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FUNCTIONAL AND OPERATIONAL ADVANCES IN
THE AFGL ROCKET TRAJECTORY SYSTEM - I

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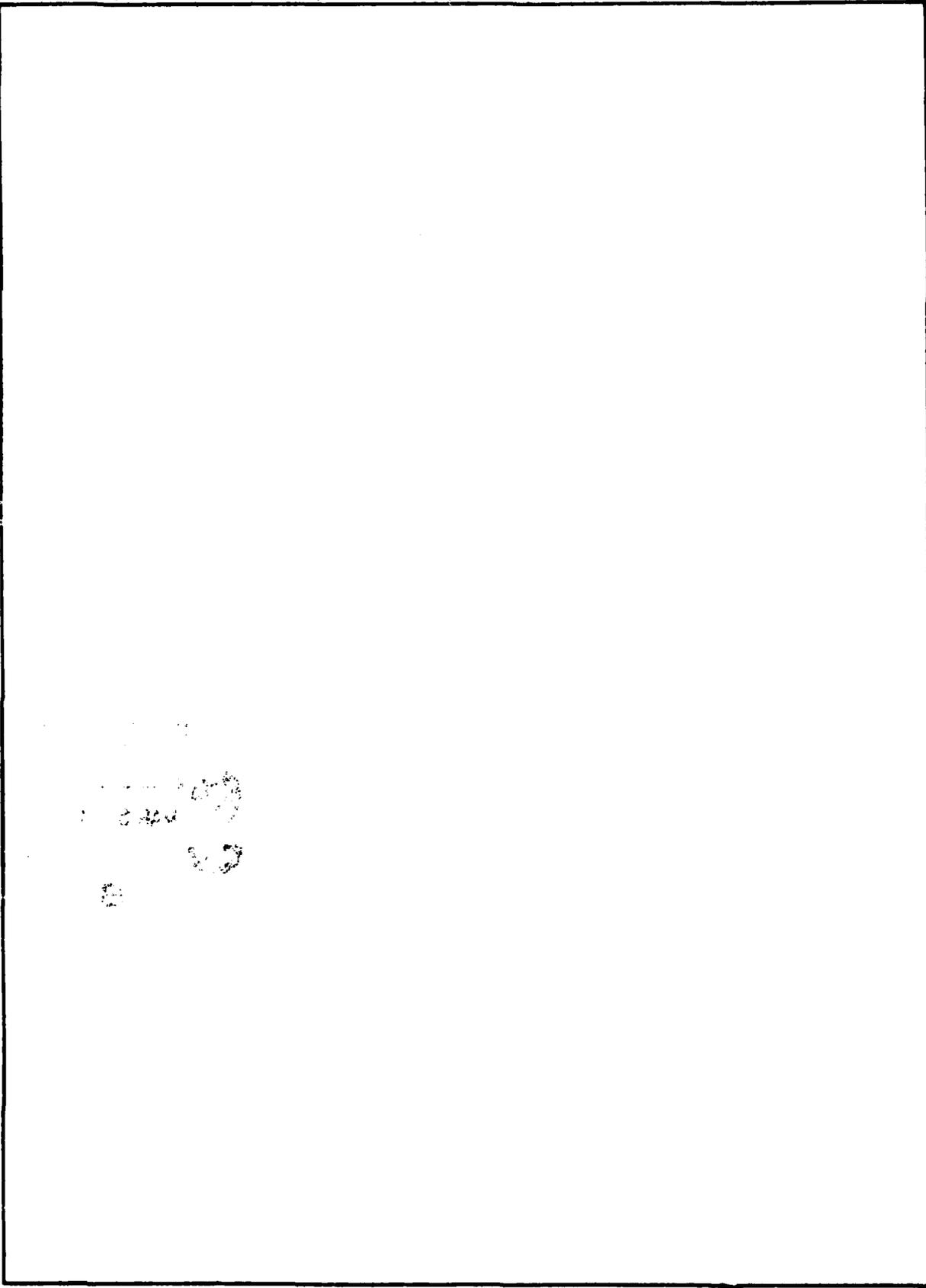
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1. INTRODUCTION

In parallel with a continuing active use of the AFGL Rocket Trajectory System, improvements and extended features have been incorporated in nearly all phases of this system. This combined report and user's guide documents current procedures and provides the analytical background underlying developments implemented since the installation of the original system.^{1, 2, 3}

The user is referred to the above referenced reports for details of the scope, analysis and implementation of these programs. Major revisions resulting in the present trajectory system include:

- 1) A combined editor and preprocessor program DRIVEA, which replaces the original DRIVE1-DRIVE2.
- 2) A comprehensive filtering program DRIVEB, which permits launch optimization estimation and segmented integration-filtering-reproducing modes.
- 3) Substantial augmentation of TAPE4, the DRIVEB output data file, to include residual, launcher referenced, ballistic, and local velocity information.
- 4) A simplified multi-radar solving program DRIVEC, which is based on more reliable estimation of the relative error covariance matrices.
- 5) Streamlining of the processing system by elimination of buffering and CPRS, elimination of the DRIVE1-DRIVE2 pass, reduction and flexible interpretation of card data input, tape processing aids in DRIVEA, standardization of common routines, and the use of CDC utilities.

This report is written so as to be maintained in ring-binder form for easy revision and updating. Section 2 presents the input card data formats and

the tape input and output formats. This information is also commented into each of the programs. A brief description of the various modes available in DRIVEB is also presented.

Section 3 outlines the organization of each program. The supporting material, which either analyzes new developments or attempts to substantiate original procedures, is presented in the Appendices.

2. USAGE GUIDE

The AFGL Rocket Trajectory System is designed to accept raw radar data from any rocket range on magnetic tapes in the format shown in Figure 1. It then generates an intermediate edited file and associated print-out using program DRIVEA. Finally, it provides a filtered best-estimate trajectory with accompanying reports, plots and summary file using program DRIVEB in one of a variety of optional filtering modes. In the event that multi-radar tracking data is available, a statistically weighted final trajectory with reports, plots, and summary file can be developed using DRIVEC. The user outputs from DRIVEB and DRIVEC are identical, and the summary file (TAPE4) can be used at any time to recreate the reports and plots using program REGEN. These four programs comprise the AFGL Rocket Trajectory System. In this section operational modes and input and output file formats are outlined for each program. The output files with their headers also serve as the input for the subsequent programs.

2.1 DRIVEA

Figure 2 presents the card input format. Details of calibration and refraction correction are unchanged from Reference 1. A number of improvements have been made over the original DRIVE1-DRIVE2 system. TOL(2) and TOL(3) control data tolerance and modification (25%) limits are described in Appendix A. NSKIP controls initial positioning of the raw data file, and NOK indicates which samples in the next record are to be used as the starting base. TLO and THI control the first and last data points to be entered on the edited output file (Figure 3); the actual times are recorded on the header as TSTART and TSTOP, and are later used automatically by DRIVEB.

Spanning of Data Gaps

An additional feature available in DRIVEA allows spanning of missing or poor data by inputting VAR, TI, and TF for smooth polynomial substitution of range, azimuth, and/or elevation samples. VAR = R, A, or E identifies the variable to be replaced over the time span TI, TF. Up to two cases are

TAPE1 - RAWDAT

This tape is a standard raw data format and consists of a 20-word header followed by a series of 96-word data blocks. Each data block contains 6 complete data samples consisting of 16 words describing the sample. The editor does not use all of these words, and only those shown below are required.

HEADER RECORD

<u>WORD</u>	<u>MODE</u>	<u>VARIABLE</u>
2	Floating	Radar latitude in radians.
3	Floating	Radar longitude in radians.
4	Floating	Radar height in feet.
5	Floating	Direction of flight line in radians.
8	Integer	Launch day.
9	Integer	Launch month.
10	Integer	Launch year.

DATA RECORD

<u>WORD</u>	<u>MODE</u>	<u>VARIABLE</u>
1	Floating	Time U.T. in milliseconds.
6	Floating	Slant range in yards.
9	Floating	Azimuth in degrees.
10	Floating	Elevation in degrees.

Figure 1 Format of Raw Range Radar Data--Tape 1--Input for DRIVEA

```

PROGRAM DRIVEA(INPUT,OUTPUT,RAWDAT,TAPE11,TAPE1=RAWDAT
1,TAPE10,TAPE9,TAPE8,TAPE6=OUTPUT,TAPE5=INPUT)
*****
DRIVEA DEVELOPED BY LCGICOM,INC. VERSION AUG.,1978
*****

```

THIS PROGRAM COMBINES FUNCTIONS PERFORMED BY TRAJECTORY ANALYSIS PROGRAMS DRIVE1 AND DRIVE2. THE DYNAMIC FILTER OF DRIVE2 HAS BEEN REPLACED BY A DATA MODIFICATION/INTERPOLATION PROCEDURE IN THE EDITOR (DRIVE1) PHASE AND A VELOCITY DIFFERENCE CLIPPER AND SMOOTHER IN THE PREPROCESSOR (DRIVE2) PHASE. THREE PARAMETERS (TCL(2),TCL(3),NOK) HAVE BEEN ADDED FOR THE POINT MODIFICATION/INTERPOLATION PROCEDURE, BUT DEFAULT VALUES ARE AVAILABLE. DRIVEA RESULTS IN FASTER COMPUTATION TIME AND AVOIDS SOME OF THE PROBLEMS OF THE ORIGINAL DRIVE2 FILTER (I.E. FILTER RINGING).

CARD INPUT REQUIREMENTS.

NO.	VARIABLE	DESCRIPTION	FORMAT
1	DATE,	RUN DATE, 20 CHARACTERS.	4A10
	FLIGHT	FLIGHT IDENTIFICATION, 20 CHARS. CHARACTERS, THESE ITEMS WILL APPEAR ON THE PRINTED OUTPUT.	
2	TITLE	80 CHARACTER TITLE FOR FURTHER RUN IDENTIFICATION. WILL APPEAR ON PRINTED OUTPUT ONLY.	3A10
3	ICODE	RANGE IDENTIFICATION CODE. 1 FOR EGLIN A.F.B. 2 FOR WOLLOPS IS. 3 FOR CHURCHILL.	15
4	DT,	DATA SAMPLING TIME INTERVAL IN MILLISECONDS.	4F10.3,2I5
	TMAX,	MAXIMUM EDITING TIME IN SECONDS. MEASURED AS A DISPLACEMENT FROM THE FIRST DATA POINT ON RAWDAT.	
	TOL(2),	TOLERANCE LIMIT - DEFAULT VALUE (J.O) = 5KFT.	
	TOL(3),	TOLERANCE LIMIT - DEFAULT VALUE (0.0) = 500G.	
	NOK,	POINTS 6-NOK & 5-NOK ARE TAKEN AS VALID BASE POINTS IN THE FIRST DATA BUFFER OF 6 SAMPLES.	
	NSKIP	NUMBER OF INITIAL BUFFER RECORDS TO BE SKIPPED.	

NOTE. THE FOLLOWING 3 DATA CARDS ARE NOT REQUIRED UNLESS THE RANGE CODE SPECIFIES CHURCHILL. (I.E. ICODE=3)

RCODE,	CODE TO DETERMINE IF A RANGE CORRECTION IS REQUIRED. = 1 FOR RANGE CORRECTION, RADAR NO.11 = 2 RADAR NO.2, = 0 SETS RANGE CORRECTION TO ZERO. IF NO CORRECTION IS REQUIRED THE REMAINING FIELDS MAY BE ZEROS.	15,5F10.3
CALRC,	RANGE CALIBRATION DISTANCE IN YARDS.	
RCE,	RANGE CALIBRATION READING IN YARDS.	
DR,	REACCN DELAY IN MICROSECONDS.	
PTM,	REACCN TRANSMITTER PULSE WIDTH IN MICROSECONDS.	
RTM	RADAR TRANSMITTER PULSE WIDTH IN MICROSECONDS.	

Figure 2a. Card Input Format for DRIVEA

6	NCOOE,	=1 FOR AZIMUTHAL CORRECTION. =0 SETS AZIMUTH CORRECTION TO ZERO.	I5,2F10.3
	AZD,	AZIMUTH PLUNGE READING IN DEGREES.	
	AZN	AZIMUTH READING IN NORMAL POSITION IN DEGREES.	
7	NCOOE,	=1 FOR ELEVATION CORRECTION. =0 SETS ELEVATION CORRECTION TO ZERO.	I5,2F10.3
	ELP,	ELEVATION READING IN THE PLUNGE PO- SITION IN DEGREES.	
	ELN	ELEVATION READING IN THE NORMAL PO- SITION IN DEGREES.	
8	KEYOPT	OPTION KEY. =1 FOR GEOCENTRIC CONVERSION. =2 FOR GEOCENTRIC CONVERSION.	I5
9	ITMO, ITB, ITY, TMS, TMIN, TSEC	CALENDAR DATE(MONTH, DAY, YEAR) AND U.T. (HOURS, MINUTES, SECONDS) OF LAUNCH.	2(I2, I4), I4, 3F10.4
10	HEIGHT	HEIGHT OF RADAR ABOVE THE SPHEROID IN METERS.	F10.2
11	PLADEG, PLCHIN, PLSECC	RADAR LONGITUDE IN DEGREES, MIN., AND SECONDS. NEGATIVE IF EAST.	3F10.4
12	PLADEG, PLAMIN, PLASEC	RADAR LATITUDE IN DEGREES, MIN., AND SECONDS. NEGATIVE IF SOUTH.	3F10.4
13	TLO, TMI	TIME LIMITS IN SECONDS U.T. FOR GEOCENTRIC CONVERSION. DEFAULT VALUES(0.0) TLO=TIME OF FIRST RAW DATA POINT. TMI=TIME OF LAST RAW DATA POINT PROCESSED BY PROGRAM.	2F10.3
14	IREF	REFRACTION CORRECTION KEY. IREF LESS THAN 3 SPECIFIES NO REFRACTION NEEDED, I.E., THE DATA IS ALREADY CORRECTED.	I5
15	PROGID	THREE WORD PROGRAM IDENTIFIER FOR THE OUTPUT PLOTS. SAMPLE 5CH02, 3502 EIN=SC PPIVA	3A10
16	VAR,	VARIABLE (R, A, E) FOR WHICH SMOOTH POLYNOMIAL SUBSTITUTION IS DESIRED. UP TO TWO CASES ALLOWED PER VARIABLE.	A1, 5X, 2F10.3
	TI, TF	START AND END OF TIMES FROM LAUNCH (SECS.) FOR SPANNING WITH SMOOTHED DATA.	

