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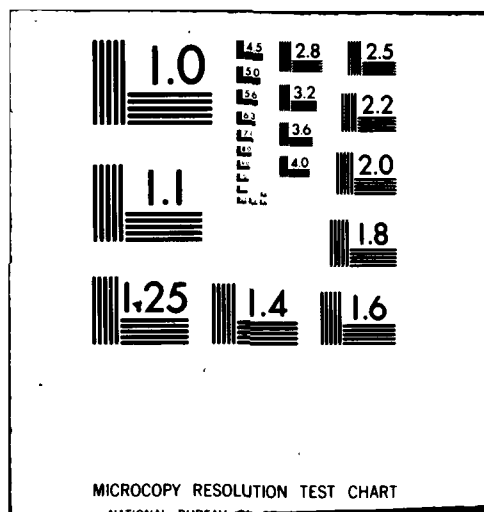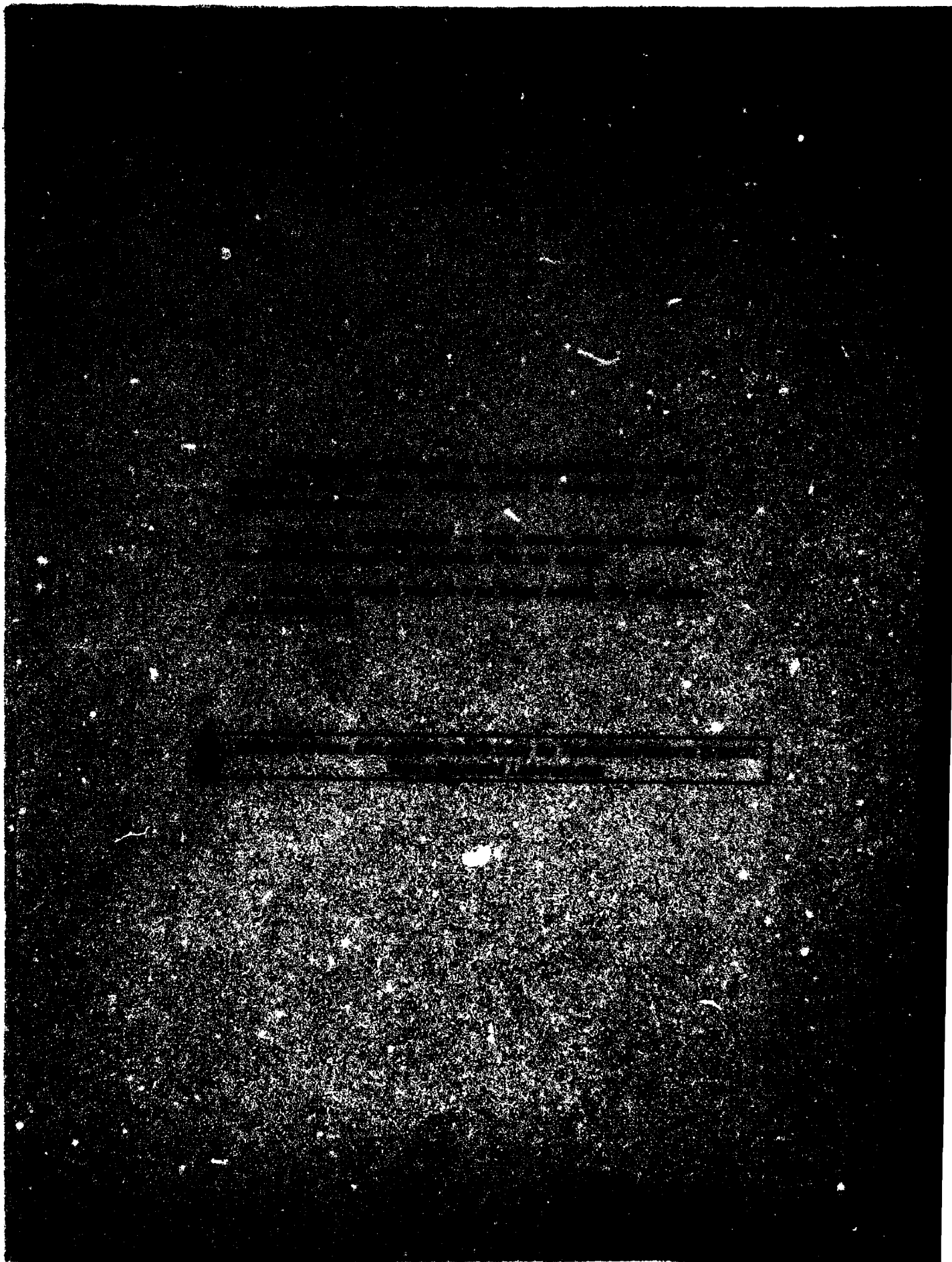   OCT 79   C S KELLEY
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The damage function for extended targets composed of discrete points exposed to the effects of a nuclear burst must be calculated point by point across the target area. Mathematical aids are developed here that reduce the number of calculations needed to find the target damage and that have excellent modeling potential for low-level aggregation of battlefield items at, or below, company size. Implications of data and modeling uncertainties are

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discussed. The accuracies of these aids are considered by applying
them to the problem of determining the incapacitation of the troops
of a 155-mm howitzer battery. In general, these aids are accurate
to within 10 percent for most damage probabilities.

