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COST-EFFECTIVENESS MEASURES OF REPLENISHMENT STRATEGIES FOR SYS--ETC(U)
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(11) December 1979

(6) COST-EFFECTIVENESS MEASURES OF REPLENISHMENT STRATEGIES FOR SYSTEMS OF ORBITAL SPACECRAFT

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A Rand Note

prepared for the

United States Air Force

(15) F49620-77-C-0023

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PREFACE

For many years the U.S. Air Force has employed large-scale, discrete-event digital simulation models for evaluating the cost-effectiveness of various replenishment strategies for operating satellite systems. The extensive use of computer simulation presupposes a problem complexity intractable to closed form or analytical solutions. This statement is true if analysis requires a great amount of detailed information. This Note demonstrates that there is a level of aggregation of the data inputs at which closed form tractability may be attained. Moreover, given this input aggregation representing failure patterns and replenishment strategies, the exact closed forms for approximating cost-effectiveness are derived.

The closed form expressions for the cost-effectiveness of satellite systems are not meant to replace the large-scale simulation programs. The aggregate level measures can be most effectively employed to check the computer programs for internal consistency and to narrow the focus of acceptable inputs into the large-scale simulation. Proper use of the analytical tools presented herein can reduce the computational effort by significantly reducing the number of simulation runs necessary to identify the most attractive replenishment methods.

This work was done under the Project AIR FORCE project "Concept Formulation and Exploratory Research (Acquisition)."

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1.0 INTRODUCTION

Planning the operational strategy for a system of joint military/civilian satellites maintained at a high-earth orbit is a difficult task. The problems involved stem from two major sources. Maintenance requirements are rather complex. Moreover, the ability to maintain and to operate the satellite systems is extremely sensitive to the predicted scheduling of budget funding.

The maintenance concepts for a system in orbit are significantly nontraditional. If a critical subsystem fails, no remove-and-replace capability exists, as yet. Therefore, maintenance cannot be accomplished according to traditional norms. The satellite possessing the nonoperative subsystem is said to have failed and must be replaced. For orbiting space systems, replacement usually is accomplished either by launching a new satellite or transitioning an on-orbit spare. In either case, certain additional considerations emerge as to the maintenance of the alternative system.

No matter which alternative is chosen or how the alternative is itself maintained, costs will be engendered. As one would suspect, each complex maintenance concept produces a cost measure and an effectiveness measure. The complexity of the relationships among operations strategies, costs, and effectiveness or performance measures naturally leads the analyst to a simulation modeling approach for planning and evaluating operations scenarios within given operational contexts. Simulation models are valuable tools for investigating micro-level interactions.

However, one might reasonably suspect that an aggregation level exists at which may be formulated a formal theory concisely characterizing the basic relationships among operations strategies, costs, and effectiveness measures. Section 2.0 establishes the appropriate aggregation level and develops the underlying theoretical constructs. The resultant output should be analytical expressions regarding the basic relationships among operations strategies, costs, and effectiveness as a function of input parameters describing the operations strategies at the appropriate level of aggregation.

Once the basic relationships have been specified, test cases are made comparing simulation model outputs with analytical model outputs, at the same aggregation levels. The necessity to aggregate simulation model inputs for use in the analytical forms increases the likelihood that the aggregate outputs from each will not match exactly. However, these aggregate outputs should be relatively close to each other. The most widely used simulation models are described in Section 3.0. The input aggregation methodologies are described and performed in Section 4.0, which also contains comparisons of simulation and analytical computations.

The comparative computations, while strongly suspect because of the aggregation requirement, do indicate that the aggregation level and closed forms derived are appropriate to the cost-effectiveness evaluation of operations strategies for orbiting satellite systems.

Finally, the Appendixes provide illustrations of the actual computations employed to derive Fig. 2: Comparison of Simulation Outputs and Closed Form Outputs. Appendix A provides the aggregation computations used to transform the simulation inputs to analytical inputs. Appendix B presents two representative computations of analytical outputs using aggregated inputs.

The general approach employed is to consider a system of orbiting satellites as a processing type of system. Under certain general assumptions (which are not critical), the inputs and outputs of processing may be balanced. Thus the state of the system is defined, and state balancing equations lead to the analytic forms desired.

2.0 A GENERAL MODEL OF AN ORBITING SATELLITE SYSTEM

A satellite system is defined as a configuration of M satellites on orbit, performing a single mission. A configuration of satellites is any number of satellites sharing a common orbital plane. The state of the satellite system is specified according to the number of operational satellites in the configuration.

The state of the satellite configuration is altered by satellite failures and satellite replenishments and, thus, may be described by the number of operational satellites on orbit. The exact form of replenishment operations to the satellite configuration is presently unspecified. Analysis of changes in the state of the on-orbit system assume the existence of a time increment at the end of which the state of the system may be clearly discerned. This time increment must possess several properties.

First, the state-changes in one time increment must not be affected by failure and replenishment activities of previous time increments. Additionally, the various activities which may change the orbital configuration state must be described by the same rate of occurrence, regardless of which time increment is considered. Finally, the length of the time increment must be chosen such that a failure and a replenishment activity have a negligible probability of joint occurrence during the time increment. For orbital systems, these assumptions are not particularly critical, especially since the stochastic modeling approach which is employed has usually proved to be the most robust approach to modeling uncertainty in state-spaces. Moreover, since the overall space segment is a relatively small population (compared to fleets of airplanes, for example), determination of an appropriate time increment (say one day) is a relatively simple task.

A common performance measure for evaluating the failure models and the replenishment strategies employed to operate the on-orbit system is the expected availability of the system. Theoretically, the concept of availability is construed to mean the probability that a system will be in an operable condition to perform its mission if required to do so. This traditional orientation to availability assumes a bistate situation: the system is available or it is not available. Satellite systems, however, have forced a different approach to measurement of availability; availability is defined as a multistate measure. A satellite configuration is deemed to be available if K or more of the $M (K \leq M)$ satellites are operational. In other words, if all M satellites are operational, the system is operating to its maximum capability; however, a minimal capability is still provided if between

K and M satellites in the orbital configuration are operational. This is an attempt to impose a form of redundancy upon the operational system.

The criterion of expected life cycle costs should also be related to the same state-changing activities and their rates of occurrence. For this analysis, life cycle costs are interpreted to mean the accumulation of expenditures over the operating horizon of the satellite system in the following categories: (a) research, development, testing, and evaluation (RDTE) activities; (b) production activities; (c) launch activities; and (d) inventory carrying activities. Note that the traditional operations and maintenance costs are simply represented as inventory carrying costs. Other operations costs do exist (data monitoring and analysis) but are not included for simplicity. The models presented could be easily adapted to include more detail as desired; however, since the primary orientation is focused upon aggregation of information, the inventory costs are assumed to be the primary component of operations and maintenance costs when considering the operation of orbital configurations.

Finally, two special cases are of interest: K of M operational satellites with no orbital spares, and K of M operational satellites with N orbital spares. Each of these cases may be deduced from the general models formulated. The major impact of the use or nonuse of orbital spares is obviously focused upon the expected service or replenishment rates employed in the analytical forms.

2.1 The Expected Availability Model

Consider a satellite system consisting of a configuration of M satellites sharing a common orbital plane. The system state and the parameters affecting the state change process may be defined as follows:

n = number of operational satellites on-orbit;

λ_n = constant expected failure rate per unit time of individual satellites, as a function of the state n of the system;

μ_n = constant expected replenishment rate per unit time of individual satellites, as a function of the state n of the system.

The values of λ_n, μ_n represent operational strategies defined at an appropriate level of aggregation, as stated earlier. The actual forms of λ_n, μ_n depend upon the specific context being considered. Section 2.3 will describe some specific forms.

Using the terminology above, the assumptions previously described may be more precisely stated. The key to this formalization is definition of the time increment in question, as follows:

Δt = a time increment which satisfies the following conditions:

(a) successive passages of Δt are stationary and independent;

(b) the probability of zero replenishments during Δt, given n operational satellites on orbit is given by $1 - \mu_n \Delta t$; the probability of zero failures during Δt, given n operational satellites on orbit is given by $1 - \lambda_n \Delta t$;

(c) the probability of one replacement during Δt, given n operational satellites on orbit is described by $\mu_n \Delta t$; the probability of one failure during Δt, given n operational satellites on orbit is described by $\lambda_n \Delta t$;

(d) the probability of two or more state changes during Δt is given by $0(\Delta t)$ [powers of $\Delta t \geq 2$], which in limit as $\Delta t \longrightarrow 0$, becomes 0 and thus, may be effectively ignored, for purposes of this analysis.

The state transitions may be completely characterized by the following categorical descriptions:

State at t	Activities During t	State at $(t + \Delta t)$
0	(No Failures, No Replenishments)	0
1	(One Failure, No Replenishments)	0
$0 < n < M$	(No Failures, No Replenishments)	$0 < n < M$
$n - 1$	(No Failures, One Replenishment)	$0 < n < M$
$n + 1$	(One Failure, No Replenishments)	$0 < n < M$
M	(No Failures, No Replenishments)	M
$M - 1$	(No Failures, One Replenishment)	M

The major concern associated with this state-space approach is to formalize the uncertainty associated with various states of the orbiting satellite system. The uncertainty may be reflected by the following notations:

$P_n(t)$ = the probability that n operational satellites are on-orbit at some time t.