TAPE 11 IS A SEQUENTIAL FILE OF THE EDITED RAW DATA, AND SERVES AS A
LINK BETWEEN DRIVE1 AND DRIVE2 PHASES. PAGES 8, 9, AND 11 ARE
BINARY FILES HOLDING THE FINAL GEOCENTRIC VELOCITIES, GEOCENTRIC
COORDINATES, AND EDITED RAW DATA RESPECTIVELY.

FIGURE 2b. CARD INPUT FOR DRIVEA

TAPE 10 INCLUDES A HEADER RECORD.

WORD(S)	SYMBOL	DESCRIPTION
1-2	DATE	RUN DATE, BLANK
3-4	DATE (3-4)	FLIGHT IDENTIFICATION
5	LDAT	LAUNCH DATE (MO/DA/YEAR)
6	TLNGH	LAUNCH TIME UT (SECS)
7-9	PROGID(1-3)	PROGRAM ID FOR PLOTTED OUTPUT
10	HRAD	RADAR ALTITUDE (KM)
11	RLODEG	RADAR LONGITUDE (DEG)
12	RLOMIN	RADAR LONGITUDE (MIN)
13	RLOSEC	RADAR LONGITUDE (SEC)
14	RLADEG	RADAR LATITUDE (DEG)
15	RLAMIN	RADAR LATITUDE (MIN)
16	RLASEC	RADAR LATITUDE (SEC)
17	TSTART	DATA STARTING TIME UT (SECS)
18	TSTOP	FINAL DATA TIME UT (SECS)
19	DT	SAMPLING TIME INTERVAL (SEC)
20	ICOTE	1 FOR EGLIN, 2 FOR WALLEOPS, 3 FOR CHURCHILL
21	IFEF	IF .GT. 0 DATA WAS CORRECTED FOR REFRACTION

FOLLOWED BY RANGE, AZ, EL (KM,RAD) LOGICAL RECORDS.

Figure 3

FORMAT OF DRIVEA OUTPUT FILE, TAPE 10.
 THIS IS A BINARY FILE GENERATED BY DRIVEA
 WHICH HOLDS THE EDITED RAW DATA USED AS INPUT
 FOR DRIVEB.

allowed for each. Good samples at TI-5, TI, TF and TF+5 seconds are used to define the spanning cubic. If samples are unavailable at TI-5 or at TF+5, all times are readjusted so that at least 2-second separations are maintained. Special care is required in selecting each TI, TF pair, as described in Appendix A.3.

2.2 DRIVEB

Before the most satisfactory best-estimate trajectory can be determined, the multi-option filtering program, DRIVEB, is usually run on the edited radar data file, TAPE 10, for a variety of choices of filtering time limits and radar parameters. Figure 4 presents the card input format.

Radar Parameters

Radar parameters are generally prescribed for any given radar, but the investigator's knowledge of the evident quality of the raw radar data may be imparted through adjusted values (see Appendix G) to more appropriately influence the statistical filtering equations.

Ballistic Parameters

Rocket drag, thrust factor and delay time may be input to override nominal values which are built into rocket models in the THRUST routine. Rocket models presently included are shown in Table 1; the key is the identification number contained in DAT(5). Interpretation of filter time limits TB...TX, KEYOPT, and the geocentric position and velocity P1(1-6), are described with the aid of Figure 5.

Run Options

Except in the pure integration mode (KEYOPT=0), TSTART is redefined by the TAPE10 header. The type of DRIVEB run determined by KEYOPT refers to the main filtering segment TC to TE. The geocentric position and velocity values refer to time TC and may initially be obtained as the rough estimates from the DRIVEA print-out. The exception to this is for KEYOPT=1 or -1 when launch

PROGRAM DRIVEB (INPUT, OUTPUT, TAPE10, TAPE4, TAPE7
 1, TAPE6=OUTPUT, TAPE5=INPUT)

 DRIVEB DEVELOPED BY LOGICON, INC. VERSION SEP., 1978

I/O DEFINITIONS.

TAPE10 RAW DATA INPUT TAPE, INCLUDING HEADER.
 THIS TAPE IS GENERATED BY DRIVEA.
 TAPE4 PRINTED AND PLOTTED OUTPUT DATA CONTAINS
 SELECTED POINTS IF IOPT=0, ALL POINTS IF IOPT=1
 TAPE7 FILTERED AND RAW DATA OUTPUT-MASS STORAGE ACCESS
 ORGANIZATION FOR FILE MANIPULATION
 TAPE6 SYSTEM PRINTER.
 TAPE5 SYSTEM CARD READER.

DRIVER FILTER CARD INPUT DATA.

5	10	21	59
CARD	ITEM	DESCRIPTION	FORMAT
DATE AND TITLE INFORMATION.			
1	TITLE1	ANY 80 CHARACTER TITLE	
2	TITLE2	FIRST 20 AND LAST 20 CHARACTERS ARE IGNORED. THE REMAINING 40 CHARACTERS MAY BE USED FOR FURTHER IDENTIFICATION.	
3	JAT (3-6)	LAUNCH DATE AND ROCKET NUMBER; USE UP TO 20 CHARACTERS EACH. TWO DIGITS IN COLUMNS 21-29 FOLLOWED BY A PERIOD DETERMINE ROCKET TYPE; IF THE PERIOD IS ABSENT, AN ARBITRARY UNPOWERED ROCKET IS ASSUMED.	
4	PROGID	PROGRAM ID FOR PLOTTED OUTPUT. 30 CHARACTERS. FIRST 5 ARE THE ID FROM THE JOB CARD, NEXT 4 ARE THE CHARGE NO. REMAINING CAN BE USED FOR FURTHER IDENTIFICATION. IE. (SCH03,3502,SC40NBEIN,FEB.28,72)	
RADAR POSITION DATA.			
5	HRA0M	HEIGHT OF THE RADAR ABOVE THE SPHEROID IN METERS.	3F10.4
6	RLODEG, RLOMIN, RLOSEC	LONGITUDE IN DEGREES, MINUTES AND SECONDS. IF THE LONGITUDE IS EAST ENTER AT LEAST ONE OF THE QUANTITIES AS A NEGATIVE NUMBER.	
7	PLADEG, PLAMIN, PLASEC	LATITUDE IN DEGREES, MINUTES AND SECONDS. IF THE LATITUDE IS SOUTH ENTER AT LEAST ONE OF THE QUANTITIES AS A NEGATIVE NUMBER.	
RADAR PARAMETERS.			
8	RPV (1),	PEAK TRANSMITTED POWER IN KILO- WATTS.	FF10.4
	RPV (2),	ANTENNA GAIN IN DB. (DOES NOT INCLUDE ANTENNA EFFICIENCY)	
	RPV (3),	TRANSMITTED WAVELENGTH IN METERS.	
	RPV (4),	RECEIVER NOISE FIGURE IN DB.	
	RPV (5).	SKIN TRACK LOSS FACTOR IN DB.	

Figure 4a. Card Input Format for DRIVEB

```

9  RPV(6), BEACON TRACK LOSS FACTOR IN DB.
   RPV(7), ANGLE VELOCITY LAG COEFFICIENT
           IN MILS/MILS/SEC.
   RPV(8), RANGE VELOCITY LAG COEFFICIENT
           KXDS/KYDS/SEC.
   RPV(9), BEACON POWER IN WATTS.
   RPV(10), BEACON ANTENNA GAIN IN DB.
10  RPV(11), ANTENNA BEAMWIDTH IN DEGREES.
   RPV(12), ANGLE TRACKING SERVJ BANDWIDTH
           IN HZ.
   RPV(13), TARGET CROSS SECTION IN DB.
   RPV(14), BASE ANGULAR VARIANCE IN MILS**2.
   RPV(15), BASE RANGE VARIANCE IN YARDS**2.
11  RPV(16), RANGE TRACKING SERVJ BANDWIDTH IN FT.
   RPV(17), PULSE LENGTH IN MICROSECONDS.
   RPV(18), I.F. BANDWIDTH IN MHZ.
   RPV(19), TIME OF LOSS OF BEACON TRACK SIGNAL
           IN SECONDS U.T.
   RPV(20) CONICAL SCAN(=1.0) OR MONOPULSE(=0.0)
ROCKET PAYLOAD AND DRAG DATA. 5F10.4
12  PAYLD, PAYLOAD MASS IN KILOGRAMS.
   QAREA, DRAG CROSS SECTION IN METERS**2.
   COIMP, IF ZERO, USE VALUE FROM ROCKET TABLE.
   DRAG COEFFICIENT AT 1404 VELOCITY=3.
   THFACT, IF ZERO, USE VALUE FROM ROCKET TABLE.
   THRUST FACTOR (DEFAULT=NOMINAL=1.)
   OFACT  DELAY FACTOR, START OF THRUST IN SECS.
           AFTER LAUNCH. (SHOULD NOT BE NEGATIVE)
LAUNCH DATE AND TIME. 2(I2,1X),I4,3F10.4
13  LMO, MONTH,
   LDA, DAY,
   LYR, YEAR.
   UHRS, UNIVERSAL TIME AT LAUNCH IN
   UMIN, HOURS, MINUTES AND SECONDS.
   USEC.
DATA STARTING TIME AND SAMPLING TIME INTERVAL. 6F10.4
14  THRS, TIME, UTC, OF THE FIRST DATA POINT
   TMIN, ON TAPE10 IN HOURS, MINUTES,
   TSEC, AND SECONDS. DT IS THE TIME INCREMENT
   DT BETWEEN SAMPLES.
NOTE. TIME & DT FROM CARD 14 ARE USED ONLY FOR KEYOPT=0,
      OTHERWISE THEY ARE OBTAINED FROM THE TAPE10 HEADER.
INTEGRATION, FILTERING AND REPRODUCING TIME LIMITS. 6F10.4
15  TB, BACKWARDS INTEGRATION LIMIT. IF
   NO BACK INTEGRATION IS REQUIRED SET TO
   TO A VALUE GREATER THAN OR EQUAL TO F1.
   TC, COMMENCEMENT TIME FOR THE FILTER.
   U.T. MEASURED IN SECONDS.
   TE, ENDING TIME FOR FILTERING.
   U.T. MEASURED IN SECONDS.
   TF, FORWARD INTEGRATION TIME LIMIT. IF
   NO INTEGRATION IS REQUIRED SET TF
   TO A VALUE LESS THAN OR EQUAL TO TE.
   TG, DATA REPRODUCING TIME LIMIT. IF
   NO REPRODUCING IS REQUIRED SET TG
   TO A VALUE LESS THAN OR EQUAL TO TF.
   TX  TIME SPAN FROM TC ON FOR IMPROVING LAUNCH
       CONDITIONS WHEN KEYOPT=1 (DEFAULT=5 SEC.)

```

Figure 4b. Card Input Format for DRIVEB

FILTERING, INTEGRATION, REPRODUCING OPTION KEY. T5

15 KEYOPT ONE(1) TO PROVIDE NORMAL INTEG.-FILT. RUN,
MINUS ONE(-1) FOR ONE PASS INTEG.-FILT. RUN,
ZERO(0) TO PROVIDE INTEGRATION WITHOUT DATA,
TWO(2) TO PROVIDE DATA REPRODUCING RUN,
THREE(3) TO PROVIDE SINGLE FILTERING RUN,
FOUR(4) TO PROVIDE DOUBLE FILTERING RUN,
USING INITIAL CONDITIONS AT TIME T1. 3F20.8

INITIAL CONDITIONS.

17 P1(1-3) INITIAL CONDITIONS IN GEOCENTRIC
COORDINATES AT TIME T1.
ALL VALUES IN KILOMETERS.
OR, IF P1(1)=P1(2)=P1(3)=0.,
OBTAIN INITIAL CONDITIONS AT LAUNCH
FROM LAUNCHER LOCATION DATA.

18 P1(4-6) CORRESPONDING VELOCITY CONDITIONS.
ALL VALUES IN KILOMETERS/SECOND.
OR- ONLY IF P1(6)=0.,
AZSL, AZIMUTH OF ROCKET CENTER-LINE AT LAUNCH DEG.
ELCL, ELEVATION OF ROCKET CENTER-LINE AT LAUNCH DEG.