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FIGURES

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1. BACKGROUND AND PURPOSE

It has been found[1-3] that the probability, P_d, of achieving a given degree of damage to a point target is a log-normal function of the environment intensity, E (total dose, in rads tissue, for example), produced by the nuclear burst:

$$P_d = \frac{1}{2}\left\{ 1 + \text{erf}\left[\frac{\ln(E/E_{50})}{\sqrt{2}\,\sigma} \right]\right\} \quad , \tag{1}$$

where $\text{erf}(x)$ is the error function and E_{50} and σ are the two parameters that characterize the vulnerability of the given system to the environment in question. The vulnerability parameter E_{50} is that value of the environment intensity that produces a value of $P_d = 0.5$, and σ is a measure of the steepness of the $P_d(E)$ curve. Thus, σ and E_{50} depend on the physical nature of the system as well as on the identity of the environment that induces the damage—neutron fluence; peak gamma dose rate; total dose; blast peak static overpressure, impulse, etc; and electromagnetic pulse (EMP).

Present methods for treating extended targets require either point-by-point calculation of damage probability for each point target within the extended target (see below) or that the analyst resort to existing manuals[4,5] that deal with specific strategic targets and tactical troop and equipment responses to the burst.

For new extended targets, or for extended targets that have not been previously treated in these manuals, the calculation of the damage to an extended target (composed of a number of discrete point targets and of finite extent) that is exposed to nuclear environments requires a weighted average of the P_d values representing each discrete point target. This calculation can prove excessively lengthy for those extended targets that are composed of many point targets; indeed, assigning appropriate weights is a problem itself. These calculations are lengthy even if one resorts to nomograms or a slide rule that have been developed to speed the calculational process.[6,7] It would be convenient to be able to treat such an extended target as a single target that requires a single P_d calculation.

The purpose of this study is to develop techniques that mathematically treat extended targets as point targets. The analysis develops appropriately accurate expressions for calculating the damage to extended targets and methods for reducing the number of required calculations.

[1] W. L. Vault and W. E. Sweeney, Vulnerability Data Array Progress Report FY76, FY7T (U), Harry Diamond Laboratories, HDL-PR-77-3 (December 1977). (CONF)

[2] D. L. Durgin, D. R. Alexander, and R. N. Randall, Hardening Options for Neutron Effects, Final Report, BDM Corporation, Harry Diamond Laboratories, HDL-CR-74-052-1 (15 November 1976).

[3] K. N. Stevens, Neutron Failure Fluence Distribution for Semiconductor Devices, IEEE Annual Conference on Nuclear and Space Radiation Effects, Seattle, WA (24 to 27 July 1972).

[4] Department of the Army, Staff Officer's Field Manual, Nuclear Weapons Employment (U), Field Manual 101-31-2 (February 1963). (SECRET)

[5] Defense Intelligence Agency, Physical Vulnerability Handbook—Nuclear Weapons (U), AP-550-1-2-INT (June 1969). (CONF)

[6] C. Stuart Kelley, Stacey E. Gehman, John H. Wasilik, and William D. Scharf, Nuclear Damage to Point Targets, Harry Diamond Laboratories, HDL-TR-1876 (December 1978).

[7] C. Stuart Kelley, John H. Wasilik, and William D. Scharf, Supplement to Nuclear Damage to Point Targets (U), Harry Diamond Laboratories, HDL-TR-1876-S (November 1978). (SECRET)

2. METHODS FOR CALCULATING DAMAGE TO EXTENDED TARGETS COMPOSED OF DISCRETE POINTS

To perform an average of a number of values of P_d using equation (1), the analyst adds terms of the form $erf(x_1) + erf(x_2) + \ldots$. Unfortunately, a basic calculational problem arises that involves the presence of the error function in equation (1). The error function is not a closed-form expression and must be evaluated either by reference to a table or by the use of a series expansion.[*] It obeys no simple mathematical properties of addition. Thus, there is no simple way to express $erf(x_1) + erf(x_2)$, an operation that is needed for the treatment of extended targets. The remainder of this section deals with approximations that can be applied to circumvent this problem.

2.1 An Approximation of P_d

The main span of interest to the analyst in damage calculations is $0.1 < P_d < 0.9$ because, for the tactical analyst, damage probability less than 0.1 implies almost complete effectiveness of the unit, while $P_d > 0.9$ implies almost complete ineffectiveness of the unit in carrying out its mission. The FM-101-31 manual series,[+] in fact, considers values of just $P_d > 0.3$ as operationally useful. Likewise, a probability greater than 0.9 that an item has suffered a given degree of damage is adequate characterization; resolution as to whether P_d is 0.93 or 0.94 is not necessary. There are occasions, such as in sensitivity analyses, where altering an input parameter of the scenario may shift the P_d value from inside to outside the range of 0.1 to 0.9. In such cases, the approximations (and their limits) developed as follows should be kept in mind.

