FINAL REPORT

Passive Ring Resonator Laser Gyroscope

United States Air Force - Office of Scientific Research
Grant AFOSR 76-3042

covering the period
June 1, 1977 - May 31, 1979

Submitted by
Shaoul Ezekiel

January 3, 1980

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Research Laboratory of Electronics
Cambridge, Massachusetts 02139

Approved for public release; distribution unlimited.
We have studied the behavior of a passive cavity optical gyroscope using (a) a small, 17 cm on a side, aluminum block and (b) a large, 70 cm on a side, cavity having discretely mounted components. In both cases, short-noise-limited performance has been reached for short periods of time, on the order of 20 minutes. Long term behavior has not been studied in much detail, but several factors that can influence the long term performance have been identified.
SUMMARY OF WORK ACCOMPLISHED

We have studied the behavior of a passive cavity optical gyroscope using (a) a small, 17 cm on a side, aluminum block and (b) a large, 70 cm on a side, cavity having discretely mounted components. In both cases, shot-noise-limited performance has been reached for short periods of time, on the order of 20 minutes. Long term behavior has not been studied in much detail, but several factors that can influence the long term performance have been identified.

Below is a summary of work accomplished in various areas.

(a) Performance of the 17 cm gyroscope

The drift performance of a square cavity 17 cm on a side is presented in this section.

Figure 1 is a schematic diagram of the setup. A 1 mW laser, frequency $f_0$, is split into two beams, one of which is shifted by a constant frequency $f_1$ by means of an acousto-optic shifter and is then coupled in a clockwise (cw) direction into the passive cavity. The cavity consists of 2 plane 99% reflectivity mirrors and 2 curved maximum reflectivity mirrors. One of the curved mirrors is mounted on a PZT crystal for cavity-length adjustment. The cw resonance frequency of the cavity is locked to the laser frequency $f_0 + f_1$ by means of a feedback loop which automatically adjusts the cavity length via the PZT crystal to maintain the lock. To achieve shot-noise-limited performance the cavity resonance is modulated at a rate of 36 kHz so that shot noise is the dominant noise source at this modulation frequency. For optimum sensitivity, the modulation amplitude is made large enough to reach points of maximum slope on either side of the cavity resonance. In our case, the width

1
of the cavity resonance is 2.5 MHz so that a peak modulation amplitude of about 0.6 MHz is needed. The detector output is demodulated in a phase-sensitive demodulator (PSD #1) at 36 kHz and then passed through a servo amplifier followed by a high-voltage summing amplifier which drives the PZT crystal. The performance of this primary lock is demonstrated in Figure 2 which shows the output of the phase-sensitive demodulator (PSD #2) when the lock is operating. The observed fluctuations in this output are essentially the residual relative frequency fluctuation between $f_0 + f_1$ and the cw cavity resonance frequency. The peak-to-peak excursion in a time constant of 1 second is about 0.3 Hz.

The second part of the laser beam in Figure 1 is shifted in a similar way by an adjustable frequency $f_2$ and is then coupled into the counterclockwise direction (ccw) of the cavity. The coupling-in and coupling-out mirrors are the same for both cw and ccw directions in this setup. A second feedback loop adjusts $f_2$ by means of a voltage-controlled oscillator (VCO) so that $f_0 + f_2$ is locked to the center of the ccw resonance of the cavity. This secondary, or rotation-rate measuring, loop takes advantage of the same modulation frequency used for the primary loop. The output of detector #2 is, therefore, also demodulated at 36 kHz and then passed through a servo amplifier to drive a voltage-controlled oscillator (VCO #2) to adjust $f_2$ as required. (By subtracting the output of PD #1 from PD #2 before demodulation in PSD #2, it is possible to substantially reduce any residual relative jitter between laser and cavity from affecting the rate-measuring loop. Moreover, this subtraction compensates for any offset in the primary loop lock caused by, for example, cavity drift since the d.c. gain in the primary servo is not infinite.)