NOISE COVARIANCE DATA. F10.4

19 IFACT MULTIPLICATION FACTOR FOR ASSUMED
DRIVING NOISE COVARIANCE.

PRINTED OUTPUT CONTROL CARD. T5

20 IUNIT, +1 FOR FEET AND FEET/SEC.
+2 FOR KILOMETERS AND KM./SEC.
ISKIP, INCREMENT AT WHICH DATA POINTS
ARE TO BE PRINTED AND PLOTTED.
ISAVE, SELECTS THE VELOCITIES AND STANDARD
DEVIATIONS TO BE PRINTED.
+1 FOR GEOCENTRIC VELOCITIES AND DEVIATIONS.
+2 FOR VELOCITIES REFERENCED TO THE LAUNCHER.
+3 FOR VELOCITIES REFERENCED TO THE RADAR.
IDCM, COMMON DUMP SKIP PARAMETER.
POSITIVE TO GET ABBREVIATED DUMP,
NEGATIVE TO GET EXTENDED DUMP.
IOPT TAPE4 WRITE OPTION SELECTION
D OR BLANK FOR EVERY ISKIP POINT
1 FOR EVERY POINT

LAUNCHER LOCATION. 3F10.4

21 AALTH LAUNCHER ALTITUDE ABOVE THE
SPHEROID IN METERS.
22 ALODEG, LONGITUDE IN DEGREES, MINUTES
ALOMIN, AND SECONDS. SAME CONVENTIONS
AL0SEC, AS RADAR LONGITUDE.
23 ALADEG, LATITUDE IN DEGREES, MINUTES
ALAMIN, AND SECONDS. SAME CONVENTIONS
ALASEC, AS RADAR ALTITUDE.

PLOTTED OUTPUT CONTROL CARDS. T2, 8X, 7A10

24 NVAR, NVAR SELECTS THE VARIABLE TO BE
YTITLE(4) PLOTTED VS TIME FROM LAUNCH FROM
THE CONTENTS OF TAPE4. THE VALUES
ALLOWED FOR NVAR CORRESPOND TO THE WORD
LOCATIONS GIVEN BELOW FOR TAPE4.
YTITLE IS THE ORDINATE TITLE AND IS
LIMITED TO 40 CHARACTERS.

25 NG, NG SPECIFIES THE NUMBER OF CHARACTERS OF
GTITLE(7) GTITLE TO BE PLOTTED. GTITLE MAY
CONTAIN UP TO 70 CHARACTERS.

26 ON SELECTS OTHER VARIABLES TO BE PLOTTED
EACH WITH NVAR, YTITLE AS FOR CARD 24.

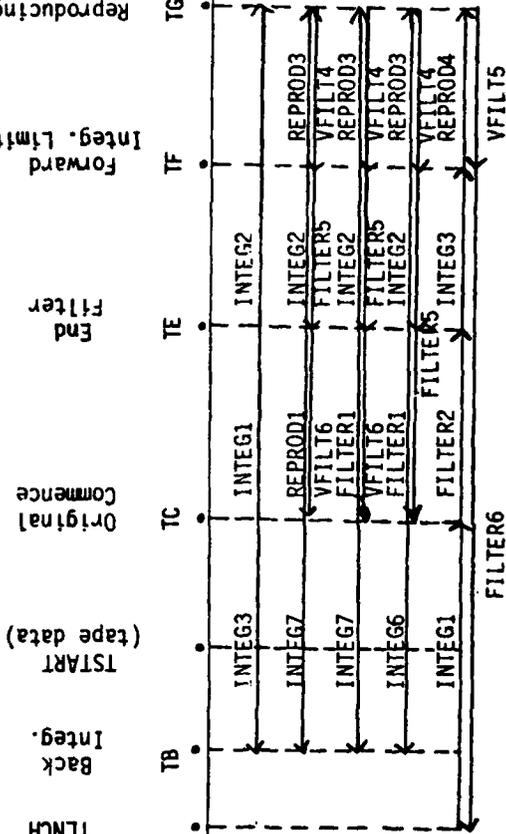
NOTE. ONLY ONE CARD OF TYPE 25 IS REQUIRED. THE PLOT SEQUENCE
IS AUTOMATICALLY TERMINATED WHEN THE EOF IS ENCOUNTERED.

FIGURE 4c. CARD INPUT FOR DRIVEB

Table 1. Rockets Modeled for DRIVEB.

Rocket	Key
AEROBEE 150	03
AEROBEE 170	04
NIRO	07
NIKE TOMAHAWK	08
UTE TOMAHAWK	09
PAIUTE TOMAHAWK	10
NIKE HYDAC	11
NIKE ORION	12
BLACK BRANT IV-B	16
BLACK BRANT V -A	17
BLACK BRANT V -B, C	18
JAVELIN	19
NIKE JAVELIN	20
ARIES	24
SERGEANT-EXCEDE	25
ASTROBEE D	30
AEROBEE 350	35
CASTOR	40
CASTOR LANCE	41
TALOS CASTOR	51
----- ARBITRARY UNPOWERED	----- NONE

KEYOPT	TYPE OF RUN	T0	T1	T2	T3	T4	Geoc. Position PV(1-3) km.	Geoc. Velocity PV(4-6) km/sec.
0	Pure Integ.	TB	TC	TE	TF	TG		
2	Data Reprod.							
3	Single Filter							
4	Double Filter							
1	Normal Integ-Filter (optimize launch conditions to time TC)							
-1	Same as KEYOPT=1, but leave launch conds. unchanged							



If PV(6)=0, use launch coords. In
 PV(4)=Launch Azimuth
 PV(5)=Launch Elevation } Degrees

If PV(1)=PV(2)=PV(3)=0
 use launched coords.

NOTE 1: For KEYOPT=1 or -1, launch conditions are required, and TB is ignored.

If PV(6) ≠ 0, PV(1-3) at time TC are used to estimate launch AZ,EL; and PV(4-6) are ignored.

NOTE 2: Filter time points are trimmed internally to correspond to data sample times; except when pure integrating, when the time points correspond to TC.

NOTE 3: Forward integrating results are backward filtered only when followed by reproducing.

Figure 5. Description of Filter Time Limits, KEYOPT, Geocentric Position and Velocity Cards in DRIVEB.

conditions are required. The numerals after the process INTEG, FILTER, REPROD, or VFILT indicate the sequence in which the trajectory is computed. Following the end of the main segment at TE, integration and/or reproducing may be selected by TF and TG. In all reproducing cases, and for the single filtering case from TC to TE, the velocity estimate only is smoothed in the backward pass. The final segment goes backward from TC to the back computation limit TB. KEYOPT=3 provides a run identical to the original DRIVE3 run, and KEYOPT=4 matches the original DRIVE5. KEYOPT=1 is normally used now. When KEYOPT=1 or -1 apparent radar data is generated from launch to TC, where good actual radar data becomes available. The subsequent backward filtering to TLNCH results in a consistent trajectory from the launch pad and through the radar data. When KEYOPT=1, launch conditions THFACT, DFACT, AZCL and ELCL are optimized to obtain the smoothest connection at TC, TC + TX/5 and TC + TX seconds. If this search procedure is unsatisfactory for some reason (such as negative thrust delay), it may be necessary to select a different connection time TC, or to fix launch conditions and inhibit optimization by setting KEYOPT=-1. Notice that for KEYOPT=1 or -1 and P1(6)=0, P1(4) and P1(5) are taken to be the launch azimuth and elevation respectively in degrees; further, P1(1-3) are ignored and the launch coordinates are derived by DRIVEB.

Model Uncertainty - QFACT

QFACT offers a general method for controlling the significance of the radar data relative to the model. The usual range for QFACT is 0.01 to 0.1, with the higher values indicating greater uncertainty in the model and therefore a greater weight on the data.

Diagnostics

A diagnostic print-out is provided every ICOMth computational step, typically once every 10 seconds by setting ICOM=100 for a 0.1 sec data rate, and at a few points about every transition stage such as TC, TE, etc. The abbreviated dump includes geocentric, radar and launcher referenced position, velocity, and acceleration data in cartesian or polar coordinates, as well as ballistic and geographic data. The extended dump with negative ICOM also includes various transformation and covariance matrices.

Output TAPE4

Complete data for recreating printed and plotted reports and for further statistical computing, if required, are written in 104-word header and data binary records on TAPE4. The format and contents are described in Figure 6. Launcher referenced coordinates, ballistic and local velocity data, and observed-computed residuals provide useful additional information that was not available originally.¹ TAPE4's resulting from various radar DRIVEB solutions of a rocket trajectory may be combined two at a time in DRIVEC, each pass producing a multi-radar TAPE4 trajectory estimate. The printed summary report is designed for 8-1/2"x11" binding, and includes a variety of information contained on TAPE4 under partial control of ISAVE.

2.3 DRIVEC

Two TAPE4 solutions of the same trajectory are input to DRIVEC as TAPE10 and TAPE11. The reference trajectory for header and residual purposes is TAPE10. Information on the header is extensive, so that only minimal print and plot control data needs to be entered on cards (Figure 7) to initiate the multi-radar solution. Note that ISKIP here refers to the increment TSKIP at which TAPE10 was generated, and not to the original computational step DT. There is presently no provision for unequal TSKIP increments on TAPE10 and TAPE11. Another item to note is that DRIVEC computes the position-velocity state vector by statistical weighting, and not according to any trajectory dynamics. Therefore, discontinuities are possible where TAPE10 or TAPE11 filtering modes change, and these can be improved by revised DRIVEB runs with modified statistical parameters (see Appendices C, F and G). The final TAPE 4 and printed and plotted output are identical to DRIVEB.

2.4 REGEN

The printed and plotted output of DRIVEB or DRIVEC can be fully reproduced using TAPE4 in program REGEN. Header information is comprehensive so that the print and plot report generation is accomplished with minimal card data (Figure 8) that is similar to the DRIVEC input. ISKIP here refers to the original or apparent computational step DT.

OUTPUT REPORT DATA FILE WRITTEN ON TAPE4. THE FIRST RECORD CONTAINS FLIGHT INFORMATION FOR USE BY PROGRAM REGEN, WHICH REPRODUCES PRINTED AND PLOTTED OUTPUT. THE REMAINING RECORDS CONTAIN DATA TO BE PRINTED AND PLOTTED BY EITHER DRIVER ROUTINES OR BY REGEN. IF IOPT=1, ALL DATA POINTS ARE WRITTEN ON TAPE4, IF IOPT=0, EVERY ISKIPTH POINT IS WRITTEN.

FIRST RECORD

WORD(S)	SYMBOL	DESCRIPTION
1-8	TITLE(1-8)	80 CHARACTER ALPHANUMERIC IDENTIFICATION
9-10	DAT(1-2)	RUN DATE, BLANK
11-14	TITLE2(1-4)	FURTHER ALPHANUMERIC IDENTIFICATION
15-16	TITLE2(5-6)	PAYLOAD (KG), AREA (KM**2)
17	JAT(3)	CO
18	JAT(4)	LAUNCH DATE (MO/JA/YR)
19-20	DAT(3-6)	ROCKET ID
21-23	PROGID(1-3)	PROGRAM ID FOR PLOTTED OUTPUT
24	HRAD	RADAR ALTITUDE (KM)
25	RLODEG	RADAR LONGITUDE (DEG)
26	RLOMIN	RADAR LONGITUDE (MIN)
27	RLOSEC	RADAR LONGITUDE (SEC)
28	PLADEG	RADAR LATITUDE (DEG)
29	PLAMIN	RADAR LATITUDE (MIN)
30	PLASEC	RADAR LATITUDE (SEC)
31	AALT	LAUNCHER ALTITUDE (KM)
32	ALODEG	LAUNCHER LONGITUDE (DEG)
33	ALOMIN	LAUNCHER LONGITUDE (MIN)
34	ALOSEC	LAUNCHER LONGITUDE (SEC)
35	ALADEG	LAUNCHER LATITUDE (DEG)
36	ALAMIN	LAUNCHER LATITUDE (MIN)
37	ALASEC	LAUNCHER LATITUDE (SEC)
38	TLNCH	LAUNCH TIME UT (SECS)
39	THRAT	THRUST RATIO TO NOMINAL
40	DELTH	DELAY IN THRUST (SECS)
41	AZGL	ROCKET LAUNCH AZIMUTH (DEGS)
42	ELGL	ROCKET LAUNCH ELEVATION (DEGS)
43	DT	COMPUTATION TIME STEP SIZE (SECS)
44	TLI	INITIAL PRINT TIME FROM LAUNCH
45	TLF	FINAL PRINT TIME FROM LAUNCH
46	TSKIP	TAPE WRITTEN EVERY TSKIP SECS.
47-104	ZEROES	ZERO-FILLED.

SUBSEQUENT RECORDS.

