Theta_q}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} \tag{83}$$

$$\frac{dM_{1_q}}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} = \left[\frac{dM_{1_q}}{d\Delta_{1_q}} \frac{d\Delta_{1_q}}{d\Delta_q} + \frac{dM_{1_q}}{d\Theta_{1_q}} \frac{d\Theta_{1_q}}{d\Delta_q}\right] \frac{d\Delta_q}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)}$$
$$+ \left[\frac{dM_{1_q}}{d\Delta_{1_q}} \frac{d\Delta_{1_q}}{d\Theta_q} + \frac{dM_{1_q}}{d\Theta_{1_q}} \frac{d\Theta_{1_q}}{d\Theta_q}\right] \frac{d\Theta_q}{d(\delta_x,\delta_y,\Theta_x,\Theta_y)} \tag{84}$$

The derivatives of Δ_q and Θ_q with respect to the inner-ring displacements are, from Equations 24 and 25.

$$\frac{d\Delta_q}{d\delta_x} = \cos\phi_q \tag{85}$$

$$\frac{d\Delta_q}{d\delta_y} = \sin\phi_q \tag{86}$$

$$\frac{d\Delta_q}{d(\Theta_x,\Theta_y)} = 0 \tag{87}$$

$$\frac{d\Theta_q}{d(\delta_x,\delta_y)} = 0 \tag{88}$$

$$\frac{d\Theta_q}{d\Theta_x} = \sin\phi_q \tag{89}$$

$$\frac{d\Theta_q}{d\Theta_y} = \cos\phi_q \tag{90}$$

Some rollers may be out-of-contact with the inner race and are loaded against the outer race by centrifugal force. Such rollers contribute to the bearing's reactions but have no effect on the derivatives of P_{1_q} and M_{1_q}.

The complete matrix for stiffness calculations is then:

$$\begin{bmatrix} \frac{dF'_x}{d\delta_x} & \frac{dF'_x}{d\delta_y} & 0 & \frac{dF'_x}{d\Theta_x} & \frac{dF'_x}{d\Theta_y} \\ \frac{dF'_y}{d\delta_x} & \frac{dF'_y}{d\delta_y} & 0 & \frac{dF'_y}{d\Theta_x} & \frac{dF'_y}{d\Theta_y} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{dM'_x}{d\delta_x} & \frac{dM'_x}{d\delta_y} & 0 & \frac{dM'_x}{d\Theta_x} & \frac{dM'_x}{d\Theta_y} \\ \frac{dM'_y}{d\delta_x} & \frac{dM'_y}{d\delta_y} & 0 & \frac{dM'_y}{d\Theta_x} & \frac{dM'_y}{d\Theta_y} \end{bmatrix} \tag{91}$$

As discussed earlier, the axial components of stiffness are zero for a cylindrical roller bearing.

SECTION III

APPLICATION OF COMPUTER PROGRAM

The analysis of Section II has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in the Appendix.

3.1 Sample Test Case

To illustrate a typical case consider the cylindrical roller bearing in Figure 9. This is a cylindrical bearing typical of a design employed in high speed applications. The geometry of this sample bearing is summarized below.

Number of rollers, n	32
Roller diameter, d	.6299 in.
Pitch diameter, E	7.75 in.
Total length of roller, ℓ_T	.6299 in.
Length of flat portion of roller ℓ_F	.2 in.
Roller crown radius	17 in.
Roller corner break	.05 in
Crown drop gage point	0

The operating conditions for the sample case are:

Diametrical clearance, P_D = .0058 in.

Rotational speed, N_2 = 13,230 rpm

Radial load, F_x = 1,465 lbs.

3.2 Input Format

Figure 10 presents the input data format and Figure 11 shows the actual input data for the sample case operating conditions.

3.3 Output Format

Figure 12 presents the output data for the sample case. The input data

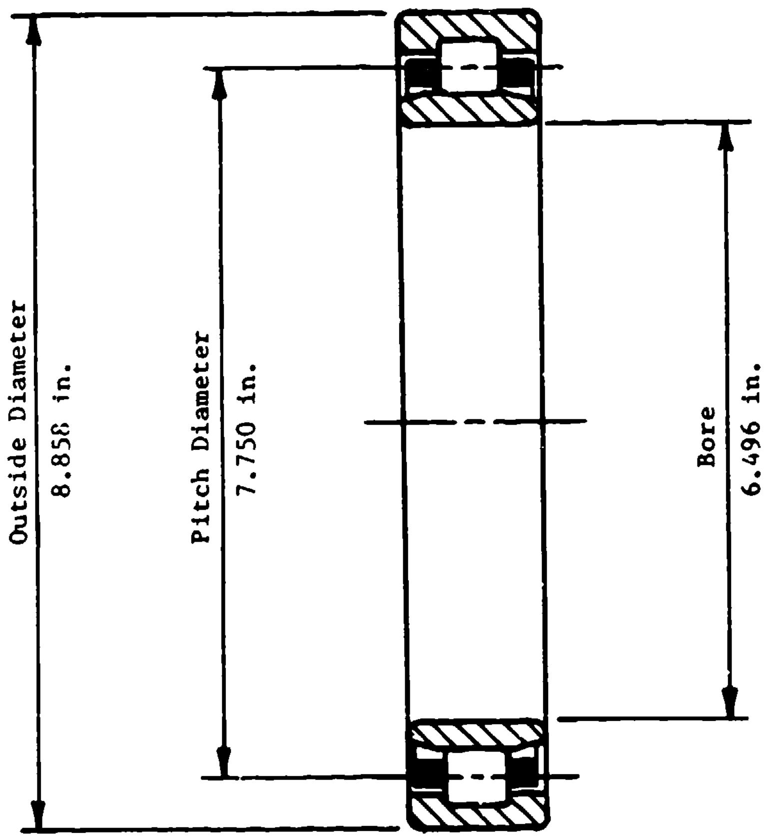

Figure 9 Sample Cylindrical Roller Bearing

Number of Rollers (60 maximum)	Roller Diameter (inches)	Pitch Diameter (inches)	Total Length of Roller (inches)	Effective Length of Roller (inches) (A)	Length of Flat Portion of Roller (inches)	Crown Radius (inches)	Crown Drop (inches) (B)
(C) TITLE							
BLANK TITLE							
Gage Point (inches)	Corner Break (inches)	Modulus of Elasticity of Outer Race (lb/in**2) (D)	Modulus of Elasticity of Inner Race (lb/in**2) (D)	Modulus of Elasticity of Rollers (lb.in**2) (D)	Poisson's Ratio for Outer Race (if blank - assumes .25)	Poisson's Ratio for Inner Race (if blank - assumes .25)	Poisson's Ratio for Rollers (if blank - assumes .25)
Roller Material Density (lb/in**3) If blank assumes .283	Tolerance for Elastic Approach of Roller & Race If blank assumes 1.E-7	Tolerance for Angular Deflection of Roller If blank assumes 1.E-7	Tolerance for Bearing Deflection Along x If blank assumes 5.E-7				
RPM of Outer Ring	RPM of Inner Ring	Load Along x (lb)	Initial Displacement Along x (inches)	Initial Displacement Along y (inches)	Initial Displacement About x (Radians)	Initial Displacement About y (Radians)	Diametral Clearance (inches)

E 10.0 Format

(A) If total length is given omit effective length.
If effective length is given omit total length.