If there is no inertial rotation (or any other nonreciprocal phase shift) the cw and ccw resonance frequencies of the cavities are identical, i.e.,
\[ f_0 + f_1 = f_{\text{CW}} = f_0 + f_2 = f_{\text{CCW}} \text{ and therefore } f_1 = f_2. \]

In the presence of inertial rotation, \( f_{\text{CW}} \) and \( f_{\text{CCW}} \) will be separated by a frequency \( \Delta f \) given by

\[ \Delta f = f_{\text{CW}} - f_{\text{CCW}} = \frac{4A}{\lambda P} \Omega \]

where \( A \) = area enclosed by the cavity, \( \lambda \) is the average wavelength of the light in the cavity, \( P \) is the perimeter of the cavity, and \( \Omega \) is the inertial rotation rate about the axis perpendicular to the plane of the cavity. This rotation-induced separation between \( \text{cw} \) and \( \text{ccw} \) cavity frequencies activates both primary and secondary feedback loops. The primary loop will change the cavity length so that \( f_{\text{CW}} \) is held at \( f_0 + f_1 \) in the presence of inertial rotation and the secondary loop will change \( f_2 \) so that \( f_0 + f_2 \) is now held at the new \( f_{\text{CCW}} \). In this way, the measurement of \( f_1 - f_2 \) by, say, an up/down counter \( f_{\text{CW}} - f_{\text{CCW}} \) which is related to \( \Omega \) by the scale factor \( 4A/\lambda P \).

The uncertainty in measuring \( \Omega \) depends on \( \delta f \), the uncertainty in measuring \( f_{\text{CW}} - f_{\text{CCW}} \), given by

\[ \delta \Omega = \frac{\lambda P}{4A} \delta f \]

For shot-noise-limited detection,

\[ \delta f = \frac{\sqrt{2} \Gamma}{\sqrt{N_{\text{ph}}} \eta_D \tau} \]

where \( \Gamma \) is the cavity-resonance width, \( N_{\text{ph}} \) is the number of photons per second at the detector, \( \eta_D \) is the photodetector quantum efficiency, and \( \tau \) is the averaging time. The factor \( \sqrt{2} \) comes from the addition of the uncertainties in the measurement of \( f_{\text{CW}} \) and \( f_{\text{CCW}} \).

Figure 3 shows the output of PSD #2 (secondary feedback loop open) as a function of time for \( \tau = 1 \) second. The rms fluctuation in the data, assuming
the data is random-noise, corresponds to 0.6 Hz rms or \(3 \times 10^{-2} \Omega_E\) or 0.45°/hour (\(T = 1\) sec) (\(\Omega_E\) is the earth-rotation rate). This compares well with the expected shot-noise-limited value of 0.32 Hz (\(T = 1\) sec) based on measured peak intensity at the detectors. The open-loop data in Figure 3 is an accurate measure of \(\Delta f\) because \(f_1\) and \(f_2\) are derived from highly stable frequency synthesizers and, in addition, all relevant electronic sources of error are small compared with 0.6 Hz.

It should be mentioned that the present setup is by no means optimum. By using 99.5% \(R\) coupling mirrors that have low loss, it should be possible to reduce the cavity linewidth to less than 1 MHz and obtain a transmission through the cavity of over 30%. Calculations indicate that the optimum performance that can be obtained with a 1 mW laser a 17 cm square cavity is 5 \(\times\) \(10^{-2}\) Hz, or 2.5 \(\times\) \(10^{-3}\) \(\Omega_E\) or 0.04°/hour (\(T = 1\) second). With \(T = 1\) hour the optimized shot-noise-limited performance should be 4.2 \(\times\) \(10^{-5}\) \(\Omega_E\) or 0.0006°/hour.

(b) Preliminary Results with Discrete Component Gyroscope

The passive ring resonator gyroscope discussed in Section (a) consists of a solid block of aluminum which supports all optical components with the exception of the laser which is mounted external to the block. The mirrors and other optical components are placed in aluminum holders which are secured in place by means of shims. Such an arrangement has been adequate for our early work, but is not too convenient for studying error sources such as those due to misalignment.