WORD(S)	SYMBOL	DESCRIPTION
1	GMT	UNIVERSAL TIME (HOURS)
2	TIME	TIME AFTER LAUNCH (SECONDS)
3	SN	SIGNAL TO NOISE RATIO
4	RAG	RIGHT ASCENSION OF GREENWICH (R17)
5-7	P2(1-3)	FILT. GEOCENTRIC POSITION VECTOR (KM)
8-10	P2(4-6)	FILT. GEOCENTRIC VELOCITY VECTOR (KM/SEC)
11-13	P2(7-9)	FILT. GEOCENTRIC ACCEL. VECTOR (KM/SEC**2)
14-16	PVL(1-3)	FILT. LAUNCHER REF. POSITION (KM)
17-19	PVL(4-6)	FILT. LAUNCHER REF. VELOCITY (KM/SEC)
20-22	PVL(7-9)	FILT. LAUNCHER REF. ACCEL. (KM/SEC**2)
23-25	PVL(1-3)	FILT. LAUNCHER REF. RANGE,AZ,EL (KM,DEG)
26-28	PVL(4-6)	FILT. LAUNCHER REF. RANGE,AZ,EL RATES (KM/SEC,DEG/S)
29-64	XSI(1-36)	COMPLETE ERROR COVARIANCE MATRIX
65-67	PVR(1-3)	FILT. RADAR RANGE,AZ,EL (KM,DEG)
68-70	PVR(4-6)	FILT. RADAR RANGE,AZ,EL RATES (KM/S,DEG/S)
71-73	PVR(1-3)	FILT. RADAR REF. POSITION VECTOR (KM)
74-75	SPV(1-3)	GEODETIC ALT., LONG(M), LAT. (KM,DEG)
77	VR	MAG. OF LOCAL VELOCITY VECTOR (KM/SEC)
78-79	AZR,ELR	AZ,EL OF LOCAL VELOCITY VECTOR (DEG)
80	DP	DPAG COEFFICIENT
81	JEN	ATMOSPHERIC DENSITY (KG/KM**3)
82-83	JRAG,TH	DPAG, THRUST FORCES (KG/KM/SZ)
84-86	JVR(1-3)	RAW RADAR DATA RANGE,AZ,EL (KM,DEG)
87-89	JVG(1-3)	GEOCENTRIC RAW RADAR DATA VECTOR (KM)
90	GRANGE	GROUND RANGE ALONG SPHEROID (KM)
91	ACL MAG	LAUNCHER REF. ACCEL. MAGNITUDE (KM/SEC**2)
92	PRESALT	ALTITUDE RESIDUAL (KM)
93-95	GRSG,YRSG,ZRSG	GEOCENTRIC POSITION VECTOR RESIDUALS (KM)
96	GRSG	GEOCENTRIC VECTOR MAGNITUDE RESIDUAL (KM)
97-99	YRSL,YRSL,ZPSL	LAUNCHER POSITION VECTOR RESIDUALS (KM)
100	GRSL	LAUNCHER VECTOR MAGNITUDE RESIDUAL (KM)
101-103	GRSR,YRSR,ZPSR	RADAR POSITION VECTOR RESIDUALS (KM)
104	GRSR	RADAR VECTOR MAGNITUDE RESIDUAL (KM)

NOTE: ALL RESIDUALS ARE (PAW-FILTERED)

Figure 6. TAPE4 Header and Data Records--Output by DRIVEB or DRIVEC, Input for DRIVEC or REGEN

```

PROGRAM DRIVEC(INPUT,OUTPUT,TAPE4,TAPE7,TAPE10,TAPE11,
1 TAPE5=OUTPUT,TAPE6=INPUT)
****
DRIVEC DEVELOPED BY LOGICON, INC. VERSION SEP., 1978
****
DESCRIPTIONS
I/O DEFINITIONS.
TAPE10 DATA INPUT REPRODUCED AS OUTPUT FROM DRIVER
TAPE11 DATA INPUT REPRODUCED AS OUTPUT FROM DRIVER
TAPE4 PRINTED AND PLOTTED OUTPUT DATA CONTAINS
SELECTED POINTS IF IOPT=0. ALL POINTS IF IOPT=1
TAPE7 SCRATCH FILE FOR DATA MANIPULATION. MASS STORAGE
ACCESS ORGANIZATION.
TAPE6 SYSTEM PRINTER.
TAPE5 SYSTEM CARD READER.
DATA INPUT DESCRIPTION
5 10 21 56
CARD ITEM DESCRIPTION FCNMT
CARD 1 PRINT AND OUTPUT CONTROL DATA. 515
IUNIT, +1 FOR FEET AND FEET/SECOND
+2 FOR KILOMETERS AND KM./SEC.
ISKIP, INCREMENT AT WHICH DATA POINTS
ARE TO BE PRINTED AND PLOTTED.
ISAVE, SELECTS THE VELOCITIES AND STANDARD
DEVIATIONS TO BE PRINTED.
+1 FOR GEOCENTRIC VELOCITIES AND DEVIATIONS.
+2 FOR VELOCITIES REFERENCED TO THE LAUNCHER.
+3 FOR VELOCITIES REFERENCED TO THE RADAR.
ISOM, COMMON DUMP SKIP PARAMETER. SERVES SAME
FUNCTION AS ISKIP PARAMETER.
IOPT TAPE4 WRITE OPTION SELECTION
0 OR BLANK FOR EVERY ISKIP POINT
1 FOR EVERY POINT
CARD 2 NAMELIST DATA. TLI,TLP, THE PRINT TIME LIMITS (AS TIME
AFTER LAUNCH) MAY BE RESET. FOR EXAMPLE,
1 1 1 1 1 1
3NAME TLI=15.0,TLP=50.02
IF NOT RESET, THE ORIGINAL LIMITS WILL BE USED.
TITLE INFORMATION IS INPUT FROM THE HEADER RECORD ON TAPE10
WHICH WAS THE TAPE4 OUTPUT FROM DRIVER.
PLOTTED OUTPUT CONTROL CARDS. FCNMT I2,3X,7013
3 NVAR, SELECTS THE VARIABLE TO BE PLOTTED FROM THE
XTITLE(1-4) CONTENTS OF TAPE4. XTITLE APPEARS AS THE
ORDINATE TITLE. (LIMIT OF 40 CHARACTERS)
4 NG, SPECIFIES THE NUMBER OF CHARACTERS IN THE
GTITLE(1-7) GENERAL TITLE. GTITLE APPEARS ON EACH OUTPUT
GRAPH. (MAXIMUM OF 70 CHARACTERS)
CARD 5 ON SELECTS OTHER VARIABLES TO BE PLOTTED
EACH WITH NVAR, XTITLE AS FOR CARD 3.
NOTE. ONLY ONE CARD OF TYPE 4 IS REQUIRED. THE PLOT SEQUENCE
IS AUTOMATICALLY TERMINATED WHEN THE EOF IS ENCOUNTERED.
OUTPUT REPORT DATA FILE WRITTEN ON TAPE4. THE FIRST RECORD
CONTAINS FLIGHT INFORMATION FOR USE BY PROGRAM REGEN, WHICH
REPRODUCES PRINTED AND PLOTTED OUTPUT. THE REMAINING RECORDS
CONTAIN DATA TO BE PRINTED AND PLOTTED BY EITHER DRIVEC ROUTINES
OR BY REGEN. IF IOPT=1, ALL DATA POINTS ARE WRITTEN ON TAPE4.
IF IOPT=0, EVERY ISKIP=1 POINT IS WRITTEN.

```

Figure 7. Card Input Format for DRIVEC

PROGRAM REGEN(INPUT,OUTPUT,TAPE3,TAPE4,TAPE5=INPUT,TAPE6=OUTPUT)

 REGEN DEVELOPED BY LOGICON, INC. VERSION SEP., 1978

~~REPORT GENERATOR. GENERATES OUTPUT TRAJECTORY REPORTS
 USING TAPE4. INPUT DATA PRODUCED BY DRIVES~~

THE FOLLOWING CARD INPUT IS REQUIRED

CARD 1 ~~PRINT CONTROL DATA~~ --- FORMAT ~~4I=~~

IUNIT, +1 FOR FEET AND FEET/SECOND

~~+2 FOR KM AND KM/SECOND~~

ISKIP, DATA POINTS WILL BE PRINTED EVERY ISKIP*DT SECONDS

~~ISAVE --- SELECTS THE VELOCITIES AND RESIDUALS TO BE PRINTED~~

+1 FOR GEOCENTRIC

~~+2 FOR LAUNCHER~~

+3 FOR RADAR

CARD 2 NAMELIST DATA. TLI,TLF, THE PRINT TIME LIMITS(CAS TIME
 AFTER LAUNCH) MAY BE RESET. FOR EXAMPLE,

0 1 2 3 4 5

1 1 1 1 1 1

SNAMA TLI=15.0,TLF=450.09

~~IF NOT RESET, THE ORIGINAL LIMITS WILL BE USED.~~

~~TITLE INFORMATION IS INPUT FROM THE HEADER RECORD ON TAPE4.~~

~~PLOTTED OUTPUT CONTROL CARDS. --- FORMAT I2,8X,7A10~~

3 NVAR, SELECTS THE VARIABLE TO BE PLOTTED FROM THE

~~YTITLE(1-4) CONTENTS OF TAPE4. YTITLE APPEARS AS THE~~

ORDINATE TITLE. (LIMIT OF 40 CHARACTERS)

4 NG, SPECIFIES THE NUMBER OF CHARACTERS IN THE

GTITLE(1-7) GENERAL TITLE. GTITLE APPEARS ON EACH OUTPUT

~~GRAPH. (MAXIMUM OF 70 CHARACTERS)~~

CARD 5 ON SELECTS OTHER VARIABLES TO BE PLOTTED

~~EACH WITH NVAR, YTITLE AS FOR CARD 3.~~

NOTE. REFER TO THE USER'S MANUAL OR THE DRIVES LISTING FOR
~~FURTHER DETAILS ON THE SETUP FOR THESE LAST PLOT CARDS.~~
 ALL FILE DEFINITIONS ARE GIVEN IN THE DRIVES LISTING.

Figure 8. Card Input Format for REGEN

2.5 Operational System

The rocket trajectory operating system has been modified to reduce the need for complete binary programs of DRIVEA, DRIVEB, DRIVEC, and REGEN. The SCOPE-EDITLIB facility is used to provide one binary library for all subroutines. The main program with the LIBRARY control card then causes unsatisfied externals to be loaded. Program maintenance thus requires only one file of source routines in UPDATE form.

3. PROGRAM GUIDE

In this section the organization and program flow of the four separate programs are described. The subroutines that are called are generally modular and self-explanatory. Most of these routines are described in reference 1; revisions and new routines are described below and in the appendices.

3.1 DRIVEA

This is the raw radar data editing-preprocessing program. The original system used a separate statistical preprocessor DRIVE2 to clean up data values for the next stage in DRIVEB, and prepare initial plots of the trajectory. The estimator-filter proved to be an excessive user of CP time and memory. Further, the predictive filter tended to ring, giving poor position and velocity estimates. All editor and preprocessor functions have been combined in DRIVEA. A five-point running average of accepted data serves as the velocity estimator. A complete report on the editing and estimating is printed. This program produces results that are consistently superior to the original DRIVE1 and DRIVE2 while requiring less memory and about 40 percent of the computer time. Additional features have also been added to facilitate processing tapes with unreadable data segments by the use of spanning polynomials. Special buffering and mass-storage to tape file conversions which were carried out with CPRS are unnecessary and have been eliminated.

Pre-edit Scan

Data on cards in the format described in Section 2.1 are read and printed. Pages III-3,4 of Vol. I of Reference 1 are reproduced in Figure 1, and describe the raw input TAPE1(=RAWDAT) header and data records. NSKIP 6-sample records are skipped and the (5-NOK)th and (6-NOK)th samples, i.e., for default NOK=0, the 5th and 6th samples, are taken as the base starting points for initial position and rate. A complete scan of the input tape is first made to determine all quadrants that the azimuth covers. Logic is implemented to provide numerical continuity if the azimuth goes through zero, i.e., 360°

jump. In the unlikely event that the azimuth crosses both 0° and 180° , the present logic is inadequate and this is flagged as requiring a special fix.

Editing and Spanning

Following this pre-edit scan the tape is repositioned and the main editor pass is carried out to create a clean file, free of gross or unreasonable sample values. The operation of the editor is described in detail in Appendix A. The cleaned file and associated flags for corrections that may have been made are saved on temporary sequential file TAPE11. At this point a special pass is to be made to determine smooth spanning cubic fits over individually selectable ranges. These curves are introduced in the appropriate segments during the final preprocessing phase. (See Appendix A.3 for details)

Output and Geocentric Estimates

Creation of the edited output file TAPE10 is carried out over the range TLO-THI. Header and data format are described in Figure 3. Raw geocentric velocity and position are estimated at the same time, are written on temporary files TAPE8 and TAPE9 respectively, and are printed out along with the edited range, azimuth, elevation values. Subroutine FINDER calculates the geocentric position corresponding to the radar data, and also obtains local geodetic coordinates. Altitude is used to determine refraction corrections, if required. A simple velocity difference clipper and 5-point velocity smoother is used to estimate the geocentric velocity vector. This results in an offset of 2 samples which is not serious but is corrected for by a hold buffer. The geocentric state vector is required by DRIVEB as the starting estimate for filtering from any time after launch. The print-out for each sample includes time, edited range, azimuth, elevation with correction flags if applicable, and from TLO to THI, altitude and the estimated geocentric position and velocity vectors. Finally, subroutine ARTIST provides a plot of these variables versus time, using Tapes 8, 9 and 10.

3.2 DRIVEB

This is the main filtering and trajectory generation program. Data on cards in the format described in Section 2.2 are read except for the plot

control cards which are read after completion of processing and filtering. The header of the TAPE10 edited radar data provides creation data for this file; this overrides card input data except when purely integrating (KEYOPT=0). The maximum capabilities of DRIVEB are exercised for the launch optimizing, filtering, and terminal reproducing run.

