DEVELOPMENT OF THE FIVE DEGREE-OF-FREEDOM LINEAR MODEL FOR THE --ETCIU

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DEVELOPMENT OF THE FIVE DEGREE-OF-FREEDOM LINEAR MODEL FOR THE XR-3 SURFACE EFFECT SHIP AND INVESTIGATION OF THE ROLL BEHAVIOR OF THE CRAFT IN TURN MANEUVERS

by

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December 1979

Thesis Advisor: G. J. Thaler

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ABSTRACT

A five degree-of-freedom linear model for the XR-3 surface effect ship is developed for constant speed operation. The weight removal transient response and the roll behavior of the craft in turn maneuvers are investigated by both linear and six degree-of-freedom nonlinear models, and the linear model simulation results are compared with the nonlinear model simulations. Some of the parameters of the craft are investigated for roll motion.
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I. INTRODUCTION

A. BACKGROUND

The Surface Effect hybrid type high performance ship has been of great interest recently, and in 1970 the XR-3 test craft was delivered to the Naval Postgraduate School. As seen in [7] Leo and Boncal converted the digital simulation program for the 100-B test craft to represent the XR-3, thus providing a nonlinear model. In [1], Gerba and Thaler have developed a heave-only linear model and further studies were made by Barnes in [2] by developing a two degree-of-freedom linear pitch and heave model representing the XR-3 test craft.

B. OBJECTIVE

In order to investigate the roll behavior of the XR-3 test craft during turn maneuvers, a five degree-of-freedom linear model is developed in this thesis. Some modifications are made for pitch and heave modes, therefore test of the vertical plane by means of weight removal is reinvestigated and compared with the nonlinear simulation results.

In [3], the roll behavior of the XR-3 and the sidewall sensitivity are studied using a six degree-of-freedom nonlinear model. Similar studies are made in this thesis after developing a five degree-of-freedom linear model and comparing with the nonlinear model results. Finally, some modifications are suggested for nonlinear modeling.
II. NONLINEAR XR-3 MODELING FOR FIVE DEGREE-OF-FREEDOM

A. ASSUMPTIONS

The following assumptions and simplifications are used during development of a constant speed XR-3 test craft model:

1. Aerodynamic drag on the forward seal, and superstructure skin friction forces are neglected.

2. Momentum (ram) drag is disregarded for all modes. In other words, it is accepted that the location of the lift fans with respect to the craft C.G. could not affect significantly the roll and pitch stability. Also, air stagnation force in the z-direction is assumed to be negligible.

3. During dynamic response of the craft, only air leakage with constant leakage area at the stern seal is assumed. No other air escape is considered such as possible leakage from sidewalls or bow seal.

4. Plenum pressure and air mass thermodynamics are represented by ideal diabatic processes with the air constant specific heat ratio $\gamma = 1.4$.

5. Effective plenum roof area is developed as a function of craft weight.

6. Planing forces are concentrated at the centroid of the keel longitudinal cross section through the fluid medium.

7. Pitch and roll damping moments due to added mass are included in the craft dynamics, but their contribution to heave motion is neglected.
8. The thrust forces and all hydrodynamic resistive forces (drag) in the z-direction cancel each other, since at constant speed operation, these forces are in equilibrium. However they might contribute to the moment equations such as pitch moment, but considering their effective moment arms with respect to the C.G. are almost equal to the thrust moment arm, so that their contribution in the moment equations is also neglected.

9. For seal modeling, vertical resistive forces due to hydrodynamic response are taken into account including corresponding effective wetted seal area.

10. Simplified craft geometry is used for plenum, buoyancy and seal modeling.

B. COORDINATE SYSTEM AND SIGN CONVENTIONS

1. The cartesian coordinate system is used with its origin located on the water line corresponding to the craft center of gravity.

2. The x-axis lies on the water line with the positive longitudinal displacement being measured forward.

3. The y-axis lies in the vertical cross section plane with the lateral displacement being measured positive to starboard.

4. The z-axis lies in the vertical cross section plane with the positive vertical displacement being measured downward.
5. \( z \) represents heave C.G. displacement measured from the origin (at water line) to the center of gravity.

6. \( z_s \) is one of the craft dimensional values measured from the keel to the C.G.

7. Pitch is the rotational motion about the \( y \)-axis, angular pitch displacement is represented by \( \theta \), positive pitch angle is measured upward (boat noses up).

8. Roll is the rotational motion about the \( x \)-axis, \( \phi \) angle represents angular roll displacement, positive roll angle is measured to starboard (boat heels to starboard).

9. Similarly the yaw is the rotational motion about the \( z \)-axis, positive yaw is measured in the clockwise direction.

10. Positive roll, pitch and yaw moments are to be defined as the moments in the positive roll, pitch and yaw directions correspondingly.

11. As a reference zero pitch and roll angle provide the \( x-y \) plane parallel to the water surface.

   The craft coordinate system is shown in figure 1.

C. SIMPLIFIED CRAFT MODELING AND GEOMETRY

1. General Simplified Geometry and Dimensions of the XR-3 Test Craft

   In figure 1 the sideview simplified geometry of the XR-3 test craft is shown. Also in figure 3 the top view is illustrated. The variables and dimensions in these figures are defined as follows;
Figure 1. Craft coordinate system and simplified geometry (sideview)
Figure 2. Bow seal geometry

Figure 3. Simplified geometry (top view)
\( l_d \) = Draft of the craft at center of gravity

\( C_p \) = Plenum pressure center (the centroid of the effective plenum area)

\( l_5, l_6 \) = The longitudinal distance of the sections' centroids from the C.G. (forward and aft section centroids respectively)

\( l_3, l_4 \) = Bow and stern seals front face distances at water level

\( l_7 \) = Centroid distance of the planing force effective area

\( l_1, l_2 \) = Forward and aft sections sidewall submerged region length.

All craft dimensional values are taken from the XR-3 six degree-of-freedom model scale drawings as indicated in [2] and [3].

2. **Sidewalls**

As reported in [2], the sidewall modeling is developed from the SIDEWALL subroutine of the 6 DOF computer model. Like craft dimensions, all sidewall dimensions and their related geometric configurations are based on scale model drawings. For simplicity of calculation, the sidewalls are divided into two sections called forward and aft. During dynamic response of the sidewalls, these sections are considered individually with their own centroids, so related calculations can be done easily. Especially, the sidewall modeling is developed in order to determine buoyancy forces and moments. By means of this modeling, the average sidewall width, average draft at section centroid and effective
plenum area are also determined and used extensively in the rest of the calculations.

The sidewall vertical cross sections at the section centroid forward and aft are sketched in figures 4(a) and 4(b) respectively. From these figures, the average forward sidewall width at the buoyancy center is found to be

$$b_f = \frac{l_f}{2 \tan \beta_f} + b_f$$  \hspace{1cm} (II-1)

where

- $l_f$ = Draft at forward buoyancy center
- $\tan \beta_f$ = Tangent of the sidewall deadrise angle at the forward buoyancy center
- $b_f$ = Sidewall flat surface width at the kell (forward section)

Therefore submerged sidewall volume of the forward section is given as follows

$$V_1 = b_f \cdot l_f \cdot l_1$$  \hspace{1cm} (II-2)

$l_f$ can be formulated as

$$l_f = l_d - l_5 \tan \theta$$

then the small angle approximation $\tan \theta \approx \theta$ gives
Figure 4(a) Forward section
Figure 4(b) Aft section

Figure 5. Plenum modeling
\[ l_f = l_d - l_5 \theta \]  \hspace{1cm} (II-3)

As defined before, \( l_d \) draft at C.G. is

\[ l_d = z_s + z \]  \hspace{1cm} (II-4)

Similar expressions can be derived for the aft section,

\[ b_a = \frac{l_a}{2 \tan \beta_2} + b_a \]  \hspace{1cm} (II-5)

where

\( b_a = \) sidewall flat surface width at the keel (aft section)

\[ V_2 = b_a \frac{a_2}{l_a} \]  \hspace{1cm} (II-6)

\[ l_a = l_d + l_6 \theta \]

Note that \( l_f, l_a \) are functions of draft and pitch angle.

3. Plenum Modeling

In [2] plenum modeling is presented including plenum pressure variation due to changing of draft. In the development of a pitch, heave and roll linear model, the simplified plenum geometry and dynamics which were derived by Barnes are considered and used in our plenum dynamics representation.

The variables and dimensions for plenum modeling, as seen in figure 5, are found as
\[ l_p = l_8 - l_9 \frac{\bar{d}}{h} \]  \hspace{1cm} (II-8)

\[ A_b = b_w \frac{l_p}{l_8 - l_o} \]  \hspace{1cm} (II-9)

where

\[ l_p = \text{Length of plenum at the water line} \]
\[ A_b = \text{Effective plenum area} \]
\[ b_w = \text{Lateral distance from one sidewall to another in the plenum region.} \]

\[ l_9', l_8, \bar{d}', l_o \] and \( h \) are all constants related to craft dimensions. Only the effective plenum area \( A_b \) varies when the craft draft varies, that means variation of the plenum pressure centroid \( C_p \) is forward or backward.

4. **Bow and Aft Seals**

The seals are an involved additional dynamic system with their own degree of freedom relative to the craft, and they carry little load compared with the other components of the craft. However in the simplified model, the seal motions relative to the craft are neglected.

The assumed seal geometry is shown in figure 2, developed in [2]. Using an appropriate geometric configuration, the seal wetted area is derived as a function of craft draft and pitch angle.
From [2], the width of the wetted surface for the bow seal is:

\[ x_f = \frac{l_d - l_3 \tan \theta}{\sin 31^\circ} \]  

where

\[ \sin 31^\circ = \sin \text{ of the angle between water surface and the bow seal front face.} \]

The numerator of this ratio represents draft at the bow seal front face.

A similar expression for the stern seal is:

\[ x_a = \frac{l_d + l_4 \tan \theta}{\sin 32^\circ} \]

Multiplying \( x_f \) and \( x_a \) by \( b_w \), bow and stern seals wetted areas are obtained as follows:

\[ A_f = b_w x_f \]

\[ A_a = b_w x_a \]

5. **Rudder and Propulsion Hydrodynamics**

The rudder hydrodynamics is calculated from [4], inserting rudder geometry dimensional values into the fin equations. Actually we are interested in the lateral force (sway) on the rudders for a given rudder order deflection. This dynamic equation is given in [4] as follows;
\[ v_r = \rho u^2 A_r (1 + \frac{d_s}{h_r}) C_r [\delta_r - (1 + \frac{d_s}{h_r}) \frac{V_h}{u}] \]  \hspace{1cm} \text{(II-11(a))}

where

\begin{align*}
A_r &= \text{Rudder platform area (for one rudder only)} \\
d_s &= \text{draft at the stern} = z + z_s - x_r \theta \\
h_r &= \text{Vertical depth to bottom tip of the rudder} \\
C_r &= \text{Rudder lift coefficient} \\
&= \frac{2\pi R_a}{R_a + 3} \quad \text{(for fully wetted flow regime)} \\
R_a &= \text{The rudder aspect ratio} \\
\delta_r &= \text{Rudder deflection (the sign convention of the rudder is to be positive for port turn)} \\
V_h &= V + x_r r - z_s p \\
x_r &= \text{longitudinal coordinate of rudder geometric center (always negative value)} \\
V &= \text{lateral speed (ft/sec)} \\
r &= \text{yaw rate (rad/sec)}
\end{align*}

In order to simplify the rudder dynamic equation, the following coefficients are defined as

\begin{align*}
C_e &= 1 + \frac{d_s}{h_r} \\
C_t &= \rho u^2 A_r (1 + \frac{d_s}{h_r}) C_r
\end{align*}
Then equation (II-11(a)) becomes

\[ Y_r = C_t (\delta_r - C_e \frac{V_h}{u}) \]  \hspace{2cm} (II-11(b))

Let us define \( z_r \) to be the vertical coordinate of the rudder from the C.G. to its geometric center, the rudder roll and yaw moments can be calculated from

\[ K_r = -Y_r z_r \]  \hspace{2cm} (II-11(c))
\[ N_r = Y_r x_r \]  \hspace{2cm} (II-11(d))

During turn maneuvers, the propulsion thrust vector is also deflected according to the given rudder order. Therefore the lateral component of the thrust contributes yaw and roll moments as well as a sway mode, but this disturbance must be considered as a step input to the linear modes with constant magnitude, therefore the total propulsion sway force is

\[ Y_{pr} = 2 X_t \sin \delta_r \]  \hspace{2cm} (II-12)

where \( X_t \) is the propulsion thrust corresponding to craft speed.

