From a design standpoint, the Hydrographic surveying and charting system is a complex operation involving a great number of interacting factors. Design considerations of almost any one aspect of the sub-system—navigation, data transmission, strategy, sounding craft, depth sensor, environmental conditions, ship—soon involve basic questions about the others. The effort outlined in this research note is the author's attempt to try to isolate some aspects of the problem of equipment specifications for the sounding craft and their echo-sounding sensor.

The main approach pursued in this note grew out of a study by Prof. Philip Mandel of the MIT Department of Naval Architecture and Marine Engineering. This was described in an internal memorandum dated 2 August 1967. The trial results presented in parts 1 and 2 of Section IV are taken directly from Prof. Mandel's memorandum.
EXPLANATORY NOTE

This is one of a series of Engineering Reports that document the background studies to be used in a system design for HYSURCH (Hydrographic Surveying and Charting System). In general, these reports cover more detail than that finally necessary for a system design. Any subsystem recommendations contained in these reports are to be considered tentative. The reports in this series are:

RN-22 Soundboat Navigation Equipment and Strategy for HYSURCH by John Hovorka
RN-23 The Role of the HYSURCH Survey Ship in the Production of Nautical Charts by Edwin A. Olsson
RN-25 A Computation Center for Compilation, Revision and Presentation of Hydrographic Chart Materials by Edwin A. Olsson
RN-27 Parameters for the Evaluation of Sonar Depth Measurement Systems by Joel B. Searcy
RN-28 Tidal Measurement, Analysis, and Prediction by J. Thomas Egan and Harold L. Jones
RN-29 Applications of Aerial Photography for HYSURCH by A.C. Conrod
RN-30 Sounding Equipment Studies, by Leonard S. Wilk
RN-31 Error Analysis of a Dual-Range Navigation Fix and Determination of an Optimal Survey Pattern by Greg Zacharias

RN-32 Tethered Balloons for Sounding Craft Navigation Aids by Lou C. Lothrop

These reports were prepared under DSR Contract 70320, sponsored by the U.S. Naval Oceanographic Office Contract Number N62306-67-C-0122. The reports are meant to fulfill the reporting requirement on Sub-system selection as specified in the MIT proposal submitted in response to the Oceanographic Office Request for Quotation, N62306-67-R-005.
Section I  

Introduction

The problem of providing a recommended selection of sounding craft-sensors for the hydrographic portion of HYSURCH is two fold: 1) assuring that the design choice can meet the system objectives and 2) assuring that the design choice is optimum with respect to its economics. One method of solution to this, is to create a model of the problem against which many potential candidates for equipment selection are analyzed, and the results of each analysis is compared with the others in terms of meeting the objectives, and its economics.

This approach appears fruitful for the boat-sensor subsystem for several reasons. One is that although the broad objectives for HYSURCH are quite clear, the detailed objectives (system requirements) have some flexibility (i.e. have not been entirely defined). Hence, with a clear outline of their system capabilities and their consequent costs, a number of proposed designs can be compared. A selection can be then determined based on some final trade-off criterion between capability and cost. (This final selection would also consider the limitation and approximations of the analysis and weigh such intangible factors as impending obsolescence or wartime vulnerability that would not be evaluated in the study.)
In particular, the approach being advocated is that of creating a mathematical model of the design problem that is suitable for computer usage and analyzing a large number of configurations. The HYSURCH sounding problem is not one of difficulty in finding equipment that can perform the function at all (such as the shipboard cartographic equipment problem) but rather one of selecting the best choice of a large number of candidates, each of which can do the basic task. Further, by a process of iterative computer runs, any proposed solution can be examined for model sensitivity\(^1\) - in order to increase the confidence of the result.

These analyses if sufficiently detailed can be useful for more than just equipment selection or requirements re-evaluation. They can also provide the basis for an operational plan. For example, with the aid of the model and its computer programs, the plans for an impending survey can be derived to obtain the best (minimum-error) distribution of available navigation aids. Later, these cost analyses can also be useful for evaluating proposed modifications to an operational HYSURCH system.

These considerations provide motivation to have the computer cost-analysis programs in general terms and designed so that they can operate (as sub-routines) with other analyses.

