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USING PERSONNEL DISTRIBUTION MODELS

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INTRODUCTION

Navy policy provides for the periodic rotation of enlisted personnel between sea, shore and neutral duty types. Rotation affects the development of skills, the maintenance of morale and the retention of personnel. Rotation patterns also determine the distribution of personnel between duty types.

The objective of the Navy's rotation policy is to provide sufficient opportunity for enlisted personnel to serve ashore, but at the same time have enough personnel rotating to sea to maintain established sea manning levels. Rotation which promotes morale and retention, however, may not produce acceptable sea/shore distributions. Consequently, rotation policies must be determined which provide a compromise between these conflicting considerations.

The purpose of this paper is to describe models to assist personnel managers in making these policy decisions. Although developed specifically for incorporation in the Navy manpower management system, the models should be applicable -- with appropriate modifications -- to a variety of considerations involving the periodic
rotation of an inventory. Two models were developed. One is an aggregate model based on a simplified force structure and steady-state assumption; the other is an expanded model based on a more detailed dynamic simulation of personnel flows in a rating or detailing community. While the steady-state assumption is overly restrictive, the aggregate model provides a more comprehensive understanding of the factors influencing rotation than the larger simulation model.

Examples of the type of policy recommendations which these models provide are presented in the context of their application to the Navy. References 1 and 2 describe the operation of the computerized versions of the models.

THE AGGREGATE MODEL

The total inventory is divided into four categories C_{ij} (1 \leq i,j \leq 2). During each time period, stock flows from one category to another along the network described in figure 1.
Flows labeled $f_1$, $f_2$, $f_3$ and $f_4$ represent rotation while flows labeled $l_1$, $l_2$, represent losses to the system. The flows $a_1$ and $a_2$ are accessions. When total flows into and out of each category are equal, the inventory distribution remains constant. This will be called a steady-state system.

Let $N_{ij}$ denote the quantity of stock in category $C_{ij}$. Losses are represented in terms of two parameters $(r,c)$. The first is the ratio of the per-period flow of stock from $C_{11} \cup C_{12}$ to the total amount of stock in this category; thus the per-period loss is given by $(1-r)(N_{11}+N_{12})$. The second is the proportion of the stock in $C_{21} \cup C_{22}$ which is retained for one additional period. The rotation pattern is denoted by $a:b$, where a
is the number of periods stock remains in $C_{21}$, and $b$ is the number of periods it remains in $C_{22}$. Rotation and continuation behavior determine all the flows except for $a_1$ and $a_2$. Before giving the mathematical description of the steady-state system, we shall indicate how this formulation applies to the Navy.

The enlisted force is divided into two experience categories: (1) first-term personnel are those who have completed preliminary training and are serving their first sea or shore tour, and (2) career personnel are those who have completed their first duty tour. At the end of their first tour, personnel who remain in the Navy are assigned to the opposite duty type. Thereafter, as long as they remain, they are rotated at the end of each tour. The first-term transition behavior, $r$, is the ratio of those who remain in the Navy beyond their first tour to the total first-term force. The career continuation rate, $c$, is the proportion of career personnel who are retained each period. In this simplified formulation, the requirement that positions be filled by personnel with a specified experience level is ignored. However, the model has been extended to incorporate experience levels in the career force (reference 3).
In order to avoid trivial considerations, we shall always assume that $N_{21}$ and $N_{22}$ are non-zero, and that $0 < r, c < 1$. If we assume that $a_i = f_i$ (for $i = 1, 2$), the conditions for a steady-state system can be represented algebraically by any two of the three equations

\begin{align*}
    f_1 + f_2 &= l_1 + l_2 \\
    f_1 + f_4 &= f_3 + l_2 \\
    f_2 + f_3 &= f_4 + l_1
\end{align*}

If the per-period flow into one of the categories $C_{21}$ or $C_{22}$ is $f$, and the tour length for that category is $b$ periods, then the per-period flow of stock to the other category is $c^b f$ and the losses are $(1-c^b)f$.