Similarly the roll and yaw moments generated by the propulsion system are

\[ K_{pr} = Y_{pr} z_p \]  \hspace{2cm} (II-13(a))
\[ N_{pr} = Y_{pr} x_p \]  

(II-13(b))

where \( x_p \) and \( z_p \) are the longitudinal and vertical locations of the propeller respectively.

D. DYNAMIC FORCES FOR FIVE DEGREE-OF-FREEDOM MODES

1. Seal Forces

Considering the seal configuration and dynamics, the pressure difference across the wetted portion of the seal is

\[ \bar{P}_b = P_b - P_a \]

where

\[ \bar{P}_b = \text{Plenum gauge pressure (lb/ft}^2\text{)} \]

\[ P_a = \text{Atmospheric pressure} \]

Since the seal is in equilibrium, the vertical hydrodynamic force on the seal is

\[ Z_{sf} = -\bar{P}_b A_f \]

\[ = -\bar{P}_b \cdot b_w \left( \frac{l_d - \ell_3 \theta}{\sin 31^\circ} \right) \]  

(II-14)

For the stern seal hydrodynamics, a similar expression can be found as

\[ Z_{sa} = - (\bar{P}_b + P_i) \cdot \ell_d \left( \frac{l_d + 4 \theta}{\sin 32^\circ} \right) \]  

(II-15)
where

\[ P_i = \text{Additional pressure supplied into stern air bags in order to reduce leakage.} \]

2. Buoyant Forces

The forward section buoyant force is determined from equation (II-2), it is

\[ Z_{bf} = -2 \rho g V_1 \]

Here \( \rho \) is the density of the seawater and \( g \) is the gravitational constant.

Finally, the forward and aft section forces are

\[ Z_{bf} = -2 \rho g \left( \frac{l_f}{2 \tan \beta_1} + b_f \right) l_f \lambda_1 \]  

\[ Z_{ba} = -2 \rho g \left( \frac{l_a}{2 \tan \beta_2} + b_a \right) l_a \lambda_2 \]

3. Planing Forces

In addition to the pitch angle \( \theta \), the sidewall flat area at the keel determines the planing effect of the craft. When the craft pitches up, the frontal area (through the fluid medium) causes a lift-drag force in the \( z \)-direction.

The planing force, from Reidel's work [3], is:

\[ Z_{pl} = -\frac{1}{2} \pi \rho A u^2 \sin \theta \]
where

\[ A = \text{flat surface area at the keel} \]

\[ u = \text{speed of the craft} \]

Using the small angle approximation and including the loss coefficient \( C_{pl} \) suggested by Barnes in [2], the planing force is redetermined as

\[ Z_{pl} = -C_{pl} \pi \rho A u^2 \theta \]  

(II-18)

A similar expression is derived for sidewall dynamics in [4], but in our modelling equation (II-18) is used.

4. Pitch Damping Forces

The sidewalls of the surface effect ship influence moderately heave, pitch and roll modes, so in [4] the hydrodynamic and hydrostatic forces and moments acting on the sidewalls of uniform triangular cross section are derived with particular application to vertical plane motion and lateral motion.

Considering constant speed operation of the craft, neglecting some terms in the equations provided from [4], the vertical force acting on only one sidewall is found as;

\[ Z_{sw1} = A_{33} x_s u q \]  

(II-19)
where

\[ A_{33} = \text{vertical added mass calculated at the stern} \]

\[ = (\pi/8) \rho B_s^2 \]

\[ B_s = \text{value of sidewall waterline at the sidewall stern} \]

\[ x_s = \text{stern distance from C.G. (negative value)} \]

\[ q = \text{pitch rate (rad/sec)}. \]

For the pitch damping moment, the same equation is used by Barnes in [2].

5. Roll Damping Forces

The XR-3 test craft is dynamically similar to 100-B surface effect ship. Therefore we can use dynamic equations calculated in [4] for the XR-3 test craft. The heave-roll damping force for one sidewall can be written as;

\[
Z_{sw2} = -A_{33}u b_o p \tag{II-20}
\]

In the above equation, \( b_o \) is half of the craft width and it must be used (+) for starboard sidewall, (-) for port sidewall.

Similarly the lateral damping force due to sidewall hydrodynamics is

\[
Y_{sw1} = -A_{22}u(v + x_s r - z_s p) \tag{II-21}
\]
here $A_{22}$ is the lateral added mass calculated at the stern, $p$ is the roll rate. $A_{22}$ is calculated from [4], that is

$$A_{22} = \frac{\pi}{5} \rho d_s^2$$

6. **Plenum Pressure Lift Force**

From [2] and [4], the plenum pressure lift force is determined assuming uniform pressure distribution. This is formulated as

$$Z_p = -A_b \bar{P}_b$$

from equations (II-8) and (II-9) follow

$$Z_p = -b_w \bar{P}_b (z_8 - \frac{g \lambda d}{h})$$  \hspace{1cm} (II-22)

E. **DYNAMIC PLENUM AIR MASS, PRESSURE EQUATIONS**

Assuming constant aft seal leakage area, the leakage flow rate ($Q_{out}$) is given by Gerba and Thaler for the XR-3 heave only model in [1], then we have

$$Q_{out} = C_n A_k \left(\frac{2 \bar{P}_B}{\rho_a}\right)^{1/2}$$  \hspace{1cm} (II-24)

where

$C_n$ = Nozzle flow coefficient
$A_k$ = Leakage area
\[ \rho_a = \text{Atmospheric air density.} \]

The linearized air flow \( Q_{in} \) into the plenum is presented in [5] according to the fan properties and calculated in [1] as follows,

\[ Q_{in} = n(Q_o - k_p \bar{F}_b) \]  \hspace{1cm} (II-24)

where \( n \) is the number of fans supplying the air to the plenum, \( Q_o \) is the steady state air flow rate corresponding to steady gauge pressure, \( k_p \) is (1/slope) of the \( \bar{F}_b \) vs. \( Q_{in} \) curve at the equilibrium point.

From equations (II-23) and (II-24), the plenum air flow rate can be written as

\[ M_b = \frac{dm_b}{dt} = \rho_a (Q_{in} - Q_{out}) \]  \hspace{1cm} (II-25)

The equation (II-25) reveals that if the fan flow rate \( Q_{in} \) equals to the leakage flow rate \( Q_{out} \), then the plenum air mass \( M_b \) is constant.

The absolute plenum pressure is calculated from

\[ P_b = P_a \left( \frac{M_b}{V_b \rho_a} \right)^{\frac{\gamma}{\gamma - 1}} \]  \hspace{1cm} (II-26)

In the above equation, \( V_b \) is defined as plenum volume and its simplified form is calculated from [1],

\[ \text{30} \]
\[ V_b = (V_n - A_b \ell_d) \] (II-27)

where \( V_n \) is to be the empty plenum volume.

**F. NET HEAVE FORCE**

The resultant heave force is the summation of the all external dynamic forces, it is

\[ Z = Z_{sf} + Z_{sa} + Z_{bf} + Z_{ba} + Z_{pl} + Z_{sw} + Z_p \]

where

\[ Z_{sw} = Z_{sw_1} + Z_{sw_2} \]

\( Z_{sw} \) also includes a pair of heave buoyancy forces due to roll, but they are in opposite directions. Therefore these stabilizing forces are only taken into account in the roll motion.

**G. PITCH AND ROLL MOMENTS**

1. Seal Moments
   a. Pitch Mode

   The seal force is modeled by Barnes in [2] acting on the center of the seal wetted length. The effective pitch moment arms are;

   \[ l_{sf} = l_3 - x_f/2 \] (for forward seal) (II-28)

   \[ l_{sa} = l_4 + x_a/2 \] (for aft seal) (II-29)
From this follows the forward and aft seal moments

\[ M_{sf} = -Z_{sf} l_{sf} \]  \hspace{1cm} (II-30)

\[ M_{sa} = -Z_{sa} l_{sa} \]  \hspace{1cm} (II-31)

where \( Z_{sf} \) and \( Z_{sa} \) have already been defined.

b. Roll Mode

The flexible seals can contribute little to roll stability. Actually the seal configurations and hydrodynamics are relatively more complex. In [4], the seal roll moments are calculated by combining the roll contribution of all seal elements.

Considering the seal response, the submerged region of the seal can be calculated and its roll moment contribution can be formulated in the following equations;

\[ K_{sf} = -b_w^3/12 \rho g x_f \tan \phi \]  \hspace{1cm} (II-30)

\[ K_{sa} = -b_w^3/12 \rho g x_a \tan \phi \]  \hspace{1cm} (II-31)

For this modeling, the roll moment arm is considered as \((2/3 b_o)\), that is the centroid of the triangle cross section of the submerged portion.

2. Buoyant Moments
   a. Pitch Mode

The pitch buoyancy moments are calculated from equation (II-16) and (II-17) assuming constant moment arms
$l_5$ and $l_6$. The pitch buoyancy moments are

$$M_{bf} = -Z_{bf} l_5 \quad (II-32)$$

$$M_{ba} = Z_{ba} l_6 \quad (II-33)$$

b. Roll Mode

As discussed in the heave mode, the effect of the sidewall buoyancy due to the roll is to be a stabilizing moment. From figures 4(a) and 4(b) the buoyancy volume change can be approximated as

$$\Delta V_f = \frac{b_f + b_w}{2} + \frac{l_f}{2 \tan \theta_1} \left(\frac{l_f}{\tan \theta_1} + b_f\right) l_1 \phi \quad (II-34)$$

Finally, roll moment is

$$K_{bf} = -2 \rho g \Delta V_f \quad (II-35)$$

Similarly, the aft section roll moment is

$$K_{ba} = -2 \rho g \Delta V_a$$

3. Planing Pitch Moment

The planing effect due to the sidewall flat area at the keel contributes only pitch motion as follows;

$$M_{pl} = Z_{pl} l_7$$
4. **Pitch Damping Moment**

This is calculated from equation (II-19) as

\[ M_{sw} = -2Z_{sw1}x_s = -2A_{33}u x_s^2 \]

In the above equation, the factor "2" appears because there are two sidewall sections.

5. **Roll Damping Moment**

Combining equation (II-20) and (II-21), the roll damping component is

\[ K_{sw} = 2Z_{sw2}b_o - Y_{sw1}z_s + K_{wave} \quad (II-37) \]

where \( K_{wave} \) represents the additional roll damping term because of vertical wave generation in the roll.

From [3] and [4], \( K_{wave} \) is

\[ K_{wave} = -\frac{32}{\pi} b_o^2 B_c \rho \quad (II-38) \]

Here, \( B_c \) is the sidewall vertical added mass given as

\[ B_c = \frac{\pi}{8} \rho \ell d^2 \cot^2 \beta \cdot \]

6. **Plenum Pitch Moment**

From equation (II-22) and defining \( X_{cp} \) to be the plenum pressure centroid, the plenum pitch moment is given as
\[ M_p = -Z_p \times_{cp} = A_b (P_b - P_a) \times_{cp} \]

where \( X_{cp} \) is calculated as a function of craft draft in [2] as follows;

\[ X_{cp} = X_c + \frac{l_g l_d}{2h} \]

\[ X_c = \frac{l_g}{2} + X_{c.g}. \]

7. **Plenum Roll Moment**

The destabilizing static plenum roll moment is generated when the craft rolls to one side. Since the effective plenum pressure centroid shifts laterally to the other side tending to increase rolling of the craft. The plenum roll moment is

\[ K_p = A_b (P_b - P_a) (Z_s - l_d) \phi \]

8. **Rudder and Propulsion Roll Moments**

These roll moments have already been calculated from equation (II-11(c)) and (II-13(a)).