\(^1\)Model sensitivity being that shortcoming of a mathematical model whereby a small variation of the model (or input) has a disproportionate effect on the analysis.
The specific goal of this study is to present a method of obtaining data to support the recommendation of the following:

1. Numbers, types and size of boats
2. Depth sensor type and mounting
3. Survey configuration and strategy

Section II  Problem Definition
1) Objectives

The following items are objectives for the hydrographic sub-system that are derived from the S.O.W. or elsewhere and from the basic goals.

1. The surveying rate is to be increased by a factor of 10 from current operations.
2. The "local" survey measurements must be completed in 1 week.
3. The survey area is covered by four 1:50,000 scale charts (34" x 48") and would correspond to 100 n. mi. of coastline.
4. The subsystem must be capable of operating day and night.
5. A sounding data accuracy of 1.5 ft.

2) Equipment Candidates

The following equipments are proposed for evaluations.

1. Sounding craft
   a. Air cushion
b. Captive air bubble

c. Hydrofoil

d. Planing

e. Displacement

2. Echo sounding depth sensors
   a. Large angle vertical beam
   b. Small angle vertical beam
   c. Small angle multiple beams
   d. Small angle scanning beam
   e. Side-looking sonar

3. Hazard avoidance sensors

4. Sensor mounts
   a. Towed body
   b. Strut
   c. Hull

3) **Survey Operations**

   The following configuration and strategy will be considered:

   1. Single sounding head per craft
   2. Multiple sounding heads per craft (paravanes or bar)
   3. Mother craft-manned daughter boats
   4. Mother craft-unmanned daughter boats
   5. Craft refueling at mother ship
   6. Craft refueling (and crew change) by logistics boat.
7. Various methods of lane spacing and area deployment.

4) Constraints and further requirements

The following factors must be taken into consideration:

1. Environment of the survey area
   a. Size, shape, and inherent hazards (use Buenaventura, Columbia as a representative area)
   b. Weather and sea surface conditions (as outlined in RE-28, but with a maximum sea state of 3)
   c. Hostility or hazards to others
   d. Limitations due to other considerations
      - Personnel limitations
      - Navigation limitations
      - Data transmission limitations
   e. Further requirements and assumptions
      - Disperse and retrieve tide gauges, current meters, and navigation aids.
      - 5 day period (max) for actual sounding operation
      - 10 year system design life
      - 50% duty cycle for HYSURCH
        (e.g., 7 days transit followed by 7 days operation.)
Section III  Model

The boat-sensor analysis problem as outlined above has four main aspects in terms of setting up a model for study. They are 1) costs, 2) benefits, 3) constraints and 4) strategy.

Costs, of course, are primarily related to equipment acquisition and operation and are developed in detail in a following section.

Benefits can be considered in many lights, but for the present study, the areas of interest are: accuracy of depth measurement (which might be related to measurement resolution); completeness of area sounding; amount of area sounded; and time required to perform the sounding.\(^1\)

Constraints provide a description of: the physical characteristics of the area of operation; environmental conditions; man-made restrictions; and boundary conditions on the model.

Strategy, or conduct of operation, can obviously affect this study in a very significant manner. Undoubtedly, some equipment selections will outperform others, depending upon the operational method chosen. Types of strategies include such concepts as: boat array (daughter boats or multiple heads) operation; zones of operation dependent upon depth; zones of operation dependent upon the navigational net; and other more abstract benefits, such as reliability, adaptability to change, etc., are not considered here.
remote boat refueling; lane spacing as determined by depth, chart scale, or fixed; etc.

All of these aspects of the study model indicate that the model is quite complex and perhaps can best be handled in parts. A fruitful approach may be to split the model into two submodels: one that considers the environmental and strategic aspects and yields a schedule of boat operations¹ (numbers, types, lane spacing, speeds, times, etc.); and the other that considers this schedule and yields information concerning costs and benefits.

This interface between the two submodels should be flexible enough to cover 3 usages of boats: 1) sounding, 2) placing and retrieving buoys and instruments, and 3) logistics support (including sounding boat refueling). For the purposes of the cost-benefit submodel, it would suffice to provide a schedule of operations in terms of durations, velocities, and lane spacing; this type of schedule would be provided by the constraint-strategy submodel.