We define

$$x \equiv \frac{1}{\sqrt{2}\,\sigma}\, \ell n(E/E_{50}) \quad . \tag{2}$$

so that equation (1) becomes

$$P_d = \tfrac{1}{2}[1 + erf(x)] \quad . \tag{3}$$

From figure 1 it can be seen that over the range of interest $(0.1 < P_d < 0.9$, corresponding to $-0.8 < x < 0.8)$, the error function may be replaced by a linear function. A least-squares fitting process (from John Wicklund of the Harry Diamond Laboratories) gives a result that is (within 3 percent) the same as

$$erf(x) \approx x \quad , \tag{4}$$

which is depicted in the figure. Over the range of interest, x approximates $erf(x)$ to within a fractional error of 10 percent. With the approximation of equation (4), P_d becomes

$$P_d \approx \frac{1}{2} + \frac{1}{2\sqrt{2}\,\sigma}\, \ell n(E/E_{50}) \tag{5}$$

[+] Department of the Army, Staff Officer's Field Manual, Nuclear Weapons Employment (U), Field Manual 101-31-2 (February 1963). (SECRET)

[*] M. Abramowitz and I. A. Stegun, editors, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards, Applied Mathematics Series 55 (June 1964).
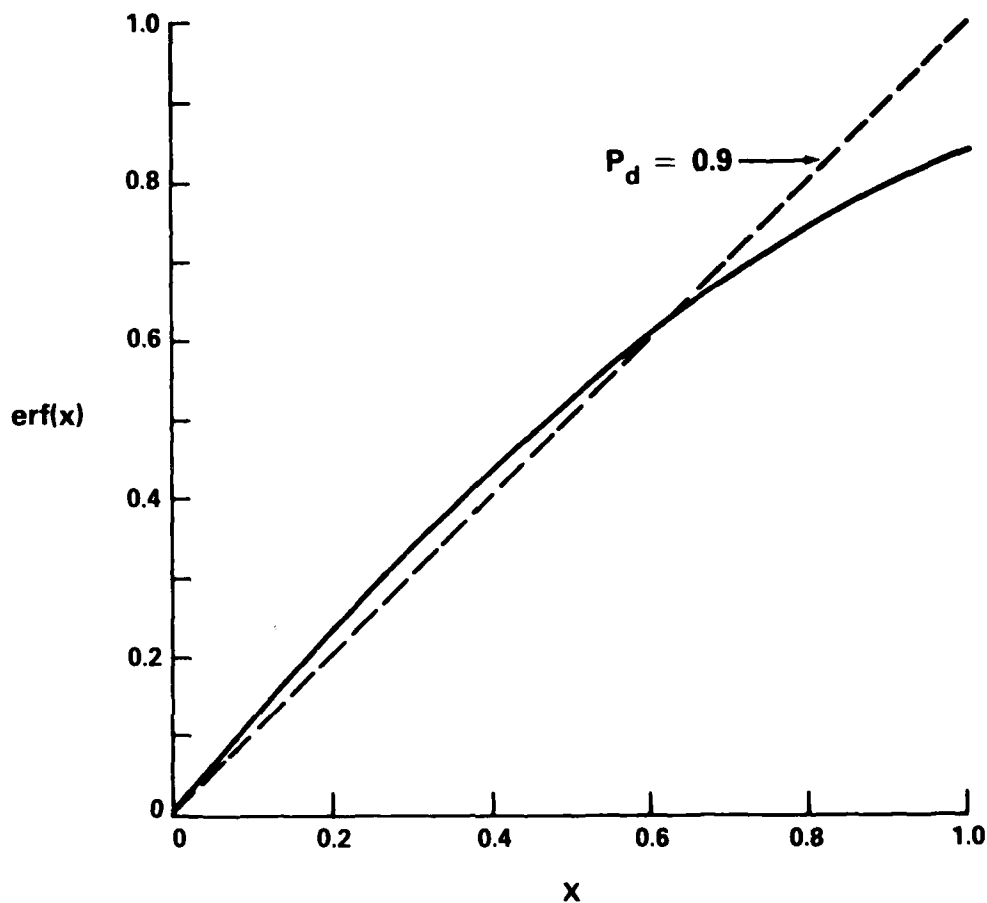
Figure 1.   Linear approximation to error function: $\text{erf}(-x) = -\text{erf}(x)$.

and is accurate to within a 10-percent fractional error over $0.1 < P_d < 0.9$. This fractional error is consistent with errors in the data to which the algorithms below were fitted to the environmental intensity, $E(r)$. Clearly, we need to carry calculations to no higher accuracy than that characterizing these data.

We can characterize all the environment intensities as

$$E = Ar^B e^{cr} \tag{6}$$

with a high degree of accuracy.[9] The uncertainty of the $E(r)$ data to which we fit these is accurate only to within a fractional error of 10 percent, sometimes only to within 30 percent.[10] These

[9] W. E. Sweeney, Jr., C. Moazed, and J. Wicklund, Nuclear Weapons Environments for Vulnerability Assessments to Support Tactical Nuclear Warfare Studies (U), Harry Diamond Laboratories, HDL-TM-77-4 (June 1977). (CONF)

[10] L. G. Mooney and R. L. French, Improved Models for Predicting Nuclear Weapons Initial Radiation Environments (U), Radiation Research Associates, Inc., DASA-2615, RRA-T93 (31 December 1969). (SRD/CNWDI)

algorithms were developed in order to provide simple, yet accurate, equations for calculating the nuclear weapon environments (static overpressure, total radiation dose, thermal radiation, neutron fluence, etc.). The constants A, B, and C vary, of course, from environment to environment. The constant A depends on weapon yield. Thus, the nuclear environments depend solely on the weapon yield and the distance from the burst.

