SOME FUNDAMENTAL PROPERTIES OF GOVERNMENTAL EXPENDITURE PATTERN--ETC(U)

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SOME FUNDAMENTAL PROPERTIES OF GOVERNMENT EXPENDITURE PATTERNS—THEORY AND EVIDENCE BASED ON MILITARY EXPENDITURES

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[Signature]

JAMES P. WITTHY, Colonel, USAF
Vice Dean of the Faculty
This paper investigates possible methods of forecasting defense expenditures in the presence of sparse data. The forecasts which are generated are for major weapon purchase categories or for standard military treasury code cost categories.
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INTRODUCTION

Economic forecasting is normally dependent on past data. The methods which are selected to analyze these data are a reflection of the researcher's estimates concerning the "best" way to deal with the historical evidence at hand. These estimates arise from a crucial decision which must be made: should the analyst deal with the data as they appear (the normal course of events), or should the analyst attempt to look beyond the observed data and base his forecasts on other, underlying structural conditions.

Expenditure forecasting in the government has, in the past, been comprised of two general types of prediction. The first is the application of some type of quantitative modeling technique. This technique is usually applied to gross data figures for some expenditure category, and it produces poor forecasts with wide variance because of the white noise generated by all of the underlying (and unconsidered) cost factors. The second type of prediction is the seat-of-the-pants guess, or a summation of guesses by various individuals at the grassroots level of an organization. These guesses run the gamut from outrageous pessimism to unbounded optimism because they are not based in theory and, instead, reflect the subjective feelings or desires of the individual forecaster.

This paper will show that in forecasting government expenditures, there are certain underlying commonalities which allow forecasting for many diverse programs at all levels of the expenditure
process. These commonalities revolve around the biological growth characteristics which are pervasive in the government's way of buying goods and services. This paper will cover the applicability of growth process forecasting to specific weapon systems and the general budgeting and expenditure process. It will then show how this same method could be used to develop military expenditure forecasts for other nations.
CHAPTER 1
THE METHOD OF FORECASTING

During the 1920s, two statisticians, Raymond Pearl and L. J. Reed, were extensively involved in research dealing with forecasting rates of population and biological growth. They discovered that cumulative data in the S-shaped curve form of Figure 1 accurately represented types of growth which are characterized by three distinct phases: (1) a slow period of development, (2) a rapid period of expansion, and (3) a tapering off at maturity.

![Growth Curve](image)

Figure 1. The Growth Curve

However, as Yamane points out, although this S-shaped curve does characterize growth in many diverse activities, "usually the
results have not been good" when the curve has been used for forecasting.¹ The failure of the two standard forms of the S-shaped curve (the Logistics and the Gompertz) to provide accurate forecasts can be attributed to two principle causes:

(1) The equation for the Gompertz curve assumes symmetry around an inflection point which is the geometric mean of the Y values, while the equation for the Logistics curve makes the same assumption for the true mean of the Y values. Neither of these assumptions is usually justified in real-world situations where the inflection point may occur at virtually any intermediate position on the curve.

(2) Because of the order of the polynomials needed to express these curves, slight perturbations of the data early in the growth process can quickly force the equation for either of the curves to forecast an unrealistically high or low figure for total growth.

The major problem in handling any S-shaped curve is thus twofold: one must keep under control the higher order polynomials necessary to express this complicated curve form while simultaneously allowing the inflection point to occur over a wide range of values. Since any S-shaped curve is merely the cumulative form of a bell curve, as shown in Figure 2, a solution to both of these problems becomes apparent.

The S-shaped curve may be separated (or "broken") at the inflection point mandated by the data to yield two simpler curves of a form which may be expressed by either a logarithmic \( y = ax^b \) or a quadratic \( y = a + b_1 x_1 + b_2 x_1^2 \) equation. Standard econometric fitting techniques may then be used to determine which of these two curve forms is the best fit for the upper and lower segments of the S-curve. The best-fitting equation for each section of the S-curve is used for all future forecasting work. The independently forecast sections of the curve are then rejoined at
the inflection point to provide a continuous expenditure pattern for whatever financial program is being investigated.  