H. **SWAY FORCES**

1. **Cross Flow Drag (Sidewall)**

During the turn maneuver, the cross flow drag is generated acting on the outboard sidewall. The direction of this force is negative while turning to the left.
Neglecting the lateral drag component inside of the sidewall, the net cross flow drag can be calculated from

\[
D_{SW} = -\frac{1}{2} \rho \text{sign}(V_r) C_d V_r^2 l(\xi_d + b_0\phi)
\]

(II-39)

where

\[
V_r = v + x_{s_r} - (z_s - \frac{b_d}{2})p
\]

\[
C_d = \text{cross flow drag coefficient}
\]

Considering the outboard surface of the submerged sidewall sections (dead rise angle effect), the lift force in the z-direction can be produced due to the lateral drag deadrise projection. In [4], the importance of the sidewall deadrise angle is mentioned, however at the hydrodynamic section in this reference, the lift component of the drag force has not been derived.

In this modeling, the lift force \((Z_d)\) and lateral drag component is calculated assuming both have the same drag coefficients \((C_d = 1.28)\). They are

\[
Y_d = D_{SW} \sin \beta
\]

(II-40)

\[
Z_d = D_{SW} \cos \beta
\]

(II-41)
From equation (II-40) and (II-21), the total hydrodynamic sway force of the sidewall is

\[ Y_{sw} = (D_{sw} + Y_{sw_1}) \sin \beta + Y_{sw_1} \quad \text{(II-42)} \]

2. **Centrifugal Force**

The centrifugal force acting on the C.G. of the craft is always in the outboard direction during the turn maneuver. \( Y_C \) is the centrifugal force, given as

\[ Y_C = -m \frac{u^2}{R} \]

R, the turning radius is equal to \((u/r)\), combining this with the above equation gives

\[ Y_C = -m u r \quad \text{(II-43)} \]

a negative sign is used due to the coordinate system sign convention. Since the right turn produces (+) yaw rate, then \( Y_C \) is calculated as (-), so outboard sway force is generated for this configuration.

3. **Rudder and Propulsion Sway Forces**

They are determined from equation (II-11(b)) and (II-12).

The sway forces due to the seals, air plenum and aerodynamic drag are neglected in this modeling.
I. YAW MOMENTS

1. Sidewall Yaw Moment

The sidewall yaw dynamic equations are given in [4].

After some simplification, the net sidewall yaw moment is calculated from

\[ N_{sw} = N_1 + N_2 + (X_{pw} - X_{sw}) b_o \]  \hspace{1cm} (II-44(a))

where

\[ N_1 = 2u(v + x_t r - z_s p) A_{22} \ell (1 - \lambda) \]
\[ - \pi/5 \cdot \rho (l_d \pm b_o \phi)^2 \ell u (v - z_s p) \] \hspace{1cm} (II-44(b))

\[ N_2 = - \rho C_d l_d (2 s_1 \ell^3 r v + s_2 \ell^2 v_r^2) \] \hspace{1cm} (II-44(c))

\[ \lambda = \frac{\ell + X_s}{\ell} \] (II-44(d))

The ratio of C.G. location of sidewall bow to the craft length

\[ s_1 = \frac{1 - 3 \lambda (1 - \lambda)}{3} \] \hspace{1cm} (II-44(e))

\[ s_2 = \frac{2 \lambda - 1}{2} \] \hspace{1cm} (II-44(f))

\[ X_{pw} = -1/2 \rho C_f A_{w_p} u^2 \] \hspace{1cm} (II-44(g))

= port wetted area surge resistive force

\[ X_{sw} = -1/2 \rho C_f A_{w_s} u^2 \] \hspace{1cm} (II-44(h))

= starbord wetted area surge resistive force
\[ A_w = [2(l_d \pm b_o \phi) + b_s]l \]  \hspace{1cm} (II-45)

\[ b_s = \frac{b_f + b_d}{2} \]

= Average sidewall flat surface width at the keel.

In equation (II-45) \( \pm \) term related to the starboard or port wetted surface calculations.

2. Rudder and Propulsion Yaw Moments

The yaw moments due to the rudder and propulsion system are determined from equation (II-11(d)) and (II-13(b)) respectively.

The seals, air plenum and aerodynamic yaw moments are neglected. Actually the aerodynamic yaw moment should be taken into account. Since during the turn maneuver, the yaw drift angle is produced, thus a positive aerodynamic yaw moment contribution to the yaw motion has been generated. From 6 DOF nonlinear model simulations, this contribution is determined at 5.2% of the total yaw moment.

J. TOTAL PITCH MOMENT

The summation of the all dynamic pitch moments gives the net pitch moment which can be written as

\[ M = M_{sf} + M_{sa} + M_{bf} + M_{ba} + M_{pl} + M_{sw} + M_p \]  \hspace{1cm} (II-46)
K. TOTAL ROLL MOMENT

Before calculating net roll moment, the roll moment equation due to the cross flow drag term must be derived. Including also the added mass effect to the equation (II-40) and (II-41), the result is

$$K_d = (D_{sw} + Y_{sw})(b_0 \cos \beta - z_0 \sin \beta) \quad (II-47)$$

Combining all roll dynamic equations, the net roll moment is calculated as

$$K = K_{sf} + K_{sa} + K_{bf} + K_{ba} + K_{sw} + K_{pr} + K_r + K_p + K_d$$

L. TOTAL SWAY FORCE

From equation (II-11(a)), (II-12), (II-42) and (II-43), the total sway force is obtained as

$$Y = Y_r + Y_{pr} + Y_{sw} + Y_c \quad (II-48)$$

M. TOTAL YAW MOMENT

The summation of the sidewall, rudder and propulsion yaw moments gives the net turning moment about the z-axis

$$N = N_{sw} + N_r + N_{pr} \quad (II-49)$$
N. FIVE DEGREE-OF-FREEDOM EQUATIONS OF MOTION

Considering equations of motion for XR-3 test craft, the yaw (ψ) is defined as the rotational motion of the craft coordinate system relative to the fixed frame and the positive ψ angle is measured in the clockwise direction. This sign convention is in agreement with the rudder order sign definition. Since positive rudder angle corresponds to turning to the left, producing negative yaw moment and finally at the equilibrium condition, the negative yaw moment is generated.

Assuming small values of pitch (θ) and roll angles and relating heave motion to the free surface, the relationship between the fixed and craft coordinate system can be written. Eliminating the terms related to the fixed coordinate system, the simplified 5 DOF equations of motion are obtained as follows:

\[
\begin{align*}
\dot{m} v &= Y \\
\dot{m} w &= Z \\
I_{xx} \dot{p} - I_{xz} \dot{r} &= K \\
I_{yy} \dot{q} &= M \\
I_{zz} \dot{r} - I_{xz} \dot{p} &= N
\end{align*}
\]

II-50

II-51

II-52

II-53

II-54
The right hand sides of the equations of motion give the forces and moments acting on the craft in terms of the craft in terms of the craft coordinate system variables. In equation (II-57) \( w \) represents heave acceleration and all other variables in these equations have already been defined.

In order to develop a 5 DOF linear model, only the surge equation of motion (x-direction) is neglected. Since from 6 DOF nonlinear simulation results, very small velocity reduction is observed during the turn maneuver of the XR-3 test craft (over the speed range 20 through 30 knots).
III. LINEARIZATION OF THE EQUATIONS

A. TAYLOR SERIES EXPANSION AND TOTAL DIFFERENTIAL

Generally, the linearization process can be performed by means of a Taylor series expansion using the linear part of the series (omitting high order terms). The same process can be easily done applying a total differential to the net force and moment equations, since the effect of small disturbances to the system is to be determined.

The total differential of a given dependent variable \( A \) with respect to the independent variables \( x, y \) and \( z \) is given as

\[
dA = \frac{\partial A}{\partial x} \, dx + \frac{\partial A}{\partial y} \, dy + \frac{\partial A}{\partial z} \, dz
\]

\[ (III-1) \]

B. LINEARIZATION OF THE EQUATIONS OF MOTION AND AIR FLOW DYNAMICS

The total differentials of the equations of motion and air flow dynamics have been derived as

\[
dw = \frac{dz}{m}
\]

\[ (III-2) \]

\[
dq = \frac{dM}{I_{yy}}
\]

\[ (III-3) \]

\[
dm_b = - \left[ \rho \frac{n}{k_p} + C_n A_\zeta \left( \frac{\rho}{2F_b(0)} \right)^{1/2} \right] d\bar{F}_b
\]

\[ (III-4) \]

\[
dv = \frac{dy}{m}
\]

\[ (III-5) \]
where $\bar{P}_b(0)$ represents the steady state value of the plenum gauge pressure according to the final equilibrium condition.

The equations (II-52) and (II-54) both involved second order yaw and roll acceleration terms. Therefore applying the total differential and solving these two equations for $dp$ and $dr$ we have

$$dr = \frac{dK}{I} + \frac{dN}{I_r} \quad (III-6)$$

$$dp = \frac{dK}{I_k} + \frac{dN}{I} \quad (III-7)$$

$I$, $I_r$ and $I_k$ are defined as

$$I \triangleq \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{xz}} \quad (III-8(a))$$

$$I_r \triangleq \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{xx}} \quad (III-8(b))$$

$$I_k \triangleq \frac{I_{xx} I_{zz} - I_{xz}^2}{I_{zz}} \quad (III-8(c))$$

Equation (III-6) and equation (III-7) contain both net incremental roll (dK) and yaw (dN) moments. This is simply a cross product inertial term ($I_{xz}$) effect in the equations of motion.

In equation (II-59), $d\bar{P}_b$ is not a state variable, it can be considered as an output variable, therefore from equation (II-26) and (II-27), $d\bar{P}_b$ must be
\[ \frac{dP_b}{dz} = \frac{\partial P_b}{\partial z} dz + \frac{\partial P_b}{\partial m_b} dm_b \]

C. FIVE DEGREE-OF-FREEDOM LINEAR MODEL SENSITIVITY COEFFICIENTS

The sensitivity coefficients related to the right hand side of the equations of motion are defined as the partial derivatives of the net forces and moments divided by craft mass or inertial terms.

In [2], Barnes has developed a 2 DOF (heave and pitch) linear XR-3 model. Therefore most of the heave and pitch sensitivity coefficients are directly used in order to develop a 5 DOF linear model. However some modification is made for plenum sensitivity coefficients and pitch mode. Also some of the steady state operating point values have been reinvestigated using the 6 DOF nonlinear model.

In all the linearized equations, the final steady state values of the variables have been used.

1. Pitch and Heave Sensitivity Coefficients

In [2] equation (III-40), the sensitivity coefficient due to \( z \) has been redefined as

\[ DHPZ = -A_{b}(0) \frac{3P_{b}}{\partial z} + P_{b}(0) \frac{b_{w} l_{9}}{h} \]

This is calculated from (II-22) where the second term does not appear in [2].

The modified form of the plenum pitch sensitivity coefficient due to state variable \( z \) is to be
The derivation of these modified equations is based upon the total differential of the plenum lift force, it is

\[
dZ_P = \frac{3Z_P}{\partial P_b} dP_b + \frac{3Z_P}{\partial A_b} dA_b
\]

From equations (II-9), (II-26) and (II-27), the total differentials \(dV_b\) and \(dP_b\) have been calculated as

\[
dV_b = -(A_b(0) - \ell_d(0) \frac{b_w \ell g}{h}) dz \quad (III-9)
\]

\[
dP_b = \gamma P_b(0) \left[ \frac{dM_{Pb}}{M_{Pb}(0)} + \frac{1}{V_b'(0)} (A_b(0) - \ell_d(0) \frac{b_w \ell g}{h}) \right] dz. \quad (III-10)
\]

d\(P_b\) and d\(V_b\) equations are also different from the equations derived in [2].

The bow and aft seal pitch moments sensitivity coefficients (DPSFTH and DPSATH) due to \(\theta\) and plenum air mass equation (AMB) are rederived and their modified forms are used for the pitch mode in the 5 DOF linear model computer program.

Considering the 5 DOF linear model state space matrix representation, the \(a_{ij}\)'s represent "A" matrix elements. Therefore the \(a_{ij}\) values relating to the sensitivity coefficients for heave motion are determined as
\[ a_{21} = DZZ \]

\[ a_{23} = DZTH \]

\[ a_{24} = DZDP \]

\[ a_{25} = DZMB \]

where DZZ, DZTH, etc., have been derived in [2]. However their modified forms can be found in the 5 DOF linear computer program.