A generalized diagramatical sketch of this interface operation is shown in Fig. 1 where a duration is specified for each of the discrete operations, a speed is specified for the transit and sounding operations, and a lane spacing specified during the sounding.² For computational purposes, this interface can be outlined as shown in Fig. 2.

¹ i.e., yields a table of parameters, some of which may be fixed constants, and others that may be left as variables.

² The times assigned to some of the operations may be zero; the times assigned to the sounding may be in terms of the available fuel; and the speed may be related to maximum design speed.
**STRATEGY - OPERATION INTERFACE**

*Fig. 1.*

- **LAUNCH BOATS**
- **TAKE ON FUEL**
- **TRANSIT A**
  - **DELIVER PAYLOAD**
  - **SOUND**
- **TRANSIT B**
- **LOGISTICS CRAFT**
- **RECOVER BOATS**
- **SURVEY CRAFT**

**INTERFACE BLOCK DIAGRAM**

*Fig. 2.*

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**Abbreviations:**
- **L** = Launch Boats
- **F** = Take on Fuel
- **TA** = Transit A
- **S** = Survey at Lane
- **Spacing Ws**
- **D** = Deliver (or Retrieve) Payload
- **TB** = Transit B
- **R** = Recover Boats
One further comment should be made about the proposed model. Acoustic sensing at high speeds is not a proven technique and may present some significant limitations on the boat-sensor operation. This problem arises from two sources: 1) decoupling of the transducer from the water (cavitation, aeration), and 2) boat noise. These sources are primarily a result of operation at high speed and/or from heavy seas. Although the model could be configured to (partially) account for these limitations, the additional complexity does not seem to be warranted. Rather, it is proposed to neglect this factor during the evaluation. The end result of the analysis, however, must be moderated by this and other sensing limitations.

The rest of this report (except the Appendix) will be primarily concerned with the cost-benefit submodel.

1) Cost Computation

System costs are generally the sum of expenditures and interest less the salvage value at end-of-life. In the following development only direct (i.e. not including R&D) expenditures will be considered.

These expenditures fall into the following categories: acquisition costs and operating expense for the boats and

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interest on expenditures, or, stated in different terms, return on an alternative investment.
and for the sensors.

a. Boat acquisition costs

The boat acquisition costs ($C_{ab}$) for the survey fleet over the lifetime of the HYSURCH system ($L_H$) are of course related to the number ($N_b$) (including spare boats) and type of boats and their lifetime ($L_B$).

For any fully developed boat type, costs are generally related to displacement.

$$C_{ab} = N_{b1}f_{d1}(\Delta)(L_H/L_B1)+N_{b2}f_{d2}(\Delta)(L_H/L_B2)$$
$$+........+N_{bn}f_{dn}(\Delta)(L_H/L_Bn)$$
$$= \sum_{n} N_{bn}f_{dn}(\Delta)(L_H/L_Bn) \quad (1)$$

The minimum displacement required is primarily a function of payload ($W_p$) endurance ($E$) and maximum required speed ($V_m$), assuming that this minimum displacement results in a tolerable sea keeping characteristic at the required speed and sea state. In this study, this assumption is made.\(^1\)

Explicitly, boat displacement for any boat type ($\Delta_n$) can be separated into terms of payload ($W_p$), hull ($W_h$), propulsion machinery ($W_m$), fuel ($W_f$), miscellaneous ($W_o$), and margin ($W_g$) weight:

$$\Delta_n = W_p + W_h + W_m + W_f + W_o + W_g \quad (2)$$

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\(^1\) The assumption is also made that the boat selected on the basis of $W_p$, $E$, and $V_m$ will also be able to meet secondary requirements such as volume, etc.
As an approximation, the hull, miscellaneous, and margin weights have been observed to be primarily directly related to the boat size (displacement),

$$W_{hn} = K_{hn} \Delta n$$  \hspace{1cm} (3)

$$W_{on} = K_{on} \Delta n$$  \hspace{1cm} (4)

$$W_{gn} = K_{gn} \Delta n$$  \hspace{1cm} (5)

Propulsion machinery weight is, of course, related to the installed shaft power and the state of art of propulsion machinery design (lbs per H.P.-Km). Installed power is sized to meet maximum requirements and is related to required speed, maximum boat and sensor drag ($D$), and propulsion efficiency ($\xi_p$).