Assuming that appropriate corrections are made for autocorrelation, this technique of splitting the S-curve at the inflection point yields excellent results in fitting curves with inflection points located over a very wide range. An additional step of normalizing the data makes this approach even more useful. If cumulative dollar costs are converted to a percentage of total program cost and, similarly, if time is converted to a percentage of total time required, all data are normalized so that they can be directly compared within any general area. This allows the calculation of general S-curves which represent whole categories of spending and, hence, provide better initial forecasts for new programs.

CHAPTER II

THE GROWTH CURVE AND MILITARY EXPENDITURES

The existence of growth curve based expenditure patterns in Department of Defense R&D programs has been verified by taking the Cost Performance Reports (CPRs) for 22 different weapon and component systems and tracing the actual costs as they were incurred throughout the life of each R&D program. Without exception, the S-shaped curve was the dominating factor in the system development as it actually occurred. The milestones associated with these programs also exhibited the same characteristics in every case for which data were available.¹

The results based on the original 22 weapon systems were then checked against new and very complete data which were provided by another source on 15 additional weapon systems. Again, the connection between R&D expenditures, the S-shaped curve, and milestone completion were firmly established.² In almost every case, the quadratic equation provided the best fit of both the upper and lower segments of the S-shaped curve. The statistics accompanying each of the curve equations indicated that the upper and lower curve segments, when rejoined, did provide an excellent proxy for the original S-shaped curve for each R&D project, and the mean

¹William J. Weida, A General Technique . . . , p. 11.
²Ibid., p. 3.
square error figures accompanying each equation assured that these reconstructed S-shaped curves were useful forecasting tools.\(^3\) The applicability of the S-shaped curve to weapon system costing was further confirmed by the adoption of this curve form by Aeronautical Systems Division for R&D cost estimating.\(^4\)

Since a general technique based on growth curve theory did apply to normalized data from individual weapon systems, later studies were devoted to an investigation of Air Force outlays in entire budget code categories using the same general format. In the USAF, outlays are the difference between disbursements and reimbursements in 31 general treasury code categories such as 3010F—Aircraft Procurement or 3500F—Military Personnel.

In 1965, Almon published research results based on capital appropriations and expenditures in private industry. These results indicated that both appropriations and expenditures exhibited typical growth curve characteristics throughout the period in which monies were expended.\(^5\) The categories of expenditures which Almon investigated were very similar to the components which make

\(^3\)Ibid., pp. 52-55.


up most of the treasury code categories in which the Air Force expends funds. However, in the military sector as a whole, past efforts to forecast expenditures have only been based on total outlay data, and no attempt has been made to look beneath this observed data to determine the structural form of the components which make up the total outlay figure.

For reasons which will become apparent, these military analyses have been quite unsuccessful. Total disbursement and reimbursement data figures in every treasury code category demonstrate seasonality patterns, but the large variance in these data creates forecasts of unacceptable reliability when any of the conventional trended data regression techniques are applied.\(^6\)

Monthly outlays in every treasury code category are actually composed of several strings of data. These strings occur because, at the start of every fiscal year, money is allocated to each treasury code category. However, in almost all major categories, this money is actually spent over the next three to five years (some categories, such as personnel, expend all funds in a single year). Thus, the observed monthly outlays in most treasury code categories are actually composed of several fiscal year subgroups,

\(^6\)Past analyses have utilized Generalized Least Squares with autoregressive schemes, Polynomial Distributed Lags, Almon Lags, Cochran-Orcutt based regression, and Box-Jenkins techniques. In every case, the variability of the data (arising from excessive "white noise") has prohibited sufficiently accurate forecasts. Navy Expenditures Forecast Project, Phase II, Development Items, Synergy Company, March 15, 1973.
each of which exhibits its own spending pattern. The possible effects of different overlapping spending patterns on the observed data in a treasury code category are shown in Figures 3, 4, and 5, page 13. Note that unless the increasing and decreasing segments of the incremental distributions for each pattern are non-linear (Figures 4 and 5), no peaks will occur in the observable data (Figure 3). Note also that as these example distributions come closer to the expected shape of an expenditure pattern (Figure 5) the observed data become more complicated, and the peaks, although present, are less pronounced. Normally, the observed data pattern in Figure 5 would be said to possess a high degree of seasonality, and forecasts would be made based on this observation. However, as Figure 5 demonstrates, causes other than seasonality (such as underlying data patterns) may also account for observed data of this type.