Time - Tape Registration

A one-to-one correspondence is first established between sample times on TAPE10 and internal TAPE7 random access record numbers (MT). Requested time ranges for filtering, integrating, etc. are re-established in terms of the actual sample times. The input tape is positioned after reading the sample for the initial or the first computation time, whichever is greater, and is read again as the processing is incremented to successive sample times.

Derivation of TAPE7 Internal Data Base

Parameters to complete a TAPE7 working record for any sample time are described in Figure 9. The derivation of these parameters which comprise the total information base is now discussed. The signal-to-noise ratio SN is obtained from RADAR. If pure integrating is being employed, SN is set to -100. If pure reproducing is used, SN is set to +100. These special values of SN control multi-radar combination modes in DRIVEC.

Statistical Filtering in the Geocentric Domain

The geocentric state position-velocity vector $P2(1-6)$ is the Kalman weighted resultant of the integrated dynamics of RK and the observed raw geocentric state vector $DVG(1-6)$ on the forward filtering pass. The observed position $DVG(1-3)$ is calculated in EULER from the raw radar range, azimuth, elevation, $DVR(1-3)$. Co-azimuth was used earlier (Ref. 1), but has been eliminated. The observed raw velocity $DVG(4-6)$ is estimated in EULER as the difference between position samples, limited to a change of 0.01 km/sec. from the existing filtered estimate. The raw velocity $DVG(4-6)$ tends to be tolerance limited, but the random access noise is smoothed very satisfactorily by the subsequent

LOGICAL RECORD STRUCTURE TAPE7 (USED INTERNALLY BY PROGRAM)

WORD(S)	SYMBOL	DESCRIPTION	
1	GMT	UNIVERSAL TIME	HOURS
2	TIME	TIME AFTER LAUNCH	SECCNDS
3	SN	SIGNAL TO NOISE RATIO	DB
4	RAG	RIGHT ASCENSION OF GREENWICH	RAD
5-7	P2(1-3)	FILT. GEOCENTRIC POSITION VECTOR	KM
8-10	P2(4-6)	FILT. GEOCENTRIC VELOCITY VECTOR	KM/SEC
11-13	P2(7-9)	FILT. GEOCENTRIC ACCEL. VECTOR	KM/SEC**2
14-22	PVL(1-9)	FILT. LAUNCHER REF. POS-VEL-ACCEL	KM, KM/S, KM/S2
23-25	OVL(1-3)	FILT. LAUNCHER REF. RANGE, AZ, EL	KM, RAD
26-28	OVL(4-6)	FILT. LAUNCHER REF. R, A, E RATES	KM/S, RAD/S
29-64	XSI(1-35)	COMPLETE ERROR COVARIANCE MATRIX	
65-70	OVR(1-6)	FILT. RADAR RANGE, AZ, EL & RATES	KM, RAD
71-73	PVR(1-3)	FILT. RADAR REF. POSITION VECTOR	KM
74-75	GFV(1-3)	GEODETIC ALT., LONG(W), LAT.	KM, DEG
77	VR	MAG. CF LOCAL VELOCITY VECTOR	KM/SEC
78-79	AZR, ELR	AZ, EL OF LOCAL VELOCITY VECTOR	RAD
80	CD	DRAG COEFFICIENT	
81	DEN	ATMOSPHERIC DENSITY	KG/KM**3
82-83	DRAG, TH	DRAG, THRUST FORCES	KGKM/S2
84-85	DVR(1-3)	RAW RADAR DATA RANGE, AZ, EL	KM, RAD
87-89	DVG(1-3)	GEOCENTRIC RAW RADAR DATA VECTOR	KM
90-92	DVG(4-6)	GEOCENTRIC RAW ESTIMATED RADAR VELOCITY VECTOR	KM/SEC
93-123	XSIP(1-35)	COMPLETE ERROR COVARIANCE MATRIX BASED ON 1 LESS POINT	

Figure 9. Description of Internal Working Records
TAPE7 - used in DRIVEB and DRIVEC.

filtering. On the backward filtering pass, maximum likelihood or interval estimation weighting of the backward integrated dynamics and the original forward estimate completes the computation of P2 in DRIVEB. The error covariance matrices XSI and XSIP are weighted, as described in Appendices B and F, and calculated in KAL during the forward pass. XSI is recomputed in DRIVEB during the backward pass. Geocentric acceleration P2(7-9) is estimated at any time by a call to FOR and WPROD using a nominal time increment of 1 second.

Radar, Launcher and Other Parameters

Filtered radar range, azimuth, elevation and rates OVR(1-6) are derived from P2(1-6) by TRACOR. The radar referenced position vector PVR(1-3) is not required, but is calculated later by GENER8 for report and TAPE4 purposes. Filtered launcher range, azimuth, elevation and rates OVL(1-6), as well as launcher referenced position, velocity and acceleration vectors PVL(1-9) are obtained from P2(1-9) by a call to ROTATE, a routine which has been extended to provide these general transformations. Local velocity and ballistic parameters VR, AZR, ELR, CD, DEN, DRAG, TH were obtained when FOR was called. The TAPE7 working record is designed to match the TAPE4 parameter list through word 89, i.e., DVG(3). The completion of parameter calculations for TAPE4 is done in GENER8.

Processing in Different Time Zones

TAPE7 parameters for record number MT1, where the filtering or integration is started, are calculated based on initial conditions. Estimates for successive times are based on a combination of the integration estimate using RK (see Appendix D) and the raw radar estimate, as discussed above and in Appendix B. Filtering is performed from MT1 to MT2, forward integration from MT2 to MT3, and reproducing of the raw radar estimate from MT3 to MT4, or until the altitude goes negative. Then, interval or maximum likelihood estimation filtering is performed backwards to MT1, with a final backward integration if requested. Figure 5 shows the processing sequence and operating modes available with the various KEYOPT options. In all reproducing and single filtering segments, the backward filtering VFILT

smooths the velocity estimate, but the position values from the forward pass are retained. Backward filtering of integrated results from MT2 to MT3 is performed if subsequent reproducing is required. Except for VFILT, all backward filtering stages are the maximum likelihood kind.

Launch Optimizing and Filtering Mode

The launch optimizing mode (KEYOPT=1) achieves the most satisfactory trajectory estimates in the usual case when good radar data is available only some time after launch. An iterative run is first made to optimize on conditions, viz, thrust ratio to nominal, delay in onset of thrust from nominal launch time, launch azimuth and elevation. The procedure is implemented in routine LEASQ which searches for the best position and velocity match to radar data near time TC (see Appendix E). In most applications radar data is erratic from launch until after burn-out, since rapid slewing and large accelerations are encountered. Therefore TC is frequently set satisfactorily at 10 to 20 secs after thrust burn-out. Radar data before time TC is ignored, using only the forward integration results. On the backward pass the filtering and integration results are uniformly refiltered.

Output TAPE4

DRIVEB results are periodically printed out in abbreviated or extended form, and include a few extra frames around each mode transition time. Following completion of the above processing under any of the KEYOPT options, TAPE7 results are converted to report and storage form on TAPE4 by GENER8. WRITER provides the printed user report and plots. The TAPE4 header and data contain sufficient information to create such reports and plots at some time in the future, using program REGEN and subroutine WRITER, with a minimum of redundant card data input.

3.3 DRIVEC

The multi-radar trajectory program DRIVEC was developed as a revision of program DRIVE4.¹ The card input requirements are greatly reduced, using instead the header information on TAPE4 generated in program DRIVEB (Figure 6).

Scope of Program

The primary purpose for the multi-radar program is to combine the results of two separate radar observations of a given rocket flight to give a composite trajectory. The mathematical and logical procedures that accomplish the combining operation on to TAPE7 internal working records are discussed in Appendix C. The methods used to accomplish this data combination are linear; therefore, an arbitrary number of different sets of radar data pertaining to the same flight may be combined by repeated execution of the program. The graphical, printed report and file outputs of this program are identical in all respects to the outputs of Program DRIVEB.

Program Functions

Specific functions of Program DRIVEC for processing two TAPE4 format input files, TAPE10 and TAPE11, include the following:

- Determine the type of data region being read for each file
- Use the region types to select proper mode for combination equations
- Evaluate combination equations, depending on the particular mode, to produce the composite observation vector and covariance matrix
- Complete the calculations for the remainder of each output file record using composite values
- Generate a printed summary of the combination process, i.e., an event log for the resulting output file
- Produce an output TAPE4 file containing the composite trajectory
- Generate printed and graphical reports in the same format as DRIVEB.

For the purpose of calculating ballistic and thrust parameters as well as residuals, TAPE10 is taken as the reference trajectory. Thrust ratio, delay, etc. are obtained from the header and used in a call to FOR. Raw components of the radar data are also obtained from TAPE10 for use when TAPE4 is completed in GENER8. Note that ISKIP and IOPT refer to the increment TSKIP at which TAPE10 was written rather than the original integration time step DT.

3.4 REGEN

REGEN is a free-standing report generator program which uses a previously created TAPE4 and provides identical reports and plots to those obtained after a DRIVEB or DRIVEC run. The card input requirements are greatly reduced from the original program, using instead the header information on TAPE4, after which WRITER is directly called. As in DRIVEB, ISKIP refers to the integration or basic time step DT. The procedure recreates conditions after a DRIVEB or DRIVEC run has called GENER8; otherwise REGEN is simply a subset of those two programs.

Appendix A

The Editor

The DRIVEA editor processes raw radar data, which may contain a variety of errors, and generates a clean file with reasonable range, azimuth and elevation readings for each sample time over the span of the original data. Errors which are handled include:

- a) Missing time points
- b) Data with grossly improper or extraneous time points
- c) Data with marginally improper time points
- d) Grossly improper data
- e) Marginally improper data (based on wide tolerances set by user)

In addition, the user may selectively cause spans of range, azimuth, or elevation data to be replaced by a smooth interpolation.

A.1 Operation of the Editor

Starting with a pair of successive observations that the user specifies as acceptable for time, range, azimuth and elevation values, the raw radar data is checked for proper time values and reasonable rates of change. If an observation is found to be improper, an entry is started in an edit buffer with the intent of replacing this data with interpolated values when acceptable data are subsequently found. The edited data for as many time points as were adjusted are then written to the output file, the edit buffer is cleared, and processing continues on new radar data with the latest values and rates of change providing the acceptance criteria.

A typical sampling rate is 0.1 sec. Hence, if the next sample time is within 0.05 sec. it is considered only marginally improper, and is accepted with the time corrected to the 0.1 sec. increment. A time value lower than this 0.05 sec. tolerance or unacceptably high (say, 6.75 secs. or more) is considered extraneous, and the complete sample is discarded as suspect. If

the time value for the next sample is moderately high, observations are probably missing, and an interpolation is carried out on all the values provided the change rates are proper and provided the next raw data sample does not reveal the time jump to have been due to an improper time reading rather than the results of missing data.

As long as the time readings are acceptable the data will not be discarded; instead the range, azimuth and elevation are individually checked against selected tolerances and adjusted if necessary. If the observation differs from the predicted value by more than the tolerance, the data will be replaced by interpolation to subsequent acceptable data. If the observation is marginally improper as determined by a tolerance factor QTOL=0.25, the data is accepted after making a compromise between the value and the prediction. If seven or more observations have been rejected, QTOL is increased to 0.5 to avoid missing a new trend. After modification, the value is treated as an accepted observation. The modification formula is simply

$$\text{Modified Obs} = (1 - \text{QTOL}) * \text{Prediction} + \text{QTOL} * \text{Actual Obs.}$$

This editing process for each variable is accompanied by count indicators which serve two purposes: (1) if non-zero, it gives the count from the previous accepted sample for interpolation when a good sample is again obtained; and (2) the print-out of the final output file uses count indicator status to flag whether a value was added, interpolated, modified, or unaltered. (see Figure A)

A.2 Implementation - Example

The implementation of the editor is described below with the help of an example for radar data containing a variety of errors.

The edit buffer is maintained for radar data and for error status indication. A zero indicates that no change was made in the data, while a 101 denotes a minor modification. If the data are all accepted (row only has 0 or 101) at any sample time, it shows that all interpolations for any unacceptable or missing data can be completed for previous observations. Then the buffers may

Raw Radar Data			Error Status Counters			Edited Radar Data					
Time	Range	Az	EI	Time	Range	Az	EI	Time	Range	Az	EI
7.1	5.71	41.9	60.3	0	0	0	0	7.1	5.71	41.9	60.3
7.2	5.78	42.0	60.3	0	0	0	0	7.2	5.78	42.0	60.4
E —8.3	5.95	41.8	60.8	1	1	1	1	7.3A	5.89A	41.85A	60.7A
7.4	6.00	41.7	61.0	0	0	0	0	7.4	6.00	41.7	61.0
D —27.8	6.10	41.8	61.3	1	1	1	1	7.5A	6.07A	41.7A	61.2A
7.6	6.14	31.7	61.4	0	0	2	0	7.6	6.14	41.7I	61.14
E —Samples Missing				1	1	3	1	7.7A	6.26A	41.7A	61.75A
7.9	6.50	41.7	73.7	2	2	4	2	7.8A	6.38A	41.7A	62.1A
C —7.92	6.51	41.4	62.0	0	0	0	3	7.9	6.50	41.7	62.45I
8.0	6.67	41.6	64.3	0	0	0	101	8.0	6.67	41.6	62.8M
8.1	6.76	41.3	62.2	0	0	0	0	8.1	6.76	41.3	62.2
A —8.2	8.51	41.6	62.5	0	101	0	0	8.2	7.29M	41.6	62.5
B —8.3	8.51	41.7	71.4	0	101	0	1	8.3	7.70M	41.7	62.6
8.42	7.13	41.4	71.6	0	0	0	2	8.4	7.13	41.4	62.7
8.5	7.32	43.0	62.8	0	0	101	0	8.5	7.32	41.8M	62.8
8.6	7.47	41.3	63.0	0	0	0	0				

Sample added - A
 Value interpolated - I
 Value modified - M

Figure A. Example of Editor Processing

be written to the output file and cleared for continued editing. In the example in Figure A, the longest retention required in the buffer is seen to be from 7.4 to 7.9 secs. or 6 samples. The present program allows up to 100 samples or 10 secs. of editing which is limited only by the size of the buffers.