Using the independence assumptions regarding Δt and ignoring all terms on $O(\Delta t)$, we can describe the probability transitions using the state transition classes described above:

$$(2.1) \quad P_o(t + \Delta t) = P_o(t) \cdot [1][1 - \mu_o \Delta t] + P_1(t)[\lambda_1 \Delta t][1 - \mu_1 \Delta t]$$

The "1" occurs above because the probability of zero failures given zero operational satellites on-orbit is 1.00;

$$(2.2) \quad P_n(t + \Delta t) = P_n(t)[1 - \lambda_n \Delta t]^n [1 - \mu_n \Delta t]$$

$$+ P_{n-1}(t)[\mu_{n-1} \Delta t][1 - \lambda_{n-1} \Delta t]^{n-1}$$

$$+ P_{n+1}(t) \binom{n+1}{1} [\lambda_{n+1} \Delta t][1 - \mu_{n+1} \Delta t]$$

for $0 < n < M$, where

$[1 - \lambda_n \Delta t]^n$ = the probability that none of the n independent operational satellites on-orbit fails during Δt; and,

$\binom{n+1}{1}$ = the combination of ways in which one of n+1 operational satellites on-orbit could fail during Δt;

$$(2.3)\quad P_M(t + \Delta t) = P_M(t)[1 - \lambda_M \Delta t]^M[1] + P_{M-1}(t)[\mu_{M-1}\Delta t][1 - \lambda_{M-1}\Delta t]^{M-1}$$

where

a) $[1 - \lambda_M \Delta t]^M$ and $[1 - \lambda_{M-1}\Delta t]^{M-1}$ are as described above; and,

b) the "1" occurs because the probability of zero launches during Δt given that the maximum number M is on-orbit is 1.00.

In order to make the equations more tractable, the power terms can be expanded, using the binomial expansion to achieve the following substitution:

$$(1 + \Theta\Delta t)^n = 1 + n\Theta\Delta t + [\text{terms with powers of } \Delta t \geq 2]$$

$$(1 + \Theta\Delta t)^n = 1 + n\Theta\Delta t + o(\Delta t) \text{ where } \Theta = \text{a constant.}$$

Using some algebra and taking the limit as Δt approaches zero. [which justifies our ignoring terms in $o(\Delta t)$], the following finite set of differential-difference equations is obtained:

$$(2.4)\qquad P_o'(t) = -\mu_o P_o(t) + \lambda_1 P_1(t) \qquad n=0$$

$$(2.5)\qquad P_n'(t) = -(n\lambda_n + \mu_n) P_n(t) + \mu_{n-1} P_{n-1}(t) + (n+1)\lambda_{n+1} P_{n+1}(t) \qquad 0 < n < M$$

$$(2.6)\qquad P_M'(t) = -M\lambda_M P_M(t) + \mu_{M-1} P_{M-1}(t) \qquad n=M$$

2.1.1 The Transient State Solution

The finite system of equations derived above may now be solved. Unfortunately, an actual solution description requires knowledge of a specific value of M and a great deal of algebraic manipulation. For completeness, a solution procedure is briefly outlined in this section.

Since the system of equations are differential in form, they must be subject to a set of boundary conditions in order to admit a finite and real solution possessing the standard properties of probability density functions. The boundary conditions are as follows:

$$P_n(0) = \begin{cases} 1 & \text{if } n=o \\ 0 & \text{if } n>o \end{cases}$$

The initial step in determining the transient-state solutions is to convert the equations to closed forms not employing derivatives. Define the function $\phi_n(\Theta)$ as

$$\phi_n(\Theta) = \int_0^\infty e^{-\Theta t} P_n(t)\, dt$$

Using integration by parts, the following expression may be derived:

$$\int_0^\infty e^{-\Theta t} P_n^1(t)\, dt = \Theta\phi_n(\Theta) - P_n(0)$$

Thus, equations (2.4), (2.5), (2.6) may be multiplied by $e^{-\Theta t}$ and integrated between zero and infinity with respect to the variable t to yield.

(2.7) $$(\Theta + \mu_o)\,\phi_o(\Theta) - \lambda_1\phi_1(\Theta) = 1$$

$$(2.8)\qquad -\mu_{n-1}\,\phi_{n-1}(\Theta) + (\Theta + n\lambda_n + \mu_n)\,\phi_n(\Theta) - (n+1)\lambda_{n+1}\,\phi_{n+1}(\Theta) = 0 \qquad 0 < n < M$$

$$(2.9)\qquad -\mu_{M-1}\,\phi_{M-1}(\Theta) + (\Theta + M\lambda_M)\,\phi_M(\Theta) = 0\ ,$$

a system of M+1 linear equations in the M+1 variables $\phi_o(\Theta), \ldots, \phi_M(\Theta)$.

These equations can be solved simultaneously for $\phi_j(\Theta)$ and then inverted for $P_n(t)$, using the method of the residues or using partial fraction expansion. Complexity of the general closed form expressions for $P_n(t)$ prohibits solution herein and, in any case, is of no interest relative to the remainder of the analysis.

2.1.2 The Steady-State Solution

Since large scale simulation models compute long-run expected availability, the steady-state solutions are of more importance to this study. The typical steady-state assumption requires a significant passage of time to allow routine and repetitive activities to describe the system. By computing expected availability over a large number of simulation trials, a computer simulation model is, in effect, allowing enough time to pass to evaluate all possible combinations of sequences of operations activities which could conceivably effect the expected availability of a satellite system.

Mathematically, one allows $t \to \infty$. The resultant effect upon equations (2.4), (2.5), and (2.6) is that $P_n'(t) = 0$ (routine activities do not allow for changes in activity levels relative to time changes) and that $P_n(t) \to P_n$ (if the rate of change of P_n is zero, the value of P_n must stabilize). Additionally, one must add the constraint that the total probability must equal to one.

$$(2.10)\qquad -\mu_o\,P_o + \lambda_1\,P_1 = 0$$

(2.11) $$\mu_{n-1} P_{n-1} - (n\lambda_n + \mu_n) P_n + (n+1)\lambda_{n+1} P_{n+1} = 0$$

$0 < n < M$

(2.12) $$\mu_{M-1} P_{M-1} - M\lambda_M P_M = 0$$

(2.13) $$P_o + --- + P_M = 1$$

The above equations contain one redundant equation in (2.11) which can be eliminated yielding a system of M+1 linear equations to be solved simultaneously for the M+1 unknowns $P_o, \ldots, P_M$

Theorem 1: The expected on-orbit availability of a satellite system requiring a minimum of K out of M ($K \leq M$) satellites on-orbit is defined by

$E(A) = P_k + \ldots + P_M \quad [1 \leq K \leq M]$

where

$$P_n = \frac{\prod_{j=1}^{n} \left(\frac{\mu_{j-1}}{\lambda_j}\right)}{n!} P_o \qquad n = 1, \ldots, M$$

and

$$P_o = \left\{ 1 + \sum_{n=1}^{M} \frac{\left[\prod_{j=1}^{n} \left(\frac{\mu_{j-1}}{\lambda_j}\right)\right]}{n!} \right\}^{-1}$$

Proof =

From (2.10), $$P_1 = \frac{\mu_o}{\lambda_1} P_o = \frac{\prod_{j=1}^{1} \left(\frac{\mu_{j-1}}{\lambda_j}\right)}{1!} P_o$$

From (2.11),

$$2\,\lambda_2\,P_2 = (\lambda_1 + \mu_1)\,P_1 - \mu_o\,P_o$$

$$2\,\lambda_2\,P_2 = (\lambda_1 + \mu_1)\,\frac{\mu_o}{\lambda_1}\,P_o - \mu_o\,P_o$$

$$P_2 = \frac{\left(\frac{\mu_1\,\mu_o}{\lambda_2\,\lambda_1}\right)}{2}\,P_o = \frac{\prod_{j=1}^{2}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{2!}\,P_o$$

Assume

$$P_{n-1} = \frac{\prod_{j=1}^{n-1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(n-1)!}\,P_o$$

and

$$P_n = \frac{\prod_{j=1}^{n}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{n!}\,P_o$$

From (2.11)

$$(n+1)\,\lambda_{n+1}\,P_{n+1} = (n\,\lambda_n + \mu_n)\,P_n - \mu_{n-1}\,P_{n-1}$$

$$(n+1)\,\lambda_{n+1}\,P_{n+1} = (n\lambda_n + \mu_n)\,\frac{\prod_{j=1}^{n}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{n!}\,P_o - \mu_{n-1}\left[\frac{\prod_{j=1}^{n-1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(n-1)!}\right]P_o$$

$$(n+1)\lambda_{n+1} P_{n+1} = \left[\frac{(n\,\lambda_n + \mu_n)}{n}\left(\frac{\mu_{n-1}}{\lambda_n}\right) - \mu_{n-1}\right]$$

$$\left[\frac{\prod_{j=1}^{n-1} \frac{\mu_{j-1}}{\lambda_j}}{(n-1)!}\right] P_o$$

$$(n+1)\,\lambda_{n+1} P_{n+1} = \left[\frac{n\,\lambda_n\,\mu_{n-1} + \mu_n \mu_{n-1} - n\,\lambda_n\,\mu_{n-1}}{n\lambda_n}\right]$$

$$\left[\frac{\prod_{j=1}^{n-1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(n-1)!}\right] P_o$$

$$P_{n+1} = \frac{\left(\frac{\mu_n}{\lambda_{n+1}}\,\frac{\mu_{n-1}}{\lambda_n}\right)\prod_{j=1}^{n-1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(n+1)\;(n)\;(n-1)!} P_o = \frac{\prod_{j=1}^{n+1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(n+1)!} P_o$$

Thus, by mathematical induction, the analysis shows that

$$P_n = \left[\frac{\prod_{j=1}^{n}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{n!}\right] P_o \qquad 1 \leqslant n < M$$

From (2.12) $\quad M\,\lambda_M P_M = \mu_{M-1} P_{M-1}$

By the induction argument, a substition for P_{M-1} is in order:

$$M\,\lambda_M P_M = \mu_{M-1}\left[\frac{\prod_{j=1}^{M-1}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{(M-1)!}\;P_o\right]$$

$$P_M = \frac{\prod_{j=1}^{M}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{M!} P_o \qquad n = M$$

completing the first part of the proof.