For pitch motion, \( a_{ij} \) coefficients are

\[ a_{41} = DTHZ \]

\[ a_{43} = DTHTH \]

\[ a_{44} = DTHDTH \]

\[ a_{45} = DTHMB \]

2. Plenum Air Flow Sensitivity Coefficients

These coefficients are calculated directly from [2]. The \( a_{ij} \)'s related to plenum air sensitivity coefficients are

\[ a_{51} = DMBZ \]
\[ a_{55} = DMBMB \]

3. **Sway Sensitivity Coefficients**

Before deriving the sensitivity coefficients for sway motion, some partial derivatives related to the rudder and sidewall hydrodynamics must be calculated. From equation (II-11(b)) \( Y_r \) is already known, this follows the partial derivatives of \( Y_r \) with respect to the state variables.

\[
R_p = \frac{\partial Y_r}{\partial p} = C_t \left( \frac{C_e}{u} \right) Z_s \quad (\text{III-11})
\]

\[
R_v = \frac{\partial Y_r}{\partial v} = - C_t \left( \frac{C_e}{u} \right) \quad (\text{III-12})
\]

\[
R_r = \frac{\partial Y_r}{\partial r} = - C_t \left( \frac{C_e}{u} \right) X_r \quad (\text{III-13})
\]

From equation (II-39), the sidewall cross flow drag terms \( D_{sw} \) partials are derived in the following equations.

Let us define first some coefficients

\[
d_1 \triangleq \frac{1}{2} \rho C_d l
\]

\[
d_r \triangleq l_d(0) + \text{sign}(V_r) b_o \phi(0)
\]

The partials are

\[
D_z = \frac{\partial D_{sw}}{\partial z} = - d_1 \text{sign}(V_r) V_r^2(0) \quad (\text{III-14})
\]
\[ D_\phi = -d_1 V_r^2(0) b \]  
(III-15)

\[ D_p = 2 d_1 \frac{d_r}{d_r} \text{sign}(V_r) V_r(0) (Z_s - x_d(0)/2) \]  
(III-16)

\[ D_v = -2 d_1 \frac{d_r}{d_r} V_r(0) \]  
(III-17)

\[ D_r = -2 d_1 \frac{d_r}{d_r} \text{sign}(V_r) V_r(0) X_s \]  
(III-18)

\( d_2 \) and \( d_3 \) are defined as

\[ d_2 \triangleq 2 A_{33} b_o^2 u \]  
(III-19)

\[ d_3 \triangleq A_{22} u \]  
(III-20)

where \( u \) is the craft speed assumed to be constant.

For sway mode, the state matrix elements are

\[ a_{96} = \frac{1}{m} \left( \frac{\partial Y}{\partial \phi} \right) = \frac{1}{m} (D_\phi \sin \beta) \]  
(III-21)

where \( D_\phi \) is given from equation (III-15).

\[ a_{97} = \frac{1}{m} \left( \frac{\partial Y}{\partial p} \right) \]  

\[ = \frac{1}{m} [R_p + (R_p + d_3 Z_s) \sin \beta + d_3 Z_s] \]  
(III-22)
\[ a_{99} = \frac{1}{m} \left( \frac{\partial Y}{\partial V} \right) \]
\[ = \frac{1}{m} [R_v + (D_v - d_3) \sin \beta - d_3] \quad (III-23) \]

\[ a_{911} = \frac{1}{m} \left( \frac{\partial Y}{\partial r} \right) \]
\[ = \frac{1}{m} [R_r + (D_r - d_3x_s) \sin \beta - d_3x_s - mu] \quad (III-24) \]

4. Roll and Yaw Sensitivity Coefficients

Recalling equations (III-6) and (III-7), the net roll and yaw moments appear in both equations of motion. In this case, to determine sensitivity coefficients is not easy, since the yaw and roll partials are involved in the dynamic equation with different inertial terms and coefficients.

In order to simplify the problem, the roll and yaw partial derivatives are calculated individually, then combining these coefficients and dividing by corresponding inertial terms give the system sensitivity coefficients for the roll and yaw modes.

a. Roll Mode

The net roll partial derivatives are defined by \( k_{ij} \) variables.

The partial derivative with respect to \( z \) is

\[ k_{71} = \frac{\partial K}{\partial z} = \frac{\partial}{\partial z} (K_{sf} + K_{sa} + K_{bf} + K_{ba}) \quad (III-25) \]

In equation (III-26) all roll components have already been derived in the preceding sections.
From equations (II-30) and (II-31), the seal partials are

\[ \frac{\partial K_{sf}}{\partial z} = - \left( \frac{b_0}{2.3} \right)^3 \rho g \left( \frac{1}{\sin 31^\circ} \right) \phi(0) \]

\[ \frac{\partial K_{sa}}{\partial z} = - \left( \frac{b_0}{2.3} \right)^3 \rho g \left( \frac{1}{\sin 32^\circ} \right) \phi(0) \]

For buoyancy partials, let us define the following coefficients;

\[ b_1 \triangleq 2 \rho g l_1 \] (III-26)

\[ b_2 \triangleq 2 \rho g l_2 \] (III-27)

\[ b_3 \triangleq \left( \frac{b_f + b_w}{2} \right) + \frac{l_f}{2 \tan \beta_1} \] (III-28)

\[ b_4 \triangleq \frac{l_f}{\tan \beta_1} + b_f \] (III-29)

\[ b_6 \triangleq \frac{l_a}{\tan \beta_2} + b_a \] (III-30)

From equations (II-35) and (II-36), the buoyancy partials are

\[ \frac{\partial K_{bf}}{\partial z} = - \frac{b_1 b_3}{\tan \beta_1} (b_5 + b_3) \phi(0) \]

\[ \frac{\partial K_{ba}}{\partial z} = - \frac{b_2 b_4}{\tan \beta_2} (b_6 + b_4) \phi(0) \]
Equation (II-26) follows;

\[
\frac{\partial \Phi}{\partial z} = -[A_c(0) \Phi_c(0) + (Z - \ell_3(0)) \frac{\partial \Phi}{\partial z}] \phi(0)
\]

Similarly for sidewall cross flow drag partial is

\[
\frac{\partial K_d}{\partial z} = D_z(b \cos \beta - Z \sin \beta)
\]

The roll partial derivatives with respect to \( \theta \) are

\[
k_r = \frac{\partial K}{\partial \theta}
\]

\[
= \frac{\partial}{\partial \theta}(K_{sf} + K_{sa} + K_{bf} + K_{ba}) \quad (III-31)
\]

where

\[
\frac{\partial K_{sf}}{\partial \theta} = \left(\frac{b}{2.3}\right)^3 \rho g(\ell_3/\sin 31^\circ) \phi(0)
\]

\[
\frac{\partial K_{sa}}{\partial \theta} = -\left(\frac{b}{2.3}\right)^3 \rho g(\ell_4/\sin 32^\circ)
\]

\[
\frac{\partial K_{bf}}{\partial \theta} = \frac{b_1 b_2}{\tan b_1} (b_5 + b_3 \ell_5) \phi(0)
\]

\[
\frac{\partial K_{ba}}{\partial \theta} = -\frac{b_2 b_4}{\tan b_2} (b_6 + b_4 \ell_3) \phi(0)
\]

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The roll partial derivative with respect to $M_b$ is

$$k_{75} = \frac{\partial K_p}{\partial M_b} = A_b(0)(Z_s - z_d(0)) \frac{\partial F_b}{\partial M_b} \phi(0) \quad (III-32)$$

The roll partials with respect to $\phi$ are

$$k_{76} = \frac{\partial K}{\partial \phi} = \frac{3}{\partial \phi}(K_{sf} + K_{sa} + K_{bf} + K_{ba} + K_p + K_d) \quad (III-33)$$

where

$$\frac{\partial K_{sf}}{\partial \phi} = -\left(\frac{b_w}{2.3}\right)^3 \rho g X_{f}$$

$$\frac{\partial K_{sa}}{\partial \phi} = -\left(\frac{b_w}{2.3}\right)^3 \rho g X_{a}$$

$$\frac{\partial K_{bf}}{\partial \phi} = -b_1 b_3^2 b_5$$

$$\frac{\partial K_{ba}}{\partial \phi} = -b_2 b_4^2 b_6$$

$$\frac{\partial K_p}{\partial \phi} = \bar{F}_b(0) (Z_s - z_d(0)) \phi b_w$$

53
\[
\frac{\partial K_d}{\partial \phi} = D_{\phi} (b_o \cos \beta - z_o \sin \beta)
\]

The roll partials with respect to \( p \) are

\[
k_{77} = \frac{\partial K}{\partial p} = \frac{\partial}{\partial p} (K_{sw} + K_d + K_r)
\]

(III-34)

where

\[
\frac{\partial K_{sw}}{\partial p} = -d_2 - z_s^2 A_{22} u - 32/\pi b_o^2 B_c
\]

\[
\frac{\partial K_d}{\partial p} = (D_p + \frac{\partial Y_{sw1}}{\partial p}) (b_o \cos \beta - z_o \sin \beta)
\]

The roll partials with respect to \( V \) are

\[
k_{79} = \frac{\partial K}{\partial V} = \frac{\partial}{\partial V} (K_{sw} + K_d + K_r)
\]

(III-35)

where

\[
\frac{\partial K_d}{\partial V} = (D_v - d_3) (b_o \cos \beta - z_o \sin \beta) + d_3 X_s Z_s
\]

\[
\frac{\partial K_{sw}}{\partial V} = d_3 Z_s
\]
The roll partials with respect to yaw rate are

\[
\frac{\partial K_r}{\partial \gamma} = C_t(C_e/u) Z_r
\]

The roll partials with respect to yaw rate are

\[
k_{711} = \frac{\partial K}{\partial \gamma}
\]

\[
= \frac{\partial}{\partial \gamma}(K_sw + K_d + K_r) \quad (\text{III-36})
\]

where

\[
\frac{\partial K_{sw}}{\partial \gamma} = d_3 X_s Z_s
\]

\[
\frac{\partial K_{sw}}{\partial \gamma} = (D_r - d_3 X_s)(b_o \cos \beta - z_o \sin \beta)
\]

\[
\frac{\partial K_r}{\partial \gamma} = C_t(C_e/u) X_r Z_r
\]

b. Yaw Mode

The following coefficients are defined for yaw rotational motion;

\[
n_1 \overset{\Delta}{=} 2 A_{22} \frac{\ell(1 - \lambda)}{5} \quad (\text{III-37})
\]

\[
n_2 \overset{\Delta}{=} -\pi/5 \rho u \ell \quad (\text{III-38})
\]

\[
n_3 \overset{\Delta}{=} -\text{sign}(v_r) \rho C_d \ell_d(0) \quad (\text{III-39})
\]
\[ C_1 \eq 2 S_1 \lambda^3 \]  

(III-40)

\[ C_2 \eq S_2 \lambda^2 \]  

(III-41)

\[ X_w \eq - \frac{1}{2} \rho C_f u^2 \lambda \]  

(III-42)

The net yaw partial derivatives are defined by \( n_{ij} \) variables.

The yaw partials with respect to \( \phi \) are

\[ n_{116} = \frac{\partial N_{sw}}{\partial \phi} \]

\[ = \frac{\partial N_1}{\partial \phi} + \frac{\partial}{\partial \phi} (X_{pw} - X_{sw}) b_o \]  

(III-43)

where

\[ \frac{\partial N_1}{\partial \phi} = 4 n_2 V(0) b_o \frac{2}{\phi(0)} \]

\[ \frac{\partial}{\partial \phi} (X_{pw} - X_{sw}) = -4 X_w \]

The yaw partials with respect to \( p \) are

\[ n_{117} = \frac{\partial N}{\partial p} = \frac{\partial}{\partial p} (N_{sw} + N_{r}) \]  

(III-44)

The \( N_{sw} \) and \( N_{r} \) partial derivatives can be calculated from equations (II-44(a)) and (II-11(d)).
The yaw moments partial derivatives with respect to $V$ and $r$ are derived similarly, the $n_{ij}$ coefficients related to these modes are

\[ n_{119} = \frac{\partial N}{\partial V} \]  
\[ n_{1111} = \frac{\partial N}{\partial r} \]  

(c. Roll Sensitivity Coefficients)

The roll sensitivity coefficients corresponding to state matrix elements $a_{ij}$'s are determined from equation (III-7). They are

\[ a_{71} = k_{71}/I_k \]
\[ a_{73} = k_{73}/I_k \]
\[ a_{75} = k_{76}/I_k \]
\[ a_{76} = k_{76}/I_k + n_{116}/I \]
\[ a_{77} = k_{79}/I_k + n_{119}/I \]
\[ a_{711} = k_{711}/I_k + n_{1111}/I \]

where inertial terms $I$ and $I_k$ are found from equations (III-8(a)) and (III-8(c)).
d. Yaw Sensitivity Coefficients

For yaw motion, the sensitivity coefficients $a_{ij}$'s have been determined from equation (III-6). These coefficients are listed below:

$$a_{111} = \frac{k_7}{I}$$

$$a_{113} = \frac{k_7}{I}$$

$$a_{115} = \frac{k_7}{I}$$

$$a_{116} = \frac{k_7}{I} + \frac{n_{116}}{I_r}$$

$$a_{117} = \frac{k_7}{I} + \frac{n_{119}}{I_r}$$

$$a_{1111} = \frac{k_7}{I} + \frac{n_{1111}}{I_r}$$

where the inertial term $I_r$ is calculated from equation (III-8(b)).