(B) If crown radius is given omit crown drop.
If crown drop is given omit crown radius.

(C) Punch 1 to start printout at top of a new page.

(D) If blank program assumes 29.E6.
To run additional load cases with same bearing repeat card 5 as required.
To run new bearing place one blank after last card 6 and repeat card 1 et seq.
To stop place two blanks after last card 6.

Figure 10 Input Data Format

IBM FORTRAN Coding Form

GX28-7327-6 U/M 050**
Printed in U.S.A.

CYLROL
J BARTLETT

AUG. 23, 1979

FORTRAN STATEMENT

```
32.        .6299      7.75       .6299                .2          17.
1165 MM CYLINDRICAL ROLLER BEARING
CROWNED ROLLERS
0.         .05
<-- BLANK CARD -->
           13230.     1465.                                                .0058
<-- BLANK CARD -->
<-- BLANK CARD -->
```

Figure 11 Sample Problem Data Input

PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLACEMENTS

DFX/DX	DFX/DY	DFX/DZ	DFX/DALX	DFX/DALY
LB/IN	LB/IN	LB/IN	LB/RAD	LB/RAD
3.4980+06	1.9336-01	0.0000	2.1193-04	1.3479-02
DFY/DX	DFY/DY	DFY/DZ	DFY/DALX	DFY/DALY
LB/IN	LB/IN	LB/IN	LB/RAD	LB/RAD
1.9531-01	2.6504+05	0.0000	9.5776-04	2.1193-04
DFZ/DX	DFZ/DY	DFZ/DZ	DFZ/DALX	DFZ/DALY
LB/IN	LB/IN	LB/IN	LB/RAD	LB/RAD
0.0000	0.0000	0.0000	0.0000	0.0000
DMX/DX	DMX/DY	DMX/DZ	DMX/DALX	DMX/DALY
LBIN/IN	LBIN/IN	LBIN/IN	LBIN/RAD	LBIN/RAD
3.0981-04	9.5084-04	0.0000	1.6056+03	9.7046-03
DMY/DX	DMY/DY	DMY/DZ	DMY/DALX	DMY/DALY
LBIN/IN	LBIN/IN	LBIN/IN	LBIN/RAD	LBIN/RAD
1.3566-02	3.0981-04	0.0000	9.7046-03	2.3117+04

Figure 12 Output Data for Sample Problem

165 MM CYLINDRICAL ROLLER BEARING
CROWNED ROLLERS

DESIGN DATA FOR BEARING NO. 1

NO. OF ROLLS	ROLL DIAMETER IN	PITCH DIAMETER IN	TOTAL LENGTH IN	FLAT LENGTH IN	EFFECTIVE LENGTH IN	CORNER BREAK IN	CROWN RADIUS IN	CROWN DROP IN	GAGE POINT IN	ROLL WEIGHT LB
3.2000+01	6.2990-01	7.7500+00	6.2990-01	2.0000-01	5.2990-01	5.0000-02	1.7000+01	1.7707-03	0.0000	5.5551-02

MODULUS OF ELASTICITY OUTER LB/IN**2	INNER LB/IN**2	ROLLS LB/IN**2	POISSON'S RATIO OUTER	INNER	ROLLS
2.9000+07	2.9000+07	2.9000+07	2.5000-01	2.5000-01	2.5000-01

INPUT DATA FOR LOAD NO. 1, BEARING NO. 1

RPM OF OUTER	RPM OF INNER	LOAD ALONG X LB	ALONG X IN	INITIAL DISPLACEMENTS ALONG Y IN	ABOUT X RADIANS	ABOUT Y RADIANS	DIAMETRAL CLEARANCE IN
0.0000	1.3230+04	1.4650+03	0.0000	0.0000	0.0000	0.0000	5.8000-03

OUTPUT DATA FOR LOAD NO. 1, BEARING NO. 1

REACTIONS OF BEARING ON SHAFT ALONG X LB	ALONG Y LB	ALONG Z LB	ABOUT X LB*IN	ABOUT Y LB*IN	TOTAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ALONG X IN	ALONG Y IN	ALONG Z IN	ABOUT X RADIANS	ABOUT Y RADIANS
-1.4655+03	4.5776-05	0.0000	-7.8035-07	-1.3067-05	-3.7195-03	0.0000	0.0000	0.0000	

ROLL NUMBER	ROLL AZIMUTH DEG	CONTACT LOAD OUTER LB	INNER LB	CONTACT MOMENT OUTER LB*IN	INNER LB*IN	OUTER PATH EXTREMITY X(1) IN	X(2) IN	INNER PATH EXTREMITY X(1) IN	X(2) IN
15	1.5750+02	4.2626+02	2.0066+02	-2.7546-07	1.2601-06	1.4773-01	-1.4773-01	1.2817-01	-1.2817-01
16	1.6875+02	5.7710+02	3.5171+02	2.1965-06	9.6610-07	1.5864-01	-1.5864-01	1.4241-01	-1.4241-01
17	1.8000+02	6.3201+02	4.0659+02	4.2638-06	1.8814-06	1.6226-01	-1.6226-01	1.4680-01	-1.4680-01
18	1.9125+02	5.7710+02	3.5171+02	2.2862-06	1.2132-06	1.5864-01	-1.5864-01	1.4241-01	-1.4241-01
19	2.0250+02	4.2626+02	2.0066+02	1.7580-06	1.0510-06	1.4773-01	-1.4773-01	1.2817-01	-1.2817-01
OTHERS		2.2564+02		0.0000		7.5189-02	-7.5189-02		