Applying this argument to both categories gives

\begin{align*}
    f_4 &= c^a(f_2 + f_3) = c^a(f_4 + l_1) \\
    f_3 &= c^b(f_1 + f_4) = c^b(f_3 + l_2)
\end{align*}

These equations allow us to eliminate $f_3$ and $f_4$ in (1) to obtain the equations

\begin{align*}
    f_1 + f_2 - l_1 - l_2 &= 0
\end{align*}
\[ f_1 + \left( \frac{c^a}{1-c} \right) l_3 - \left( \frac{1}{1-c} \right) l_2 = 0 \]
\[ f_2 + \left( \frac{c^b}{1-c} \right) l_4 - \left( \frac{1}{1-c} \right) l_1 = 0 \]

Substituting
\[ l_1 = (1-c)N_{21} \]
\[ l_2 = (1-c)N_{22} \]
\[ f_1 = rN_{11} \]
\[ f_2 = rN_{12} \]

gives necessary and sufficient conditions for a steady-state system in terms of \( a, b, c, r, \) and the \( N_{ij}. \)

We define the model by the first two of the equations thus obtained:

(2) \( r(N_{11} + N_{12}) - (1-c)(N_{21} + N_{22}) = 0 \)
(3) \( rN_{11} + c^a \left( \frac{1-c}{1-c^a} \right) N_{21} - \left( \frac{1-c}{1-c^b} \right) N_{22} = 0 \)

The apparent asymmetry is due to the fact that only two of the three equations in (1) are independent. This model has been applied in reference 4 to answer specific Navy questions. Some of the conclusions will be discussed later.

\textsuperscript{1}This derivation is due to Huntzinger (reference 2).
When a:b and (r,c) are given, equations (2) and (3) are linear; and it is not difficult to see that if any two of the components $N_{ij}$ are known, the remaining ones can be computed. If the quantities $N_{ij}$ are prescribed, another linear aspect of the system is equation (2), which defines the connection between $r$ and $c$. Therefore, the analysis of the system is reduced to studying the relation between rotation and continuation. This relationship, expressed in (3), is a little more difficult to deal with.

Assume that the $N_{ij}$ are given. We first eliminate $r$ by multiplying (2) by $N_{11}$ and (3) by $N_{11}+N_{12}$. Since $c \neq 1$, we can subtract the results and cancel $1-c$ to obtain the equation

$$(N_{21}+N_{22})N_{11} = (N_{11}+N_{12}) \left( \left( \frac{1}{1-c} \right) N_{22} - \left( \frac{c^a}{1-c} \right) N_{21} \right).$$

Now set

$$d = N_{22}N_{11} - N_{21}N_{12},$$
$$e = N_{21}N_{12} + N_{22}N_{11},$$
$$f = N_{21}N_{11} + N_{22}N_{11},$$
$$g = N_{22}N_{12} - N_{21}N_{11}. $$

Then an equivalent condition is

$$(4) \quad f_{a:b}(c) = 0. $$
where
\[ f_{a:b}(x) = d x^{a+b} + e x^a - f x^b - q. \]

For a given rotation pattern, \( a:b \), the solutions of (4) are the continuation rates which provide a steady-state system if \( r \) is defined by (2).

Our first theorem describes the solutions of (4). In order to further simplify notion, let
\[ h = \begin{cases} 
q/e & \text{if } q > 0 \\
-g/e & \text{if } q < 0 
\end{cases} . \]

Theorem 1. (i) Suppose \( g > 0 \). Then \( f_{a:b}(x) = 0 \) has a solution \( c \) with \( 0 < c < 1 \) if and only if \( a/b > N_{21}/N_{22} \). In this case \( \sqrt{h} c < 1 \). Conversely, if \( c \) is in this range, there is a unique \( b \) for which \( f_{a:b}(c) = 0 \).

(ii) Suppose \( g < 0 \). Then \( f_{a:b}(x) = 0 \) has a solution \( c \) with \( 0 < c < 1 \) if and only if \( a/b < N_{21}/N_{22} \). In this case \( \sqrt{h} c < 1 \). Conversely, if \( c \) is in this range, then there is a unique \( a \) such that \( f_{a:b}(c) = 0 \).

(iii) Suppose \( g = 0 \), but \( d \neq 0 \). Then \( f_{a:b}(x) = 0 \) has a solution \( c \) with \( 0 < c < 1 \) if and only if \( a/b \) is strictly between 1 and \( N_{21}/N_{22} \).
(iv) If \( d = g = 0 \), there is no solution unless \( a = b \). However, \( f_{a:a}(x) = 0 \) identically.

Proof: There is no loss in generality if we assume \( g > 0 \), since the other case may be derived by using the symmetry of the model to replace the vector \((a, b, N_{11}', N_{12}', N_{21}', N_{22}')\) by \((b, a, N_{11}', N_{12}', N_{21}', N_{22}')\).