D. STATE SPACE REPRESENTATION AND OUTPUT EQUATION

The state vector of the linear 5 DOF model is shown as

$$X = [dz \; dw \; d\theta \; dq \; dM_\rho \; d\phi \; dp \; dy \; dV \; d\psi \; dr]^T \quad (III-47)$$

Then, the system state equation must be

$$\dot{X} = Ax + Bu \quad (III-48)$$
And the output equation is

\[ y = C X \]  

(III-49)

"A" matrix elements have already been determined. However for "B" matrix elements, the step input to the system can be considered as a weight removal disturbance and for the roll and yaw modes, a rudder order disturbance.

For weight removal forcing function, the input is to be

\[ b_{21} = \frac{W_f - W_i}{m} \]  

(III-50)

where

- \( W_f \) = final craft weight
- \( W_i \) = initial craft weight
- \( m \) = craft mass corresponding to the final weight \( W_f \).

When the rudder order is used, sway, roll and yaw forcing functions are generated as follows;

**Sway**

\[ b_{91} = \frac{1}{m} (2 X_t \sin \delta_r + C_t \delta_r) \]  

(III-51)

**Roll**

\[ b_{71} = (2 X_t \sin \delta_r + C_t \delta_r) (-z_r/I_k + x_r/I) \]

**Yaw**

\[ b_{111} = (2 X_t \sin \delta_r + C_t \delta_r) (x_r/I_x + z_r/I) \]
Finally, defining state variables \( x_1, x_2, \ldots, \) etc.,
the state equations of the 5 DOF linear model are given as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_{21}x_1 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a_{41}x_1 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 \\
\dot{x}_5 &= a_{51}x_1 + a_{55}x_5 \\
\dot{x}_6 &= x_7 \\
\dot{x}_7 &= a_{71}x_1 + a_{73}x_3 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 \\
&\quad + a_{79}x_9 + a_{711}x_{11} \\
\dot{x}_8 &= x_9 \\
\dot{x}_9 &= a_{96}x_6 + a_{97}x_7 + a_{99}x_9 + a_{911}x_{11} \\
\dot{x}_{10} &= x_{11}
\end{align*}
\]
\[ \dot{x}_{11} = a_{111}x_1 + a_{113}x_3 + a_{115}x_5 + a_{116}x_6 \\
+ a_{117}x_7 + a_{119}x_9 + a_{1111}x_{11} \]

All coefficients of the state equations are defined in the preceding sections.

Considering outputs of the linear model, only incremental changes of the variables can be observed.
IV. LINEAR FIVE DEGREE-OF-FREEDOM COMPUTER PROGRAM DESCRIPTIONS

The Digital Simulation Language (DSL/360) is used for the linear 5 DOF XR-3 computer program. Eleven first order integrator blocks are needed for five degree of freedom simulations and for all linear runs, fourth order Runge-Kutta with fixed interval integration method is selected.

In addition to the craft dimensional values, initial and final steady state operating points are entered into the program for linearization purposes and plot outputs.

The input parameters of this program are:

\[
\begin{align*}
V &= \text{Craft speed (ft/sec)} \\
WF &= \text{Final craft weight (lb)} \\
W &= \text{Initial craft weight (lb)} \\
RUDANG &= \text{Rudder input (positive for port turn)} \\
DR &= \text{Average deadrise angle} \\
PBBAR &= \text{Final steady state value of plenum pressure (lb/ft}^2) \\
ALD &= \text{Final steady state craft draft (inches)} \\
PHI &= \text{Final steady state roll angle} \\
FYP &= \text{The propulsion system thrust corresponding to the craft speed (for each of propellers)} \\
VL &= \text{Steady state lateral speed (ft/sec)} \\
R &= \text{Steady state yaw rate (rad/sec)}. 
\end{align*}
\]
The steady state values of the variables are determined from 6 DOF nonlinear model simulations.
V. SIX DEGREE-OF-FREEDOM NONLINEAR MODEL SIMULATIONS

In this chapter 6 DOF nonlinear model simulations are discussed.

At first, some of the runs are made in order to determine craft initial steady state operating point corresponding to craft speed and thrust inputs. Since 6 DOF model weight transient response can be analyzed by means of entering proper craft initial values into the program, this is simply an initial condition response of the nonlinear dynamic system.

A. WEIGHT REMOVAL TRANSIENT

For 30 and 20 knot craft speeds, the steady state values of some variables are listed in Table I.

<table>
<thead>
<tr>
<th>Speed (knt)</th>
<th>Pitch (degree)</th>
<th>$\bar{F}_b$ (lb/ft$^2$)</th>
<th>Draft (inch)</th>
<th>Thrust (lb)</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.95</td>
<td>24.8</td>
<td>8.6</td>
<td>355.6</td>
<td>6722</td>
</tr>
<tr>
<td>30</td>
<td>0.26</td>
<td>24.8</td>
<td>5.6</td>
<td>287.8</td>
<td>6050</td>
</tr>
<tr>
<td>20</td>
<td>0.45</td>
<td>24.8</td>
<td>6.06</td>
<td>186.2</td>
<td>6050</td>
</tr>
</tbody>
</table>

For 30 knot craft speed, two different steady state pitch angles are observed corresponding to the different weights. In [2] for pitch transient response, 0.65° steady state pitch angle value has been used for both linear and nonlinear...
simulations. On the other hand as seen in Table I, 0.26° and 0.95° steady state pitch values are obtained. This disagreement may be explained by noting that different dimensional values were used or more probably a different modified 6 DOF computer program deck was used.

For 6050 lb craft weight, the steady state pitch angle 0.26° is in agreement with the value determined by Menzel in figure 6 in [6].

During the weight removal response, simulations of 6 DOF, two different stern seal base leakage areas are used. In [1] a smaller leakage area is assumed for the linear heave only model as $A_\xi = 0.384$. However $A_\xi = 3.84$ also has been simulated and it is understood that the smaller leakage area is more reasonable considering other steady state variables in the air flow dynamic equations (11-23) and (11-24). Since at the steady state condition $Q_{in} = Q_{out}$ must be satisfied, that permits only smaller $A_\xi$ for a given $Q_o$ and $P_b$ steady state values. Therefore $A_\xi = 0.384$ is preferred and used for nonlinear simulations.

B. TURNING MANEUVER

When the 3 DOF pitch, heave and roll linear model was developed, the 6 DOF nonlinear model simulations were used in order to determine all steady state roll moments during the turn maneuver. In addition to the roll moments, yaw and sway outputs were also investigated and the results are tabulated in Tables II, III and IV.
Table II

**Steady State Roll Moments**

(rudder = 5°, 30 knot speed)

<table>
<thead>
<tr>
<th>Roll Moments</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern seal FKSS</td>
<td>-</td>
<td>-2.2</td>
</tr>
<tr>
<td>Bow seal FKBS</td>
<td>-</td>
<td>-280.</td>
</tr>
<tr>
<td>Sidewall FKSW</td>
<td>-</td>
<td>-41.5</td>
</tr>
<tr>
<td>Rudder FKRUD</td>
<td>131.</td>
<td>-</td>
</tr>
<tr>
<td>Propulsion FKP</td>
<td>0.057</td>
<td>-</td>
</tr>
<tr>
<td>Aerodynamic FKAED</td>
<td>-</td>
<td>-22.7</td>
</tr>
<tr>
<td>Plenum ABPB<em>PHI</em>(-Z)</td>
<td>215.3</td>
<td>-</td>
</tr>
</tbody>
</table>

Table III

**Steady State Yaw Moments**

(rudder = 5°, 30 knot speed)

<table>
<thead>
<tr>
<th>Yaw Moments</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern seal FNBS</td>
<td>3.22</td>
<td>-</td>
</tr>
<tr>
<td>Bow seal FNSS</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Sidewall FNSW</td>
<td>-</td>
<td>-482.2</td>
</tr>
<tr>
<td>Rudder FNRUD</td>
<td>532.65</td>
<td>-</td>
</tr>
<tr>
<td>Propulsion FNP</td>
<td>2.78</td>
<td>-</td>
</tr>
<tr>
<td>Aerodynamic FNAED</td>
<td>-</td>
<td>-56.41</td>
</tr>
</tbody>
</table>
Table IV

**Steady State Sway Forces**

(rudder = 5°, 30 knot speed)

<table>
<thead>
<tr>
<th>Sway Forces</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sidewall FYSW</td>
<td>-</td>
<td>-351.8</td>
</tr>
<tr>
<td>Rudder FYRUD</td>
<td>-</td>
<td>-47.6</td>
</tr>
<tr>
<td>Aerodynamic FYAED</td>
<td>-</td>
<td>-19.07</td>
</tr>
<tr>
<td>Centrifugal -R<em>U</em>AM</td>
<td>419.24</td>
<td>-</td>
</tr>
</tbody>
</table>

From Table II, it is understood that for a given (+) rudder input, the sign convention of the craft dynamics was in agreement with turning to the left. However, a negative sway displacement plot is observed for this maneuver. As a result, in the R.H.S. subroutine of the 6 DOF program, the sign correction should be done for y-displacement.

The steady state sway forces acting on the XR-3 test craft in a turn to the left are shown in figure 6. As seen in this figure and from Table IV, the rudder sway force is in the steady state condition. For a left turn, initially the rudder sway force is generated in the reverse direction and finally a negative steady state rudder sway force is produced. Further investigation is done in the RUDDER subroutine dynamic equations of the 6 DOF computer model, but no error has been determined in these equations. Also, different craft speeds and different effective rudder angle coefficients (ENDFAC) are used but everytime the same configuration is observed.
Figure 6. Steady state sway forces configuration at the left turn
In addition to rudder dynamics, the propulsion system of the XR-3 contributes to the roll, yaw and sway motions. However this contribution is not a dominant factor compared with the rudder effect.

For the XR-3 test craft propulsion system configuration, the direction of the thrust vector is also changed as well as the rudder deflection. As seen from Tables II, III and the simulations with different rudder order, it is understood that the propulsion thrust component due to the rudder order is not calculated properly, therefore the following modification is made in the PROPULSION subroutine of the 6 DOF program,

\[ F_{YS} = -STHSTS*CD + THSS*CD \]

where \( F_{YS} \) represents the sway force due to the starboard propeller, and the others are defined in the propulsion subroutine program.