ROLL NUMBER	CONTACT LENGTH OUTER IN	INNER IN	MAXIMUM CONTACT WIDTH OUTER IN	INNER IN	MAX. CONTACT DEFLECTION OUTER IN	INNER IN	MAXIMUM HERTZ STRESS OUTER LB/IN**2	INNER LB/IN**2	LOCATION OF MAX. VALUES OUTER IN	INNER IN
15	2.9545-01	2.5633-01	1.3638-02	9.1569-03	3.4774-04	1.8903-04	1.5712+05	1.2237+05	-8.2070-02	9.9685-02
16	3.1728-01	2.8482-01	1.5768-02	1.1744-02	4.4608-04	3.0237-04	1.7903+05	1.5694+05	8.8133-02	9.4939-02
17	3.2452-01	2.9359-01	1.6387-02	1.2486-02	4.8024-04	3.3969-04	1.8606+05	1.6685+05	5.4086-02	9.7864-02
18	3.1728-01	2.8482-01	1.5768-02	1.1744-02	4.4608-04	3.0237-04	1.7903+05	1.5694+05	-7.0506-02	9.4939-02
19	2.9545-01	2.5633-01	1.3638-02	9.1569-03	3.4774-04	1.8903-04	1.5712+05	1.2237+05	9.8484-02	9.9685-02
OTHERS	1.[illegible]-01		5.6035-03		-1.2830-04		6.3623+04		0.0000	

ROLL NUMBER	CONTACT MISALIGNMENT OUTER RADIANS	INNER RADIANS
15	3.7332-11	-3.7332-11
16	-2.1512-11	2.1512-11
17	-1.6081-10	1.6081-10
18	-1.5531-10	1.5531-10
19	-7.7504-11	7.7504-11
OTHERS	0.0000	

CENTRIFUGAL FORCE OF ROLLER = 2.2564+02 LB

Figure 12 Output Data for Sample Problem (Continued) are summarized in Figure 12, followed by the output data including the internal load distribution as well as various other stress and displacement parameters. The stiffness matrix is given on the last page of Figure 12.