$$W_{mn} = K_m (SHP_n)$$  \hspace{1cm} (6)

$$SHP_n = SHP_b + SHP_s = \frac{1}{\xi_p} V_m D_b + \frac{1}{\xi_p} V_m D_s$$  \hspace{1cm} (7)

The installed power to overcome this maximum boat drag for normal sea states can be approximated by a direct relation to boat displacement and a function of required speed.

$$SHP_b = \Lambda_{pn} f_p (V_m)$$  \hspace{1cm} (8)
Maximum sensor drag is dependent on the type of sensor housing selected. Flusel-hull mounted sensors add no appreciable drag. Strut\(^1\) and tow body sensors are primarily characterized by a viscous (type) drag.

\[ D_s = \frac{C_D}{2} \rho a V_m^2 \]  

Hence installed horsepower is given by

\[ \text{SHP}_n = \Delta F_n p_n(V_m) + K_s V_m^3 \]  

where

\[ K_s = \frac{C_D \rho a}{2} \]  

Fuel weight is proportional to the specific fuel consumption rate, operating time, and fuel margin, as follows:

\[ W_{fn} = (F)(\text{SFR})(\text{OHP}_1 T_1 + \text{OHP}_2 T_2 + \ldots) \]  

where

\[ F = \text{fuel factor (1+fuel margin)} \]

\[ \text{SFR} = \text{Specific fuel rate (lb/hp-hr)} \]

\[ T_1, T_2, \ldots = \text{Operating times at different velocities} \]

\[ \text{OHP}_1, \text{OHP}_2, \ldots = \text{Operating Horsepower at different velocities} \]

\(^1\)Of course, with hydrofoils, the strut drag is assigned to the boat and not the sensors.
The operating power is related to drag in the same fashion as installed power.

\[
\frac{\text{OHP}}{\Delta n} = f_{\text{pn}}(V) \quad \text{(13)}
\]

Hence the fuel weight can be given by

\[
W_{fn} = (P)(\text{SFR}) \left[ \left( \frac{\Delta n}{f_{\text{pn}}(V_1) + K_s V_1^3} \right) T_1 + \left( \frac{\Delta n}{f_{\text{pn}}(V_2) + K_s V_2^3} \right) T_2 + \ldots \right] \quad \text{(14)}
\]

or

\[
W_{fn} = (P)(\text{SFR}) \left[ \frac{\Delta n R_1}{V_1} f_{\text{pn}}(V_1) + K_s R_1 V_1^2 + \frac{\Delta n R_2}{V_2} f_{\text{pn}}(V_2) + K_s R_2 V_2^2 + \ldots \right] \quad \text{(15)}
\]

where \( R_1, R_2, \ldots \) = distance traveled at \( V_1, V_2, \ldots \)

Although eq. (15) is useful to compute fuel consumption with operations at a variety of speeds, the purpose for calculating fuel weight in this present case is to help size the boat displacement. Hence the fuel weight at maximum capacity is required. This is usually stated in terms of operating endurance \((E)\) at maximum velocity \((V_m)\).

\[
T = E, \ V = V_m \quad \text{(16)}
\]
Hence, for calculating boat displacement

$$W_n = (F)(SFR)(\Delta_n E_{n_{f_{pn}}} V_m) + K_s E_n V_m^3$$  \hspace{1cm} (17)$$

The payload weight includes any weight not assigned to other categories. This model assumes that payload includes

Crew weight = $Kc N_{cn}$  \hspace{1cm} (18)$$

Depth sensor and housing

Navigation equipment

Communication equipment

Etc.

$$= \sum W_{ex} x$$  \hspace{1cm} (19)$$

Secondary payload requirements, such as electrical power, crew provisions, etc. are assumed to be covered under Miscellaneous (i.e., a percentage of boat size).