From (2.13)

$$P_o + \sum_{n=1}^{M}\left[\frac{\prod_{j=1}^{n}\left(\frac{\mu_{j-1}}{\lambda_j}\right)}{n!}\right] P_o = 1$$

$$P_o = \left\{1 + \sum_{n=1}^{M}\left[\frac{\prod_{j=1}^{n}\frac{\mu_{j-1}}{\lambda_j}}{n!}\right]\right\}^{-1}$$

completing the proof of Theorem 1.

<u>Corallary 1.1</u>: The parameters

$\left\{\lambda_n\right\}$ = expected number of failures per unit time as a function of the number of satellites on-orbit, and

$\left\{\mu_n\right\}$ = expected number of replenishments per unit time as a function of the number of satellites on-orbit,

are the appropriate aggregate level indicators described in the introduction from which on-orbit availability can be computed.

The result of this section has been to provide operations and budget planners with an analytical scheme for identifying the most important components of and for computing expected availability of an orbiting satellite system. The next section identifies how these same aggregate parameters may be employed to approximate expected life-cycle costs.

2.2 The Expected Life-Cycle Cost Model

Since the parameters $\left\{\lambda_n, \mu_n\right\}$ maintain the most significant impact upon expected availability, there should exist an analogous analytical expression for expected life-cycle cost which employs the aggregate information represented by the parameters $\left\{\lambda_n, \mu_n\right\}$. Moreover, since the occurrence of a rate of activity summarized by $\left\{\lambda_n, \mu_n\right\}$ can now be described according to a probability, as described in Theorem 1, a true expectation of life-cycle costs may be computed.

The major contributors to life-cycle costs may be categorized as Research, Development, Testing, and Evaluation (RTDE) costs, Production (P) costs, Launch (L) costs, and Inventory Carrying (I) costs. The production costs are a linear function of the cost per unit to produce a single satellite and the weighted replenishment rate. Since required replenishments are assumed to drive the production schedule, launch costs are also assumed to be directly proportional to the weighted replenishment rate, as well as the cost per launch. Finally, the inventory carrying costs are a function of the cost per satellite per unit time of ground storage and the rate at which inventory is accumulated. Additionally, RDTE costs are assumed to be a constant amount, from the perspective of failures and replenishments.

The rate of inventory buildup is assumed to be the difference between the replenishment rate and the failure rate, assuming that replenishments occur at a faster rate than failures. This is a quite tenable assumption since an active on-orbit complement can only occur if the number of replenishments exceeds the number of failures. Otherwise, the satellites would depart from the orbital configuration faster than they could be replenished, leaving no active orbiting satellites, and the system would never be available to a user.

The above considerations may be combined to provide the following analytical expression for expected life-cycle costs as a function of the parameter set $\{\lambda_n, \mu_n\}$:

Theorem 2: The expected life-cycle costs E(LCC) of an on-orbit satellite system characterized by the parameter set $\{\lambda_n, \mu_n\}$ may be approximated by the expression.

$$E(LCC) = RDTE + C_P T_1 \sum_{n=o}^{M} P_n \mu_n + C_L T_2 \sum_{n=o}^{M} P_n \mu_n$$

$$+ C_I T_3 \sum_{n=o}^{M} \left[P_n (\mu_n - \lambda_n) T_4 \right]$$

where

RDTE = constant Research, Development, Testing, and Evaluation Costs;

C_p = cost per unit to produce a single satellite;

C_L = cost per launch;

C_I = cost/satellite/time unit of ground storage;

T_j = appropriate normalizing factors to convert costs from a time unit basis to a life-cycle basis;

$\sum_{n=o}^{M} (\cdot)$ = weighted average of the relevant activity parameters.

The budget planner may now employ the tools in Theorems 1 and 2 to investigate the cost effectiveness of various design strategies $\left(\text{represented by } \{\lambda_n\}\right)$ and replenishment strategies $\left(\text{modeled by } \{\mu_n\}\right)$ for the procurement of satellite systems. Of course, the critical reader may question the validity of some of the assumptions made earlier. However, a careful consideration comparing Theorem 1 to the assumptions made early in Section 2.1 reveals an interesting phenomenon. While the assumptions relate to specific time increments, the results of Theorem 1 relate the state of the on-orbit system to the expected activity levels summarized by the parameter set $\{\lambda_n, \mu_n\}$. Moreover, at the means, any perturbations engendered by the assumptions will essentially be smoothed or eliminated. This is the real value of using the state-space, Markov process modelling approach. Thus, the only critical assumption is the existence of the aggregate parameters $\{\lambda_n, \mu_n\}$ and their adequacy and utility in measuring or summarizing design and replenishment decisions. This consideration is the subject of Sections 3.0 and 4.0.

2.3 Special Cases of Interest

Two major special cases are of interest: no on-orbit spares and N on-orbit spares. These are the two most common replenishment strategies modelled in the large-scale simulation models to be discussed. Each of these strategies should be reflected in the form of $\{\mu_n\}$. For simplicity, in each case, the failure rates are assumed to be constant. Additionally, the expression for expected life-cycle costs must be modified to reflect the specialized definitions of $\{\lambda_n, \mu_n\}$.

2.3.1 No On-Orbit Spares

For this case, K out of M active satellites are required on orbit to achieve availability. The expected replenishment rates are constants reflecting launch patterns and launch delays.

$$\lambda_n = \lambda \qquad n = 0, \ldots, M$$

$$\mu_n = \mu = 1/(MTBL + MLD) \qquad n \geq 0$$

where

MTBL = Mean Time Between Launches; and,

MLD = Mean Launch Delay.

Using the constants above, the expressions in Theorems 1 and 2 reduce to

$$P_n = \left[\left(\frac{\mu}{\lambda}\right)^n \Big/ n! \right] P_o \qquad n = 1, \ldots, M$$

$$P_o = \left\{ \sum_{n=o}^{M} \left[\left(\frac{\mu}{\lambda}\right)^n \Big/ n! \right] \right\}^{-1}$$

$$E(LCC) = RDTE + C_P\ T_1\ \mu + C_L\ T_2\ \mu + C_I\ T_3 \left[(\mu - \lambda)\ T_4 \right].$$

2.3.2 N On-Orbit Spares

In this operational scheme, the total number of satellites must reflect the transitioning of on-orbit spares to active satellite positions. The transitioning of on-orbit spares is characterized by a faster replenishment rate. Moreover, only the last N active satellites may be replenished from orbital spares. The spares themselves are assumed to be replenished by ground launches. For example, consider a satellite system with four active positions and two orbiting spares. Only the third and fourth active positions may be replenished from the orbital spares. All other active spacecraft and the spares themselves must be replenished by ground launches.

$$\lambda_n = \lambda \qquad n = 0, \ldots, M$$

$$\mu_n = \begin{cases} \mu_1 = \dfrac{1}{MTBL + MLD} & 0 \leq n \leq M - N \\ \mu_2 = \dfrac{1}{OTD} & M - N < n \leq M \\ \mu_1 = \dfrac{1}{MTBL + MLD} & M < n \leq M + N \end{cases}$$

where

MTBL = Mean Time Between Launches;

MLD = Mean Launch Delay; and,

OTD = Mean Orbital Transfer Delay.

The form of the expressions described in Theorems 1 and 2 now becomes

$$P_n = \begin{cases} \dfrac{\mu_1^{\,n}}{n!\ \lambda^n} P_o & 1 \le n \le M - N \\[2ex] \dfrac{\mu_1^{\,M-N}\ \mu_2^{\,n-N}}{n!\ \lambda^n} P_o & M - N < n \le M \\[2ex] \dfrac{\mu_1^{\,n-N}\ \mu_2^{\,N}}{n!\ \lambda^n} P_o & M < n \le M + N \end{cases}$$

$$P_o = \left\{ 1 + \sum_{n=1}^{M-N} \frac{\mu_1^{\,n}}{n!\ \lambda^n} + \sum_{n=M-N+1}^{M} \frac{\mu_1^{\,M-N}\ \mu_2^{\,n-N}}{n!\ \lambda^n} + \sum_{n=M+1}^{M+N} \frac{\mu_1^{\,n-N}\ \mu_2^{\,N}}{n!\ \lambda^n} \right\}^{-1}$$

$$E(LCC) = RDTE + C_P\ T_1\ \mu_1 + C_L\ T_2\ \mu_1 + C_I\ T_3 \left[\left(\mu_1 \sum_{n=0}^{M} P_n \right) - \lambda \right] T_4$$

This modified expected life-cycle cost formula reflects the fact that the orbital transition rate in μ_2 does not engender production costs (C_p) or launch costs (C_L). In other words, the use of on-orbit spares is assumed to affect only the expected inventory costs. Moreover, the expected rate of inventory accumulation must be modified to incorporate the fact that the replenishment rate μ_1 is only in effect during the states in which no on-orbit spares are available. This explains the appearance of the term $\sum_{n=o}^{M} P_n$.

3.0 COMMONLY USED SIMULATION MODELS

Procurement planning for satellite systems is accomplished by the U.S. Air Force predominantly at the Air Force Space and Missile Systems Organization (SAMSO)[1] in El Segundo, California. The Air Force personnel are familiar with three large scale simulation tools available to analyze the cost effectiveness of procurement strategies. By far, the most well known model is the General Availability Program (GAP) operated for SAMSO by the Aerospace Corporation. The GAP program (and its many versions) plays a significant role in the budgetary planning process at SAMSO. However, that role is of no interest here, since this is the subject of other current study efforts.

More recently, a large scale simulation model called the On-Orbit Spares Analysis (OOSA) has become available for analysis of procurement policies. The OOSA model was developed for SAMSO by ECON, Inc., a commercial contractor. The interactive, operational version of OOSA is housed on the Space Launch Systems Program Office Minicomputer and is available for use by all payload offices, without charge. Finally, The Rand Corporation's *Spacecraft Acquisition Strategies* Project has employed the Satellite Availability Simulation Program (SASP) for analytical purposes. The SASP model was developed by Major A. Gary Parish, USAF, assigned to Rand by the Air Force. Documentation for all of these models is generally available to the public and a listing of sources appears in the references, where the test case data is discussed.