APPENDIX

COMPUTER PROGRAM
FOR
CALCULATING STIFFNESS MATRIX
OF
CYLINDRICAL ROLLER BEARING

```
      COMMON RB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DROP,DENS,DFL11(5),
     1DFLD1(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
     2HERTZ(2,60),IRR,ILOAD,IQUIT,ITER,JPASS,N,NOLOAD,OME,PR(2),PRR,PD,
     3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
     4XLT,XLE,XMASS,XLE2,XF2,XF1(5),XYM(2,60),XYX(2,2,60),XHSAV(2,60),
     5YM(2),YMR
      COMMON ABIG,DELLX,HRZ,XBIG,XXO(2)
      COMMON CRN,FLT2D,HD,XLE2D
      DOUBLE PRECISION  CRN,FLT2D,XLE2D,HD
C CARD  COLS.  ITEM
C   1    1-10  NUMBER OF ROLLS - 60 MAXIMUM
C       11-20  ROLL DIAMETER - IN
C       21-30  PITCH DIAMETER - IN
C       31-40  TOTAL LENGTH OF ROLL - IN. MEASURED BETWEEN ROLL ENDS
C       41-50  EFFECTIVE LENGTH OF ROLL - IN. MAXIMUM WORKING LENGTH OF
C              ROLL MEASURED BETWEEN OUTER EXTREMITIES OF CROWNED
C              SURFACE
C       NOTE   IF TOTAL LENGTH IS GIVEN OMIT EFFECTIVE LENGTH
C              IF EFFECTIVE LENGTH IS GIVEN OMIT TOTAL LENGTH
C       51-60  LENGTH OF FLAT PORTION OF ROLL - IN. MAY BE ZERO FOR A
C              FULLY-CROWNED ROLL
C       61-70  CROWN RADIUS - IN
C       71-80  CROWN DROP MEASURED AT GAGE POINT - IN
C       NOTE   IF CROWN RADIUS IS GIVEN OMIT CROWN DROP
C              IF CROWN DROP IS GIVEN OMIT CROWN RADIUS
C   2    1     PUNCH 1 TO START PRINTOUT AT THE TOP OF A NEW PAGE
C        2-80  TITLE - PUNCH ANYTHING
C   3    1     LEAVE BLANK
C        2-80  TITLE - PUNCH ANYTHING
C   4    1-10  GAGE POINT - IN. THE AXIAL DISTANCE IN FROM THE END OF THE
C              CROWNED SURFACE OF THE ROLL TO WHERE THE CROWN DROP IS
C              MEASURED
C       11-20  CORNER BREAK - IN. MEASURED PARALLEL TO ROLL AXIS
C       21-30  MODULUS OF ELASTICITY OF OUTER RACE - LB/IN**2. IF BLANK
C              PROGRAM ASSUMES 29.E6
C       31-40  SAME FOR INNER RACE
C       41-50  SAME FOR ROLLS
C       51-60  POISSON'S RATIO FOR OUTER RACE. IF BLANK PROGRAM ASSUMES
C              .25
```

```
C        61-70  SAME FOR INNER RACE
C        71-80  SAME FOR ROLLS
C     5   1-10  ROLL MATERIAL DENSITY - LB/IN**3. IF BLANK PROGRAM
C               ASSUMES .283
C        11-20  TOLERANCE FOR ELASTIC APPROACH OF ROLL AND RACE. IF BLANK
C               PROGRAM ASSUMES 1.E-7
C        21-30  TOLERANCE FOR ANGULAR DEFLECTION OF ROLL RELATIVE TO RACE
C               IF BLANK PROGRAM ASSUMES 1.E-7
C        31-40  TOLERANCE FOR BEARING DEFLECTION ALONG X. IF BLANK
C               PROGRAM ASSUMES 5.E-7
C
C
C
C     6   1-10  RPM OF OUTER RING
C        11-20  RPM OF INNER RING
C        21-30  LOAD ALONG X - LB
C        31-40  INITIAL DISPLACEMENT ALONG X - IN
C        41-50  INITIAL DISPLACEMENT ALONG Y - IN
C        51-60  INITIAL DISPLACEMENT ABOUT X - RADIANS
C        61-70  INITIAL DISPLACEMENT ABOUT Y - RADIANS
C        71-80  DIAMETRAL CLEARANCE - IN. A POSITIVE VALUE INDICATES
C               LOOSENESS
C
C
C
C        TO RUN ADDITIONAL LOAD CASES WITH SAME BEARING REPEAT CARD 6 AS
C        REQUIRED
C
C
C
C        TO RUN NEW BEARING PLACE ONE BLANK AFTER LAST CARD 6 AND REPEAT
C        CARD 1 ET SEQ
C
C
C
C        TO STOP PLACE TWO BLANKS AFTER LAST CARD 6
C
C
C
```

```
      IBR=0
      C(1)=1.
      C(2)=-1.
 5    READ(5,10)XN,D,E,XLT,XLE,FLT,CROWN,DROP
      IF(XN.EQ.0.)STOP
      IBR=IBR+1
 10   FORMAT(8F10.0)
      READ(5,20)
 20   FORMAT(80H
     1                               /80H
     2                                                       )
      WRITE(6,20)
      WRITE(6,30)IBR
 30   FORMAT(30H  DESIGN DATA FOR BEARING NO.,I3)
      READ(5,10)GAGE,CORNER,YM(1),YM(2),YMR,PR(1),PR(2),PRR
     1,DENS,TOL1,TOL2,TOL3
      IF(XLE.EQ.0.)XLE=XLT-2.*CORNER
      IF(XLT.EQ.0.)XLT=XLE+2.*CORNER
      IF(YMR.EQ.0.)YMR=29.E6
      IF(PRR.EQ.0.)PRR=.25
      IF(TOL1.EQ.0.)TOL1=1.E-7
      IF(TOL2.EQ.0.)TOL2=1.E-7
      IF(TOL3.EQ.0.)TOL3=5.E-7
      DO 40 K=1,2
      IF(YM(K).EQ.0.)YM(K)=29.E6
      IF(PR(K).EQ.0.)PR(K)=.25
      EL(K)=.6366198*((1.-PR(K)**2)/YM(K)+(1.-PRR**2)/YMR)
 40   CONTINUE
      IF(DENS.EQ.0.)DENS=.283
      XMASS=D**2*XLT*DENS/491.9798
      N=XN
      ILOAD=0
      XLE2=.5*XLE
      FLT2=.5*FLT
      XLE2D=XLE2
      FLT2D=FLT2
      CRN=CROWN
      CALL RCALC(CROWN,DROP,FLT2,GAGE,HD,XLE2)
      H=HD
```

```
      WEIGHT=XMASS*386.4
      WRITE(6,50)XN,D,E,XLT,FLT,XLE,CORNER,CROWN,DROP,GAGE,WEIGHT
   50 FORMAT(129H0  NO. OF          ROLL          PITCH          TOTAL          FL
     1AT      EFFECTIVE      CORNER        CROWN        CROWN          GAGE
     2    ROLL/130H    ROLLS        DIAMETER     DIAMETER      LENGTH        L
     3ENGTH       LENGTH        BREAK         RADIUS         DROP          POINT
     4      WEIGHT/18X,9(12HIN          ),2HLB,/1P11E12.4)
      WRITE(6,60)YM(1),YM(2),YMR,PR(1),PR(2),PRR
   60 FORMAT(62H0         MODULUS OF ELASTICITY                    POISSON'S
     1 RATIO/69H    OUTER         INNER         ROLLS          OUTER          INNE
     2R        ROLLS/3(12H   LB/IN**2 )/1P6E12.4)
      GAM=D/E
      DFL11(1)=0.
      DELD1(4)=0.
      DELD1(5)=0.
      THD1(2)=0.
      THD1(3)=0.
   65 READ(5,10)RPM(1),RPM(2),XF2,DFL11(2),DFL11(3),DFL11(4),DFL11(5),PD
      IF(ABS(RPM(1))+ABS(RPM(2)).EQ.0.)GO TO 5
      OME=.5*(RPM(1)*(1.+GAM)+RPM(2)*(1.-GAM))
      ILOAD=ILOAD+1
      FC=XMASS*(.1047198*OME)**2*.5*E
      IQUIT=0
      WRITE(6,70)ILOAD,IBR
   70 FORMAT(26H1  INPUT DATA FOR LOAD NO.,I3,13H, BEARING NO.,I3)
      WRITE(6,80)RPM(1),RPM(2),XF2,(DFL11(I),I=2,5),PD
   80 FORMAT(95H0    RPM OF        RPM OF         LOAD                    INITI
     1AL DISPLACEMENTS                  DIAMETRAL/95H    OUTER          INNER
     2    ALONG X       ALONG X      ALONG Y       ABOUT X       ABOUT Y     CLE
     3ARANCE/30X,6OHLB              IN             IN           RADIANS       RADIANS
     4          IN/1P8E12.4)
      DFL2=.005*D
      IF(PD.GT.0.)DFL2=DFL2+.5*PD
      DFL2=-SIGN(DFL2,XF2)
      DO 140 ITER=1,20
      NOLOAD=0
      DO 100 K=1,5
      XF1(K)=0.
      DO 100 L=1,5
```

```
100  DTV(L,K)=0.
     DO 130 J=1,N
     JPASS=J