Cumulative Expenditure Patterns

Although incremental distributions can be used to demonstrate the manner in which the observed outlay data are generated, cumulative expenditure distributions are easier to use for modelling and forecasting. When the cumulative expenditure pattern for each fiscal year's money is investigated, it is apparent that both disbursements and reimbursements follow an S-curve pattern in every treasury code category as money is spent over a one to five year period. These
Observed Expenditure Pattern for Data

Fig. 3
Linear Increase and Decay

Expenditure Pattern for a Single Fiscal Year’s Money

Fig. 4
Nonlinear Increase and Decay

Incremental Monthly Outlays

Fig. 5
Nonlinear Increase and Decay
S-curves always show that most activity takes place in the middle part of the spendout period. For example, Figure 6, page 15, shows the cumulative pattern associated with reimbursements in the aircraft procurement area (3010). This is a typical S-curve pattern which reflects a slow start to the program followed by a rapid building phase and an even slower winddown of activity. Figure 7, page 16, shows how the cumulative expenditure patterns for each fiscal year's money in any given treasury code category will eventually yield a string of observed data. In this figure, it is important to note that the inflection point of each cumulative expenditure pattern is directly aligned with the peak of the corresponding incremental distribution. The peaks of the incremental distributions, when combined with the segments of the other incremental distributions which fall immediately below these peaks, then generate the peaks in the observed data.

In actual practice, it is the S-curves which underly the observed data which become the vehicles for forecasting. Using the techniques listed earlier in this paper, forecasts are generated for each fiscal year's money in each treasury code category based on the growth curve characteristics of the spendout of these monies. These forecasts are then combined to yield the total disbursement or reimbursement figures for any given month. For example, if a treasury code category has fiscal year monies which are spent over a five year period, then five forecasts for five different
Figure 7

Incremental Expenditures

Observed Data (Total Incremental Expenditures)

Cumulative Expenditures

Inflection Point
expenditure curves would be required to generate the total monthly figure. Outlays, the end objective of the forecast, are calculated by taking the difference between forecasted disbursements and forecasted reimbursements. The results of this technique have provided markedly improved accuracy in forecasting Air Force outlays.7

7A complete description of the method, the procedures, and the results for every Air Force expenditure category are presented in William J. Weida, Determining Governmental Disbursements, pp. 1-11.
CHAPTER III

FORECASTING MILITARY OUTLAYS

The fact that regular S-shaped cumulative expenditure patterns do exist for each fiscal year's money in every treasury code category should allow S-shaped cumulative expenditure curves to serve as the proper vehicle for forecasting most military outlays. In addition, since these curves possess detailed information concerning the actual expenditure of each fiscal year's money, they should provide better forecasting accuracy than methods which use only the observed data. This leads to two suggested paradigms for accomplishing fiscal spending research in the military outlay area. The choice between these paradigms depends on whether or not fiscal year data in each outlay category are known.

Actual Expenditure Data for Each Fiscal Year are Known

If the expenditure data for each fiscal year's money in every spending category are available, the growth curves can be directly calculated. This is the case for United States' defense expenditures, and it should generally be the case with all organizations having historical data. As shown in Figure 6, page 15, the format for plotting these S-curves should be percent cumulative expenditure versus percent total time. For disbursements, total obligation authority (or, for past data on which the spendout has been
completed, total amount spent) will determine the 100% level for expenditures. For reimbursements, the total figure will be the actual amount received.\(^1\) Total spendout time for each program will determine the 100% level for time and will generally be three or five years. This method of formatting will compensate for the effects of inflation or other factors which are dependent on the actual dollar figures involved.

**Actual Expenditure Data for Each Fiscal Year are Not Known**

If specific data for each fiscal year are not available (as is likely to be the situation when foreign defense expenditures are investigated) and only the total data stream for some general spending category can be used (i.e., only the observed data shown in Figure 7, page 16, is available), one may proceed in the following manner.

First, assume:

1. that the amount of money available for yearly defense spending is constrained to the extent that purchases must be made in accordance with the amount of money available;

2. that military expenditures will involve projects which will often span more than a single fiscal year. (This implies,\(^1\)

\(^1\)Detailed methods for calculating these figures may be found in William J. Weida, *Determining Government Disbursements*, p. 24.
regardless of the budgeting process, that funds for a project authorized in one year may be spent over the following several years.)