With some relatively significant variations, all three simulation models operate upon the basis of the same fundamental sequence of activities. During the operations phase of the satellite system's life cycle, individual satellites are launched, failed, and replenished according to some failure model which represents individual spacecraft reliability and according to an individual program's preferences regarding various aspects of replenishment. At the end of the operations phase, the availability and/or operations costs are computed. This process is repeated for a large number of operations phases creating

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[1]The Air Force Space and Missile Systems Organization (SAMSO) has been redesignated the Air Force Space Division (SD).

a statistical data base regarding costs and/or performance. From this data base, detailed statistical information can be generated. More importantly for this study, both expected availability and expected life-cycle costs may be (and usually are) computed.

In making comparisons among the various models, one encounters some significant problems, some of which are organizational in nature and some of which are methodological in nature. One of the functions in introducing the $\{\lambda_n, \mu_n\}$ as aggregate measures is to achieve a level of standardization for comparative purposes. However, comparison of the results is tenuous due to the fundamental methodological differences employed in obtaining the expected output measures--availability and life-cycle costs. However, this study does not use the simulation outputs to make value judgments concerning individual models. The comparisons to be performed are employed to lend credence to the analytical expressions in Theorems 1 and 2.

From the perspective of access, GAP is not an interactively oriented simulation program while OOSA and SASP are geared predominantly towards interactive use. In terms of aggregate output measures, OOSA deals with both costs and performance (in the form of availability); neither GAP nor SASP computes aggregate costs, both concentrating primarily upon availability. All three have assumed underlying failure and replenishment models. The remainder of this section analyzes each of the simulation programs according to these underlying models. The analyses in the section below are based upon the simplest version of the simulation program (when more than one version exists).

3.1 The General Availability Program (GAP)

The GAP program was the first major SAMSO attempt to place procurement strategy on an analytical basis. Failures are assumed to be of three types: infancy (which occurs with probability p) and steady-state and wearout (which occur jointly with probability 1-p). Thus, the failure model requires estimation of three sets of reliability parameters--one set for infancy failures, one set for steady-state failures, and one set for wearout failures--and estimation of the parameter p, leading to the failure model:

$$R_{s/c} = p R_I + (1-p)\, R_{ss} R_w,$$

where

$R_{s/c}$ = reliability of the spacecraft;

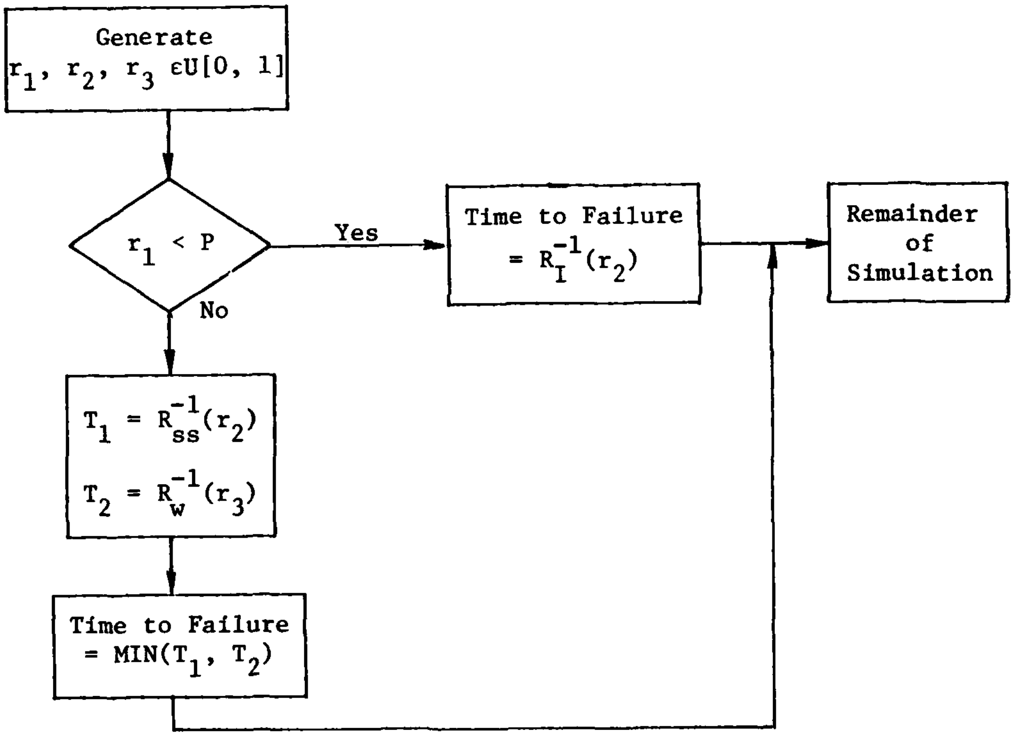
P = probability of an infancy failure;

R_I = reliability of infancy failures;

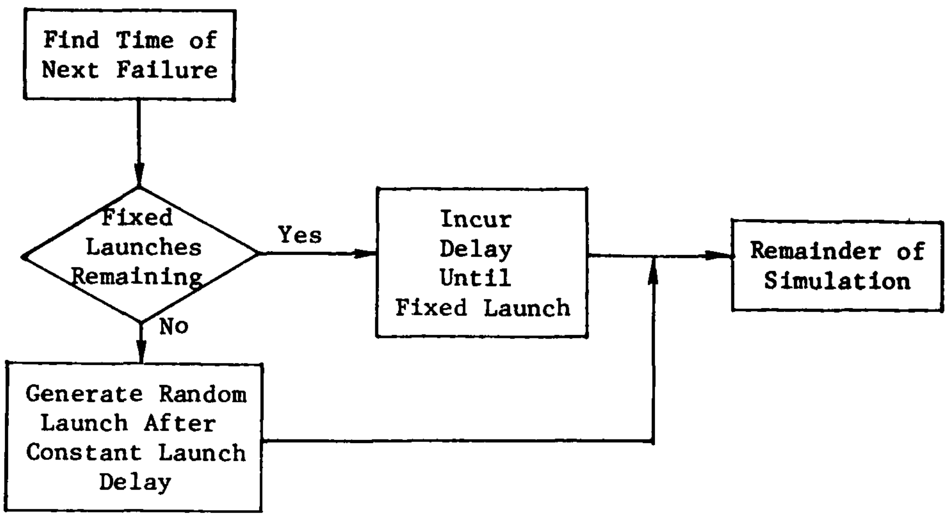
R_{ss} = reliability of steady-state failures;

R_w = reliability of wearout failures.

A variety of forms is used for each of the reliability curves above and, of course, each curve has its own parameters. The program logic uses the failure model to generate failure times according to the following algorithm (1, p. A-13):



The GAP replenishment model distinguishes between fixed and ramdom launches. In the basic version of GAP, no on-orbit spares are allowed. When an active satellite on orbit fails, the program checks to determine if a fixed launch has been scheduled. If so, a delay is incurred until the fixed launch can occur. If all fixed launches have occurred, a random launch must be generated with a constant launch delay. The replenishment model can be represented in the logic form given below (1, p. 11).



The computation of availability is performed against the time standard of initial achievement of K active satellites (out of M) required for minimal availability (1, p. 10). In essence, this approach ignores the effect of any infancy failures prior to that point in time, providing a slightly higher availability measure. The GAP program does not compute costs at all.

3.2 The On-Orbit Spares Analysis (OOSA)

The OOSA model uses similar methodological approaches to GAP with the added advantages of modelling the use of on-orbit spares as a replenishment strategy, of being interactively oriented, and of assessing relative cost impacts on a total life-cycle basis. The failure model in OOSA is defined differently from the failure model of GAP. The closed form of the OOSA failure model may be represented in the following form:

$$R_{s/c} = R_s R_p R_c$$

where

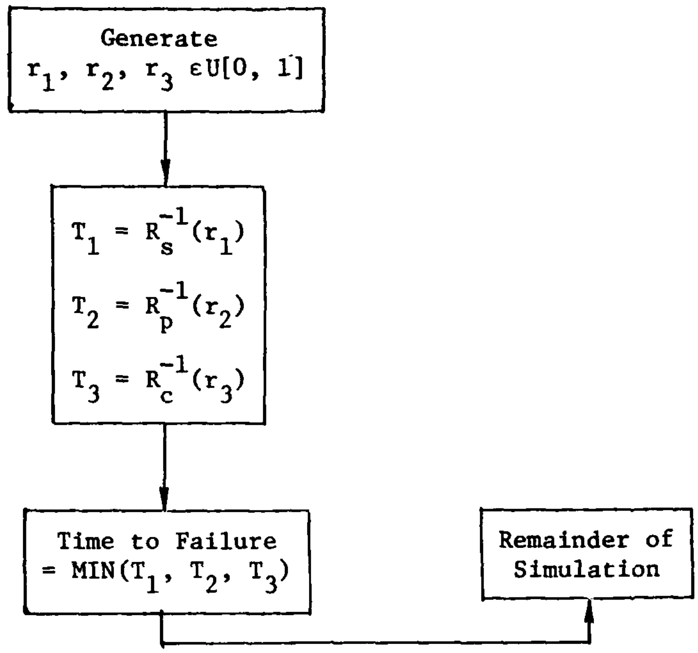
$R_{s/c}$ = reliability of the spacecraft;

R_s = reliability of supporting functions;