     XJ=J
     PHI=6.283185*(XJ-1.)/XN
     SPH=SIN(PHI)
     CPH=COS(PHI)
     CALL CYLROL
     IF(IQUIT)130,130,110
110  WRITE(6,120)IBR,ILOAD,ITER,J,DFL2
120  FORMAT(17HOMAIN PROGRAM 120,4I6,1P1E12.4)
     GO TO 65
130  CONTINUE
     IF(DFL11(2))160,135,160
135  COR2=(XF1(2)+XF2)/DTV(2,2)
     DFL2=DFL2-COR2
     IF(ABS(COR2)-TOL3)160,140,140
140  CONTINUE
     WRITE(6,150)DFL2,COR2
150  FORMAT(17HOMAIN PROGRAM 150,1P2E12.4)
     GO TO 65
160  CALL OUTPT1
     GO TO 65
     END
     SUBROUTINE CYLROL
     COMMON BB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DROP,DENS,DFL11(5),
    1DELD1(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
    2HERTZ(2,60),IBR,ILOAD,IQUIT,ITER,JPASS,N,NOLOAD,OME,PR(2),PRR,PD,
    3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
    4XLT,XLE,XMASS,XLE2,XF2,XF1(5),XMM(2,60),XXX(2,2,60),XHSAV(2,60),
    5YM(2),YMR
     COMMON ABIG,DELLX,HRZ,XBIG,XXO(2)
     COMMON CRN,FLT2D,HD,XLE2D
     DOUBLE PRECISION  CRN,FLT2D,XLE2D,HD
     DOUBLE PRECISION XTH,XDEL,X1D,X2D
     DIMENSION A(4,4),COR(4),DEL(2),DPDEL(2),DMDEL(2),DPTH(2),DMTH(2),
    1ER(4),PX(2),TH(2),XM(2)
     J=JPASS
     DELTA=(DFL2+DFL11(2))*CPH+DFL11(3)*SPH-.5*PD
```

```
      THETA=DFL11(4)*SPH+DFL11(5)*CPH
      IF(ABS(THETA).LE.1.E-9)THETA=0.
      DEL(1)=.55*DELTA
      DEL(2)=DELTA-DEL(1)
      TH(1)=.5*THETA
      TH(2)=THETA-TH(1)
      DELD1(2)=CPH
      DELD1(3)=SPH
      THD1(4)=SPH
      THD1(5)=CPH
      DO 140 IT=1,20
      DO 120 K=1,2
      P(K,J)=0.
      XMM(K,J)=0.
      PX(K)=0.
      XM(K)=0.
      DPDEL(K)=0.
      DPTH(K)=0.
      DMDEL(K)=0.
      DMTH(K)=0.
      XTH=TH(K)
      CTH=COS(XTH)
      STH=SIN(XTH)
      TANTH=STH/CTH
      XDEL=DEL(K)
      ZDEL=DEL(K)
      CALL XIREME(CRN,FLT2D,HD,KM,XTH,X1D,X2D,XDEL,XLE2D)
      IF(KM)20,20,10
10    P(1,J)=FC
      P(2,J)=0.
      NOLOAD=NOLOAD+1
      GO TO 240
20    X1=X1D
      X2=X2D
      EFL=X1-X2
      IF(EFL.LT..01*XLE2)GO TO 10
      XINC=EFL/18.
      XXX(1,K,J)=X1
      XXX(2,K,J)=X2
```

```
      XH=X1+XINC
      SMINC=1.
      RGTMP=0.
      DO 110 L=1,19
      SM=3.-SMINC
      IF((L.EQ.1).OR.(L.EQ.19))SM=1.
      SMINC=-SMINC
      XH=XH-XINC
      DELX=ZDEL+XH*TANTH
      DDTH=XH/CTH**2
      IF(ABS(XH).LE.FLT2*CTH)GO TO 60
      TEMP=XH-H*STH
      TEMP1=CROWN**2-TEMP**2
      IF(TEMP1)30,30,50
30    IQUIT=1
      WRITE(6,40)IBR,ILOAD,ITER,J,K,L,DFL2,THETA,ZDEL,DELX
40    FORMAT(10HUCYLROL 40,6I6,1P4E12.4)
      RETURN
50    TEMP1=SQRT(TEMP1)
      DELX=TEMP1-H*CTH+ZDEL
      DDTH=TEMP*H*CTH/TEMP1+H*STH
60    IF(DELX.LT.1.E-8)GO TO 110
      TEMP=5.E7*DELX**1.111111/EFL**.111111
      DO 70 LLL=1,20
      A1=SQRT(EL(K)*TEMP*D*(1.+C(K)*GAM))
      A2=1.8864+ALOG(EFL*.5/A1)
      A3=EL(K)*TEMP*A2
      A4=(A3-DELX)/((A2-.5)*EL(K))
      TEMP=TEMP-A4
      IF(TEMP.LE.0.)GO TO 110
      IF(ABS(A3-DELX)-TOL1)90,70,70
70    CONTINUE
      WRITE(6,80)IBR,ILOAD,ITER,IT,J,K,L,TEMP ,DELX ,DFL2
80    FORMAT(10HUCYLROL 80,7I6,1P3E12.4)
      IQUIT=1
      RETURN
90    PX(K)=PX(K)+TEMP*SM
      XM(K)=XM(K)+TEMP*XH*SM
      PDEL=SM/((A2-.5)*EL(K))
```

```
      DPDEL(K)=DPDEL(K)+PDEL
      DPTH(K)=DPTH(K)+DDTH*PDEL
      DMDEL(K)=DMDEL(K)+XH*PDEL
      DMTH(K)=DMTH(K)+XH*DDTH*PDEL
      IF(TEMP-BGTMP)110,110,100
100   BGTMP=TEMP
      DLX(K,J)=DELX
      HERTZ(K,J)=.6366198*TEMP/A1
      BB(K,J)=2.*A1
      XHSAV(K,J)=XH
110   CONTINUE
      IF(PX(K).EQ.0.)GO TO 10
      TEMP=XINC/3.
      PX(K)=PX(K)*TEMP
      XM(K)=XM(K)*TEMP
      DPDEL(K)=DPDEL(K)*TEMP
      DPTH(K)=DPTH(K)*TEMP
      DMDEL(K)=DMDEL(K)*TEMP
      DMTH(K)=DMTH(K)*TEMP
120   CONTINUE
      PS1=-PX(1)+PX(2)+FC
      PS2=XM(1)-XM(2)
      PS1DEL=-DPDEL(1)-DPDEL(2)
      PS1TH=-DPTH(1)-DPTH(2)
      PS2DEL=DMDEL(1)+DMDEL(2)
      PS2TH=DMTH(1)+DMTH(2)
      DET=PS1DEL*PS2TH-PS2DEL*PS1TH
      CORDEL=(PS1*PS2TH-PS2*PS1TH)/DET
      CORTH=(PS1DEL*PS2-PS2DEL*PS1)/DET
      DEL(1)=DEL(1)-CORDEL
      DEL(2)=DELTA-DEL(1)
      TH(1)=TH(1)-CORTH
      TH(2)=THETA-TH(1)
      IF(ABS(CORDEL)-TOL1)130,140,140
130   IF(ABS(CORTH)-TOL2)160,140,140
140   CONTINUE
      WRITE(6,150)IBR,ILOAD,ITER,J,DELTA,THETA,CORDEL,CORTH
150   FORMAT(11HOCYLROL 150,4I6,1P4E12.4)
      IQUIT=1
```

```
      RETURN
160   DO 170 K=1,2
      P(K,J)=PX(K)
      THSAV(K,J)=TH(K)
170   XMM(K,J)=XM(K)
      MM=1
      ER(1)=0.
      ER(2)=0.
      ER(3)=1.
      ER(4)=0.
180   A(1,1)=-DPDEL(1)
      A(1,2)=DPDEL(2)
      A(1,3)=-DPTH(1)
      A(1,4)=DPTH(2)
      A(2,1)=DMDEL(1)
      A(2,2)=-DMDEL(2)
      A(2,3)=DMTH(1)
      A(2,4)=-DMTH(2)
      A(3,1)=1.
      A(3,2)=1.
      A(3,3)=0.
      A(3,4)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(4,4)=1.
      NZ=4
      CALL SIMULT(A,NZ,ER,COR,IQUIT)
      IF(IQUIT)210,210,190
190   WRITE(6,200)IBR,ILOAD,ITER,J,MM,DELTA,THETA
200   FORMAT(11HUCYLROL 200,5I6,1P2E12.4)
      RETURN
210   GO TO(220,230),MM
220   DELDEL=COR(1)
      THDEL=COR(3)
      MM=2
      ER(3)=0.
      ER(4)=1.
      GO TO 180
```

```
230   DELTH=COR(1)
      THTH=COR(3)
      DP1DEL=DPDEL(1)*DELDEL+DPTH(1)*THDEL
      DM1DEL=DMDEL(1)*DELDEL+DMTH(1)*THDEL
      DP1TH=DPDEL(1)*DELTH+DPTH(1)*THTH
      DM1TH=DMDEL(1)*DELTH+DMTH(1)*THTH
240   XF1(2)=XF1(2)+P(1,J)*CPH
      XF1(3)=XF1(3)+P(1,J)*SPH
      XF1(4)=XF1(4)+XMM(1,J)*SPH
      XF1(5)=XF1(5)+XMM(1,J)*CPH
      IF(P(2,J).EQ.0.)RETURN
      DO 250 L=2,5
      ZZ=DP1DEL*DELD1(L)+DP1TH*THD1(L)
      DTV(2,L)=DTV(2,L)+ZZ*CPH
      DTV(3,L)=DTV(3,L)+ZZ*SPH
      ZZ=DM1DEL*DELD1(L)+DM1TH*THD1(L)
      DTV(4,L)=DTV(4,L)+ZZ*SPH
250   DTV(5,L)=DTV(5,L)+ZZ*CPH
      RETURN
      END
      SUBROUTINE CONOUT
      COMMON BB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DROP,DENS,DFL11(5),
     1DELD1(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
     2HERTZ(2,60),IBR,ILOAD,IQUIT,ITER,JPASS,N,NOLOAD,OME,PR(2),PRR,PD,
     3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
     4XLT,XLE,XMASS,XLE2,XF2,XF1(5),XMM(2,60),XXX(2,2,60),XHSAV(2,60),
     5YM(2),IMR
      COMMON ABIG,DELLX,HRZ,XBIG,XXO(2)
      COMMON CRN,FLT2D,HD,XLE2D
      DOUBLE PRECISION  CRN,FLT2D,XLE2D,HD
      EL1=EL(1)
      DELL=.005*D