We have recently assembled a setup that has discretely mounted components which are attached to the top of a vibration isolated table. This new arrangement makes it possible to perform misalignment tests rather conveniently.
and, in addition, makes it possible to change cavity size, to insert mode matching optical components, and so on. The size of the cavity in the new setup is about 70 cm on a side which increases the scale factor by 4 and the overall sensitivity, based on shot-noise-limited detection, by 16.

Figure 4 shows the narrow cavity resonance width of 200 kHz that has been obtained so far with this new cavity. Preliminary drift data is shown in Figure 5 demonstrating an rms fluctuation of about $10^{-2} \sigma_E (\tau = 1 \text{ sec})$ or 0.15°/hour ($\tau = 1 \text{ sec}$). There are a number of noise sources in this setup that have yet to be investigated. With a 1/2 mW laser we expect the shot noise limit for this cavity size to be $10^{-4} \sigma_E$ or 0.0015°/hour for an integration time of 1 second.

(c) Theoretical Calculations of Misalignment Errors

One of the major sources of error in a passive resonator optical gyroscope is due to a change in alignment of the optical beams with respect to the cavity due to mechanical or thermal stresses. A nonreciprocal misalignment causes higher order transverse modes of the ring cavity to be excited which tend to pull the resonance frequency of the fundamental or TEM$_{00}$ mode that is used as reference. Unlike a laser resonator, where higher order modes are suppressed because of insufficient gain, a passive resonator supports a large number of higher order transverse modes. In the absence of any time-dependent misalignment, the existence of higher order modes can only contribute a constant offset to $f_{\text{CW}}$ and $f_{\text{CCW}}$ which is not a problem. However, a problem does arise when there is a nonreciprocal time-dependent change in the relative amplitudes of the higher order modes caused by misalignment or any other effect and this generates a time-dependent offset or bias drift. To reduce such frequency-pulling effects, it is clear that
the ring cavity and the external optics must remain in good alignment. The degree of misalignment that can be tolerated for a given performance requirement was analytically investigated in our laboratory. By calculating the amplitudes of higher order modes that can be excited in a given resonator configuration as a function of misalignment, we could determine an optimum cavity arrangement that minimizes frequency pulling caused by misalignment. The calculation could be carried out with and without perfect matching of the mode of the external laser field into the TEM$_{00}$ mode of the ring cavity.

In order to determine the frequency pulling due to higher-order transverse modes brought about by a given change in alignment, we must first calculate the change in the amplitudes of all higher order modes TEM$_{(m,n)}$ caused by the misalignment and then calculate the frequency pulling due to each higher order mode with respect to the TEM$_{00}$ resonance.

In order to calculate the amplitude of any particular higher order mode TEM$_{(m,n)}$ $E_R(m,n)(x,y,z)$ due to a misalignment, we must integrate the product of the electric field distribution of the incoming beam $E_L(x,y,z)$ and the complex conjugate of the electric field distribution of the TEM$_{(m,n)}$ mode of the cavity $E_C^{*}_{(m,n)}(x,y,z)$ over a cross sectional plane of the cavity. Mathematically, this is represented by

$$E_R(m,n)(x,y,z) = \frac{\iint E_L(x,y,z)E_C^{*}_{(m,n)}(x,y,z) \, dx \, dy}{\iint E_C^{*}_{(m,n)}E_C^{*}_{(m,n)} \, dx \, dy}$$

where $(x,y,z)$ are coordinates in a plane normal to the optical axis of the cavity at some point $z$ on the axis. The integral in the denominator is a normalization integral. A computer program has been written to perform the above calculation for a ring cavity with its inherent astigmatism. Once $E_R(m,n)(x,y,z)$ is calculated as a function of misalignment the corresponding frequency pulling due to $E_R(m,n)$ may then be easily determined.
SUGGESTIONS FOR FUTURE RESEARCH

A. Continuation of the theoretical calculations of frequency pulling as a function of misalignment, discussed in the previous section. This will aid in determining an optimum cavity configuration that will minimize frequency pulling errors caused by beam misalignment due to (i) mirrors external to the cavity; (ii) mirrors internal to the cavity; (iii) temperature effects on the acousto-optic shifters; (iv) laser beam pointing instability, and so on.