2.2 The Effective Environment Intensity

The approximate form of P_d, equation (5), has useful calculational consequences that will now be developed. Consider an extended target consisting of m different *types* of point targets (trucks, people, radios, etc.), each type denoted by the subscript j. By type of point target is meant all items that have identical σ_j values and identical E_{50j} values. Let n_j denote the *number* of point targets of each target type j, and let the subscript i identify the particular item. Then the total number, N, of point targets within the extended target is

$$N = \sum_{j=1}^{m} n_j \quad . \tag{7}$$

We now calculate the value of P_d appropriate to the extended target for a single category of damage. As the first step in this calculation, we assume that all point targets characterized by the same j subscript have the same "military value." The jth portion of the extended target is composed of n_j items (each of these items is characterized by the subscript i and is thus identified by the subscripts i and j). We define the P_{dj} value of the extended target as the average of the P_{dji} values for the individual points. Thus, P_{dj} is given by

$$P_{dj} = \frac{1}{n_j} \sum_{i=1}^{n_j} P_{dji} \quad . \tag{8}$$

Therefore, P_{dj} represents the probability of damage to the jth portion of the extended target. Using equation (5),

$$P_{dj} = \frac{1}{n_j} \left\{ \frac{n_j}{2} + \frac{1}{2\sqrt{2}\,\sigma_j} \sum_{i=1}^{n_j} \ell n(E_{ji}/E_{50ji}) \right\} \quad . \tag{9}$$

where E_{ji} is the environment intensity at the ith item of the jth type. We replace E_{50ji} by E_{50j} because each of the ith items have the same E_{50ji} value. Equation (9) can be arranged as

$$P_{dj} = \frac{1}{2} + \frac{1}{2\sqrt{2}\,\sigma_j} \ell n \left[\left(\prod_{i=1}^{n_j} E_{ji}^{1/n_j} \right) \middle/ E_{50j} \right] \quad . \tag{10}$$

If we now define E_{ej} to be the "effective environment intensity" of the jth portion of the extended target by the relation

$$E_{ej} \equiv \prod_{i=1}^{n_j} E_{ji}^{1/n_j} \quad . \tag{11}$$

we observe that the expression for P_{dj} is

$$P_{dj} = \frac{1}{2} + \frac{1}{2\sqrt{2}\,\sigma_j} \ell n(E_{ej}/E_{50j}) \quad . \tag{12}$$

Thus, because the approximation $\mathrm{erf}(x) = x$ [equation (4)] is accurate for cases of interest to the tactical nuclear warfare analyst, we can avoid the necessity of using series expansions or

tables to calculate erf(x). This approximation led us to the concept of the effective environment intensity. This concept has important and useful mathematical properties that are described below.

2.3 Incorporation of Shielding Factors and Military Values

We now redefine the number of target types by grouping those having identical shielding factors and identical military value in addition to having identical vulnerability parameters (σ_j and E_{50j}). The shielding factor, T_j, is the ratio of the shielded environment intensity to the unshielded environment intensity. Then, using equation (12), one can write

$$P_{dj}(\text{shielded}) = \frac{1}{2}\left\{ 1 + \frac{1}{\sqrt{2}\,\sigma}\ \ell n(T_j E_{ej}/E_{50j}) \right\} \tag{13}$$

$$= \frac{1}{2}\left\{ 1 + \frac{1}{\sqrt{2}\,\sigma}\ \ell n(E_{ej}/E_{50j}) + \frac{1}{\sqrt{2}\,\sigma}\ \ell n(T_j) \right\}\ , \tag{14}$$

which reduces to

$$P_{dj}(\text{shielded}) = P_{dj}(\text{unshielded}) + \frac{\ell n(T_j)}{2\sqrt{2}\,\sigma_j}\ . \tag{15}$$

Conversely, one can define a shielded effective environment intensity

$$E_{ej}(\text{shielded}) = T_j E_{ej}(\text{unshielded}), \tag{16}$$

and state that, in effect, inclusion of T_j is mathematically equivalent to replacing E_{50j} by E_{50j}/T_j.

We now assign the "military value" V_j to each item of the jth type. (One means for arriving at the military value is addressed by BDM.[11]) The V_j values are normalized such that

$$\sum_{j=1}^{m} V_j = 1\ . \tag{17}$$

The V_j is a linear weighting factor to the P_{dj} values. Thus, the P_d value for the extended target as a whole is

$$P_d = \sum_{j=1}^{m} P_{dj} V_j\ , \tag{18}$$

where shielding has been considered in calculating P_{dj}. Inserting equation (10) into equation (18) and simplifying, we find that P_d is given by

$$P_d = \frac{1}{2} + \frac{1}{2\sqrt{2}}\ \ell n\left[\prod_{j=1}^{m} (T_j E_{ej}/E_{50j})^{V_j/\sigma_j} \right]\ . \tag{19}$$

Equation (19) can be compared to equation (5), the probability of damage to a point target, and we obtain

$$P_d = \frac{1}{2} + \frac{1}{2\sqrt{2}}\ \ell n[(E_e/E_{50})^{1/\sigma}]\ . \tag{20}$$