Equation (4) is clearly equivalent to the simultaneous system

\[
\begin{align*}
y &= \frac{(f + g)}{(dx + e)} \\
y &= \frac{x^a}{b} \\
x &= \frac{c}{b}
\end{align*}
\]

The first equation in (5), which is independent of the continuation rate \( c \), is a hyperbola which degenerates to a straight line when \( d = 0 \). Otherwise its asymptotes are \( x = -e/d \) and \( y = f/d \). It is concave up if \( d < 0 \) and concave down if \( d > 0 \), and it always passes through the two points \((1,1)\) and \((0,h)\), since \( d + e = f + g \). When \( d = g = 0 \), the hyperbola degenerates to the line \( y = x \), and we shall say the system is degenerate. Using implicit differentiation, we have

\[
(dx+e) \frac{dy}{dx} = f - dy.
\]

Hence at the point \((1,1)\) \( dy/dx = N_{21}/N_{22} \).
The second equation represents a convex curve, and increases monotonically for $0 < x < 1$. Its derivative at the point $(1,1)$ is $a/b$. By convexity, these two arcs can have at most one intersection point, and if $g > 0$ it will occur if and only if $a/b < N_{21}/N_{22}$. Since the $y$-coordinate of this point must satisfy $h < y < 1$, we have the restriction $\sqrt{h} < c < 1$. On the other hand, if $c$ satisfies this inequality, there is an intersection point, for as $b$ tends to $+\infty$, the intersection point moves from $(1,1)$ to $(0,h)$. Clearly, the strict inequality is necessary.

Suppose that $g = 0$, but the system is non-degenerate. If $N_{21}/N_{22} > 1$, then $d > 0$ so the hyperbola is concave up. The second curve, $y = x^{a/b}$, is concave down if $a/b < 1$, and in this case there is no non-trivial intersection. If $1 < a/b < N_{21}/N_{22}$, however, the curve lies between $y = x$ and the hyperbola for values of $x$ near 1. Since the hyperbola has a positive derivative, and the derivative of $y = x^{a/b}$ is zero at $x=0$, the two curves must have exactly one non-trivial intersection. Similarly, if $N_{21}/N_{22} < 1$, an intersection occurs if and only if $N_{21}/N_{22} < a/b < 1$. 

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Finally, suppose \( d = g = 0 \). Then the hyperbola reduces to \( y = x \), and no non-trivial intersection occurs unless \( a = b \). But if \( a = b \), the two curves coincide. The proof is complete.

Theorem 1 provides an interpretation for \( h \). Since stocks would not generally be rotated after less than one period, we may assume that \( a, b \geq 1 \). Then the theorem shows that a steady-state system is possible if and only if the continuation rate, \( c \), satisfies \( h < c < 1 \). However, even if this condition is satisfied, the corresponding rotation patterns may not be desirable. Therefore practical considerations will further restrict the feasible range for \( c \).

The theorem also provides a method for computing rotation patterns. Fractional values are possible. For example, a solution of the form \( a = 11/4, b = 13/4 \) is interpreted as a quarterly rotation pattern 11:13 associated with a quarterly continuation rate \( 4\sqrt{c} \).

The results obtained so far show that policies which alter the distribution of stocks, continuation rates or rotation patterns affect every aspect of the system. In order to get a better understanding of these effects, consider the Navy example again. Seas billets must be
filled. However, since this duty is arduous, and involves separation from normal community ties, long sea tours have a negative effect on morale and retention. On the other hand, short tours involve significant increases in personnel moving costs. Table 12 gives the number of shore billets required for some common rotation patterns when the sea billet structure is fixed at its present level. Rotation has been expressed in quarters. A 12:12 rotation pattern is desired; but, as the data indicates, it would require 172,509 shore billets. Currently there are 87,708 shore billets. Therefore the Navy would need 85,801 additional shore billets. A change of this magnitude is not feasible. Apart from economic considerations, the Navy is a sea-going force; historically the number of sea billets has far exceeded the number of shore billets.

\[ A \text{ steady-state system with an } a:a \text{ rotation pattern does not necessarily require } N_{11} + N_{21} = N_{12} + N_{22}. \] In fact, the 12:12 rotation corresponds to a shore total of 179,509 and a sea total of 219,447.

\[ \text{Reference 4.} \]
Table 1: Shore Billet Requirements for Current Sea Billet Structure