      DO 50 ITR=1,20
      P1=0.
      P1DEL=0.
      SMC=1.
      X1=SQRT(CROWN**2-(  H-DELL)**2)
      IF(X1.GT.XLE2)X1=XLE2
      EFL=2.*X1
```

```
      XNC=EFL/10.
      XS=X1+XNC
      XYO(1)=Y1
      XYO(2)=-X1
      DO 40 L=1,11
      XS=XS-XNC
      SMM=3.-SMC
      SMC=-SMC
      IF((L.EQ.1).OR.(L.EQ.11))SMM=1.
      DLTX=SQRT(CROWN**2-XS**2)-  H+DELL
      IF(DLTX.LT.1.E-8)GO TO 40
      TEMP=5.E7*DLTX**1.111111/EFL**.111111
      DO 10 IK=1,20
      IF(TEMP.LT.0.)GO TO  40
      A1=SQRT(EL1*D*TEMP*(1.+GAM))
      A2=1.8864+ALOG(EFL*.5/A1)
      A3=EL1*A2*TEMP
      A4=(A3-DLTX)/((A2-.5)*EL1)
      TEMP=TEMP-A4
      IF(ABS(A3-DLTX)-1.E-7)30,10,10
10    CONTINUE
      WRITE(6,20)IBR,ILOAD,DLTX,TEMP,DEL2
20    FORMAT(10HUOUTCNT 20,2I6,1P3E12.4)
      IQUIT=1
      RETURN
30    P1=P1+TEMP*SMM
      PDL=SMM/((A2-.5)*EL1)
      P1DEL=P1DEL+PDL
40    CONTINUE
      P1=.6666667*P1*XNC+TEMP*FLT
      P1DEL=.6666667*P1DEL*XNC+PDL*FLT
      CRR=(P1-FC)/P1DEL
      DELL=DELL-CRR
      IF(ABS(CRR)-1.E-7)70,50,50
50    CONTINUE
      WRITE(6,60)DELL,CRR
60    FORMAT(10HUOUTCNT 60,1P2E12.4)
      IQUIT=1
      RETURN
```

```
70    ABIG=A1*2.
      HRZ=.6366198*TEMP/A1
      DELLX=DELL
      XBIG=0.
      RETURN
      END
      SUBROUTINE OUTPT1
      COMMON BB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DROP,DENS,DFL11(5),
     1DELD1(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
     2HERTZ(2,60),IBR,ILOAD,IQUIT,ITER,JPASS,N,NOLOAD,OME,PR(2),PRR,PD,
     3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
     4XLT,XLE,XMASS,XLE2,XF2,XF1(5),XMM(2,60),XXX(2,2,60),XHSAV(2,60),
     5YM(2),YMR
      COMMON ABIG,DELLX,HRZ,XBIG,XXO(2)
      COMMON CRN,FLT2D,HD,XLE2D
      DOUBLE PRECISION  CRN,FLT2D,XLE2D,HD
      DFL2=DFL2+DFL11(2)
      WRITE(6,10)ILOAD,IBR
10    FORMAT(27H0  OUTPUT DATA FOR LOAD NO.,I3,13H, BEARING NO.,I3/1H0,1
     15X,29HREACTIONS OF BEARING ON SHAFT,21X,50HTOTAL DISPLACEMENTS OF
     2INNER WITH RESPECT TO OUTER/118H   ALONG X     ALONG Y     ALONG Z
     3     ABOUT X     ABOUT Y     ALONG X     ALONG Y     ALONG Z     A
     4BOUT X     ABOUT Y/118H     LB          LB          LB          LB*
     5IN       LB*IN          IN          IN          IN       RADIANS
     6  RADIANS)
      WRITE(6,20)XF1(2),XF1(3),XF1(1),XF1(4),XF1(5),DFL2,DFL11(3),
     1DFL11(4),DFL11(5)
20    FORMAT(1P11E12.4)
      WRITE(6,30)
30    FORMAT(119H0    ROLL        ROLL           CONTACT LOAD          C
     1ONTACT MOMENT        OUTER PATH EXTREMITY    INNER PATH EXTREMITY/1
     217H    NUMBER      AZIMUTH      OUTER       INNER       OUTER
     3 INNER        X(1)        X(2)        X(1)        X(2)/115H
     4           DEG          LB          LB       LB*IN       LB*IN
     5     IN          IN          IN          IN)
      DO 50 J=1,N
      IF(P(2,J).EQ.0.)GO TO 50
      XJ=J
      PHI=360.*(XJ-1.)/XN
```

```
      WRITE(6,40)J,PHI,P(1,J),P(2,J),XMM(1,J),XMM(2,J),XXX(1,1,J),XXX(2,
     11,J),XXX(1,2,J),XXX(2,2,J)
40    FORMAT(I7,5X,1P10E12.4)
50    CONTINUE
      IF(NOLOAD.EQ.0)GO TO 100
      CALL CONOUT
      IF(IQUIT)80,80,60
60    WRITE(6,70)
70    FORMAT(9HOUTPT1 70)
      RETURN
80    TEMP=0.
      WRITE(6,90)FC,TEMP,XXO(1),XXO(2)
90    FORMAT(10H     OTHERS,14X,1P1E12.4,12X,1P1E12.4,12X,1P2E12.4)
100   WRITE(6,110)
110   FORMAT(132H0    ROLL            CONTACT LENGTH       MAXIMUM CONTACT W
     1IDTH  MAX. CONTACT DEFLECTION   MAXIMUM HERTZ STRESS  LOCATION OF
     2MAX. VALUES/129H    NUMBER       OUTER        INNER        OUTER
     3  INNER        OUTER        INNER        OUTER        INNER        OUTE
     4R        INNER/18X,2HIN,10X,2HIN,10X,2HIN,10X,2HIN,10X,2HIN,10X,2HI
     5N,7X,8HLB/IN**2,4X,8HLB/IN**2,7X,2HIN,10X,2HIN)
      DO 120 J=1,N
      IF(P(2,J).EQ.0.)GO TO 120
      TEMP=XXX(1,1,J)-XXX(2,1,J)
      TEMP1=XXX(1,2,J)-XXX(2,2,J)
      WRITE(6,40)J,TEMP,TEMP1,BB(1,J),BB(2,J),DLX(1,J),DLX(2,J),HERTZ(1,
     1J),HERTZ(2,J),XHSAV(1,J),XHSAV(2,J)
120   CONTINUE
      IF(NOLOAD.EQ.0)GO TO 140
      TEMP=XXO(1)-XXO(2)
      WRITE(6,130)TEMP,ABIG,DELLX,HRZ,XBIG
130   FORMAT(10H    OTHERS,1P1E14.4,12X,1P1E12.4,12X,1P1E12.4,12X,1P1E12
     1.4,12X,1P1E12.4)
140   WRITE(6,150)
150   FORMAT(34H0     ROLL      CONTACT MISALIGNMENT/33H    NUMBER      OU
     1TER        INNER/15X,19HRADIANS      RADIANS)
      DO 160 J=1,N
      IF(P(2,J).EQ.0.)GO TO 160
      WRITE(6,40)J,THSAV(1,J),THSAV(2,J)
160   CONTINUE
```

```
      IF(NOLOAD.EQ.0)GO TO 180
      TEMP=0.
      WRITE(6,170)TEMP
170   FORMAT(10H    OTHERS,1P1E14.4)
180   WRITE(6,185)FC
185   FORMAT(32H0  CENTRIFUGAL FORCE OF ROLLER =,1P1E11.4,3H LB)
      WRITE(6,190)
190   FORMAT(65H0  PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISP
     1LACEMENTS)
      WRITE(6,200)
200   FORMAT(59H0    DFX/DX       DFX/DY       DFX/DZ       DFX/DALX     DFX/D
     1ALY/58H    LB/IN        LB/IN        LB/IN        LB/RAD       LB/RAD)
      WRITE(6,20)DTV(2,2),DTV(2,3),DTV(2,1),DTV(2,4),DTV(2,5)
      WRITE(6,210)
210   FORMAT(59H0    DFY/DX       DFY/DY       DFY/DZ       DFY/DALX     DFY/D
     1ALY/58H    LB/IN        LB/IN        LB/IN        LB/RAD       LB/RAD)
      WRITE(6,20)DTV(3,2),DTV(3,3),DTV(3,1),DTV(3,4),DTV(3,5)
      WRITE(6,220)
220   FORMAT(59H0    DFZ/DX       DFZ/DY       DFZ/DZ       DFZ/DALX     DFZ/D
     1ALY/58H    LB/IN        LB/IN        LB/IN        LB/RAD       LB/RAD)
      WRITE(6,20)DTV(1,2),DTV(1,3),DTV(1,1),DTV(1,4),DTV(1,5)
      WRITE(6,230)
230   FORMAT(59H0    DMX/DX       DMX/DY       DMX/DZ       DMX/DALX     DMX/D
     1ALY/59H   LBIN/IN      LBIN/IN      LBIN/IN      LBIN/RAD     LBIN/RAD
     2)
      WRITE(6,20)DTV(4,2),DTV(4,3),DTV(4,1),DTV(4,4),DTV(4,5)
      WRITE(6,240)
240   FORMAT(59H0    DMY/DX       DMY/DY       DMY/DZ       DMY/DALX     DMY/D
     1ALY/59H   LBIN/IN      LBIN/IN      LBIN/IN      LBIN/RAD     LBIN/RAD
     2)
      WRITE(6,20)DTV(5,2),DTV(5,3),DTV(5,1),DTV(5,4),DTV(5,5)
      RETURN
      END
      SUBROUTINE RCALC(CROWN,DROP,FLT02,GAGE,HD,XLE02)
      DOUBLE PRECISION CRD,DRP,DSQRT,FLT2D,GAG,HD,XLE02D
      FLT2D=FLT02
      GAG=GAGE