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[11] BDM, Inc., Theater Nuclear Force Combat Capability Degradation Methodology, Development, and Demonstration, BDM/W-78-167-TR (April 1978). (SECRET)

where the parameters E_{50} and σ represent the extended target by a single entity and E_e represents the target/burst characteristics (weapon yield, distance from burst, and deployment of the point targets within the extended target). This definition of E_e means that

$$(E_e/E_{50})^{1/\sigma} = \prod_{j=1}^{m} (T_j E_{ej}/E_{50j})^{V_j/\sigma_j} \quad . \tag{21}$$

The presence of V_j in equation (21) merely serves to scale the value of the corresponding σ_j. Thus, in addition to combining the transmission factors T_j into the definition of E_{50j}, one could similarly incorporate the V_j factor into the definition of $\sigma_j (\sigma_{ej} \equiv \sigma_j/V_j)$. That is to say, the vulnerability parameters may be easily altered to include the effects of transmission as well as military value.

To calculate P_d, the coordinates and yield of the weapon in question would be given, as would be the locations of the sub-target items and the values of V_j, σ_j, E_{50j}, and T_j for each item. Environment algorithms, developed by Sweeney et al,[9] would then be used to find E_{ej} from equation (11). Then the value of the right-hand side of equation (21) would be calculated, and the result inserted in equation (20) to give P_d. This procedure would then be repeated for all bursts, environments, targets, and damage categories. This methodology provides an excellent basis for low-level aggregation of targets.

2.4 The Vulnerability Center

Corresponding to E_{ej}, one can define an effective distance r_{ej} that can be determined from the $E(r)$ relations, equation (6). The distance r_{ej} is not exactly given by the distance between the target geometric centroid and the burst point, but depends upon the particular sub-target distribution. This is illustrated by considering two identical point targets equidistant (r_d) from a burst, but not on opposite sides of the burst. Their P_d values are the same, and $r_{ej} = r_d$ is located on an arc of radius r_d centered at the burst point. Since $r_e = r_d$ is located at the intersection of the arc with the perpendicular bisector of the two targets, r_e does not lie at the target centroid, which is not on that arc. Similarly, irregular target shapes are cases where the target centroid is not colocated with r_e. The interest in r_e is for cases when its location can be closely approximated by the target centroid, for then the extended target can be replaced by a point target located at the centroid. To calculate the effective distance r_{ej}, a point-by-point weighting by the environment intensity is required. Because this function is not linear in distance, r_{ej} differs from the target centroid (which is found by a linear weighting of distances). Because we are concerned with extended targets having a limited extent, it may prove possible to approximate r_{ej} by the target centroid. We now consider the accuracy of such an approximation.

If all items are of the same sub-target type, $V_j = 1$, $\sigma_j = \sigma$, and equation (11) reduces to

$$E_e = \prod_{i=1}^{n} E_i^{1/n} \quad . \tag{22}$$

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[9] W. E. Sweeney, Jr., C. Moazed, and J. Wicklund. Nuclear Weapons Environments for Vulnerability Assessments to Support Tactical Nuclear Warfare Studies (U), Harry Diamond Laboratories, HDL-TM-77-4 (June 1977). (CONF)

where the subscript j has been deleted, and T has been included in E_{50}. For all environments, r_e is found by inserting equation (6) into equation (22).

$$r_e^B e^{Cr_e} = \prod_{i=1}^{n} r_i^{B/n} e^{Cr_i/n} \quad . \tag{23}$$

Thus, r_e is independent of the values of the weapon yield and the vulnerability parameters of the individual sub-targets. Because V and T just serve to scale the sub-target vulnerability parameters, r_e is also independent of V and T.

From equation (23), representative of a single sub-target type, the value of r_e differs from that of the arithmetic mean $\sum_{i=1}^{n} r_i/n$. The geometric mean r_g is always less than or equal to the arithmetic mean r_a:[*]

$$\prod_{i=1}^{n} r_i^{1/n} \leq \frac{1}{n} \sum_{i=1}^{n} r_i \tag{24}$$

or

$$r_g \leq r_a \quad . \tag{25}$$

Then the bounds on r_e,

$$\prod_{i=1}^{n} r_i^{1/n} \leq r_e \leq \frac{1}{n} \sum_{i=1}^{n} r_i \tag{26}$$

or

$$r_g \leq r_e \leq r_a \quad , \tag{27}$$

may be demonstrated by a two-step process based on equation (23).

We rewrite equation (23) in the following form

$$r_e^B e^{Cr_e} = \left(\prod_{i=1}^{n} r_i^{1/n} \right)^B \left(\exp \frac{C}{n} \sum_{i=1}^{n} r_i \right) \tag{28}$$

and reduce it to

$$r_e^B e^{Cr_e} = r_g^B e^{Cr_a} \quad . \tag{29}$$

In the first step in the process, we insert r_g for r_e in the left-hand side of equation (29) to find that

$$r_g^B e^{Cr_g} \gtrless r_g^B e^{Cr_a} \quad . \tag{30}$$

However, since $r_g \leq r_a$, the left-hand side is smaller than the right-hand side, implying that our choice of r_g for r_e is too small. Thus,

$$r_g \leq r_e \quad . \tag{31}$$

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[*] M. Abramowitz and I. A. Stegun, editors, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards, Applied Mathematics Series 55 (June 1964).