The observations are seen to be clean at 7.1 and 7.2 secs. Change rates are re-estimated for every observation with a 10 sample time constant. The predicted values are compared with the raw values using tolerance limits, usually 5 km and 5°, for complete rejection. A lower tolerance limit (25%) is used to fully accept the data, or if the error exceeds this amount to make compromise modifications as discussed earlier. Illustrations of typical editing actions A thru H follow: (A) In the example this type of modification is seen to have been made in the range, azimuth and elevation where the status counters are 101. (B) The sample at 8.4? secs. is judged to actually be at 8.4 secs. for consistent time incrementing. (C) The sample at 7.92 secs. however is less than 0.05 secs. from the earlier sample and is discarded as extraneous. (D) The sample after 7.4 secs. has an excessive time jump and is completely discarded even though the associated observations are reasonable. These values will be interpolated. (E) The jump in time after 7.2 secs. and 7.6 secs. also call for interpolation. This is done in both cases, but in the former case the next sample at 7.4 secs. shows that the interpolation between 7.2 and 8.3 secs. was a false alarm. This is replaced by a simple interpolation for the 7.3 sec. sample only. (F) Grossly improper data such as the elevation values at 8.3 and 8.4 secs. must be replaced by interpolation. The status counters are correspondingly incremented from 1 thru 99 (the buffer size limit) until a good sample is encountered (0 or 101). (G) & (H) Since this interpolation is identical to the case for missing data, connected instances of bad data and missing data cause the appropriate counters to continue incrementing.

The modification procedure bears some further discussion. A slow adjustment band is necessary between the limits of complete rejection and complete acceptance so as to be influenced by possible new trends without being permanently and incorrectly thrown off by a few offset samples. In fact, if 7 samples in succession are completely rejected, the modification tolerance

factor is increased from 0.25 to 0.5 so as to react faster to diverging trends. The example shows the rapid adjustment of the range at 8.2 and 8.3 secs. to allow for the 8.51 km readings. But the subsequent value of 7.13 km at 8.4 secs. is still fully acceptable. The wide variation has been greatly reduced, but its smoothed elimination is possible only with the subsequent filtering program DRIVEB. Some improvements may be made however by resetting the tolerance limits of 5 km and 5° to better reflect particularly clean or noisy radar data.

A.3 Spanning over Data Gaps

A special procedure is available in DRIVEA to selectively override obvious biases and extended deviations in range, azimuth or elevation readings by specifying the time spans over which it is desired to bridge this data by smooth spanning cubic polynomials. This function is implemented following the editor, and the print-out of the edited data includes indication of these spanning replacements in addition to the data added, interpolated, or modified. Each cubic satisfies readings at the two times specified plus two readings 5 secs. (and no less than 2 secs.) outside the span of discarded readings.

The user must specify the initial (TI) and final (TF) times over which one of the variables (R, A, or E) is to be spanned. The program then solves for the cubic which matches values of this variable at TI-5, TI, TF, TF+5 seconds. Note that the program requires data to be available for at least 2 secs. before and after the interval to be spanned. Otherwise TI is increased and/or TF is decreased by DRIVEA to satisfy these minimum requirements. Finally the user must verify that the variable values at the four specific times are proper for generating a satisfactory spanning function.

Appendix B

Qualitative Operation of the Forward and Backward Filtering Equations

Notation:

- $P(k/n)$ forward filtering position-velocity vector estimate for time k based on observations to time n .
- $P(k/N)$ backward filtering final estimate for position-velocity vector.
- $D(k)$ observed data vector for time k .
- $\psi(k/n)$ forward filtering error covariance matrix associated with corresponding estimate for P .
- $\psi(k/N)$ backward filtering final error covariance matrix for $P(k/N)$.
- $\phi(k+1,k)$ state transition matrix from k to $k+1$. Thus, when purely integrating, $P(k+1) = \phi(k+1,k) * P(k)$. Note: for backward integration, $\phi(k+1) = \phi^{-1}(k+1,k)$ is obtained by using $-DT$ in the Runge-Kutta calculations.
- $W(k)$ covariance matrix of the estimated radar measurement white noise errors at time k .
- $Q(k)$ covariance matrix of the unknown factors in the physical model at time k .
- $G(k)$ Kalman filter gain for forward filtering pass.
- $M(k)$ weight matrix for backward filtering pass.

The various rotation and state transition matrices are necessary to connect the trajectory in space and time, but the noise and optimum filtering estimation is described below ignoring these transformations. The operation is also best understood in terms of a single rather than a multi-variable process.

Operation:

During the forward filtering pass the error covariance matrices $\psi(k+1,k)$ and $\psi(k+1/k+1)$ are calculated and stored for time $k+1$ from time k .

$$\psi(k+1/k) = \psi(k/k) + Q(k+1) \quad (B-1)$$

i.e., if use is not made of observations to update the estimated position-velocity vector P, this estimate will be poorer with time. Note that except at launch, Q is a few orders of magnitude less than ψ .

$$G(k+1) = \frac{\psi(k+1/k)}{[\psi(k+1/k) + W(k+1)]} \quad (B-2)$$

This gain matrix determines how much confidence to place in the observed data. If the radar measurements are assumed to be noisy relative to the current estimation process, reliance on each data point should be reduced. If physical events are taking place which are unaccounted for in the model, or if the radar data is of high quality, reliance on the model estimation should be reduced.

$$P(k+1/k+1) = [1 - G(k+1)] P(k) + G(k+1) D(k+1) \quad (B-3)$$

$$\psi(k+1/k+1) = [1 - G(k+1)] \psi(k+1/k) \quad (B-4)$$

i.e., by placing some reliance on the recent observation the estimated error in P is reduced.

Besides P(k+1), i.e. P(k+1/k+1), both $\psi(k+1/k)$ and $\psi(k+1/k+1)$ are stored on a file for later use on the backward filtering or smoothing pass.

Since forward estimates were made without knowledge of future observations, these estimates can reasonably be expected to be improved by a "hindsight" backward pass.

$$M(k) = \frac{\psi(k/k)}{\psi(k+1/k)} \quad (B-5)$$

This weight matrix will effectively range between 0 and 1 by virtue of

equation (B-1).

$$\text{Let } y(k+1/k) = \phi \cdot P(k/k)$$

Then

$$P(k/N) = P(k/k) + M(k)[P(k+1/N) - y(k+1/k)] \quad (B-6)$$

This backward filtering adjustment of P will be greater, i.e. more influenced by future values, when M approaches 1. This is normally the case in mid-trajectory when the error covariance matrix is not changing rapidly. However, close to launch the state vector of the rocket is known reliably and the ψ matrix builds up from zero. Hence on the backward pass M approaches 0 near launch and the trajectory smoothly assumes the original forward estimates P(k/k).

Finally,

$$\psi(k/N) = \psi(k/k) + M(k)[\psi(k+1/N) - \psi(k+1/k)] M(k)^T \quad (B-7)$$

which parallels the P(k/N) logic since if P(k/k) is greatly influenced by future values in obtaining P(k/N), $\psi(k/N)$ will be substantially lower than $\psi(k/k)$. Notice that $\psi(k/N)$ is naturally less than $\psi(k/k)$ and is equal only for k=N.

In practice, the error covariance matrix that is computed on the backward pass sometimes failed to remain positive definite. This problem was solved by limiting this matrix to 2% of the forward pass error covariance matrix whenever the computed value for a diagonal term becomes less than 1% of the forward estimate.

Appendix C

Qualitative Operation of the Multi-Radar Combining Equations

Notation:

- P(k) Position-velocity final estimate at time point k.
 $\psi(k)$ Final error covariance matrix associated with P(k)
 at time point k

Subscripts 1 and 2 will identify the two input trajectory estimates.

The composite best estimate state vector is given by

$$P(k) = [\psi_1^{-1}(k) P_1(k) + \psi_2^{-1}(k) P_2(k)][\psi_1^{-1}(k) + \psi_2^{-1}(k)]^{-1} \quad (C-1)$$

The associated joint error covariance matrix is given by

$$\psi(k) = [\psi_1^{-1}(k) + \psi_2^{-1}(k)]^{-1} \quad (C-2)$$

Section VI of Vol. 1 of Reference 1 discusses the basis of the original DRIVE4 multi-radar estimating program. In addition to the primary equations, C-1 and C-2, prior weighting and scaling was available to de-emphasize integration results further away in time from filtered radar data. The choice of weighting was subjective, and could be avoided if the error covariance matrices reflected the confidence in the estimate of the state vectors more exactly. Improvements in generating error covariance matrices have been achieved in DRIVEB, and are further advanced by the use of backward filtering. The desired objective of using the above equations exclusively is implemented, with the exception of the case when a segment combines radar filtered and integrated data; in this case the radar filtered results are copied and the integrated results are ignored.

Subroutine COMBIN is called during each time step to generate the composite covariance matrix, state and acceleration vectors under the control flag MODE. The reduced applicable MODE values are described below.

- MODE=2 Trajectory 1 covariance matrix, state and acceleration vector are copied to obtain the composite estimate.

- MODE=4 The composite trajectory is calculated in EVAL using the parameters from both trajectories according to equations C-1 and C-2. The composite acceleration estimate is obtained by using the same relative weights as applied to position.

- MODE=11 Trajectory 2 covariance matrix, state and acceleration vector are copied to obtain the composite matrix.

Table C summarizes the mode selection algorithm in effect, as described above.

The condensed procedure described above has not been extensively tested, and the potential exists for jumps in the composite state vector when switching between trajectories 1 and 2. The original code for weighting and scaling in COMBIN using other MODE values has therefore been retained, though it is presently non-functional. As noted before, the desirable direction is to have the separate error covariance matrices better reflect the results of filtering and integrating.

Table C Combination Mode Table

STATE OF TRAJECTORY FILE 1

STATE OF TRAJECTORY FILE 2

	NULL	RADAR	INTEGRATING
NULL	NULL	2	2
RADAR	11	4	11
INTEGRATING	11	2	4

Appendix D

Modeling of the Rocket Dynamics

The modeling of the rocket dynamics was extended considerably from the situation when only after burn-out free flight trajectories were estimated in DRIVE3. A straightforward representation was developed for the thrust, mass loss, and drag characteristics of most of the rockets encountered. Concurrently, the force equations were developed and the integration procedure was extended to include the thrusting stages from take-off to burn-out.

Assuming thrusting in the axial direction of the rocket only, the thrust components in the geocentric inertial x, y, z system are given by

$$\begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} = (C) (B) \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix}$$

where the axial thrust T is known, B rotates this thrust to the local ENR (east-north-vertical) system using the local azimuth Az_r and elevation El_r of the rocket center-line; C rotates the ENR thrust components of the geocentric x, y, z values by the right ascension α and declination δ of the rocket (see Appendix A, Ref. 3).

Adjusting elevation for the angle of attack λ , the azimuth and elevation of the rocket is determined by the rocket position \bar{r} and relative velocity vector \bar{v}_r .

$$Az_r, El_r = f(\bar{r}, \bar{v}_r)$$

\bar{v}_r is obtained from geocentric quantities and the earth rotation rate w

$$\bar{v}_r = \bar{v} - \bar{w} \times \bar{r}$$

For the low relative velocities near lift-off, the launch azimuth and elevation values are maintained constant.

D.1 Representation of the Rocket Characteristics

A piecewise linear approximation is made for each significantly different thrusting phase. Thrust and mass are expressed as straight line functions of time which apply until the next time phase is reached, when new straight line functions apply. For the drag coefficient, which is a function of Mach number, a multiplier is applied on a universal C_D curve for each time segment. This universal curve is given by:

$$C_D = 1.0 + V_{\text{mach}}^4 \quad (\text{for } V_{\text{mach}} \leq 1)$$

$$C_D = \frac{4.0}{V_{\text{mach}} + 1.0} \quad (\text{for } V_{\text{mach}} > 1)$$

For a particular configuration the drag coefficient is usually 10-25% less during the power-on stage than when power is off.

V_{mach} is given by $\sqrt{V_r} / V_s$ where V_s is the speed of sound. For the U.S. standard atmosphere, 1962, the speed of sound is given as a function of altitude (Ref. 6). This relationship is reproduced in Figure D1.