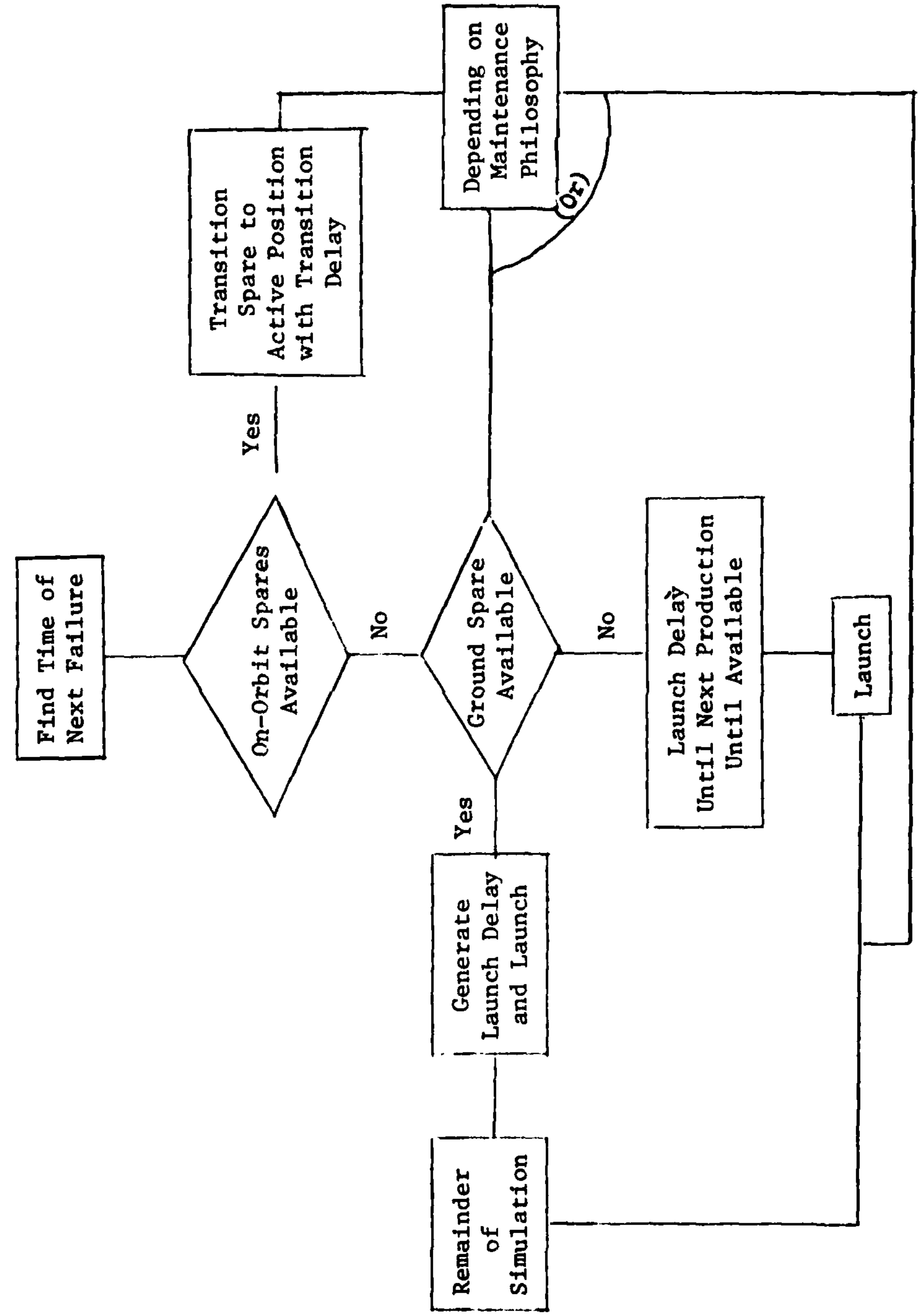
R_p = reliability of the payload;

R_c = reliability of the consumables.

For this model, three sets of parameters must also be estimated--one set for each of R_s, R_p, and R_c. However, the reliability model appears to be more realistic than the mixed model employed in GAP. The failure model above is implementing using the logic below (2, p. 2-31):



Since OOSA specifically addresses the use of on-orbit spares, the replenishment model is quite complex. When an active satellite on orbit fails and the simulation user has specified the allowance of on-orbit spares, a check must first be accomplished to see if any orbiting spares may be transitioned to active positions. The replenishment mechanism may then take one of several courses of action depending upon which maintenance philosophy has been adopted by the user (the model identifies thirteen maintenance philosophies). In general, assuming on-orbit spares are employed, the replenishment model may be represented as follows (2, p. 2-17).


Find Time of Next Failure
On-Orbit Spares Available
Yes
No
Transition Spare to Active Position with Transition Delay
Ground Spare Available
Yes
No
Depending on Maintenance Philosophy
(Or)
Generate Launch Delay and Launch
Launch Delay Until Next Production Until Available
Launch
Remainder of Simulation

Since the failure model does not distinguish infancy failures, availability of the satellite system is computed upon the basis of the time initialization of the operations phase (2, p.2-37). However, the user specifies production and launch availability inputs relative to an absolute zero time labeled Full Operational Capability (FOC) (2,p.2-5). The effect of this time referencing allows some satellites to be produced and launched prior to the initialization of the operations phase of the life cycle, the ultimate impact being that availability estimates are slightly inflated since a steady-state will have been achieved before availability computations are performed.

As mentioned earlier, the OOSA program is the only program of the three presented which actually performs an economic analysis. The cost analysis in the RDTE and production phases consists of spreading total costs over the years in which they were actually incurred and applying learning curve effects to the production costs. Thus, the RDTE and production phases of OOSA are deterministic in nature. The operations phase cost analysis, however, must track the costs--launch and ground storage--as a function of the stochastic activities generated during the operations simulation trials. Since all cost expenditures are attributed to the years incurred, the costs for each simulation trial may be discounted and inflated, both of which are automatically performed by the OOSA program. A large number of detailed cost information printouts are available, providing the simulation user with an amazing amount of insight into the sensitivity of budget expenditures to a variety of operational and development strategies. Of course, the expected on-orbit availability and expected life-cycle costs are automatically output as a result of a simulation execution.

3.3 The Satellite Availability Simulation Program (SASP)

The final simulation model to be presented is The Rand Corporation's SASP program. Although written independently from the GAP model, the SASP program operates in a manner similar to GAP. The SASP program, however, has not been subjected to an extensive modificaiton process induced by a variety of users. For this reason, the failure and replenishment models are somewhat simpler than the GAP models. The form of the SASP failure model is a mixture process described by

$$R_{s/c} = (1-p)\ R_{ss} + pR_w$$

where

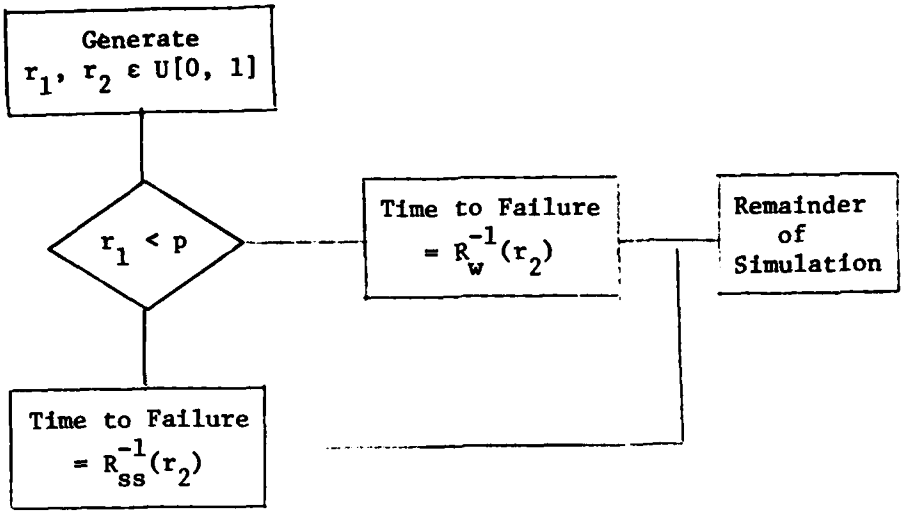
$R_{s/c}$ = reliability of the spacecraft;

p = percent of time the wearout reliability dominates the piece-part reliability;

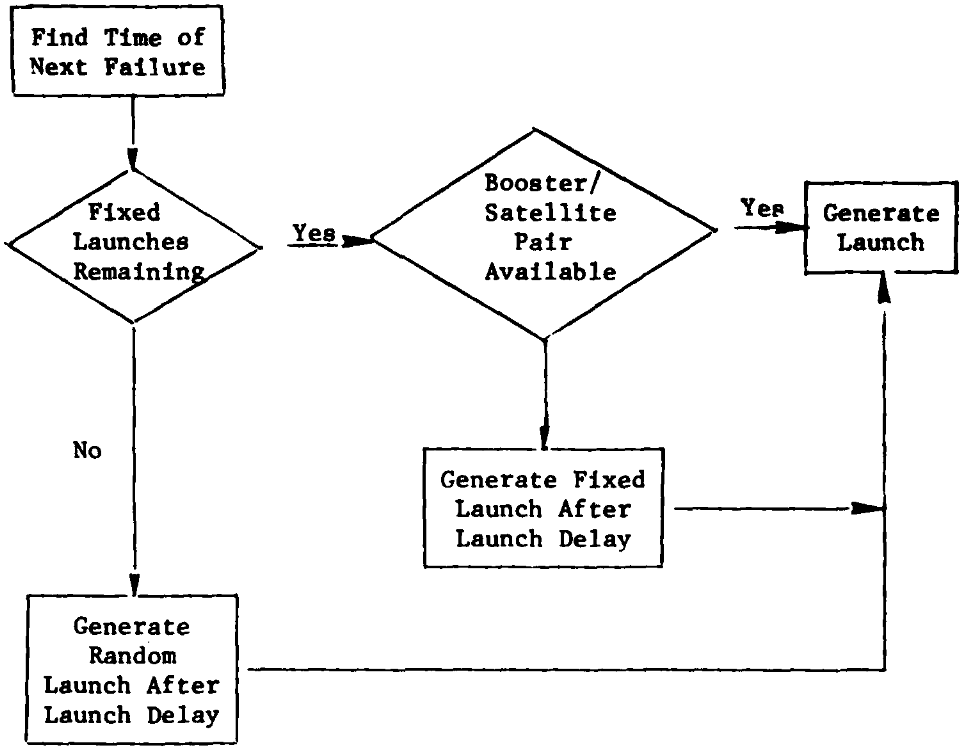
R_{ss} = steady-state or piece-part reliability function derived by standard techniques explained in Mil HdbK 217B; and,

R_w = wearout reliability function.