C. DEADRISE ANGLE EFFECT

As reported in [4], the deadrise angle has a major effect on the surface effect ship's roll behavior and therefore is an important design consideration. The deadrise angle effect on the roll motion has been investigated by Reidel in [3]. In this investigation, some arbitrary coefficients (PROMO 1, 2, etc.) are used for the vertical force component of the sidewall hydrodynamics. From this investigation, the conclusion reported by Reidel was that more damping could be obtained for the pitch and roll motions by changing the deadrise angle.
The deadrise effect is reinvestigated by simulating the system with different rudder orders including zero disturbance, then the heave forces due to sidewall hydrodynamics were compared to each other. As a result, no significant sidewall heave force change was observed. For instance, for 0° and 5° rudder inputs, the sidewall heave forces (FZSW) are found to be -568.8 and -568.05 respectively. This result is not acceptable, since in a turn maneuver, the cross flow drag is generated at the outboard sidewall, so the deadrise projection of cross flow drag must be contributed to the sidewall heave force. More investigation has been made in the SIDEWALL subroutine of the 6 DOF program and it has been determined that only the force component due to added mass given by equation (II-21) is projected as a lift force. Actually the cross flow drag component must be included to this projection, since the frontal area for cross flow drag component is outboard of the submerged section of the sidewall. Therefore it is concluded that the XR-3 test craft 6 DOF program SIDEWALL subroutine does not include effectively the lift force due to deadrise. On the other hand, the lift force and horizontal drag component can have different drag coefficients, therefore the modification of the sidewall subroutine is left for future studies.
VI. COMPARISON OF NONLINEAR AND LINEAR MODEL RESULTS

A. C.G. ACCELERATION (see figs. 9 and 23)

A 10% weight removal disturbance is used for both models. The linear and nonlinear model showed very rapid response with small rise time. As seen in figure 9, the 6 DOF model transient is relatively more damped compared with the linear model response. The linear model showed a small overshoot but it is also sufficiently damped. Small oscillatory behavior is observed in both figures which is due to pitch-heave coupling. In general the two models agree in transient response and steady state behavior.

Both of the C.G. acceleration curves were normalized by a factor "g", the gravitational constant.

B. PITCH TRANSIENT (see figs. 10 and 24)

As seen in these figures, both models showed an oscillatory transient response with the same frequency, \( w \approx 3.12 \) rad/sec. However their steady state values are different. In fact, the pitch moment due to plenum pressure and the effective plenum pressure center play a dominant role for pitch motion. In [2] Barnes developed plenum modeling including effective plenum roof area calculations and plenum pressure centroid shift due to draft variation. Therefore the linear model plenum modeling is different from the nonlinear modeling, so that a greater plenum moment arm is calculated in the linear model.
The steady state pitch angle values are 0.5° and 0.75° for nonlinear and linear models correspondingly.

C. PLENUM PRESSURE TRANSIENT (see figs. 7 and 21)

As seen in these figures, both models are exactly in agreement with each other. In each curve, the plenum pressure suddenly drops to 21.6 (lb/ft²) then increases up to its steady state value with sufficient damping. This is expected behavior because the plenum volume increases when 10% weight is removed, obviously this disturbance causes a sudden drop in plenum pressure. As seen before, pitch-plenum pressure coupling had been observed during the exponential increasing phase of the plenum pressure. At the fifth second of simulation time, nonlinear model plenum pressure is about 24.29 while the linear model pressure is 24.25.

D. DRAFT TRANSIENT (see figs. 8 and 22)

Excellent agreement was achieved between linear and nonlinear model draft responses. As expected, the draft decreases exponentially when the weight is removed from the craft. The draft is found to be 5.7 inches for the linear model, 5.85 inches for the nonlinear model at the fifth second of simulation time.

As a result, both models exhibited the same draft responses.

E. ROLL RESPONSE

As mentioned before, at first a 3 DOF linear model (pitch-heave-roll) had been developed. 3 DOF linear and 6 DOF
nonlinear models roll behavior comparison is done by means of a step input roll moment disturbance to the linear model and a turning command to the nonlinear model. In figure 13, 6 DOF nonlinear model roll angle response is shown for a 5° rudder order (left turn), in this simulation 1.18° outboard steady state roll angle is generated. From the R.H.S. subroutine of the 6 DOF program, all the steady state roll moment components are determined as seen in Table II and the sum of these disturbance moments applied to the linear model roll mode as a step input. Roll behavior is analyzed from figure 27. As expected, the linear model showed a greater overshoot and more oscillatory response compared with the nonlinear roll transient. Since the linear model has been disturbed by a step roll moment corresponding to the nonlinear model steady state roll moments. On the other hand, its steady state roll angle is found to be 1.28°. The roll steady state difference of two models is about 8.4%. For a 3° rudder input, nonlinear and linear simulations are obtained in figures 11 and 25 with the steady state values 0.62° and 0.65° respectively. Hence the difference is 4.8% in the steady state values for these simulations.

Figures 12 and 14 show XR-3 yaw rate plots and corresponding linear simulations are shown in figures 26 and 28.

As a result, the 3 DOF linear model and 6 DOF program roll simulations are in agreement with each other within 10% difference.
As discussed in the previous chapter, some problems are observed in the 6 DOF nonlinear program. Therefore developing two more modes for the 3 DOF model, the 5 DOF linear model has been generated. By means of this model, roll behavior of the XR-3 in a turn, deadrise angle and cross product inertial term effects have been investigated for 20 knot craft speed with different rudder deflections.

The roll response can be seen in figure 29 omitting the cross product inertial term \((I_{xz} = -2800.)\) in the equations of motion, applying a 10° rudder order and assuming 70 degree average deadrise angle of the craft. For this particular test, the craft starts inboard rolling with the oscillatory transient then goes to 0.47° steady state roll. Using the same input parameters and also including the cross product term \(I_{xz}\) in the equations of motion, smaller overshoot (18%) and a more damped roll response has been observed in figure 31. As seen in this figure, at the initial phase, secondary overshoot has been generated. The steady state roll angle is 0.40° and the oscillation frequency \(w = 3.5\) rad/sec., for this simulation. On the other hand, the corresponding 6 DOF nonlinear simulation is shown in figure 17 and also a roll rate plot is shown in figure 18. The nonlinear simulation in figure 17 shows a more damped transient but greater steady state roll behavior (1.32°) compared with the corresponding linear simulation shown in figure 31.
Figures 30 and 32 present linear simulation curves for 5 and 15 degree rudder inputs respectively. Both curves exhibit the same linear model response discussed before. Finally the linear and nonlinear model steady state responses are summarized as follows.

<table>
<thead>
<tr>
<th>Rudder</th>
<th>6 DOF Nonlinear</th>
<th>5 DOF Linear</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.57</td>
<td>0.24</td>
<td>figures 15 and 30</td>
</tr>
<tr>
<td>10</td>
<td>1.32</td>
<td>0.40</td>
<td>figures 17 and 31</td>
</tr>
<tr>
<td>15</td>
<td>2.2</td>
<td>0.45</td>
<td>figures 19 and 32</td>
</tr>
</tbody>
</table>

The deadrise angle effect is investigated by using different deadrise angle values for 10 degree rudder input and 20 knot craft speed simulations. The results are tabulated in Table V.

**Table V**

*Sidewall Deadrise Angle Effect*

(rudder = 10°, 20 knot speed)

<table>
<thead>
<tr>
<th>Deadrise angle (degree)</th>
<th>Roll angle (degree)</th>
<th>Lateral Speed (ft/sec)</th>
<th>Yaw rate (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>-1.01</td>
<td>4.18</td>
<td>-0.064</td>
</tr>
<tr>
<td>50</td>
<td>-0.84</td>
<td>4.04</td>
<td>-0.069</td>
</tr>
<tr>
<td>55</td>
<td>-0.62</td>
<td>3.90</td>
<td>-0.076</td>
</tr>
<tr>
<td>60</td>
<td>-0.30</td>
<td>3.83</td>
<td>-0.080</td>
</tr>
<tr>
<td>65</td>
<td>0.02</td>
<td>3.73</td>
<td>-0.085</td>
</tr>
<tr>
<td>70</td>
<td>0.40</td>
<td>3.60</td>
<td>-0.100</td>
</tr>
</tbody>
</table>
For the linear model, the lateral velocity and yaw rate plots are shown in figures 33 and 34. Both of them showed sufficiently damped transient response. From figure 33, the lateral speed steady state value is found 3.6 (ft/sec) and as expected for turning to the left, the negative yaw rate is generated as -0.1 (rad/sec).
Figure 7. 6 DOF Plenum pressure transient
Figure 8. 6 DOF draft transient

X-SCALE=1.00E+00 UNITS INCH.
Y-SCALE=2.00E+00 UNITS INCH.
XR-3 30 KNOTS SPEED
PLOT IS Z DISPLACEMENT
X-SCALE = 1.00E+00 UNITS INCH.
Y-SCALE = 2.00E-02 UNITS INCH.
XR-3 30 KNOTS SPEED
PLOT IS C.G. ACCELERATION

Figure 9. 6 DOF C.G. Acceleration
Figure 10. 6 DOF pitch angle transient
Figure 11. 6 DOF Roll angle plot, $\delta_r = 30^\circ$
30 knot speed
Figure 12. 6 DOF roll rate plot, $\delta_r = 3^\circ$

30 knot speed
X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=5.00E-01 UNITS INCH.
XR-3 30 KNOTS SPEED RD=5
PLOT IS ROLL ANGLE

Figure 13. 6 DOF roll angle plot, $\delta_r = 5^\circ$
30 knot speed
Figure 14. 6 DOF roll rate plot, $\delta_r = 5^\circ$
30 knot speed

X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=1.00E+00 UNITS INCH.
XR-3 30 KNOTS SPEED RD=5
PLOT IS ROLL RATE
Figure 15. 6 DOF roll angle plot, $\delta_r = 5^\circ$
20 knot speed

X-SCALE = 5.00E+00 UNITS INCH.
Y-SCALE = 1.00E-01 UNITS INCH.
XR-3 20 KNOTS SPEED RD = 5
PLOT IS ROLL ANGLE
Figure 16. 6 DOF roll rate plot, $\delta_r = 5^\circ$
20 knot speed
Figure 17. 6 DOF roll angle plot, $\delta_r = 10^\circ$
20 knot speed
Figure 18. 60 DOF roll rate plot, $\delta_r = 10^\circ$
20 knot speed
Figure 19. 6 DOF roll angle plot, $\delta_r = 15^\circ$

20 knot speed
X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=1.00E+00 UNITS INCH.
XR-3 20 KNOTS SPEED RD=15
PLOT IS ROLL RATE

Figure 20.  6 DOF roll rate plot, $\delta_r = 15^\circ$
20 knot speed

90
Figure 21. Linear 5 DOF plenum pressure transient
Figure 22. Linear 5 DOF draft transient
Figure 23. Linear 5 DOF C.G. acceleration
Figure 24. Linear 5 DOF pitch angle transient
Figure 25. Linear 3 DOF roll angle plot, $\delta_r = 3^\circ$
30 knot speed
DEVELOPMENT OF THE FIVE DEGREE-OF-FREEDOM LINEAR MODEL FOR THE --- ETC

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Figure 26. Linear 3 DOF roll rate plot, $\delta_r = 3^\circ$
30 knot speed
Figure 27. Linear 3 DOF roll angle plot, $\delta_r = 5^\circ$
30 knot speed
PLOT IS ROLL RATE
XR-3 30 KNOTS SPEED RD=5

Figure 28. Linear 3 DOF roll rate plot, $\delta_r = 5^\circ$
30 knot speed
Figure 29. Linear 5 DOF roll angle plot, $\delta = 10^\circ$
20 knot speed (omitting $I_{xz}$ term)
PLOT IS ROLL ANGLE
XR-3 20 KNOTS SPEED RD=5

Figure 30. Linear 5 DOF roll angle plot, $\delta_r = 5^\circ$
20 knot speed
Figure 31. Linear 5 DOF roll angle plot, $\delta_x = 10^\circ$

... 20 knot speed
PLOT IS ROLL ANGLE
XR-3 20 KNOTS SPEED RD=15

Figure 32. Linear 5 DOF roll angle plot, $\delta_r = 15^\circ$
20 knot speed
Figure 33. Linear 5 DOF lateral velocity plot
Figure 34. Linear 5 DOF yaw rate plot
VII. CONCLUSIONS AND RECOMMENDATIONS

In the preceding sections, the 5 DOF linear XR-3 model was developed, then weight removal response and roll behavior are compared with 6 DOF nonlinear simulations. Also some of the craft sensitivity parameters are investigated for the XR-3 test craft.

The results of this study are summarized and recommendations are made as follows.

1. The system response is investigated by both a linear model and a 6 DOF nonlinear model by means of sudden weight removal from the C.G. and by sway force disturbance in a turn.

Comparison of results shows that the difference is within 10% for both models. In fact, assuming accuracy in measurements of about 10%, the linearized model gives acceptable results as compared with the nonlinear model. Obviously the linear model simulations take less computer CPU time.