(3) that the planning periods which are utilized for this spending and for weapon system development are uniform and hence, predictable.

The third assumption is critical if defense outlays are to be forecasted in an economy where little actual data is available. If this assumption can be made, then certain analyses which can be performed on the observed data will allow the location and general shape of the underlying growth curves to be specified. Therefore, the following section of this paper will provide the intuitive rationale behind the premise that the planning periods for weapon expenditures will not only be uniform, they will be approximately the same between any two competing countries.

The Weapon Development Cycle

Consider, for example, the group of weapons which may be called the "air group." Fighters must be developed to counter enemy fighters, bombers to counter enemy fighters and vice-versa, missiles to counter enemy bombers, or fighters, or other missiles, etc. In other words, weapon systems in this group are inter-related since the mission and characteristics of each are dependent on all of the competing countries' other systems in this generic group.
Now consider the plight of the military planner. Figure 8 shows that the planner faces a cost curve (A) which is dependent on the amount of time he allocates for the development cycle of the weapon system he desires. The planner's problem is that cycle time cannot be shortened more than the time required for normal state-of-the-art technology to develop [Point (D)]. If the planner tries to push the level of technology, he incurs rapidly increasing costs. This gives the kinked cost curve (A). Plotted along this curve as examples are some recent weapon systems and their probable locations. However, this curve, in itself, does not specify the cycle length a planner will choose. Figure 9, which introduces risk, shows why, in an atmosphere of competition, two competing planners will usually produce at the same place.

Note that Figure 9 again shows the cost and cycle time axis and the kinked cost curve (A). However, general risk is introduced on the third axis as a twofold problem: too long a cycle time creates an increased chance of enemy surprise (B) while too short a cycle time creates an increased chance of project failure (C). In both cases, risk expands rapidly as one departs (D), the cycle time which corresponds to state-of-the-art technology development. Each planner is thus caught in a "prisoner's dilemma" situation. In order to minimize risk, each will choose to produce at (D), with each accepting higher costs than would be necessary if both planners, through collusion, were to extend the cycle time and develop weapon systems at a slower, less costly pace.
Figure 8

Kink Cost Curve and Weapons Development

INCREASING COST

F-15 ENGINE
F-15 AIR FRAME
A-10
CARGO A/C

INCREASING CYCLE TIME

DEVELOPMENT TIME OF STATE-OF-THE-ART LEVEL TECHNOLOGY
Figure 9
A Comparison of Risk versus Cost and Cycle Time
Thus, if one can assume that at least one member of the military competition is developing weapons on a cycle which is approximately matched with advances in state-of-the-art design, it is logical to assume that the other member, for the reasons just listed, will also follow that cycle.

It is interesting to note that in past years the U. S., in the bomber field, has found an unusual way of using the short cycle to some advantage. Both the B-70 and the B-1 were aircraft which pushed hard against the constraints caused by state-of-the-art techniques and hence, incurred vastly increased costs. Both systems were discontinued before the full increased costs of production were felt, but the USSR, working on a normal development cycle, became involved in building a fighter to counter at least the B-70. This fighter (the MiG 25) was then deployed on schedule to meet a threat which had been cancelled many months prior. One would hope that the B-1 has spawned a similar reaction.

Forecasting with Sparse Data

If, as the foregoing analysis indicates, similar weapon systems do develop along similar cycles in competing countries, then the following points should apply for a country about which only gross or incomplete data exist:

(A) As shown in Figure 6, page 15, the spending of each fiscal year's monies should resemble a growth curve (S-curve) when data is displayed in a cumulative format.
(B) For any given constant pattern of S-curves, a vertical line cannot intersect more than a single S-curve in its period of maximum growth (Figure 7, page 16). This period of maximum growth in the area of the S-curve located closest to the inflection point.

(C) Any intersection of a vertical line and an S-curve at a period of maximum growth will hit all other curves at periods of much lower growth. This characteristic will invariably be generated from a series of incremental expenditure distributions as shown in Figure 7.

(D) The points of maximum growth (inflection points) and hence, the peaks of the individual incremental distributions, will only occur at intervals of the same length as those on which the growth curves are generated. [If uniform S-curves are generated at the start of each planning period, the time between the inflection points must match the interval between curve generations, no matter where the inflection points are located in each specific S-curve].