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In the second step in the process, we insert r_a for r_c in the left-hand side of equation (29) to find that

$$r_a^B e^{Cr_a} \overset{?}{\geq} r_g^B e^{Cr_a} \quad . \tag{32}$$

Now, since $r_a \geq r_g$, the left-hand side is too large, implying that

$$r_e \leq r_a \quad . \tag{33}$$

Combining equations (31) and (33), we obtain the desired relation, equation (27).

We now define Δr_i, the separation of the individual point targets from the target centroid, by the scalar relation

$$r_i = r_c + \Delta r_i \quad . \tag{34}$$

where r_c denotes the target centroid. If $\Delta r_i / r_c$ is small (about 10 percent or less), then r_e is located at the geometric centroid (arithmetic mean)

$$r_e \approx \frac{1}{n} \sum_{i=1}^{n} r_i \tag{35}$$

to within an error that is given by

$$\frac{-r_c}{2n^2} \sum_{i=1}^{n} (\Delta r_i / r_c)^2 \quad . \tag{36}$$

using the bounds of equation (26). The negative sign of equation (36) implies that the vulnerability center is always on the burst side of the centroid. That is, the closer sub-target items are the more heavily weighted. With a target extent of less than one-tenth the distance from the target centroid to the burst point, r_e is equal to the centroid-to-burst point distance to within a fractional error of 5×10^{-3}. This small error means that the vulnerability center (the location of the point target that mathematically replaces the extended target) is well approximated by the target centroid.

3. APPLICATION TO BATTERY-SIZED TARGET

In this section we investigate the accuracy of the approximations developed in section 2 by applying them to an appropriately sized extended target (100 by 200 m) that is, indeed, composed of discrete points. This unit is the nuclear-capable 155-mm Field Artillery Firing Battery, Self Propelled. The prime reasons for this choice are that an idealized deployment (see fig. 2) of this unit has been developed, the sub-target locations have been identified, and this unit is currently of interest in applications of tactical nuclear warfare analyses.[12] A second reason for the choice of this unit is that it has been found[12] that this battery can be accurately characterized as a point target located at its "vulnerability center" when the nuclear damage mechanism is radiation-induced incapacitation of the unit's personnel (as well as vehicle overturn, transient radiation effects to radio and electronics failure, etc.).

The four categories[13] of incapacitation due to total radiation dose, and the vulnerability

[12] C. E. Spyropoulos and J. Wicklund, A Method for Assessing the Vulnerability of Small Units in Tactical Nuclear Engagements (U), Harry Diamond Laboratories, HDL-TR-1851 (June 1978). (CONF)

[13] C. Stuart Kelley, Distribution of Nuclear Probability with Distance, Harry Diamond Laboratories, HDL-TR-1866 (August 1978).
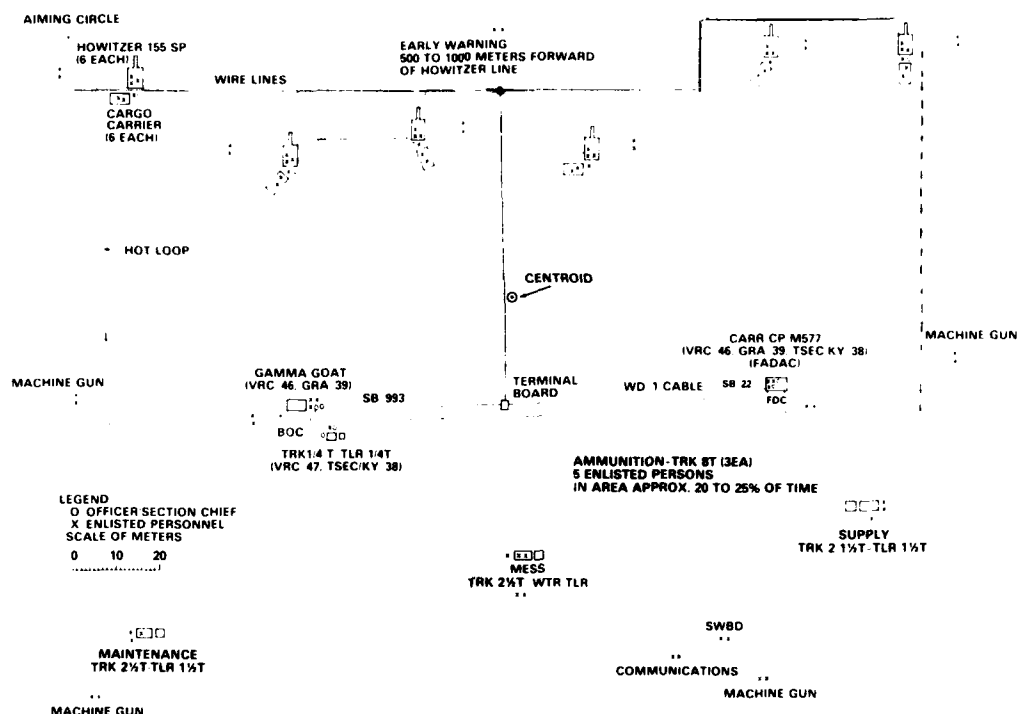
Figure 2.   Deployment of the field artillery battery,155 mm self propelled.

parameters associated with these categories, are shown in table 1. The criteria for the incapaci-
tation of the personnel will vary from position to position inside the unit's boundary because the
various troops perform different functions; some are physically demanding, some are undemand-
ing. Table 2 shows the coordinates (in km) of the troops as measured from the centroid of the
battery [the center of the outline of the battery is displaced $(-0.0075, -0.0215)$ from the battery
centroid]. Incidentally, for this example, the vulnerability center found by Spyropoulos and
Wicklund[12] is within 5 m of the centroid.