Hence,

$$\Delta_n = K_{hn}' \Delta_n + K_{on}' \Delta_n + K_{gn}' \Delta_n$$

$$+ K_{m_{f_{pn}}} (V_m) \Delta_n + K_{m} K_{V_m}^3$$

$$+ F(SFR) E_{n_{f_{pn}}} (V_m) \Delta_n + F(SFR) K_s E_n V_m^3$$

$$+ Kc N_{cn} + \sum W_{ex} x$$  \hspace{1cm} (20)$$
or,

\[
\Delta_n = \frac{(K_s K_m + F(SFR)K_e E_n)V_m^3 + K_c N + \sum W_{ex}}{1 - K_h n - K_{on} - K_g n + F(SFR)E_n f(V_m)}
\]  

(21)

Costs, again are

\[
C_{ab} = \sum N_{bn} f_{dn}(\Delta)(L_H/L_{Bn})
\]  

(1)

and if boat costs are linearly related to displacement

\[
f_{dn}(\Delta) = K_{dn}
\]  

(22)

then boat acquisition costs are

\[
C_{ab} = \sum N_{bn} K_{dn} \Delta_n (L_H/L_{Bn})
\]  

(23)

b. Boat operating costs

Boat operating costs can be enumerated as follows:

1. Wage and subsistence - boat crew \( (W_{cb}) \)
2. Wage and subsistence - boat maintenance crews \( (W_{mb}) \)
3. Cost of Spare parts \( (S_b) \)
4. Fuel costs \( (F_b) \)

Hence,

\[
C_{ob} = W_{cb} + W_{mb} + S_b + F_b
\]  

(24)
It is clear that

$$W_{cb} = \sum_{n} N_{bn} S_{n} N_{cn} W_{an} L_{n}$$

(25)

Where

- $N_{bn} =$ Number of boats of type $n$
- $S_{n} =$ Number of daily crew-shifts required
- $N_{cn} =$ Boat complement, including sensor operators
- $W_{an} =$ Average Annual wage and subsistence of boat complement
- $L_{n} =$ HYSURCH operating life in years

The annual maintenance and repair of these boats is generally related to the boat acquisition costs.

$$W_{mb} + S_{b} = K_{m} C_{ab} L_{n}$$

Fuel costs for individual boats in terms of operating velocities and time (see eq. 14) are given by

$$F_{bn} = \frac{S_{n} K_{f} W_{fn}}{F}$$

$$= S_{n} X_{f} (SFR) \left[ (\Delta_{n} f_{p} (V_{1}) + K_{s} V_{1}^{3}) T_{n1} + (\Delta_{n} f_{p} (V_{2}) + K_{v} V_{2}^{3}) T_{n2} + \ldots \ldots \right]$$

(26)
where

\[ K_f = \text{fuel cost to weight ratio} \]
\[ S_n = \text{Number of surveys in HYSURCH design life} \]
\[ T_{n1}, T_{n2} = \text{time spent at } V_1, V_2 \]

\( c. \) Sensor costs

Since the sensor operator and the drag of the sensor housing were included in the boat costs, the sensor operating costs can be neglected, provided that the repair, maintenance and spares are included in the sensor acquisition costs.

The sensor costs, then are simply

\[ C_s = \sum_{n} K_m C_{sn} L_n / L_{sn} \]  \( (27) \)

where

\[ L_{sn} = \text{Useful life of sensor} \]
\[ K_m = 1.0 \text{ plus fraction of cost allocated to repair, maintenance and spares} \]
\[ C_{sn} = \text{Cost of sensor and housing including acquisition and installation} \]

\( 2) \) Benefit Computations

The explicit product of the boat-sensor operation is the sounding of an area. Over the lifetime of the HYSURCH system, this area will simply be:
\begin{align}
A_s &= \sum_n S_n \sum W_n N_n R_n \\
R_{sn} &= \sum_a V_a T_a
\end{align}

where

\begin{align*}
A_s &= \text{Total area surveyed during HYSURCH design life} \\
N_{sn} &= \text{Number of boat (arrays)} \\
W_{sn} &= \text{Sweep width of boat (array)} \\
R_{sn} &= \text{Distance swept in one survey period} \\
V_a &= \text{Sounding velocity} \\
T_a &= \text{Time at velocity } V_a
\end{align*}

The sounding rate (sq. n. mi. per survey period) is simply

\begin{equation}
SR = \frac{A_s}{S_n}
\end{equation}

The completeness of the area covered is a function of the strategy of operation and is given by

\begin{equation}
\text{Coverage} = \frac{\text{Sounded Area}}{\text{Chart Area}}
\end{equation}

Accuracy of the depth measurement is strictly a function of the equipment selection and is not determined by the model.