<table>
<thead>
<tr>
<th>Rotation a:b</th>
<th>First-term shore (N_{12})</th>
<th>Career shore (N_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:8</td>
<td>54,827</td>
<td>58,662</td>
</tr>
<tr>
<td>12:12</td>
<td>90,221</td>
<td>82,288</td>
</tr>
<tr>
<td>13:8</td>
<td>49,920</td>
<td>55,387</td>
</tr>
<tr>
<td>14:9</td>
<td>54,088</td>
<td>58,169</td>
</tr>
<tr>
<td>16:8</td>
<td>38,942</td>
<td>48,059</td>
</tr>
<tr>
<td>17:9</td>
<td>43,508</td>
<td>51,107</td>
</tr>
<tr>
<td>20:8</td>
<td>29,538</td>
<td>41,782</td>
</tr>
<tr>
<td>20:12</td>
<td>54,748</td>
<td>58,610</td>
</tr>
</tbody>
</table>

Since the present personnel inventory distribution is:

Shore

\[ N_{11} = 35,550 \]
\[ N_{21} = 51,158 \]

Sea

\[ N_{12} = 144,835 \]
\[ N_{22} = 74,612 \],

from table 1, we see that the steady-state rotation pattern is more nearly 16:8. Thus, the Navy must adjust to sea-tours which are longer than desired.

Fluctuations in continuation, due either to Navy policy decisions or external factors, can affect the stability of the system. The present continuation behavior is
\( r = 0.0935 \) and \( c = 0.8600 \). Table 2 shows how shore billet requirements for various rotation patterns are affected by projected changes in continuation. Changes of the magnitude indicated clearly have important rotation consequences. If continuation drops to the low estimate, the 16:8 rotation pattern would require 37,696 more shore billets; a steady-state system could also be attained by adding 20,573 shore billets and changing to an 20:8 rotation pattern. But 5-years at sea could cause further decreases in retention.

Table 2: Changes in Career Continuation

<table>
<thead>
<tr>
<th>Rotation pattern (quarters)</th>
<th>Shore billets needed for rotation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Current</strong></td>
<td><strong>Low</strong></td>
<td><strong>Medium</strong></td>
<td><strong>High</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( C = 0.860 )</td>
<td>( C = 0.824 )</td>
<td>( C = 0.857 )</td>
<td>( C = 0.878 )</td>
<td></td>
</tr>
<tr>
<td>12:12</td>
<td>172,509</td>
<td>213,575</td>
<td>176,068</td>
<td>150,592</td>
<td></td>
</tr>
<tr>
<td>14:12</td>
<td>151,223</td>
<td>190,489</td>
<td>154,619</td>
<td>130,341</td>
<td></td>
</tr>
<tr>
<td>16:12</td>
<td>135,353</td>
<td>173,336</td>
<td>138,631</td>
<td>115,221</td>
<td></td>
</tr>
<tr>
<td>12:8</td>
<td>113,489</td>
<td>153,559</td>
<td>116,915</td>
<td>92,574</td>
<td></td>
</tr>
<tr>
<td>17:10</td>
<td>106,436</td>
<td>143,885</td>
<td>109,645</td>
<td>86,810</td>
<td></td>
</tr>
<tr>
<td>18:10</td>
<td>101,475</td>
<td>138,508</td>
<td>104,645</td>
<td>82,093</td>
<td></td>
</tr>
<tr>
<td>16:8</td>
<td>87,001</td>
<td>124,404</td>
<td>90,190</td>
<td>67,566</td>
<td></td>
</tr>
<tr>
<td>17:8</td>
<td>82,365</td>
<td>119,327</td>
<td>85,515</td>
<td>63,181</td>
<td></td>
</tr>
<tr>
<td>20:8</td>
<td>71,320</td>
<td>107,281</td>
<td>74,378</td>
<td>52,714</td>
<td></td>
</tr>
</tbody>
</table>
It is clear from this example that the effect of continuation change is critical to an understanding of rotation dynamics. We shall now give an analytic formulation of the relationship between continuation and rotation. Again assume $g > 0$.