The reference cross-sectional area is included per thrust phase. The thrusting stage (1, 2, etc.) is also given. An example of linearized characteristics for the BLACK BRANT IV-B (MOD 1) without payload is given in Figure D2. This rocket is represented in six phases as follows:

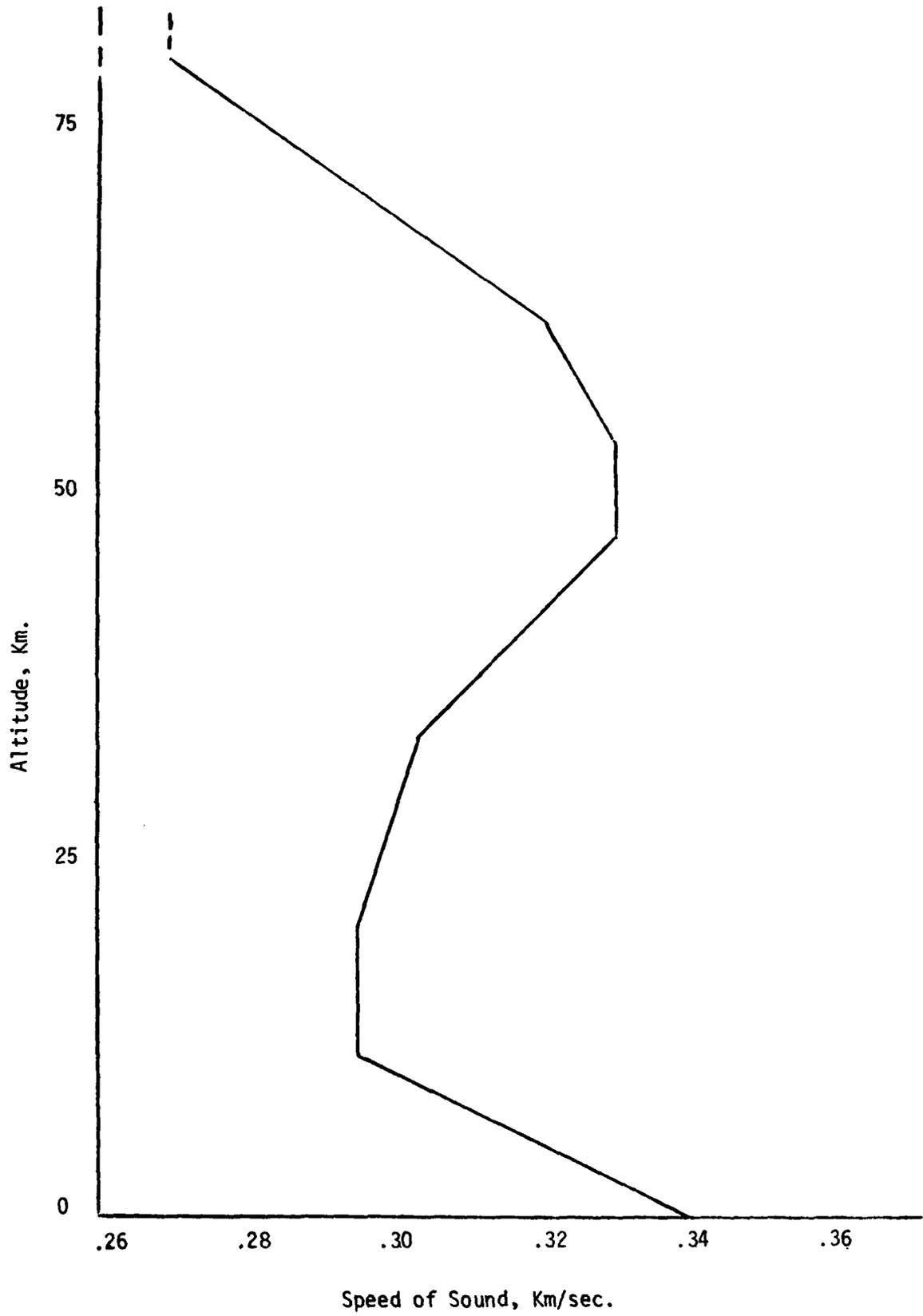


FIGURE D1: SPEED OF SOUND AS A FUNCTION OF ALTITUDE.

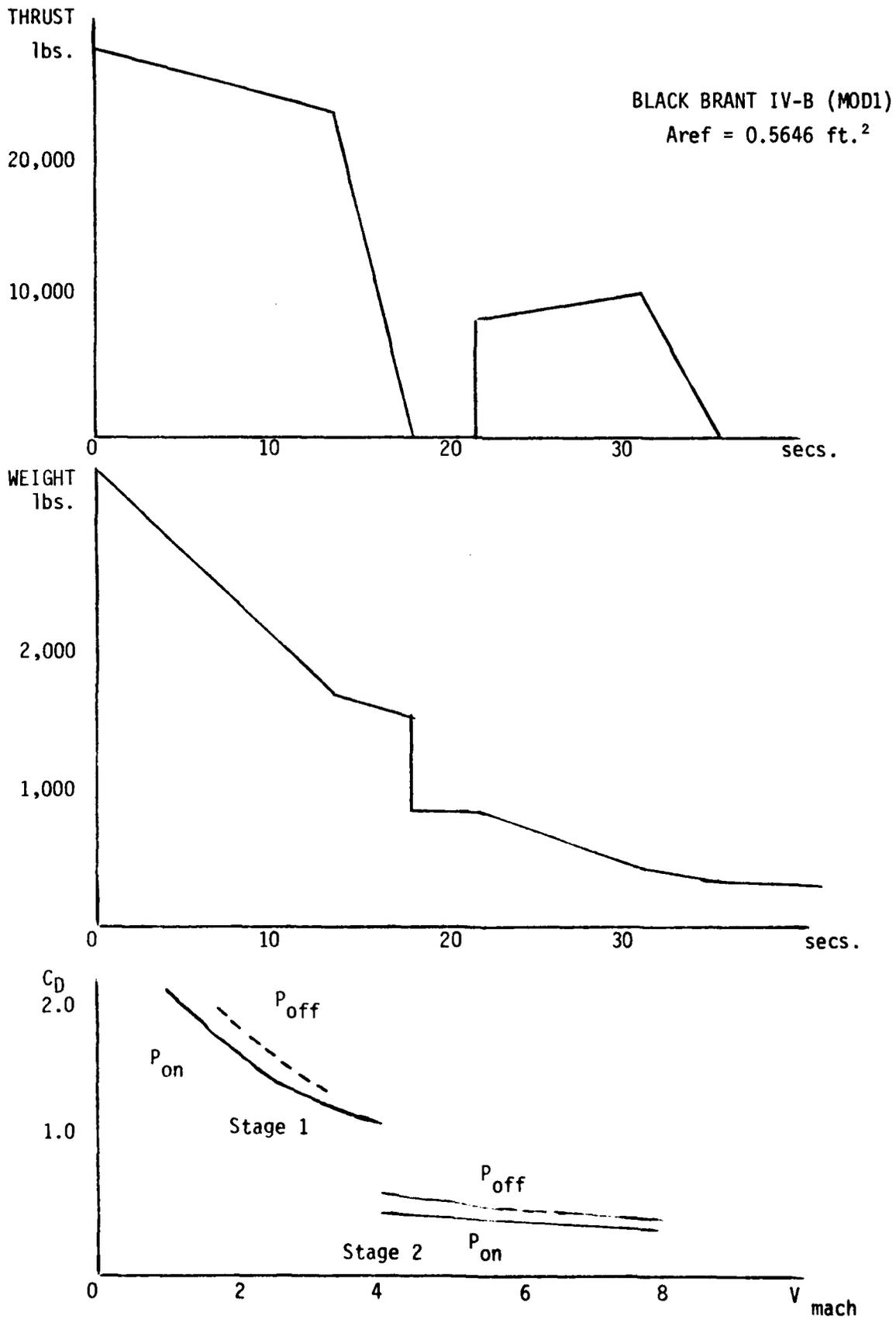


FIGURE D2: PIECEWISE LINEAR ROCKET CHARACTERISTICS.

Time Secs.	Thrust Lbs.	Δ Thrust per sec.	Weight Lbs.	- Δ Wt. per sec.	Area Ft. ²	C_D Mult.	Stage No.
0	27,500	-333	3294	121	0.56	1.20	1
13.5	23,000	-5111	1660	35	0.56	1.20	1
18.0	0	0	850	0	0.56	.85	2
21.5	8,400	200	850	45	0.56	.70	2
31.0	10,300	-229	420	18	0.56	.70	2
35.5	0	0	340	0	0.56	.85	2

35.5 secs. from lift-off the rocket is in free flight.

The representation of the thrust history needs to cover pre-launch conditions since it turns out in many instances that the actual thrusting and lift-off may not occur until a few seconds after launch time. This requirement is conveniently implemented by providing a pre-launch thrust equal to the total rocket and payload weight, i.e. a force sufficient to maintain the rocket on the pad.

D.2 Integration Techniques

Section 4.0 (Ref. 3) details the adaptation of integration techniques to include powered flight. Only an outline of the approach will be presented here.

The Runge Kutta method is used with the equations of motion for forward or backward time integration.

$$\frac{d\bar{p}}{dt} = A[\bar{p}] \cdot \bar{p} + \bar{B}(\bar{p})$$

where \bar{p} is the position vector

$\bar{B}(\bar{p})$ is the thrust vector

and $A(\bar{p})$ represents the free flight dynamics

Then,

$$\bar{p}_{n+1} = \bar{p}_n + 1/6[\bar{K}_1 + 2\bar{K}_2 + 2\bar{K}_3 + \bar{K}_4]$$

where

$$\bar{K}_1 = \Delta T A[\bar{p}_n] \cdot \bar{p}_n + \Delta T \bar{B}(\bar{p}_n)$$

$$\bar{K}_2 = \Delta T A[\bar{p}_n + 1/2 \bar{K}_1] \cdot (\bar{p}_n + 1/2 \bar{K}_1) + \Delta T \bar{B}(\bar{p}_n + 1/2 \bar{K}_1),$$

$$\bar{K}_3 = \Delta T A[\bar{p}_n + 1/2 \bar{K}_2] \cdot (\bar{p}_n + 1/2 \bar{K}_2) + \Delta T \bar{B}(\bar{p}_n + 1/2 \bar{K}_2),$$

$$\bar{K}_4 = \Delta T A[\bar{p}_n + \bar{K}_3] \cdot (\bar{p}_n + \bar{K}_3) + \Delta T \bar{B}(\bar{p}_n + \bar{K}_3),$$

and ΔT is the sampling interval in seconds. \bar{K}_1 is evaluated at time t_n , \bar{K}_2 and \bar{K}_3 at $t_n + \Delta T/2$, and \bar{K}_4 at $t_n + \Delta T$ or t_{n+1} .

In the above recurrence relationships if the thrust matrix \bar{B} is represented by dividing and multiplying by the position vector \bar{p} , the resulting normalized thrust matrix may be added to $A(\bar{p})$. A procedure identical to the free-flight calculations is then carried out to give the desired state transition

matrix ϕ such that:

$$\bar{p}_{n+1} = \phi(n+1, n) \bar{p}_n$$

Under integration or filtering, good correspondence to expected rocket performance has been obtained unless the rocket malfunctioned or experienced large wind effects.

D.3 Integration Step Size

Some raw data tapes were received with samples at 1 second rather than the usual 0.1 second intervals. It was found that if the integration step size was allowed to increase to this data rate, there was a significant build-up of error in the fourth order Runge-Kutta integration procedure. Step size less than 0.1 second resulted in negligible change. Subroutine RK was consequently modified to loop a selectable number of times with a specified step size. The maximum step size is set at 0.1 second.

Appendix E

Least Squares Solution for an Overdetermined Set of Equations

The classical problem in trajectory determination when ample observations are available relative to the variable model parameters is to determine these unknowns so as to minimize the sum of all the squares of the separations between the observations and the computed trajectory at the corresponding times. These unknowns are usually the initial position and velocity vectors plus any adjustable model parameters such as drag coefficient and geopotential terms which, once fixed, uniquely determine the trajectory to within the accuracy of the model.

This optimization problem can be solved directly in one pass if the process is a system of linear equations. The theory is discussed below. However, the process is generally non-linear; the procedure then is to use the same theory using a linearized set of equations for the partials of the variables (unknowns) being optimized. Minimization now requires an iterative improvement of the variables, possibly with a recomputation of the partials at the observation times for each new pass.

E.1 Linear Equations

Let x be the set of unknowns to be estimated, and b the set of observations at different times. At each sample time the value of the observation may be estimated by the linear equation ax where a is a fixed set of coefficients corresponding to the x 's and the observation instant. The complete set of equations is thus represented by $Ax=b$ where each line of this equation applies to an observation, and there are many more observations (rows) than unknowns (columns).

Each observation may in general be a vector, as for instance in trajectory determination where the geocentric rectangular coordinates are usually taken as the equally weighted metrics. The solution follows the treatment of Lanczos (Ref. 5, p. 156). The square of the length of the residual

vector r in the equation

$$Ax - b = r \quad (E-1)$$

has to be minimized.

The adjoint matrix \tilde{A} is obtained by transposing the rows and columns of A . For example, if

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad \text{then,} \quad \tilde{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

The solution to the least squares problem is then elegantly given by the even determined system

$$\tilde{A} Ax = \tilde{A} b \quad (E-2)$$

which has just as many equations as unknowns. It should be noted that the matrix $(\tilde{A} A)$ is symmetric and that its eigenvalues are positive real.