3. Output Format

A meaningful presentation of the initial results of the boat-sensor cost-benefit computation described in the previous sections would be a graph of unit surveying costs.
as a function of unit surveying rate. (Specifically a plot of dollars per square nautical mile surveyed vs. square nautical miles surveyed per survey period.) These plots would be developed with numbers of craft, survey velocity, craft endurance (for example) as independent parameters. This graph would need to be accompanied by a fact sheet describing the specific equipment represented and including an outline of the survey strategy, along with an evaluation of the depth accuracy and coverage.

This presentation would be especially useful to evaluate and put into economic perspective the first three Objectives in Section II and support the basic decisions of craft and sensor selection. As these basic design parameters are committed, other factors become of primary interest, and lead to similar graphs, but with different independent parameters (e.g., crew size, course uncertainty, sea state). In other words, the simulation effort would progress to finer levels of detail.

Section IV Trial Results

As explained in the foreword, the initial basis for this study was a preliminary internal MIT Memorandum written by Professor Mandel of the Department of Naval Architecture and Marine Engineering. The results of that report are entirely applicable to this study (as outlined below) and are presented as a representative output. Additionally, some Model Sensitivity Studies by the author on Professor Mandel's
results are also presented as a representative output.

1. **System Description**

   The results presented below are based on a simplified system with the following characteristics:

   a. The study applies to Air Cushion Vehicles and results are given in terms of linear distance surveyed.

   b. Sensor costs and drag are neglected.

   c. The strategy considers N boats surveying at maximum velocity, \( V \), with an endurance, \( E \), between refueling. The refueling takes one hour at the survey site by the Mother Ship.

   d. The boats have a 10 year life and survey for 5 days every other week.

   e. 6-man crew per boat.

   f. $67,000 per crew annual wage and subsistence.

   g. 3 crews per boat.

   h. $29.80 per long ton fuel cost.

   i. 2 spare boats.

   j. Maintenance and repair annual cost is 4% of the acquisition cost.

   k. Total payload is 2 long tons.

   l. \( K_{hn} = .33; K_{on} = .10; K_{gn} = .03; K_{m} = 2 \text{ lb/H.P.}; \)
     \( F = 1.25; SFR = .5 \text{ lb/H.P.-hr.} \)

   m. \( f_{pn}(V) = (0.167V^2 - 7.5V + 100)10^{-3} \text{ H.P./lb.} \)

   n. \( K_{dn} = $30,000/\text{ton}. \)
2. Cost Benefit Results

The results of the computation with the above parameters is shown in Fig. 3. Survey costs ($ per linear nautical mile) are shown as a function of survey rates (linear nautical miles per survey period of 5 days); with endurance, velocity, and numbers of boats as independent parameters.

To survey on the order of 10,000 n. mi. in the 5 day period, it is clearly shown that the optimum operation would be with 3 boats surveying at 35 knots. This operation would require an ACV displacement of 7.03 tons, have an endurance of 7.4 hours, and would actually survey at the rate of 11,100 n. mi./s.p. It would cost $2.74 per n. mi. or $7,900,000 during the 10 year period.

3. Model Sensitivity

By evaluating the model at the optimum point selected above with variations in the assumptions, the following items of interest were calculated.

1. Reducing the crew cuts the survey cost by approximately 12% per man.
2. Changing the acquisition costs by 10% changed the survey costs by approximately 2%
3. Reducing the spares from 2 boats to 1 boat reduced survey costs by approximately 4%.
4. Changing the payload by 100 lbs changed the survey costs by approximately 1/2%.
**ACV COST-BENEFIT STUDY**

**FIG. 3**

- **N-V** = No. of Boats
- **V** = Velocity in Knots

**SURVEY RATE** (n. mi./5-day Survey Period)
5. Reducing the crew to 3, cutting the payload to 1 long ton and reducing the spares to 1 boat had the effect of cutting the survey costs, and also the boat displacement, in half.

6. Reducing the survey activity from 3-8 hour shifts per day to 1-8 hour shift per day of course cut the survey rate by 3 (from 11,100 mi. per week to 3700 mi. per week) and increased the survey costs by 35%.