If $y = a(x)$ is the equation of the hyperbola in (5), then the equations

$$a = \frac{\log(a(x))}{\log(c)}, \quad b = \frac{\log(x)}{\log(c)} \quad 0 < x \leq 1,$$

define a functional relation $b = \phi_c(a)$ between the components of steady-state rotation patterns. The parameter, $x$, varies from 0 to 1 as the corresponding point $(a,b)$ moves toward the origin. Examples of such rotation curves are illustrated in figure 2.
Clearly, as $x \to 0$, we have $a \to \frac{\log(h)}{\log(c)}$ and $b \to +\infty$. Therefore, these curves have asymptotes $a_o = \frac{\log(h)}{\log(c)}$. Moreover, the slope at $(a,b)$ is given in terms of the parameter $x$ by

$$u(x) = \frac{db}{dx} / \frac{da}{dx} = \frac{\alpha(x)}{x\alpha'(x)}.$$ 

Note that $u$ is a function of $x$ alone. In order to analyze the dependence of these rotation curves on continuation, we first determine the sign of $\frac{du}{dx}$ as a function of $x$. 

Figure 2: Rotation Curves for $d > g > 0$. 

\[ a_o = \frac{\log(h)}{\log(c)} \]

\[ \frac{\log(h)}{\log(c')} \]
Lemma. Suppose $g > 0$. Then if $g > d$, $\text{sgn} \left( \frac{du}{dx} \right) = -1$ on $0 < x < 1$. Otherwise, $\frac{du}{dx}$ is negative for $x$ near zero, and changes sign exactly once as $x$ approaches 1.

Proof. We have

$$\frac{du}{dx} = \frac{x(a'(x))^2 - a(x)(xa''(x) + a'(x))}{(xa'(x))^2}$$

Suppose $d \neq 0$, then $y = a(x)$ is a hyperbola whose asymptotes intersect at the point $(-e/d, f/d)$ and are parallel to the coordinate axes.

Let $r = e/d$, $s = f/d$, $t = (dg-ef)/d^2$, and set

$$\sigma(z) = rt^2 + 2rstz - stz^2.$$ 

The transformation

$$x_1 = x+r, \quad y_1 = y-s$$

reduces the hyperbola to $y_1 = \beta(x_1) = t/x_1$. Clearly, for all $n$-th derivatives ($n>0$), we have $a^{(n)}(x) = \beta^{(n)}(x)$. If this substitution is made in the expression for $du/dx$, we have (after some calculation) $\text{sgn}(du/dx) = \text{sgn}(\sigma(x_1))$. If $d = 0$, then $y = a(x)$ reduces to a straight line, and a similar argument gives $\text{sgn}(du/dx) = -\text{sgn}(g) = -1$. 

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Now, again suppose \( d \neq 0 \). As \( x \) varies between 0 and 1, \( x_1 \) varies between \( r \) and \( r+1 \). Therefore we must determine the sign of the parabola \( \sigma(z) \) on \( r < z < r+1 \). Since the critical point occurs at \( z=r \), the curve is either strictly increasing or decreasing on \( r < z < r+1 \). Furthermore, \( \sigma(r) = rtg/d \), and \( \sigma(r+1) = t(eg-df)/d^2 \).

We can easily show that \( t < 0 \). For \( d+e = f+g \), and so \( dg+eq = fg+g^2 \). If \( t > 0 \), we would have \( (f+g)e < (f+g)g \). But \( f+g > 0 \), so the last inequality would imply \( e < g \). However, \( e > g \) by definition, and the case \( e = g \) is excluded since \( g > 0 \). A similar argument shows that \( eg-df < 0 \) if and only if \( g < d \).

Suppose \( d > g \). Then according to the preceding remarks, \( \sigma(r) < 0 \) and \( \sigma(r+1) > 0 \). This establishes the second case. Similar arguments complete the proof in case (1).

The lemma implies that when \( g > d \), \( \phi_c \) is concave up; however, if \( d > g \), there is one inflexion point at which the concavity changes. The second case is illustrated in figure 2.

The lemma can be used to compare rotation curves for continuation rates \( c < c' \).
Theorem 2. Suppose $g>0$, and $c<c'$. 

(i) If $g>d$, then $\phi_c(a) > \phi_{c'}(a)$ for $0<a<a_o$. 

(ii) If $d>g$, then there is a point $a^*$ between $0$ and $a_o$ such that $\phi_c(a) < \phi_{c'}(a)$ if $0<a<a^*$, and $\phi_c(a) > \phi_{c'}(a)$ if $a^*<a<a_o$.