B. Examination in more detail of the behavior of acousto-optic frequency shifters with respect to (i) temperature-induced misalignment; (ii) temperature-induced birefringence; (iii) driver amplifier noise; (iv) driver amplifier power fluctuations; (v) scattering phenomena within the acousto-optic medium; (vi) temperature-dependent spatial distortion of the optical beam propagating through the A/O medium, and so on.

C. Studying the performance and limitations of the electronic feedback loops, particularly with respect to time-dependent offsets. This may lead to the development of digital techniques for phase-sensitive demodulation and integration.

D. Investigation of new approaches to the determination of the resonance frequencies of the cw and ccw cavities with minimum contributions from higher order modes. One such approach is to use a third derivative lock which promises to reduce the effect of background slope on the
center frequency of the TEM\(_{00}\) resonance.

E. Studying the spatial inhomogeneities in the beams coupled out of the resonator caused by mirror imperfections, etc., such inhomogeneities can contribute significantly to errors in the determination of the true center frequency of the cavity resonance.

F. Studying the effect of the spatial inhomogeneities in the photo detectors, any detector surface inhomogeneity coupled with beam inhomogeneities discussed in E above can contribute to errors in the determination of the true center frequency of the cavity resonance.

G. Investigation of mirror backscattering effects. Mirror backscattering generates an output at the phase sensitive detector that is at the difference frequency between cw and ccw beams. The magnitude of the signal is as expected. In our experiments so far we have substantially reduced the error due to backscattering by operating at a large offset e.g., 200 Hz so that with a time constant of 1 second we saw very little of the backscattering beat frequency. Otherwise it would be necessary to externally jitter one or both of the frequencies or their phases before entering the cavity.

H. Investigation of sources of optical coupling between laser and cavity. A number of possibilities exist for coupling light other than the primary beam into the cavity. For example, the primary beam reflected out of the cavity gets recoupled into the cavity after reflection at the output mirror of the laser.
Figure 1. Schematic diagram of passive ring cavity inertial rotation sensor.

Figure 2. Output of PSD $f_1$ demonstrating the residual relative frequency fluctuation between $f_0 + f_1$ and $f_{cw}$ (time constant is 1 second). Scan duration about 10 minutes.
Figure 3. Output of PSD #2 demonstrating fluctuation in the measurement of $f_{cw} - f_{ccw}$ or $\Omega$ in a time constant of 1 second. Scan duration is about 5 minutes (calibration spike is also shown).

Figure 4. Resonance of passive ring cavity 70 cm on a side. Full width at half intensity is 200 kHz.
Figure 5. Output of PSD #2 for discrete component cavity 70 cm on a side. Scan duration 10 minutes. Time constant $\tau = 1$ second.
Paper Publications


Paper Presentations


Theses


G. A. Sanders, SM/PhD Thesis (in progress)

R. P. Schloss, SM Thesis (in progress)
<table>
<thead>
<tr>
<th>Personnel</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. Shaoul Ezekiel</td>
<td>Professor</td>
</tr>
<tr>
<td>Salvatore R. Balsamo</td>
<td>Graduate Student</td>
</tr>
<tr>
<td>James A. Cole</td>
<td>Graduate Student</td>
</tr>
<tr>
<td>Vaughn E. Dobalian</td>
<td>SB Thesis</td>
</tr>
<tr>
<td>James Harrison</td>
<td>SB Thesis</td>
</tr>
<tr>
<td>Glen A. Sanders</td>
<td>SM/PhD Candidate</td>
</tr>
<tr>
<td>Robert P. Schloss</td>
<td>SM Candidate</td>
</tr>
<tr>
<td>Jack Wolosewicz</td>
<td>Graduate Student</td>
</tr>
</tbody>
</table>