7. Reducing from 3-8 hour shifts to 2-6 hour shifts cut the survey rate by 2 (from 11,100 to 5,550 mi. per week) and increased the survey costs by 42%. 

- 23 -
APPENDIX

SURVEY RATE OF A BEAMED, PULSED SENSOR

If the sounding craft is using a pulsed, sweeping sensor with a conical beam, then the constraint-strategy model must account for the relation between craft operation and the sensor characteristics. Derived below is the survey rate of a number of craft which are proceeding at the maximum velocity that still permits 100% coverage. The derivation assumes that the operation is not limited by the transit time of the pulse.
Definition of Terms

A = Area Survey by one craft
SA = Area surveyed by all craft
T = Time to survey SA
C = Number of craft to survey SA
V_c = Velocity of craft
R_s = Sweep rate
t_s = time for one sweep
\theta_b = Beam illumination half-angles, excluding overlap
\theta_s = Sweep half angle, excluding overlap
a = Altitude of craft
d = depth of water
h = Height of craft above bottom
L = Length of square illuminated, excluding overlap
W_s = Width of strip illuminated, excluding overlap
n = Pulses per sweep
R_p = Pulse rate

Clearly
(1) A = V_c W_s T  
    (assumes 100% coverage)

(2) A/T = V_c W_s = 2V_c h \tan \theta_s

(3) t_s = \frac{n}{R_p} = \frac{\theta_s}{R_p}  
    (assumes constant sweep and pulse rate)

= \frac{L}{V_c}  
    (Maximum craft velocity for 100% coverage; note increased overlap at ends)

Therefore, from eq. (3)
(4) \[ V_c = \frac{L_i b R_p}{\theta_s} \]

and from eq. (2) and (4)

(5) \[ \frac{r}{T} = 2LR_p b^2 \frac{\tan \theta_s}{\theta_s} = R_p b^2 \frac{\tan \theta_s}{\theta_s} \]

(assumes small \( \theta_p \))

Note that

<table>
<thead>
<tr>
<th>( \theta_s )</th>
<th>( \frac{\tan \theta_s}{\theta_s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1</td>
</tr>
<tr>
<td>20°</td>
<td>1.04</td>
</tr>
<tr>
<td>30°</td>
<td>1.10</td>
</tr>
<tr>
<td>45°</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Therefore eq. (1) will be in error by no more than 16.4% for \( \theta_s < 15° \) if written as

(6) \[ \frac{A_s}{T} = 1.14 L^2 \]

Note that

(7) \[ \theta_s = (\pi/2) CT \]

(10) \[ y = \frac{L_i b}{b_s} = \frac{R_p b^2}{\theta_s} \]

Hint

(1) \[ b = 3b_s \]

and

(11) \[ V_c = \text{potentials} \]
then

\begin{equation}
\left(11\right) \frac{w_s}{L} = \frac{\tan \theta_s}{\varepsilon_b}
\end{equation}

For small \( \varepsilon_s \) (or for first iteration)

\begin{equation}
\left(12\right) \frac{w_s}{L} = \frac{\varepsilon_s}{\varepsilon_b} = n
\end{equation}

These relations can be conveniently summarized as shown on Fig. A2.
Survey 800 sq. n. mi.  
with 2 craft in 1 day 

CT = 2 x 20 x 1 = 40  

Survey rate = 20 sq. n. mi./hr. 

with a sweep width of 1000 feet 

Craft velocity = 120 kts 

and a pulse which illuminates a 

square (excluding overlap) of 20 ft.  

on a side (under the craft) 

Pulse rate = 440 pps
Note that for a first approximation

\[ n = \frac{W_s}{L} = \frac{1000}{20} = 50 \]  \hspace{1cm} (H)

Sweep rate = 9 sps \hspace{1cm} (I)

Given a beam of 10 m. rad. half-angle \( \theta_b \) the required craft height above the bottom is

\[ h = a + d \]

\[ = \frac{L}{2\theta_b} = \frac{20}{.02} = 1000' \]  \hspace{1cm} (J)

and the sweep angle must be

\[ \tan \theta_s = \frac{W_s}{2h} = \frac{1000}{2000} = .5 \]

\[ \theta_s = 26 \frac{1}{2}^\circ \]  \hspace{1cm} (K)