This simplified failure model requires the estimation of two sets of parameters--one set for R_{ss} and one set for R_w--and estimation of the parameter p. However, the SASP User's Guide specifically states that unless the user desires to test alternative hypotheses about the validity of piece-part reliability functions, the parameter p should be set to zero, eliminating the need to estimate p and the parameters for R_w and reducing the SASP model to a steady-state dominated failure process (3, p.2). The algorithmic representation of the failure model is easily visualized as follows:



The approach to replenishment employed by SASP is also relatively simple compared to both GAP and OOSA. The SASP program, like GAP, makes no provision for on-orbit spares. However, the approach is unique in the sense that both the satellite and the launch mechanism must be available for a replenishment to occur (3, p. 5). This mating schedule is input into the program by the user. Additionally, the SASP program distinguishes between fixed and random launches, as does the GAP program. The replenishment model may be characterized in the following flowchart format (3, p. 5):



Examination of the actual SASP computer code reveals that the expected availability is computed after an initial orbital configuration is established (3, p. 20,23). This is equivalent to the approaches employed by GAP and OOSA. Moreover, this is consistent with a failure model which essentially treats operations phase activities as steady state in nature (the parameter p is set by zero). The criterion employed to compute availability is that at least K out of M operational satellites must be maintained on orbit in order for the system to be classed as available. As with the other models, a large number of output plots are accessible to the user. The expected availability measure is the only output relevant to this study. Costs are not considered in any form.

The chief virtue of the SASP model is its relative simplicity, both from an algorithmic perspective and from a user access orientation. The user can trace the derivation of output measures from the input parameters, inviting a relative level of confidence in the outputs.

The algorithmic simplicity insures relatively simple inputs which are easily organized and understood.

This section has attempted to summarize some of the major aspects of large scale simulation models employed to investigate the impact of replenishment strategies upon the cost effectiveness of a satellite system maintained on-orbit. In each of the three models examined, only the basic version of the model was represented. For each program, the underlying failure and replenishment models were presented and the availability measure was analyzed. While basic differences do exist regarding the fundamental modeling approaches of each of the three alternatives, some commonality is apparent. For the most part, the expected on-orbit availability is computed according to the requirement that at least K out of M satellites must be maintained on orbit. Additionally, certain of the parameter inputs from which expected availability may be computed are required in all three models. For a comparative input/output analysis, see Figure 1. Specifically, reliability distribution parameters, and production or launch schedules are used as required inputs. The reliability parameters may be employed to compute the aggregate $\{\lambda_n\}$ while the launch schedules may be utilized to derive the replenishment parameters $\{\mu_n\}$, with some variations required to represent the use of on-orbit spares.

Using these aggregated $\{\lambda_n, \mu_n\}$ in the analytical forms of Theorems 1 and 2 should yield expected costs and performance measures which are roughly approximate to the corresponding simulation outputs. This comparison is the subject of the remainder of this study.

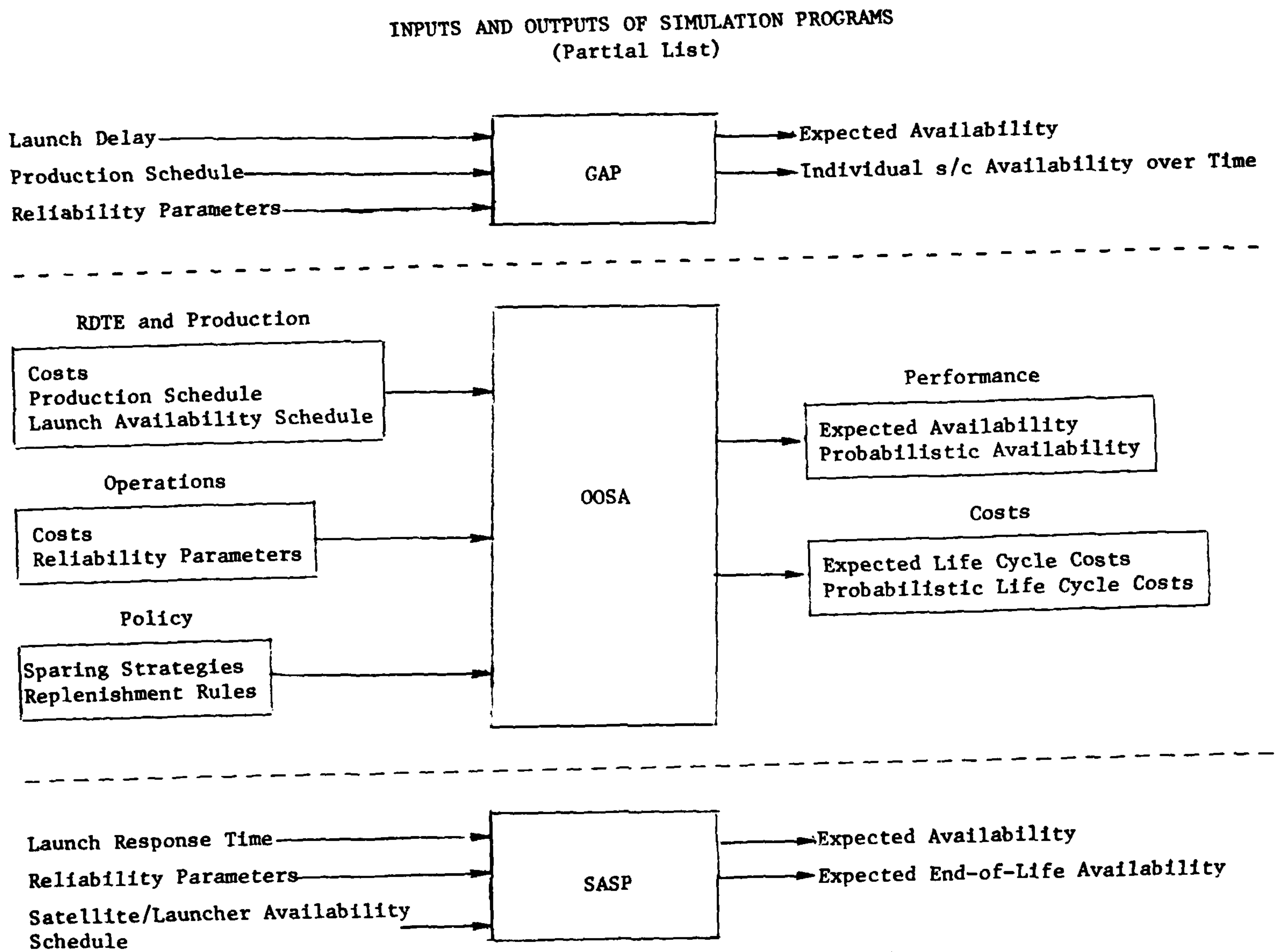


Fig. 1--Comparative Simulation Model Inputs and Outputs

4.0 TEST CASES

The purpose of this section is to compare the closed form approximations of expected availability and expected life-cycle costs with the simulated results. **Five cases are** presented. Derivations of the aggregate parameters $\{\lambda_n, \mu_n\}$ appear in Section 4.1. A brief discussion comparing closed form and simulation outputs is the content of the second subsection, Section 4.2. Finally, Section 4.3 contains some general comments regarding the applicability of simulation and closed form expressions. This section of the study is both important and interesting because of the evidential strength lent to the claim regarding the appropriateness of the aggregate level approach to modelling.

4.1 Derivation of Aggregate Inputs

The major inputs used to derive GAP aggregate parameters $\{\lambda_n, \mu_n\}$ are MMD = Mean Mission Duration = $\alpha \Gamma \left(1 + \frac{1}{\beta}\right)$

where α, β = parameters of Wiebull reliability function for R_{ss};

PS = $\{t_1, \ldots, t_p\}$ = a sequence of production times;

MLD = deterministic launch delay time.

M = maximum number of allowable active satellites in a constellation;

Using these inputs, the aggregate parameters may be computed as

$$\lambda_n = \lambda = \frac{1}{MMD} \qquad \text{for} \qquad n = 0, \ldots, M$$

$$\mu_n = \mu = \frac{1}{MTBL + MLD} \qquad \text{for} \qquad n = 0, \ldots, M$$

where $MTBL = \frac{t_p - t_1}{P}$ = mean time between launches.

Since no on-orbit spares are allowed, one can see that the GAP program is simply an instance of Special Case 2.3.1 No On-Orbit Spares

Since the failure and replenishment models are significantly different for OOSA, different inputs are employed for determination of the aggregate parameter set, as follows:

DL = design life of the satellite in hours;

$LS = \{ t_1, \ldots, t_p \}$ = launch availability schedule in days relative to Full Operational Capability (FOC);

MLD = mean launch delay;

OTD = mean orbital transfer delay;

M = maximum number of allowable active satellites in a constellation;

N = maximum number of allowable on-orbit spares.