(E) Then the periods of maximum growth (inflection points) of the S-curves will be coincident with the peaks in the observed data, and the observed data stream would appear as shown in Figure 7. Seasonality would normally be suspected as the cause of the peaks in this observed data, but points (A) through (D) above show that this pattern will also result from a family of S-curves. In addition, for defense expenditures, the element of seasonality is probably a very minor consideration.
Constructing Curves When Fiscal Year Data are Unknown

Since the S-shaped curves play such an important role in outlay forecasting, it is desirable to be able to construct these curves even when fiscal year data are missing. When this is the case, the following methodology may be employed.

Assume a string of observed data, as shown in Figure 7, in which neither the underlying pattern of the S-curves nor the specific fiscal year outlays are known (but which, one had reason to believe, is composed of this type of spending pattern). If sufficient observations are available, analyze the data using an appropriate lagging procedure for trended data. The results of this analysis will specify the location of and interval between the peaks in the data. If these peaks are found by this lagging procedure, the inflection points and their associated S-curves, if they exist, must be in a regular pattern.

The distance between the peaks is the time distance between the formation of individual S-curves. This distance should correspond to known planning periods for the system being investigated.

If all expenditures are handled in a percentage format (as previously suggested in this chapter), the following items will have been determined.

(1) the upper level of expenditure is defined to be 100%.

The actual monetary value may be calculated later using the method shown below.
(2) the location of the S-curve inflection points is coincident with the peaks in the observed data.

(3) the time between generation of the individual S-curves is the time between the peaks in the observed data. These three items plus a knowledge of the planning intervals in the economy being studied (as developed in the previous section) will allow one to infer the time between the start of each S-curve expenditure pattern and the inflection point in that pattern. This, in turn, should lead to reasonable inferences as to the likely ending points and probable shape of the S-curves.

The shape of the S-curve then becomes a very powerful forecasting tool. Since weapon development cycle times can be determined based on the rationale in the previous section of this chapter, the 100% of time figure for the curve will be known. Any specific data on spending, no matter how sparse, can then be broken down into the amounts spent in each fiscal year in the following manner:

Assume that the planning cycle (weapon development cycle) for aircraft runs three years.

Assume also that intelligence determines a spending level of $10 in this weapon category at time $t$.

Figure 10 shows this situation.

Now, if the amounts of money allocated in each fiscal year are approximately equal, the likely amount which would be spent
Figure 10. Expenditure of Fiscal Year Monies
from each fiscal year's money in time period $t$ can be determined. Assume, for example purposes, that these incremental amounts were:

- 2% in Fiscal Year 1
- 10% in Fiscal Year 2
- 1% in Fiscal Year 3

Then $\left(\frac{10}{13}\right) \cdot \$10 = \$7.70$ of the observed amount came from fiscal year 2 money. This figure will, in turn, allow calculation of the total amount programmed for this fiscal year's outlays over the total 3-year period. Similar calculations for the other two fiscal year's monies will give the ability to determine spending trends in this weapon category as well as the ability to determine approximate incremental outlays in this weapons group. Obviously, additional data will allow additional refinements of the S-curves and hence, better accuracy.\(^2\)

\(^2\) Complete methods for developing expenditure forecasts based on these curves can be found in William J. Weida, Determining Governmental Disbursements, p. 37.
This paper proposes the thesis that there is some common element which links all military expenditures. That element is the growth curve—the idea that government expenditures proceed in a logical, well-ordered pattern, building on what has happened in the past and predetermining what will happen in the future. Considered rationally, there is simply no other way to do things when large amounts of money and effort are expended.

And yet, for reasons which are more based on the academic mystique than logic, this important, underlying factor which determines the way in which money is spent is seldom exploited. The result is poor forecasts, and the solution to the problem is simple. The research in this paper argues that the growth process is the driving force behind both individual weapon systems costs and the expenditure of monies in entire weapon cost categories. This research has also shown that a knowledge of the growth curve will allow useful forecasting in the presence of data so sparse that time series techniques may not be able to function. In sum, the research cited in this paper would seem to indicate that, given the proper methods of normalizing the data and calculating the curve form, the growth curve could serve as the major tool in forecasting this and other countries' military expenditures.