The procedure for assessing the accuracy of the approximations of section 2 is to detonate
nuclear weapons at various distances (R) and angles (θ) from the battery centroid, perform the
exact and approximate P_d calculations for each detonation, and compare the resulting values of
P_d. The cases considered here use weapon yields of $W = 1, 10,$ and 100 kT detonated at distances
of $R = 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.2,$ and 1.5 km from the battery centroid and at angles
(measured in a clockwise sense from the $+y$ axis) of $\theta = 0, 45, 90, 135,$ and 180 deg. In each of
these cases, we consider the total radiation dose experienced by each of the 21 groups of troops
described in table 2.

<hr>

[12] C. E. Spyropoulos and J. Wicklund, A Method for Assessing the Vulnerability of Small Units in Tactical Nuclear
Engagements (U), Harry Diamond Laboratories, HDL-TR-1851 (June 1978). (CONF)

TABLE 1
Description of Troop Incapacitation Categories

Category type	Incapacitation[a]	D_{50} in rads, including shielding	σ	Troops in this category
0	IPU	17,700	0.581	12
1	IPU (shielded)	22,125	0.581	5
2	IPD	8,490	0.638	59
3	IPD (shielded)	10,613	0.638	18

[a] I = Immediate
P = Permanent
U = Incapable of performing physically undemanding tasks
D = Incapable of performing physically demanding tasks

TABLE 2
Coordinates of Troops Measured from Battery Centroid

Number of men	Category type	$x(km)$	$y(km)$
5	2	−0.1	−0.099
2	2	−0.115	−0.029
6	2	−0.1	0.051
3	3	−0.1	0.051
6	2	−0.06	0.031
3	3	−0.06	0.031
6	2	−0.025	0.036
3	3	−0.025	0.036
6	2	0.015	0.031
3	3	0.015	0.031
6	2	0.065	0.056
3	3	0.065	0.056
6	2	0.1	0.056
3	3	0.1	0.056
10	0	−0.05	−0.039
5	2	0	−0.079
6	2	0.055	−0.099
3	2	0.09	−0.059
2	0	0.065	−0.029
5	1	0.065	−0.029
2	2	0.11	−0.039

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The total radiation dose (in rads tissue) experienced by the ith group of the 21 groups is

$$D_i = 8528 W r_i^{-2.485} e^{-3.572 r_i} \quad , \tag{37}$$

where r_i is the distance between a detonation and the ith group in the battery. The equation for P_d that we consider to be exact is the appropriate form of equation (1); namely,

$$P_{de} = \frac{1}{94} \sum_{i=1}^{21} m_i \frac{1}{2} \left\{ 1 + \text{erf} \left[\frac{\ell n(D_i/D_{50i})}{\sqrt{2} \, \sigma_i} \right] \right\} \quad . \tag{38}$$

The number of troops m_i in the ith group is listed in table 2.

We first consider the approximation to equation (38) that occurs when $\text{erf}(x)$ is replaced by x. Then P_d takes the form

$$P_{d1} = \frac{1}{94} \sum_{i=1}^{21} m_i \frac{1}{2} \left\{ 1 + \frac{1}{\sqrt{2} \, \sigma_i} \ell n(D_i/D_{50i}) \right\} \quad . \tag{39}$$

Values of P_{de} and P_{d1} were calculated for combinations of W, R, and θ, and a plot was made of the P_{d1} versus P_{de} data points. Although there was some scatter in the data points, they are very well represented by the solid curve of figure 3. The dashed line on that figure represents $P_{d1} = P_{de}$. The extent to which the solid line deviates from the dashed line measures the extent to which P_{d1} deviates from P_{de}. It will be seen from figure 3 that P_{d1} most accurately reproduces P_{de} near $P_{de} = 0.4$, but diverges widely at the extremes of $P_{de} = 0$ and 1.0. Over the range of $0.2 \gtrsim P_{de} \gtrsim 0.8$, however, P_{d1} equals P_{de} to within 10 percent, and over this range P_{d1} is a useful approximation.

Consider next the concept of the effective environment intensity, here the effective dose, D_e, to the battery. The effective dose is defined by the left-hand side of equation (21), but can only be determined once the value of σ has been found for the area target as a whole. The quantity of interest, however, is the right-hand side of equation (21): applying it to a P_d calculation is exactly the same as calculating P_d by equation (39), and the results are the same as described in the previous paragraph. Therefore, over $0.2 \gtrsim P_d \gtrsim 0.8$, the concept of the effective environment intensity is both valid and accurate.