2. Concerning roll response during the turn maneuver, the product inertia term ($I_{xz}$) affects transient response such as overshoot, damping, etc. More overshoot and oscillatory response have been observed without $I_{xz}$. However, this term is found to be -2800. from the 6 DOF program. It is suggested that this number be verified.
3. The importance of deadrise is observed for roll steady state behavior. The XR-3 test craft steady state roll angle depends upon the average deadrise angle of the sidewall. Different average deadrise angles have been simulated by means of the linear model and it is observed that the craft rolls inboard or outboard according to the sidewall deadrise configuration.

The 6 DOF nonlinear model SIDEWALL SUBROUTINE must be modified by including a cross flow drag component deadrise projection (lift force). However a new drag coefficient is needed for this lift component.

4. The aerodynamic effect of the craft contributes about 5% of the total yaw moment.

5. The XR-3 failure mode operation (such as fan failure) can be simulated by using either the linear or nonlinear model. But, the nonlinear model is preferable for failure mode simulations, since it includes sidewall and stern seal leakage configurations.
APPENDIX A.

COMPUTER PROGRAM LISTING

XK-3 5 D.O.F. LINEAR MODEL

***
**
** INTEG RKSFX
** INCCK A=0.0
** INTEGER NPLCT
** CONST NPLCT=1
** INITIAL
**
** PHYSICAL CONSTANTS
**
G=22.2
GAMMA=1.4
PA=2116.0
PI=3.141553
RHO=2.0
RHOA=0.002378

CRAFT DIMENSIONS

ZS=2.5
BUHGT=1.515
XCPC=0.4634
XS=10.65
YSW=5.
ZSW=2.3
PLARM=4.23
PLAREA=6.78
PLCOEF=C.16
CDSW=1.28
DR=70.
L=20.
WDF=10.0
A1=10.3
A2=9.56
A3=11.52
A4=8.26
A5=A1/2.0
A6=A2/2.0
A7=4.227
A8=17.1
D1=61.6
D2=68.5
WS1=0.52
WS2=0.52
CN=0.9
ALPF=15.72
ALPWL=15.65

**
ANGULAR CONVERSION FACTORS AND CONVERSIONS

**

RACEG=360.0/(2.0*PI)
DEGRAD=(2.0*PI)/360.0
CR1=CR1*DEGRAD
CR2=CR2*DEGRAD
BETA=CR*DEGRAD
CR=BETA

**

INPUT FINAL OPERATING POINT

V=33.73
PBAR=24.6
THETA=0.26
ALD=5.34
VL=3.6
R=-0.1
PHI=0.47
PHI=0.25
W=6.722
WF=6.056

**

RUDDER CROSER INPUT

RUCANG=10.

**

VELOCITY REDUCTION

V=SQRT(V**2-VL**2)

**

AMASS=MF/G
ALE=ALC/2.4
THETA=THETA*DEGRAD
RUDDANG=RUCANG*DEGRAD
** INPUT INITIAL OPERATING POINT

PHI=0.0
THI=0.5
PB1=24.5E
LD1=8.5E
FPY=18.6E
PHI=PHI*CEGRAD

** CALCULATE INTERMEDIATE VARIABLES AND CONVERSIONS

ALC1=ALC-AL5*THETA
ALC2=ALD+AL6*THETA
\( S1=\sin(\frac{\pi}{2} \cdot \tan(\text{DR1})) \)
\( S2=\sin(\frac{\pi}{2} \cdot \tan(\text{CR2})) \)
AK1=2.0*\sin(\theta)*AL1
AK2=2.0*\sin(\theta)*AL2
AK3=\sin(32*\text{DEGRAD})
AK4=\sin(32*\text{DEGRAD})
XSEAL1=(ALC*AL3*THETA)/\sin(32.0*\text{DEGRAD})
XSEAL2=(ALC*AL4*THETA)/\sin(32.0*\text{DEGRAD})
ASEAL1=\sin(32*\text{DEGRAD})
ASEAL2=\sin(32*\text{DEGRAD})
XPC=\sin(32*\text{DEGRAD})
AB=\sin(32*\text{DEGRAD})
VB=\sin(32*\text{DEGRAD})
P=\sin(32*\text{DEGRAD})
AMB=-\sin(32*\text{DEGRAD})

**
* OUTPUT FORCES, MOMENTS, AND RESIDUALS AT FINAL OPER POINT

\begin{align*}
\text{PPRES} &= \text{AB} \times \text{PBBAR} \\
\text{PPRES} &= \text{HPR} \times \text{XCP} \\
\text{HBA} &= \text{AK1} \times \text{ALD01} \times \text{WS1} \\
\text{PBA} &= \text{AK2} \times \text{ALD02} \times \text{WS2} \\
\text{HSF} &= \text{PBBAR} \times \text{ASEAL1} \\
\text{PBA} &= \text{PLC} \times \text{PBBAR} \times \text{ASEAL2} \\
\text{HPF} &= \text{PLC} + \text{PBBAR} \times \text{PPLC} \\
\text{PBL} &= \text{AL5} \times \text{BF} \\
\text{PBB} &= \text{ALC} + \text{EA} \\
\text{PAB} &= \text{PLS1} \times \text{XSEAL1} \times 2 \times \text{O} \\
\text{PAB} &= \text{PLS1} \times \text{XSEAL2} \times 2 \times \text{O} \\
\text{PPL} &= \text{HSF} \times \text{PLF} \\
\text{PLA} &= \text{PLC} \times \text{PLP} \times \text{PLC} \times \text{THT} \\
\text{PLA} &= \text{PLP} \times \text{PLC} \times \text{PLC} \times \text{THT} \\
\text{PRP} &= \text{HPR} \times (\text{RS} - \text{ALD1} \times \text{PHI} \\
\text{RRP} &= \text{AK1} \times (\text{H} - \text{PHI} \\
\text{RBA} &= \text{AK2} \times \text{B12} \times 2 \times \text{BW} \times \text{PHI} \\
\text{RSF} &= (\text{HS} \times 2 \times 2.52) \times \text{HSEAL1} \times \text{RHO} \times \text{G} \times \text{PHI} \\
\text{PES} &= \text{SEAL} \times \text{SEAL2} \times \text{RHO} \times \text{G} \times \text{PHI} \\
\text{RES} &= \text{H} \times \text{BF} + \text{HBA} \times \text{HF} + \text{HSA} \times \text{HPR} \times \text{PHRES} \\
\text{RES} &= \text{PBB} \times \text{PBB} \times \text{PSF} \times \text{PSA} \times \text{PLP} \times \text{PPESP} + \text{PPRES} \\
\text{RES} &= \text{RFE} \times \text{RBB} \times \text{RBA} \times \text{RSF} \times \text{RSA} \\
\text{ACC} &= \text{RES} \times \text{PHI} \times \text{AMASS} \\
\text{PACC} &= \text{RES} \times \text{PHI} \times \text{AV} \\
\text{RACC} &= \text{RES} \times \text{PHI} \times \text{AX} \\
\text{WSTF} &= \text{W} - \text{T} \\
\text{SIN}31 &= \text{SIN31} (31.0 \times \text{DEGRC}) \\
\text{SIN}32 &= \text{SIN32} (32.0 \times \text{DEGRC}) \\
\end{align*}

** COMPLETE SENSITIVITY COEFFICIENTS **

** DERIVATIVES OF PBBAR AND MB **

\begin{align*}
\text{DPBB} &= \text{GAMMA} \times (\text{AB}1 - \text{ABC1F} \times \text{ALD/UBHGT}1 \times \text{PB/VB} \\
\text{CPBB} &= \text{GAMMA} \times \text{PB} \times \text{AMB} \\
\text{DMBB} &= \text{RHOA} \times (\text{CN} \times \text{CN} \times \text{AL/RTQA} \times \text{SORT}(2.0 \times \text{PBBAR} \times \text{RHQA})) \\
\text{DMBB} &= \text{DPBB} \times \text{DPBB} \\
\text{EMBB} &= \text{CPBB} \times \text{DPBB} \\
\end{align*}

** DERIVATIVES OF HULL FORCES W/RESPECT TO Z **
CHBFZ=-AK1*((ALD-AL5*THETA)/TAN(DR1)+WS10)
CHBAZ=-AK2*((ALD+AL6*THETA)/TAN(DR2)+WS20)
DHBZ=(CHBFZ+CHBAZ)
CHSFZ=-AK3*(PBS+AK3*(ALD-AL3*THETA)*DPBDZ
DBZ=PB+ADIFF/BS*GT
DHSZ=AK4*(PB+Z)*-AK4*(ALD+AL4*THETA)*DPBDZ
DPSZ=PSF+DHSZ
DPSBZ=-AB*DPBDZ*DPBDZ
CZZ=(CHBZ+DHSZ+DPBDZ)/ANASS

**
DERIVATIVES OF HEAVE FORCES W/RESPECT TO THETA

DHBFT=AK1*((ALD-AL5*THETA)/TAN(DR1)+AL5+WS10)
CHBFTH=-AK2*((ALD+AL6*THETA)/TAN(DR2)+AL6+WS20)
DHSTH=AK3*(PBS+AK3*(ALD-AL3*THETA)*DPBDZ
DHSSTH=AK4*(ALD+AL4*THETA)*DPBDZ
DSWH=2*(2*(A33S+THETA)*(2*RHO*PI*BS*X5/TAN(DR2))/8)
CPFT=FLC*0.5*DSWH
DHPATH=FLC*0.5*DSWH
DHSPTH=CPFTHDXPHATH
DPTH=CPFTH+DHPATH
CZTH=(CHBTH+DHSSTH+DPSTH)/ANASS

**
DERIVATIVE OF HEAVE FORCE W/RESPECT TO MB

DHPBMB=-AB*CPMB
DHSFBMB=-ASEALD*DPBMB
DHSAMB=-ASEAL2*DPBMB
DZMB=(CPMB+DHSFMB+DHSAMB)/ANASS

DPEP=2*(BC2+A33S*X5)*V
DZDP=DFCP/ANASS

**
DERIVATIVES OF HEAVE FORCES W/R TO PHI

CZPH=0.0

**
DERIVATIVES OF PITCH MOMENTS W/RESPECT TO THETA

DPBFTH=-AL5*DBFTH
DPATH=AL6*DBATH
CPATH=CPBFT+DPATH
DPPLTH=-2*(PLC+PLAR)*DSWH*X5
DPSTF=PS+PBAR*(PLS*AL3*WIDTH/SIN31+ASEAL1*AL3/(2.*SIN31))
DPSATH=1*(PB+Z1)*(PLS*AL4*WIDTH/SIN32+ASEAL2*AL4/(2.*SIN32))
DPRPTH=C-C
CPSTH=(DFSPTH+DPSATH)

**
DTTH=(CPBTH+DPPLTH+DPSTH+DPPRTH)/AIYY

**DERIVATIVES OF PITCH MOMENTS W/R TO THETACOT**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPPN8 = -2.0<em>P35</em>X5**2*V</td>
<td>S502290</td>
</tr>
<tr>
<td>DTHTH=CFDPM/IAYY</td>
<td></td>
</tr>
</tbody>
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**DERIVATIVES OF PITCH MOMENTS W/RESPECT TO Z**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPFBZ=AL5*DMFBZ</td>
<td>S502340</td>
</tr>
<tr>
<td>DPBZ=AL6*CPBZ</td>
<td>S502350</td>
</tr>
<tr>
<td>DPPZ=XCP*(AB<em>DPBDZ-PPBAR</em>ABDIFF/BUBHGT)+AB<em>PPBAR</em>ALDIFF/...</td>
<td>S502370</td>
</tr>
<tr>
<td>(2.*BUBHGT)</td>
<td></td>
</tr>
<tr>
<td>DPSFZ=PPBAR*(PLSF<em>WIDTH/SIN31-ASEAL1/(2.0</em>SIN31))</td>
<td>S502390</td>
</tr>
<tr>
<td>DPSAZ=-(PPBAR+2.0)<em>(PLSA</em>WIDTH/SIN32+ASEAL2/(2.0*SIN32))</td>
<td>S502410</td>
</tr>
<tr>
<td>DPSFMZ=ASEAL1<em>PLSF</em>CPBCZ</td>
<td>S50242C</td>
</tr>
<tr>
<td>DPSAPZ=ASEAL2<em>PLSA</em>CPBDZ</td>
<td>S502430</td>
</tr>
<tr>
<td>CPSFZ=CPFSZ+DPSFPZ</td>
<td>S502440</td>
</tr>
<tr>
<td>DPSAZ=CPSAZ+DPSAPZ</td>
<td></td>
</tr>
<tr>
<td>CPBZ=CPBFZ+DPBZ</td>
<td></td>
</tr>
<tr>
<td>CPZ=0.0</td>
<td></td>
</tr>
<tr>
<td>DTHZ=(CPZ+DPZ+DPPZ+DPFZ)/AIYY</td>
<td></td>
</tr>
</tbody>
</table>