The Navy example corresponds to (i). In this case, theorem 2 implies that a decrease in continuation will **always** require an increase in the sea/shore ratio for a steady-state system (table 2 gives empirical data on the magnitude of these alterations). This is not true in case (ii). If $b$ is fixed at a value less than $\phi_{c'}(a^*)$, a decrease in continuation from $c'$ to $c$ will require an increase in the $a$-component of rotation, but if $b$ is fixed at a value greater than $\phi_{c'}(a^*)$, the opposite is true.

The analogue of theorem 2 for $g<0$ can be obtained by symmetry.

Proof of theorem 2. Suppose $d>g$. For any $b>0$, let $x$ and $x'$ be chosen so that 

$$b = \frac{\log(x)}{\log(c)} = \frac{\log(x')}{\log(c')}$$

then $x<x'$. The slopes of $\phi_c$ at $(a,b)$ and $\phi_{c'}$ at $(a',b)$
are, respectively, u(x) and u(x'). Therefore if b is sufficiently close to 0, \( \frac{du}{dx} \) will be negative on an interval containing x and x', and consequently \( \phi_c(a) > \phi_c'(a) \) for all a sufficiently close to the origin.

However, the asymptote of \( \phi_c \), is to the right of that of \( \phi_c' \), and so there must be a point where the two curves cross. Let \( a^* \) be the a-coordinate of that point. Since \( \frac{du}{dx} \) changes sign only once, this point must be unique.

The case \( g>d \) is treated similarly.

Remark: Some of these results can be extended to include \( g=0 \). In the non-degenerate case with \( g=0 \), the curve \( \phi_c \) has no asymptote, however, it must be bounded by the two rays with slopes 1 and \( \frac{N_{22}}{N_{21}} \).

For a specific distribution of stocks \( N_{ij} \), the quantities \( \frac{N_{22}}{N_{21}} \), d, e, f, g and h can be calculated. Together with theorems 1 and 2, these provide insight into the specific rotation requirements of the system.
THE EXPANDED MODEL

The foregoing analysis suggests that maintaining a fixed inventory size and distribution in a steady-state system imposes severe constraints on rotation patterns, since they are so sensitive to changes in continuation. Continuation is affected both by policy decisions and factors beyond management control. Therefore, if a fixed rotation pattern is maintained over a period of time, the system response to these fluctuations in continuation will be an unstable inventory structure.

In order to extend management control, a generalized rotation pattern which varies from period-to-period and which distributes rotating stocks among several possible tour lengths is suggested. This approach was taken in reference 1, where a dynamic, real-time simulation model was developed and applied to the problem of determining a rotation pattern which would produce a pre-determined inventory distribution based on future expected continuation behavior.

This expanded model is a network flow model in which the nodes represent the inventory classified by paygrade (or experience level), length of service (LOS) and rotation date. The variable flow paths are defined by the rotation pattern; and the magnitude of flows along these
paths are determined by continuation rates, promotion probabilities and the rotation patterns. The user defines the number of experience levels, LOS categories and the maximum time spent in each of the three rotation categories (corresponding to sea shore and neutral duty types). For a more detailed description, the reader may consult reference 1. Our purpose here is to present an example which illustrates how the model can be used to find rotation patterns producing desired inventory distributions.

Table 3 shows a 5-year projection for a paygrade in a sample rating community using the prescribed rotation pattern of 48 months at sea and 24 months ashore.\(^4\) This projection indicates that if the anticipated continuation behavior is realized, the Navy will have to purchase 100 additional shore billets for this rating over the next five years. In order to avoid this

\(^4\)There is no prescribed tour length for neutral duty since it consists of only a small portion of the inventory.
Table 3: Paygrade 3 Inventory Projection

<table>
<thead>
<tr>
<th>Year</th>
<th>Sea</th>
<th>Shore</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,301</td>
<td>788</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1,416</td>
<td>846</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1,447</td>
<td>872</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1,454</td>
<td>867</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1,447</td>
<td>875</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1,421</td>
<td>900</td>
<td>4</td>
</tr>
</tbody>
</table>

expense, and to further increase sea manning levels, the expanded model was used to determine a new rotation pattern that would produce a stable shore total near the present level of 788. The results, shown below, illustrate the suggested type of generalized rotation pattern.

<table>
<thead>
<tr>
<th>Years</th>
<th>Tour lengths (in months)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>.21</td>
</tr>
<tr>
<td>2</td>
<td>.31</td>
</tr>
<tr>
<td>3</td>
<td>.79</td>
</tr>
</tbody>
</table>
The columns on the right give the percentage of the sea force rotating to shore at each future year who receive the indicated tour length of 18 or 21 months. The remaining 1 percent are rotated to neutral duty.