The remaining portion of this section concerns the troops in category 2. These troops comprise most (63 percent) of the battery's troops; therefore, it is worthwhile to consider the results of approximating the entire battery by just those troops in category 2 alone. As a first step in this analysis, the exact P_d values for the entire battery were compared to the corresponding exact P_d values that result when only the troops of category 2 are included. The category 2, P_d values were found to reproduce the exact values to within 8 percent, and are accurate near $P_{de} = 0$ and 1.0.

For category 2, the effective dose is given by the appropriate modification of equation (22)

$$D_e = \prod_{i=1}^{12} D_i^{m_i/59} \tag{40}$$

that corresponds to an effective distance r_e that is bounded by the appropriate modification of equation (26).

$$\prod_{i=1}^{12} r_i^{m_i/59} \leq r_e \leq \frac{1}{59} \sum_{i=1}^{12} m_i r_i \quad . \tag{41}$$

For the cases considered, the two bounds are very close to each other and differ by less than 1
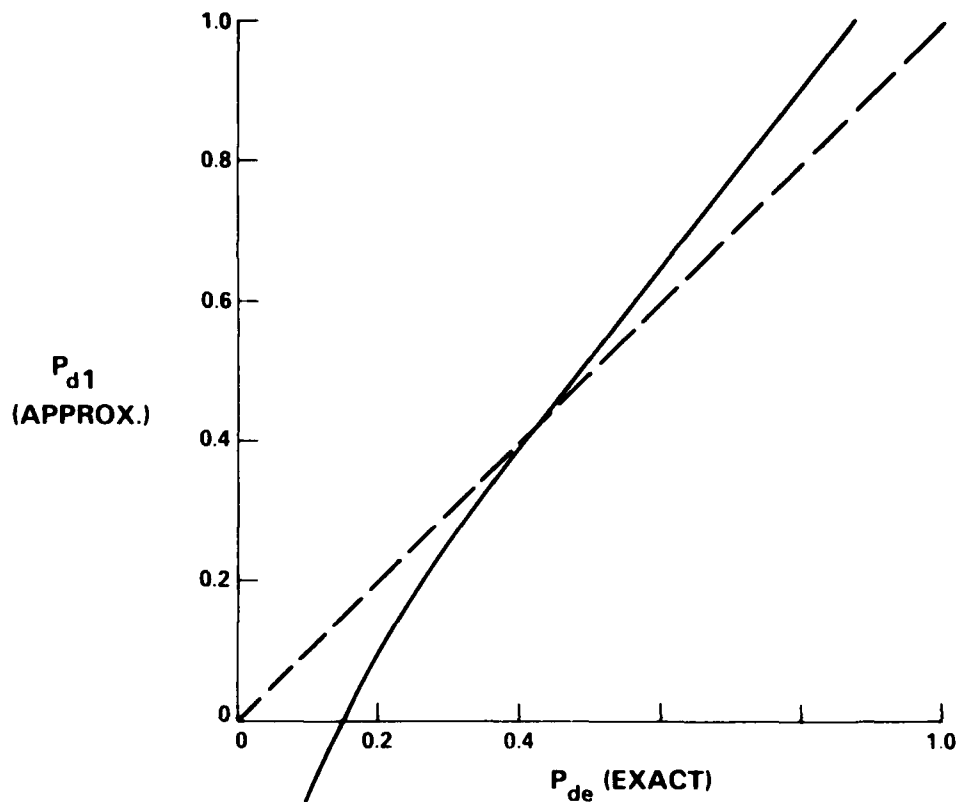
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Figure 3. Accuracy of approximation. equation (29). to exact probability of damage: deviation of solid line from dashed line measures inaccuracy of equation (29) in reproducing exact result. equation (28).

percent of the value of r_e. Accordingly, r_e is accurately specified by either of these two bounds. Also, to within 1 percent, the effective distance is equal to the separation of the burst from the target centroid.

The final approximation to be evaluated consists of making the point target approximation using D_e as the environment intensity, and replacing erf x by x.

$$P_{d2} = \frac{1}{2}\left\{ 1 + \frac{1}{\sqrt{2}\,\sigma_2} \quad \ell n(D_e/D_{502}) \right\} \quad . \tag{42}$$

The results of the calculations are quite similar to those shown in figure 3. As in figure 3. this approximation gives P_{de} to within 10 percent over $0.2 \gtrsim P_{de} \gtrsim 0.8$. but is inaccurate outside this range.

4. SUMMARY

This report deals with the question of how extended targets composed of discrete points can be mathematically treated as single point targets when the analyst calculates target damage caused

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by a nuclear burst. The answer involves uncertainties in the data that originally gave rise to the mathematically complex form of the damage function. Given these uncertainties and the damage function values that are of interest, the damage function can be greatly simplified, thereby allowing a savings in the calculations needed to determine damage to an extended target.

The procedure for calculating damage involves sub-dividing the target into sub-targets that have similar vulnerability parameters. These parameters can be easily scaled to account for transmission and "military value" factors. The resulting formalism permits the calculation of "surviving military value" of any such target. The procedure is especially appropriate to low-level aggregation schemes.

The accuracy of this procedure is assessed by applying it to the problem of determining the incapacitation of the troops of a 155-mm howitzer battery that is caused by a nuclear detonation. Consistent with the results of other analyses, the concept of effective distance is shown to be appropriate, and therefore this extended target can be considered as a point target that is located at the centroid of the extended target. The simplified damage function is found to have acceptable accuracy for most damage probabilities.

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