The aggregate parameters may now be computed from the above numbers as

$$\lambda_n = \lambda = \frac{1}{DL} \quad (24 \text{ hours/day}) \quad \text{for} \quad n = 0, \ldots, M$$

$$\mu_n = \begin{cases} \mu_1 = \dfrac{1}{MTBL + MLD} & \text{for} \quad 0 \leq n \leq M - N; \\ & \phantom{\text{for} \quad} M < n \leq M + N \\ \mu_2 = \dfrac{1}{OTD} & \text{for} \quad M - N < n \leq M \end{cases}$$

$$\text{where MTBL = mean time between launches} = \frac{t_p - t_1}{p}.$$

Moreover, the same parameter set $\{\lambda_n, \mu_n\}$ yields an expression for expected life-cycle costs from Theorem 2 =

$$E(LCC) = RDTE + C_P\, T\, \mu_1 + C_L\, T\, \mu_1 + C_{IF}\, (24 \text{ hours/day})\, T \left(10^{-6}\right)$$
$$+ C_{IV}\, (24 \text{ hours/day})\, T \left(10^{-6}\right) \left[\left(\mu_1 \sum_{n=o}^{M} P_n\right) - \lambda\right] T_4$$

where some additional OOSA inputs are necessary. The inputs are listed below:

RDTE = fixed research, development, testing, and evaluation costs $\left(\text{in } \$10^6\right)$;

C_P = production costs per satellite $\left(\text{in } \$10^6\right)$

= (Total Production Costs)/(Number of Satellites);

C_L = cost per launch vehicle $\left(\text{in } \$10^6\right)$;

C_{IF} = Fixed inventory carrying costs per hour;

C_{IV} = Variable inventory carrying cost per hour per satellite;

T = 3600 days per ten-year operation's phase of a life cycle;

10^{-6} = necessary to convert inventory costs to same level of magnitude as other costs.

Since on-orbit spares are incorporated into the analysis, the OOSA approach is a specific example of Special Case 2.3.2 On-Orbit Spares

Aggregating the SASP inputs to determine $\{\lambda_n, \mu_n\}$ also requires a different strategy. The input parameters used from SASP are listed as

$$MMD = \text{mean mission duration} = \alpha\, \Gamma\left(1 + \frac{1}{\beta}\right)$$

where α, β = parameters of the Wiebull reliability function for R_{ss};

L = length of satellite operation's simulation;

M = maximum number of allowable active satellites on orbit.

The computations for $\{\lambda_n, \mu_n\}$ are

$$\lambda_n = \lambda = \frac{1}{MMD} \qquad \text{for} \qquad n = 0, \ldots, M$$

$$\mu_n = \mu = \frac{1}{MTBL} \qquad \text{for} \qquad n = 0, \ldots, M$$

where $MTBL = \frac{L}{MMD}$ = mean time between launches.

Thus, the SASP model can be represented as an application of Special Case 2.3.1 No On-Orbit Spares.

4.2 Comparison of Simulation and Closed Form Outputs

Five basic cases were chosen for comparing simulation results and analytical results. OOSA-1 is a case allowing zero on-orbit spares. OOSA-2 analyzes the use of, at most, two on-orbit spares. GAP-1 and GAP-2 compare the effect of increased Mean Mission Duration (reflecting higher reliability satellites). The case from SASP is supplied to provide additional support for the closed form solution. The results of the analysis are summarized in Figure 2 in which $\hat{A}_s$, $\widehat{LCC}_s$ are simulation outputs and $\hat{A}_c$, $\widehat{LCC}_c$ are closed form estimates derived by the formulas in Theorems 1 and 2.

The OOSA and GAP cases demonstrate the internal consistency of the expressions for P_n. Specifically, the addition of on-orbit spaces changes $\hat{A}_c$ from .994 to .9999 (increased availability). Additionally, the $\widehat{LCC}_c$ from Theorem 2 is sensitive to cost savings accrued from the use of on-orbit spares, decreasing from \$889M to \$828M from the decision to use two on-orbit spares. This is a direct result of an assumption made earlier in which production and launch costs remain constant with respect to the use of on-orbit spares and only ground storage costs were assumed to effect the expected life-cycle costs. A comparison of the cost categories as computed in Appendix B is presented in Figure 3. The simple analytic computations reveal a significant

SIMULATION AND CLOSED FORM INPUTS AND OUTPUTS

Simulation	$\hat{A}_s$	$\widehat{LCC}_s$	Source	Inputs	Source	$\{\lambda_n, \mu_n\}$	$\hat{A}_c$	$\widehat{LCC}_c$
OOSA-1	.991	956	2, Vol. III	DL = 61386 HOURS LS = $\{t_1 = -459, \ldots, t_{10} = 918\}$ MLD = 720 HOURS = 30 DAYS OTD = 168 HOURS = 7 DAYS M = 4 N = 0 C_P = 504.5 FOR 20 SATELLITES C_L = 8 x 10^6 PER L/V C_{IF} = \$174 PER HOUR C_{IV} = \$35 PER HOUR PER S/C Min = 2	2, p. 4-39 2, p. 4-32 2, p. 4-40 2, p. 4-34 2, Vol. III 2, Vol. III 2, p. 4-22 2, p. 4-40 2, p. 4-41 2, p. 4-41	λ = .000391/DAY μ = .006/DAY	.994	889
OOSA-2	1.00	814	2, p. 5-17-18	SAME AS ABOVE EXCEPT N=2	SAME AS ABOVE 2, p. 4-31	λ = .000391/DAY μ_1 = .006/DAY μ_2 = .14/DAY	.9999	828
GAP-1	.526	--	1, p. 30	MMD = 12. 95 MONTHS PS = $\{t_1 = 0, \ldots, t_5 = 30\}$ MLD = 1 MONTH M = 6; Min = 2	1, p. 30 1, p. 13 1, p. 13 1, p. 13	λ = .007/MONTH μ = .143/MONTH	.552	--
GAP-2	.641	--	1, p. 30	SAME EXCEPT MMD = 17.32 MONTHS	1, p. 13	λ = .059/MONTH μ = .143/MONTH	.692	--
SASP	.8907	--	3, p. 29	MMD = 58.02 MONTHS - if α = 62.4, β = 6 L = 120 MONTHS M = 4; Min = 4	3, p. 12 3, p. 8 3, p. 8	λ = .017/MONTH μ = .46/MONTH	.858	

Fig. 2—Comparison of Simulation Outputs ($\hat{A}_s$, $\widehat{LCC}_s$) and Closed Form Outputs ($\hat{A}_c$, $\widehat{LCC}_c$)

Category \ Strategy	No Spares (OOSA-1)	2 Spares (OOSA-2)
RDTE	638	638
Production	173	173
Launch	16	16
Operations	62	1
TOTAL	889	828

Fig. 3--Detailed Cost Comparisons (Millions of Dollars)

savings in ground operations costs which may be directly attributable to the use of on-orbit spares. The two GAP cases reflect increased availability incurred by building more reliable satellites. Again, the analytical form demonstrates internal consistency by increasing $\hat{A}_c$ from .552 to .692 as a result of increasing mean mission duration from 12.95 months to 17.32 months.

A percentage comparison between $\hat{A}_c$ and $\hat{A}_s$ relative to $\hat{A}_c$ yields a percentage difference range of -3.8 percent to +7.3 percent with several values being extremely close. A similar comparison for $\widehat{LCC}_s$ and $\widehat{LCC}_c$ yields a difference range of -7 percent to + 2 percent. While this information is not conclusive, the fact that $\hat{A}_s$ values are derived from extremely complex and detailed computer simulations lends strong support to the appropriateness of the closed forms in Theorems 1 and 2. The difference range is relatively small, indicating that the closed-form equations provide some useful approximations. Moreover, since the difference range includes both positive and negative values, no bias is indicated in either direction. More important are the reasons for the differences. The analytical solution adopted $\lambda_n = \lambda$ whereas the simulation models employ more detailed failure mechanisms, as displayed in Sections 3.1 - 3.3. In each of the simulations, booster failures were incorporated into the analysis. Other details regarding replenishment strategies (such as the multitude of sparing philosophists in OOSA) provide a certain variance in the estimates. The next subject to be addressed is the utility of the approximations obtained from Theorems 1 and 2.

4.3 Relationships Between Simulation Solutions and Closed-Form Solutions

As with any modelling effort, certain advantages and disadvantages accrue to the ultimate user. The evaluation of this approach's utility lies in the realm of the aggregation level employed. The concept of aggregating inputs at the level of the mean serves to identify the major contributors to satellite system cost effectiveness. The analytical models may be used with great care to establish the internal consistency of the simulation models. The expected analytical outputs can effectively be employed to eliminate large ranges of values for certain replenishment input parameters. The budget analyst can rapidly perform cost effectiveness tradeoffs with a hand calculator. The design engineer can investigate the impact of improved or lesser reliability upon the system cost effectiveness. Since a basic input is Mean Time Between Failures, a procurement office might successfully use the simpler models in negotiations involving MTBF warranties. Ideally, in either case, the proper use of the closed forms is to sufficiently narrow the focus of the failure and replenishment inputs in order to minimize the number of simulation runs required.

However, the aggregate model is quite insensitive to certain detailed inputs that may be critical. A specific example is the aggregation of the launch schedule into a Mean Time Between Launches (MTBL). Consider the two schedules below:

Launch Time (Months)	Launch Time (Months)
0	0
6	2
12	10

In both cases, MTBL = 6 months. Thus, the aggregate model makes no distinction between the schedules. However, a budget analyst would desire to convey the sensitivity of the two schedules to corresponding funding allocations. For this purpose, he would be required to use a detailed simulation to measure cost effectiveness. But, the models from Theorems 1 and 2 could have been used to determine that the 6-month MTBL was the most cost effective, given the reliability or failure model for his satellite. The proper use of the analytical model served to eliminate a large range of input values for MTBL. The OOSA simulation could now be used to determine which of the two schedules above with 6-month MTBL provided the most acceptable budget allocation (a subjective decision, to be sure). Other factors not included in the analytical models, but usually included in the detailed simulation, are booster reliability, number of satellites per launch, detailed failure models by subsystem, cost and funding profiles, expected availability over time, and the probabilistic nature of both costs and availability (reflecting the risk of nonattainment). Thus, the utility of the analytical forms is to provide a convenient and effective starting point for detailed simulation analysis which minimizes the computational effort of multiple simulation runs. However, the power of dealing with complex failure and replenishment interactions at the level of means or expected rates should not be minimized.