The new inventory projection using this generalized pattern is shown in table 4. Note that the shore inventory has been stabilized at the third year.

Table 4: New Inventory Projection

<table>
<thead>
<tr>
<th>Years</th>
<th>Sea</th>
<th>Shore</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,301</td>
<td>788</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1,416</td>
<td>844</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1,487</td>
<td>827</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1,528</td>
<td>787</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1,530</td>
<td>782</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1,528</td>
<td>781</td>
<td>4</td>
</tr>
</tbody>
</table>

A detailed derivation of this rotation pattern is presented in reference 1. This "solution" is not unique, and it was not obtained by an optimization procedure. Since similar rotation models incorporate least-cost or allocation algorithms (e.g., reference 5), it is appropriate to amplify our rationale for use of the expanded rotation model.
The initial projection did not satisfy the community planner's criteria for a satisfactory solution. These criteria reflect his expectations and desires; and while some of these are objective and quantifiable others are not. Moreover, the relative importance of his criteria may shift from time to time; e.g., least-cost solutions may not be as attractive in a crisis environment as they would be in a stable period. In our illustration, the planner wished to develop a rotation pattern which would re-distribute the personnel inventory so that shore billets would remain at this current level, but which would not further extend an already long sea tour and which would not increase the need for neutral billets. A different set of criteria would have produced another rotation pattern. For instance, extending sea tours or increasing the percent of the rotating force sent to neutral would attenuate the progressively shortened shore tour characteristic of the original solution. Planners must also be able to assess the impact of policy alternatives before implementation (e.g., longer sea tours could produce declining continuation rates, and this would have important implications for future rotation and personnel distributions).

In view of these considerations, we advocate an approach similar to that suggested by Brill in reference 6. The
models should be used as planning tools for defining rotation patterns compatible with management policy and expected continuation behavior. If these change, the model should be flexible enough to allow the planner to alter his strategy. Also, the models should be used to generate alternative solutions and analyze the consequences of each. In this way the planner gains a qualitative understanding of the relationship and constraints involved. This increased insight can, in turn, be used to operate the models more effectively. The process can be enhanced, for example, by using the two models together or by using small scale optimization models at various stages to help in the analysis. These secondary models can be developed by the user to suit his particular needs.

The expanded model was developed for interactive real-time use at a terminal. The community manager can ask "what if" questions and receive an almost immediate answer. The solution for our example was obtained in this way by a series of inventory projections alternated with selective modification of the initial 24:48 rotation pattern. The modifications were determined from information generated by the model indicating the effect of changes in a given direction and magnitude.
Thus as the planner's criteria change, his strategy for using the various subroutines can change accordingly.

SUMMARY
The aggregate model provides analytical insight into the rotation process in the steady-state. It shows that an inventory distribution is sensitive to the interrelation between rotation and continuation. Because fluctuations in continuation are due to many factors beyond management control, a long-term stable inventory distribution will require a generalized rotation pattern which allows controlled variation in tour lengths, and which can be altered from period to period in response to expected changes in continuation.

The expanded model can be used as a tool to help the analyst define these generalized rotation patterns. It provides a useful method for stabilizing desired distribution of stock, or for gradually changing the distribution. Other appropriate models can be used in conjunction with the rotation models to assist the analyst at various stages in this process.
REFERENCES


CNA Professional Papers — 1973 to Present

PP 103

PP 104

PP 116

PP 117

PP 106

PP 109

PP 107
Stoloff, Peter H., "Ratering Factor Analytically Derived Measures to Exogenous Variables," 17 pp., Mar 1973, AD 758 820

PP 108

PP 109

PP 110

PP 111

PP 112
Ginsberg, Lawrence H., "ELF Atmosphere Noise Level Statistics for Project SANGUINE," 29 pp., Apr 1974, AD 776 969

PP 113
Ginsberg, Lawrence H., "Propagation Anomalies During Project SANGUINE Experiments," 5 pp., Apr 1974, AD 786 968

PP 114
Maloney, Arthur P., "Job Satisfaction and Job Turnover," 41 pp., Jul 1973, AD 786 410

PP 115

PP 116

PP 117

*University of Florida

Research supported in part under Office of Naval Research Contract N00014-66-0273 (017)

PP 118

PP 119

PP 120

"Economics: North Carolina State University

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PP 125

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