**DERIVATIVES OF PITCH MOMENT W/RESPECT TO MB**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
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<tbody>
<tr>
<td>DPFB<em>B=AB</em>XCP*DPFBMB</td>
<td>S502500</td>
</tr>
<tr>
<td>DPSFB=FLSF*DFSFMB</td>
<td>S502510</td>
</tr>
<tr>
<td>DPSAMF=PLSA*DPSAMF</td>
<td>S50252C</td>
</tr>
<tr>
<td>DTME=(DPFBMB+DPSFB+DPSAMF)/AIYY</td>
<td></td>
</tr>
</tbody>
</table>

**DERIVATIVES OF PITCH MOMENTS W/R TO PHI**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPPH=0.0</td>
<td></td>
</tr>
</tbody>
</table>

**RUDDER HYDRODYN.**

| RAREA=0.68                                    |          |
| RSPAN=1.21                                    |          |
| XR=-11.173                                    |          |
| ZR=2.338                                     |          |
| RASPB=2.15                                    |          |
| CSR=AL*-XR*THETA                              |          |
| RCLB2.0*FI*RASPB/(RASPB+3.0)                 |          |
| ENDFAC=1.0*DSR/(DSR+RSPAN)                   |          |
| VH=V*XR*R-Z/RMPH                             |          |
| QQ=0.5*RHC*V**2*RAREA                        |          |
| RDC=2.4*CC*ENDFAC*RCLE                       |          |
FRD=RDC*RLCANG
PH=1.0
IF(VL.LT.0.0) GO TO 10
PM=-1
10 CONTINUE

***

DERIVATIVES OF ROLL MOMENTS W/R TO Z

DRAZ=-(WIDTH/2.28)***3*RHO*G*PHI/SIN3
DREZ=-(WIDTH/2.28)***3*RHO*G*PHI/SIN3
CRAFT=CRSFT+DRASTH
DBRFTH=AK1*PHI**(2.*BL1*BW1/AL512.*TAN(DR1))+BL1**2*AL5/TAN(DR1))
DRAZ=AK2*PHI**(2.*BL2*BW2/AL212.*TAN(DR2))+BL2**2/TAN(DR2))
CREZ=CFZFR2+CRBZ
DRPRZ=PHI*(AB*PBAR+(ZS-ALD)*DPMB)
CFYDZ=PP**2*VREL**2
DRPLZ=CFYDZ*(COS(DR)*YSW-SIN(DR)*ZSW)
DRZ=(DREZ+DRBZ+DRPRZ+DRPLZ)/AIXX

***

DERIVATIVES OF ROLL MOMENTS W/R TO THETA

CRSFT=-(WIDTH/2.28)***3*RHO*G*PHI*(AL3/SIN3)
DREATH=-(WIDTH/2.28)***3*RHO*G*PHI*(AL4/SIN3)
CRAST=CRSFT+DRASTH
DBRATH=AK1*PHI**(2.*BL1*BW1*AL512.*TAN(DR1))+BL1**2*AL5/TAN(DR1))
DRAST=AK2*PHI**(2.*BL2*BW2*AL212.*TAN(DR2))+BL2**2*AL2/TAN(DR2))
CRETH=CFZFR2+CRBATH
DRTH=0.0
DRTH=CRST+DRBTH+DRPTH/AIXX

***

DERIVATIVES OF ROLL MOMENTS W/R TO MB

DRPRPB=PHI*(ZS-ALD)*PHI*DPMB
DRMB=DRPRPB/AIXX

***

DERIVATIVES OF ROLL MOMENTS W/R TO PHI

DREPH=-(WIDTH/2.28)***3*SEAL1*RHO*G
DREPH=-(WIDTH/2.28)***3*SEAL2*RHO*G
CRAPH=CRSFP+DRSAPH
CRBFP=AK1*BL1**2*EGW
CREFP=AK2*EL2*EGW
CRBPH=CRBFP+DRBAPH
CRARPH=PBAR*(ZS-ALD)*AB1
CFVDP=PP**2*VREL**2*PHI*YSW
DRPLPH=CFVDP*(COS(DR)*YSW-SIN(DR)*ZSW)
CRFH=CRSFP+DRBPH+DRPRPH+DRPLPH)/AIXX
DERIVATIVES OF ROLL MOMENTS W/R TO PHIDOT

\[ \begin{align*}
\text{CFYP} & = A22S*VZS \\
\text{ORDC} & = ALC-\text{FP}+\text{YSW}+\text{PHI} \\
\text{CFYDP} & = F\text{M1}+\text{ORD}+2.0*\text{VREL}*(2S-\text{ALD}/2) \\
\text{CF} & = \text{DFYLCFD}+\text{CFYP} \\
\text{ORDPHI} & = -D2+\text{DF}*(\text{COS}DR)*\text{YSW}+\text{SIN}DR)*ZSW1-\text{DFYP}+\text{ZS} \\
\text{CRNAV} & = -3.0/\text{PI}+\text{YSW}+2*8C2 \\
\text{CRRPD} & = \text{CFEPRH}+\text{ORDAV} \\
\text{CRRUPD} & = \text{REC}*(\text{ENDFAC}/V)*ZR**2 \\
\text{CRPC} & = \text{CRCFD}+\text{DORRUPD}/\text{AIXX} \\
\end{align*} \]

ROLL MOMENTS W/R TO YDOT

\[ \begin{align*}
\text{DFYDV} & = F\text{P}+\text{D1}+\text{ORD}+2.0*\text{VREL} \\
\text{DRSWV} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)*ZSW1+D3*ZS \\
\text{DLYD} & = \text{ICSR}+\text{YD}+\text{DURD}+\text{AIAX} \\
\text{ROLL MOMENTS W/R T} \\
\text{R} \\
\text{CFYDR} & = -F\text{P}+\text{D1}+\text{ORD}+2.0*\text{VREL}+XZS \\
\text{DRSWR} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)*ZSW1-D3*ZS \\
\text{DRRUCR} & = \text{REC}+\text{ENDFAC}/V)*ZR+XZR \\
\text{CRF} & = \text{DFSFR}+\text{CRRUDR}/\text{AIAX} \\
\text{SWAY FORCES W/R TO YDOT} \\
\text{AMASS} & = 5*10.0^6 \\
\text{DVRDV} & = -\text{CRUVDV}/ZR \\
\text{CYSWV} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)-D3 \\
\text{DLYD} & = \text{ICSR}+\text{YD}+\text{DYSWV}/\text{AIXS} \\
\text{SWAY FORCES W/R TO R} \\
\text{DVROR} & = -\text{CRRLDR}/ZR \\
\text{CFY} & = V*\text{AMASS} \\
\text{DRSWR} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)+D3*ZS \\
\text{DVR} & = \text{DVROR}+\text{DTRW}+\text{DFY}+\text{AMASS} \\
\text{SWAY forces W/R TO PHI} \\
\text{CYRDO} & = -\text{CRRUPD}/ZR \\
\text{DYSWPD} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)+D3*ZS \\
\text{CYPD} & = \text{ICYRCPD}+\text{DYSWPD}/\text{AMASS} \\
\text{SWAY FORCES W/R TO PHI} \\
\text{DYYPH} & = \text{efs}+\text{D3}+1+\text{cc}+\text{DR})*\text{YSW}+\text{SIN}DR)+D3*ZS \\
\text{DYDP} & = \text{ICYRCPD}+\text{DYSWPD}/\text{AMASS} \
\end{align*} \]
\[
\begin{align*}
SW1 &= 0.5E2 \\
SW2 &= 0.5E5 \\
LAT &= 0.3E5 \\
MLAN &= 0.4E5 \\
AVEM &= 0.4E5 \\
CDF &= 0.6E5 \\
N1 &= 2.5E5 \times 2S \times MLAM \\
N2 &= -7.5E5 \times 2R \times SVM \\
N3 &= 5R \times 5 \times 2S \times CDSW \times [(ALD + YSW \times PHI) + (ALD - YSW \times PHI)] \\
N4 &= (ALD + YSW \times PHI) \times 2 + (ALD - YSW \times PHI) \times 2 \\
C1 &= SW1 \times 2N3 \times 2 \\
C2 &= SW2 \times 2C3 \times 2 \\
XX &= 0.5 \times RH0 \times CDF \times V \times 2 \times L \\
\text{yaw moments w/r to ydot} \\
CNRCYC &= CYRCYD \times XR \\
DNSWYD &= N1 \times 2N4 \times N3 \times (C1 \times R + 2 \times C2 \times VREL) \\
DNYD &= CARLYD + DNSWYD \\
\text{yaw moments w/r to r} \\
DNRD &= CYRDO \times XR \\
DNSR &= -N1 \times 2 \times N3 \times (C1 \times VL - 2 \times C2 \times VREL \times XS) \\
CN &= CARTDF + DNSWR \\
\text{yaw moments w/r to phi} \\
DNRDPD &= CYRDPD \times XR \\
DNSWPC &= -N1 \times 2S \times N2 \times N4 \times 2 \times N3 \times C2 \times VREL \times (ZS - ALD / 2) \\
DNFD &= CARDCF + DNSWPD \\
\text{yaw moments w/r to phi} \\
DNSWPH &= 2 \times N2 \times (VL - 2 \times SPHD) \times (ALD + YSW \times PHI) \times YSW - (ALD - YSW \times PHI) \times YSW \\
CNXP &= -4 \times XX \times YSW \\
CNPH &= CSWPH + DNXPH \\
\text{DERIVATIVE} \\
X1 &= INTL (A \times X2) \\
X2 &= INTL (A \times X2) \\
X3 &= INTL (A \times X4) \\
X4 &= INTL (A \times XP) \\
X5 &= INTL (A \times XMB) 
\end{align*}
\]
X6 = IATCRFL(A,X7)
X7 = IATCRFL(A,XROLL)
X8 = IATCRFL(A,X9)
X9 = IATCRFL(A,XY)
X10 = IATCRFL(A,XY)
X11 = IATCRFL(A,XY)
X12 = CMZ*XI*DMB*X5
X13 = CMZ*XI*DMB*X5+DTH*DTH*X4*DPPH*X6
X14 = ZYC*XI*DMB*X5+DPPH*X6
X15 = ZYC*XI*DMB*X5+DPPH*X6
X16 = ZYC*XI*DMB*X5+DPPH*X6

DPB = DPECZ*X1+DP8MB*X5
FBB = PB1+CPB
ID = D1*TA*12
TH = Th1+X3*RADEG
THDC = XF*RACEG
ZD = Z/G
FHD = XRCELL*RADEG
PH = P*1*X6*RADEG
XO = V*TIME
X = XG*CCS(X10)+X8*SIN(X10)
Y = -XG*SIN(X10)+X8*CCS(X10)
YAWRT = X11
YDOT = X5

* SAMPLE
PREPARE 0.05; F=;Y; YAWRT; YDOT
CONTROL FINIT=25; DEL=0.01; DELS=0.031
PRINT 0.05; PH, X; Y; YAWRT; YDOT
GRAF; TPE, PF
PRPLCT ONLY
CALL CGRGL(1,1; TIME; PH)
CALL CGRGL(2,1; X, Y)
TERMINAL
CALL EACRw(NPLOT)
END
STOP
//PLCT SYSIN DD *
PLCT IS ROLL, ANGLE
XR-3 20 KNCTS SPEED RD=10
0.0 5.0 0.0
PLCT IS Y VS. X DISPLACEMENT
XR-3 20 KNCTS SPEED RD=10
0.15 5.0 6.0
LIST OF REFERENCES


<table>
<thead>
<tr>
<th>No.</th>
<th>Distribution List</th>
<th>Copies</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Defense Technical Information Center</td>
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