5.0 CONCLUSIONS AND RECOMMENDATIONS

This analysis has produced two major results: a theoretical base to the major computer simulation models for assessing satellite system cost effectiveness and a set of simple tools for guidance in the use of the large-scale models. The theoretical base has been to express the complex relationship between failures and replenishments as a simple expression involving the mean or expected levels of these activities. The same expressions representing the theoretical constructs provide the useful tool, as long as the user recognizes the limitations inherent in the aggregation process.

Expansion of the theoretical base could take one of several directions. The failure parameters $\{\lambda_n\}$ could be modified to reflect launch boosters and to incorporate the effect of workarounds. Detailed subsystem modelling could be included as part of the failure mechanism. Aggregated replenishment strategies $\{\mu_n\}$ should provide for launches with multiple payloads. Shuttle activities might be handily included. The latter are really simple modifications. Finally, the most general form of the models should address the interaction among multiple satellite systems competing for a common replenishment pool (i.e., an analytical model for the entire space segment), while including all information previously modelled.

A common usage of the simulation models at SAMSO is <u>not</u> addressed within this paper. Generating production and launch schedules from probabilistic curves relative to the state of the satellite system may not be accomplished with the steady-state solutions. In order to perform scheduling allocation, the transient state equations of Section 2.1.1 must be solved. At present, the current development of applied stochastic processes does not admit an exact solution to the transient equations, given a maximum number (M) of satellites on-orbit. In fact, approximation formulas using diffusion processes have not progressed that far either. Whether alternative approximation techniques are available is not yet known. A future study should attempt to numerically evaluate the extent and duration of transient activities. For small numbered systems, transient analysis may be quite revealing.

Appendix A

DERIVATION OF AGGREGATE PARAMETERS FROM SAMPLE DATA

NOTES: (a) All data are documented in Figure 2: Comparison of Simulation Outputs and Closed Form Outputs;

(b) The formulas employed are derived in Section 4-1.

A-1: On-Orbit Spares Analysis (OOSA)

OOSA-1: (a) $\lambda = \frac{1}{DL} \cdot (24 \text{ hours/day}) = \frac{1}{61386}(24) = .000391/\text{day}$

(b) $\text{MTBL} = \frac{t_p - t_1}{p} = \frac{918 + 459 \text{ days}}{10} = 137.7 \text{ days}$

$$\mu = \frac{1}{\text{MTBL} + \text{MLD}} = \frac{1}{137.7 \text{ days} + 30 \text{ days}} = .006/\text{day}$$

OOSA-2: (a) $\lambda = \frac{1}{DL} \cdot (24 \text{ hours/day}) = \frac{1}{61386}(24) = .000391/\text{day}$

(b) $\text{MTBL} = \frac{t_p - t_1}{p} = \frac{918 + 459 \text{ days}}{10} = 137.7 \text{ days}$

$$\mu_1 = \frac{1}{\text{MTBL} + \text{MLD}} = \frac{1}{137.7 + 30} = .006/\text{day}$$

$$\mu_2 = \frac{1}{\text{OTD}} = \frac{1}{7 \text{ days}} = .14/\text{day}$$

A-2: General Availability Program (GAP)

GAP-1: (a) $\lambda = \frac{1}{MMD} = \frac{1}{12.95 \text{ months}} = .077/\text{month}$

(b) $MTBL = \frac{t_p - t_1}{p} = \frac{30 - 0}{5} = 6 \text{ months}$

$$\mu = \frac{1}{MTBL + MLD} = \frac{1}{6 \text{ mo.} + 1 \text{ mo.}} = .143/\text{month}$$

GAP-2: (a) $\lambda = \frac{1}{MMD} = \frac{1}{17.32 \text{ months}} = .059/\text{month}$

(b) $MTBL = \frac{t_p - t_1}{1} = \frac{30 - 0}{5} = 6 \text{ months}$

$$\mu = \frac{1}{MTBL + MLD} = \frac{1}{6 + 1} = .143/\text{month}$$

A-3: Satellite Availability Simulation Program (SASP)

SASP: (a) $\lambda = \frac{1}{MMD} = \frac{1}{58.02 \text{ months}} = .017/\text{month}$

(b) $MTBL = \frac{L}{MMD} = \frac{120}{58.02} = 2.06 \text{ months}$

$$\mu = \frac{1}{MTBL} = \frac{1}{2.06 \text{ months}} = .46/\text{month}$$

Appendix B

REPRESENTATIVE ANALYTICAL COMPUTATIONS

FOR ILLUSTRATIVE PURPOSES, the cases OOSA-1 and OOSA-2 represent the most general set of alternatives available.

OOSA-1: (a) $A_c = P_2 + P_3 + P_4$ (No On-Orbit Spares)

$$P_n = \frac{\mu_1{}^n}{n!\lambda^n} \text{ Po} \qquad 1 \leq n \leq 4$$

$$P_o = \left\{ 1 + \sum_{n=1}^{4} \frac{\mu_1{}^n}{n!\lambda^n} \right\}^{-1}$$

$$P_o = \left\{ 1 + \frac{(.006)^1}{1!(.000391)^1} + \frac{(.006)^2}{2!(.000391)^2} + \frac{(.006)^3}{3!(.000391)^3} + \frac{(.006)^4}{4!(.000391)^4} \right\}^{-1}$$

$$P_o = .000328$$

$$P_1 = \frac{(.006)^1}{1!(.000391)^1} (.000328) = .005$$

$$P_2 = \frac{(.006)^2}{2!(.000391)^2} (.000328) = 038$$

$$P_3 = \frac{(.006)^3}{3!(.000391)^3}(.000328) = .198$$

$$P_4 = \frac{(.006)^4}{4!(.000391)^4}(.000328) = .758$$

$$\hat{A}_c = .024 + .161 + .81 = .994$$

(b) $$L\hat{C}C_c = RDTE + C_p T\mu_1 + C_L T\mu_1 + C_{IF}(24)T(10^{-6})$$

$$+ C_{IV}(24)T(10^{-6})\ (\mu_1 - \lambda)T_4$$

$$L\hat{C}C_c = (\frac{594}{20})3600(.006) + 8(3600)(.006) + 174(24)(3600)(10^{-6})$$

$$+ 35(24)(3600)[.006 - .000391](10^{-6})(3600)$$

$$L\hat{C}C_c = 889$$

OOSA-2: (a) $\hat{A}_c = P_2 + P_3 + P_4 + P_5 + P_6$ [Two On-Orbit Spares]

$$P_n = \begin{cases} \dfrac{\mu_1^{\,n}}{n!\ \lambda^n} P_o & 1 \le n \le 2 \\[2ex] \dfrac{\mu_1^{\,2}\ \mu_2^{\,n-2}}{n!\ \lambda^n} P_o & 2 < n \le 4 \\[2ex] \dfrac{\mu_1^{\,n-2}\ \mu_2^{\,2}}{n!\ \lambda^n} P_o & 4 < n \le 6 \end{cases}$$

$$P_o = \left\{1 + \sum_{n=1}^{2} \frac{\mu_1^{\,n}}{n!\ \lambda^n} + \sum_{n=3}^{4} \frac{\mu_1^{\,2}\ \mu_2^{\,n-2}}{n!\ \lambda^n} + \sum_{n=5}^{6} \frac{\mu_1^{\,n-2}\ \mu_2^{\,2}}{n!\ \lambda^n}\right\}^{-1}$$

$$P_o = \left\{1 + \frac{(.006)^1}{1!(.000391)^1} + \frac{(.006)^2}{2!(.000391)^2} + \frac{(.006)^2(.14)}{3!(.000391)^3}\right.$$

$$\left. + \frac{(.006)^2(14)^2}{4!(.000391)^4} + \frac{(.006)^3(.14)^2}{5!(.000391)^5} + \frac{(.006)^4(.14)^2}{6!(.000391)^6}\right\}^{-1}$$

$$P_o = (.666)(10^{-7})$$

$$P_1 = \frac{(.006)^1}{1!(.000391)}(.666)(10^{-7}) = (.102)(10^{-5})$$

$$P_2 = \frac{(.006)^2}{2!(.000391)}(.666)(10^{-7}) = (.784)(10^{-5})$$

$$P_3 = \frac{(.006)^2(.14)}{3!(.000391)^3}(.666)(10^{-7}) = (.936)(10^{-3})$$

$$P_4 = \frac{(.006)^2(.14)^2}{4!(.000391)^4}(.666)(10^{-7}) = (.838)(10^{-1})$$

$$P_5 = \frac{(.006)^3(.14)^2}{5!(.000391)^5}(.666)(10^{-7}) = .257$$

$$P_6 = \frac{(.006)^4(.14)^2}{6!(.000391)^6}(.666)(10^{-7}) = .658$$

$$\hat{A}_c = (.784)(10^{-5}) + (.936)(10^{-3}) + (.838)(10^{-1}) + .257 + .658$$

$$\hat{A}_c = .9999$$

(b) $$\hat{LCC}_c = RDTE + C_p T\mu_1 + C_L T\mu_1 + C_{IF}(24)\ T\ (10^{-6})$$

$$+ C_{IV}(24)\ T\ (10^{-6})[(\mu_1 \sum_{n=0}^{4} P_n) - \lambda](3600)$$

$$\hat{LCC}_c = (\frac{594}{20})(3600)(.006) + (8)(3600)(.006) + (174)(24)(3600)(10^{-6})$$

$$+ (35)(24)(3600)[(.006)(.085) - .000391](10^{-6})(3600)$$

$$\hat{LCC}_c = 828$$

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