      XLE02D=XLE02
      IF(CROWN)10,10,20
```

```
10    DRP=DROP
      CRD=DSQRT((((XLEO2D-GAG)**2-FLT2D**2+DRP**2)/(2.D0*DRP))**2+
     1FLT2D**2)
      HD=DSQRT(CRD**2-FLT2D**2)
      CROWN=CRD
      RETURN
20    CRD=CROWN
      HD=DSQRT(CRD**2-FLT2D**2)
      DROP=HD-DSQRT(CRD**2-(ALEO2D-GAG)**2)
      RETURN
      END
      SUBROUTINE SIMULT(AA,N,BB,XX,KX)
      DIMENSION AA(4,4),BB(4),XX(4),KOL(4)
      DOUBLE PRECISION A(4,4),B(4),X(4),ROW(4),AMPY,TEMP
      DO 10 I=1,N
      DO 5 J=1,N
5     A(J,I)=AA(J,I)
10    B(I)=BB(I)
      DO 50 J=1,N
      TEMP=0.D0
      DO 30 K=1,N
      IF(DABS(A(J,K))-TEMP)30,30,20
20    TEMP=DABS(A(J,K))
30    CONTINUE
      DO 40 K=1,N
40    A(J,K)=A(J,K)/TEMP
      B(J)=B(J)/TEMP
50    CONTINUE
5000  KOL(1)=1                                                          SIMU0190
5001  DO 5002  IROW=2,N                                                 SIMU0200
5002  KOL(IROW)=KOL(IROW-1)+1                                           SIMU0210
5004  DO 5025  KOUNT=1,N                                                SIMU0220
      LARGST=N-KOUNT+1                                                  SIMU0230
      IERASE=KOL(1)                                                     SIMU0240
      JCOL=1                                                            SIMU0250
5005  IF (N-KOUNT)  5035,5014,5006                                      SIMU0260
5006     AMPY =DABS ( A(1,1) )                                          SIMU0270
5007  DO 5010  IROW=2,LARGST                                            SIMU0280
5008     IF ( AMPY -DABS ( A(IROW,1))) 5009, 5010, 5010                 SIMU0290
```

```
5009 JCOL=IROW                                                    SIMU0300
        AMPY =DABS ( A(IROW,1) )                                  SIMU0310
     IERASE=KOL(IROW)                                             SIMU0320
5010 CONTINUE                                                     SIMU0330
5011 IF (KOL(1)-IERASE)  5012,5014,5012                           SIMU0340
5012 KOL(JCOL)=KOL(1)                                             SIMU0350
     KOL(1)=IERASE                                                SIMU0360
5014 IF(A(JCOL,1))5015,5035,5015                                  SIMU0370
5015 AMPY=A(JCOL,1)                                               SIMU0380
5017 DO 5018  IROW=2,N                                            SIMU0390
     ROW(IROW-1)=A(JCOL,IROW)/AMPY                                SIMU0400
5018 A(JCOL,IROW-1)=A(1,IROW-1)                                   SIMU0410
     ROW(N)=1.D0/AMPY
     A(JCOL,N)=A(1,N)                                             SIMU0430
5019 DO 5022  IROW=2,N                                            SIMU0440
     AMPY=A(IROW,1)                                               SIMU0450
5020 DO 5021  JCOL=2,N                                            SIMU0460
5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(JCOL-1)               SIMU0470
5022 A(IROW-1,N)=-AMPY*ROW(N)                                     SIMU0480
5023 DO 5024  JCOL=1,N                                            SIMU0490
     KOL(JCOL)=KOL(JCOL+1)                                        SIMU0500
5024 A(N,JCOL)=ROW(JCOL)                                          SIMU0510
5025 KOL(N)=IERASE                                                SIMU0520
5026 DO 5034 KOUNT=1,N                                            SIMU0530
5027 IF (KOL(KOUNT)-KOUNT)  5035,5034,5028                        SIMU0540
5028 DO 5032  IROW=KOUNT,N                                        SIMU0550
5029 IF (KOL(IROW)-KOUNT)  5035,5030,5032                         SIMU0560
5030 DO 5031  JCOL=1,N                                            SIMU0570
     ROW(1)=A(JCOL,IROW)                                          SIMU0580
     A(JCOL,IROW)=A(JCOL,KOUNT)                                   SIMU0590
5031 A(JCOL,KOUNT)=ROW(1)                                         SIMU0600
     IERASE=KOL(KOUNT)                                            SIMU0610
     KOL(KOUNT)=KOL(IROW)                                         SIMU0620
     KOL(IROW)=IERASE                                             SIMU0630
     GO TO 5034                                                   SIMU0640
5032 CONTINUE                                                     SIMU0650
     GO TO 5035                                                   SIMU0660
5034 CONTINUE                                                     SIMU0670
997  IF(KX-3)998,9000,998                                         SIMU0680
```

```
9000 KY=0                                                           SIMU0690
     RETURN                                                         SIMU0700
998   DO 5042 IROW=1,N                                              SIMU0710
5040 X(IROW)=0.D0                                                   SIMU0720
5041 DO  5043 JKOL=1,N
5043 X(IROW)=X(IROW)+A(IROW,JKOL)*B(JKOL)
5042 XY(IROW)=X(IROW)
     KY=0                                                           SIMU0750
     RETURN                                                         SIMU0780
5035 KY=1                                                           SIMU0790
     RETURN                                                         SIMU0800
     END                                                            SIMU0810
     SUBROUTINE XTREME(CRN,FLT2D,HD,KM,XTH,X1D,X2D,XDEL,XLE2D)
     DOUBLE PRECISION CRN,CTH,DSIN,DCOS,DSQRT,DNABLA,FLT2D,HD,STH,TEMP,
    1XTH,X1D,X2D,XDEL,XLE2D,XTHAB,XSTAR1,XSTAR2
     XTHAB=DABS(XTH)
     KM=0
     STH=DSIN(XTHAB)
     CTH=DCOS(XTHAB)
     TEMP=CRN**2-(HD*CTH-XDEL)**2
     IF(TEMP)10,10,20
10   KM=1
     RETURN
20   TEMP=DSQRT(TEMP)
     X1D=TEMP+HD*STH
     X2D=-TEMP+HD*STH
     DNABLA=HD-DSQRT(CRN**2-XLE2D**2)
     XSTAR1=XLE2D*CTH+DNABLA*STH
     XSTAR2=-XLE2D*CTH+DNABLA*STH
     IF(X2D.GT.XSTAR1)GO TO 10
     IF(X1D.LE.FLT2D*CTH)GO TO 10
     IF(X1D.GT.XSTAR1)X1D=XSTAR1
     IF(X2D.LT.XSTAR2)X2D=XSTAR2
     IF(XTH.EQ.0.D0)RETURN
     IF((X2D.GT.-FLT2D*CTH).AND.(X2D.LT.FLT2D*CTH))X2D=-XDEL*CTH/STH
     IF(XTH.GE.0.D0)RETURN
     TEMP=-X1D
     X1D=-X2D
     X2D=TEMP
```

REFERENCES

1. Lewis, P. and Malanoski, S.B., "Rotor Bearing Dynamics Design Technology. Part IV: Ball Bearing Design Data Technical Report," AFAPL-TR-65-45, Part IV. Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio.

2. Mauriello, J.A., LaGasse, and Jones, A.B., "Rolling Element Bearing Retainer Analysis," DAAJ02-69-C-0080, TR105.7.10, USAAMRDL-TR-72-45.

3. Crecelius, W.T. and Pirvics, J., "Computer Program Operation Manual on "SHABERTH" a Computer Program for the Analysis of the Steady State and Transient Thermal Performance of Shaft Bearing Systems," AFAPL-TR-76-90, Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio, October 1976.

4. Jones, A.B. and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide -- Part II: Ball Bearings," AFAPL-TR-78-6, Part II, February 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

5. Pan, C.H.T., Wu, E.R., and Krauter, A.I., "Rotor Bearing Dynamics Technology Design Guide: Part I, Flexible Rotor Dynamics," AFAPL-TR-78-6, Part I, June 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

6. Jones, A.B. and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide - Part III: Tapered Roller Bearings," AFAPL-TR-78-6, Part III, February 1979, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

7. Lundberg, G., "Elastische Berührung Zweier Halbräume," Forschung auf dem Gebiete des Ingenieurwesens, September/October, 1939.

☆U.S.Government Printing Office: 1980 — 657-084/510