E.2 Non-linear but Differentiable System

Rather than express results in the general notation of an arbitrary number of unknowns, the procedure is outlined below for three unknowns that are to be optimized. The technique will be seen to extend without restriction to any number of independent unknowns.

Assume reasonable original estimates $x_0 = (x_1, x_2, x_3)$ for the three unknowns. Four sets of equations are required, one for the system and residuals at this base setting, and one each with one of the three variables differentially adjusted. (This is the simplest form, but all three variables could be varied jointly as long as a mutually independent set of adjustments is achieved). Let these adjustments of x_1, x_2, x_3 one at a time be given by $\Delta x_1, \Delta x_2, \Delta x_3$.

The residual vector for n observations with the original setting is r_0 , and the new residuals following the adjustments are given by r_1, r_2, r_3 which may be determined either from analytic partials of the system model or from tests on the model with the adjustments $\Delta x_1, \Delta x_2, \Delta x_3$ applied one by one. Thus we have the vector

$$x = \begin{bmatrix} x_0 \\ x_0 + \Delta x_1 \\ x_0 + \Delta x_2 \\ x_0 + \Delta x_3 \end{bmatrix}$$

representing four trials and the associated residual matrix

$$R = \begin{bmatrix} r_{01} & r_{11} & r_{21} & r_{31} \\ - & - & - & - \\ - & - & - & - \\ r_{0n} & r_{1n} & r_{2n} & r_{3n} \end{bmatrix}$$

for n observations with each of the trials. Expressed in q units of x, where we wish to make the "observed" residuals (i.e., all of the b's of Eqn. B-1) zero, the problem becomes that of minimizing

$$Rq = Aq \tag{E-3}$$

Hence R becomes both the coefficients of the unknowns q and the Ax-b of equation (E-1). The procedure for minimizing the residuals in a least squares sense is then suggested by equation (E-2)

$$\tilde{R} R \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = 0 \tag{E-4}$$

where q_j is in units of x_j , and

$$q_0 + q_1 + q_2 + q_3 = 1 \quad (E-5)$$

since the optimization is to be an adjustment about x_0 .

The four equations of (E-4) constitute three differential relationships such that changes to x_1, x_2, x_3 in the proper combination $q_1 \Delta x_1, q_2 \Delta x_2, q_3 \Delta x_3$, will minimize the residual vector r_0 . The solution uses the property that the q_i are in the ratio of the row sums of the cofactors of $\tilde{R} R$.

In practice the solution may be implemented by obtaining $W = [\tilde{R} R]^{-1}$, the row sums w_0, w_1, w_2, w_3 , and their total $T = w_0 + w_1 + w_2 + w_3$.

Then $q_1 = w_1/T, q_2 = w_2/T, q_3 = w_3/T$.

An iterative procedure may be incorporated using the revised values for x_0 as the base point. The partials determined before for the same observation times remain applicable, but if x_0 underwent a large change recomputation of the partials may be necessary. It should be noted that monotonic minimization of the residual metric or even its convergence is not assured if the system is sufficiently non-linear and if the original estimates are too far off the actual. Criteria for stability are beyond the scope of this report.

E.3 Example

The process being tested is

$$X = 1.1A (T - .9B)^{0.85} + 0.9C$$

$$Y = A/2.1 (T - .9B)^{1.95} - 1.9C$$

where A,B,C are actually 2,1,1 and X,Y are the observations at any time T.

This process has been formulated by us as

$$X = A (T-B) + C$$

$$Y = A/2 (T-B)^2 - 2C$$

that is, the expressional form is approximately known and the associated optimum values for A,B,C are to be determined from observations on the process.

The procedure described in section E.2 is shown in operation on the following pages. A,B,C are originally estimated to be 1,0,0 and observations are carried out at times T=1,2,3 and 4. The residual vector (Observed-Computed) for X and Y at each sample time is thus obtained. The residual vectors are also obtained for the cases when A,B,C one by one are estimated to be 0.1 larger. This gives the residual R-matrix.

The square root of the sum of the squares of the residuals before each iteration is the measure of the closeness of the estimate. In this example the original estimate and sample times result in relatively wide variations of the residuals, but the system is seen to approach the true values of A,B,C after three iterations. The fourth iteration is essentially superfluous. The optimum obtained for A,B,C is 1.78, 0.84, 1.02. Further optimization is not possible unless the model is formulated more closely to the actual process.

LEAST SQUARES OPTIMIZATION ITERATION 1

SQRT(SUM (RO)2)
.576830E+01

R-MATRIX	(O-C)	TRIALS	
ORIG.	DA= .1	DB= .1	DC= .1
.210758E+00	.110758E+00	.310753E+00	.110753E+00
-.238931E+01	-.243931E+01	-.229431E+01	-.218931E+01
.128565E+01	.108565E+01	.138565E+01	.118565E+01
-.275310E+01	-.295310E+01	-.255810E+01	-.255310E+01
.203342E+01	.173342E+01	.213342E+01	.193342E+01
-.235295E+01	-.280295E+01	-.205795E+01	-.215295E+01
.265546E+01	.225546E+01	.275546E+01	.255546E+01
-.125100E+01	-.205100E+01	-.855998E+00	-.105100E+01

W-MATRIX	(RADJ)P		
.332733E+02	.340526E+02	.319397E+02	.309055E+02
.340526E+02	.360169E+02	.321265E+02	.314848E+02
.319397E+02	.321265E+02	.309363E+02	.297279E+02
.309055E+02	.314848E+02	.297279E+02	.287377E+02

1/W			
.320495E+04	-.837149E+03	-.148173E+04	-.996749E+03
-.837149E+03	.220866E+03	.392330E+03	.252471E+03
-.148173E+04	.392330E+03	.703230E+03	.436207E+03
-.996749E+03	.252471E+03	.436207E+03	.344130E+03

ROW SUMS:			
-.110681E+03	.285178E+02	.500377E+02	.360569E+02

	DA	DB	DC
	.724973E+00	.127205E+01	.916678E+00

NEW	A	B	C
	.172497E+01	.127205E+01	.916673E+00

LEAST SQUARES OPTIMIZATION ITERATION 2

SQRT(SUM (R0) 2)
 .335262E+01

R-MATRIX	(O-C)	TRIALS	
ORIG.	DA= .1	DB= .1	DC= .1
.763352E+00	.790557E+00	.935849E+00	.663352E+00
-.119790E+00	-.123491E+00	-.175342E+00	.882095E-01
.111327E+01	.104047E+01	.128577E+01	.101327E+01
.623211E+00	.596715E+00	.740156E+00	.823211E+00
.113606E+01	.963268E+00	.130856E+01	.103606E+01
.140517E+01	.125588E+01	.169461E+01	.160517E+01
.103314E+01	.760341E+00	.120563E+01	.933136E+00
.216396E+01	.179188E+01	.262590E+01	.236396E+01

W-MATRIX	(RADJ)R		
.112401E+02	.967061E+01	.134238E+02	.116500E+02
.967061E+01	.837293E+01	.115517E+02	.100193E+02
.134238E+02	.115517E+02	.160405E+02	.139273E+02
.116500E+02	.100193E+02	.139273E+02	.122599E+02

1/W			
.174349E+03	-.998558E+01	-.143003E+03	.493760E+01
-.998558E+01	.193347E+02	-.634042E+01	.890337E+00
-.143003E+03	-.634042E+01	.133111E+03	-.101441E+02
.493760E+01	.890337E+00	-.101441E+02	.618572E+01

ROW SUMS:			
.262978E+02	.389908E+01	-.263767E+02	.166956E+01

DA	DB	DC
.685277E-01	-.463580E+00	.328582E-01

NEW A	B	C
.179350E+01	.908466E+00	.949536E+00

LEAST SQUARES OPTIMIZATION ITERATION 3

SQRT(SUM (R0) 2)
 .625550E+00

R-MATRIX	(O-C)	TRIALS	
ORIG.	DA= .1	DB= .1	DC= .1
-.822941E-01	-.101448E+00	.970563E-01	-.182294E+00
-.231400E-01	-.249743E-01	.224415E-02	.176863E+00
.199096E+00	.799422E-01	.378446E+00	.990955E-01
-.127190E+00	-.198178E+00	.775438E-01	.728096E-01
.153362E+00	-.657912E-01	.332712E+00	.533622E-01
-.260812E+00	-.500953E+00	.123272E+00	-.608123E-01
-.180930E-01	-.337246E+00	.161257E+00	-.116093E+00
-.486126E+00	-.995420E+00	.773084E-01	-.286126E+00

W-MATRIX	(RADJ)F		
.391313E+00	.660615E+00	.358204E-01	.186652E+00
.660615E+00	.141646E+01	-.209997E+00	.359154E+00
.358204E-01	-.209997E+00	.316533E+00	-.505336E-02
.186652E+00	.359164E+00	-.505306E-02	.181991E+00

1/W			
.490695E+02	-.268635E+02	-.233423E+02	.204152E+01
-.268635E+02	.164273E+02	.138667E+02	-.448314E+01
-.233423E+02	.138667E+02	.149523E+02	-.301107E+01
.204152E+01	-.448314E+01	-.301107E+01	.121649E+02

ROW SUMS:			
.905149E+00	-.105261E+01	.246562E+01	.671225E+01

DA	DB	DC
-.116563E-01	.273035E-01	.743294E-01

NEW	A	B	C
	.178184E+01	.835769E+00	.102387E+01

LEAST SQUARES OPTIMIZATION ITERATION 4

SQRT(SUM (RO) 2)
.333392E+00

R-MATRIX	(O-C)	TRIALS	
ORIG.	DA= .1	DB= .1	DC= .1
-.105740E+00	-.122163E+00	.724441E-01	-.205740E+00
.134387E+00	.133039E+00	.154741E+00	.334387E+00
.187306E+00	.708826E-01	.365490E+00	.873057E-01
.870478E-01	.192762E-01	.285586E+00	.287043E+00
.153229E+00	-.631945E-01	.331413E+00	.532296E-01
.217935E-01	-.212401E+00	.398516E+00	.221794E+00
-.657027E-02	-.322993E+00	.171614E+00	-.106579E+00
-.123496E+00	-.624114E+00	.431411E+00	.765043E-01

W-MATRIX	(RADJ)F		
.111150E+00	.110636E+00	.111515E+00	.112274E+00
.110636E+00	.580969E+00	-.387120E+00	.175431E-01
.111515E+00	-.387120E+00	.728553E+00	.271470E+00
.112274E+00	.175431E-01	.271470E+00	.313398E+00

1/W			
.583953E+02	-.268821E+02	-.236074E+02	.103385E+01
-.268821E+02	.163443E+02	.141040E+02	-.350156E+01
-.236074E+02	.141040E+02	.142094E+02	-.464063E+01
.103385E+01	-.350156E+01	-.464063E+01	.703624E+01

ROW SUMS:			
.893968E+01	.646347E-01	.654569E-01	-.721034E-01

	DA	DB	DC
	.718349E-03	.727488E-03	-.801357E-03
NEW	A	B	C
	.178256E+01	.836497E+00	.102306E+01

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Appendix F

Formation of the Driving Noise or Uncertainty Matrix Q

Due to our inexact knowledge of the physical forces influencing the rocket trajectory, the state vector error covariance matrix must inevitably increase with time in the absence of additional radar observations for updating this vector. This phenomenon is mathematically represented by adding a diagonal matrix Q to the error covariance matrix at each step of the integration process.

In the original DRIVE5 implementation the Q element corresponding to the Ith velocity vector was given by $Q(4-6) = (ACC(I) \cdot QFACT \cdot DT)^2$ where ACC is the acceleration at the time, DT is the integration step size, and QFACT is a judiciously chosen factor usually between .01 and .1. The corresponding position vector element was given by $Q(1-3) = Q(4-6) \cdot DT^2/4$. This form makes the reasonable assumption that model uncertainties are greatest at times when the greatest accelerations are experienced. Operation has usually been satisfactory except at re-entry when model and data sometimes deviate.

The present DRIVEB in the normal integration-filtering mode replaces radar data near launch by an optimum trajectory which connects smoothly with subsequent radar data. This feature plus the need for greater emphasis on data at re-entry led to the formulation of the Ith velocity vector as $Q(4-6) = OVR \cdot |ACC(I)| \cdot (.01 \cdot QFACT \cdot DT)^2$ where OVR is the vehicle range at the time, and the absolute value of the acceleration is used. The .01 multiplier gives values comparable to the earlier DRIVE5 version for 20-30 km range with QFACT relatively unchanged. The position vector elements are as before $Q(1-3) = Q(4-6) \cdot DT^2/4$. This form has proved to be an improvement.

Appendix G

Significance of Radar Parameters in the Filtering Equations

Radar error estimates determine the measurement error covariance matrix $W(k)$ which in turn directly affects the forward pass Kalman filtering gain. A prime concern in our applications is to obtain the best transition to radar data as it settles down from launch fluctuations to smoother down range readings.

The critical choice of radar parameters for most rockets is therefore where the velocity is high and the range low, and where the data is first considered reliable. At 20 to 30 seconds from launch, calculations of variances for various radars have established the following factors to be predominant over other noise sources by two or more orders of magnitude:

Range velocity lag coefficient for range error

Base angular variance for azimuth and elevation error

For flights where the radar data is particularly good or poor, the above parameters may be appropriately adjusted from nominal to better represent the operating conditions and correspondingly the forward filtering gain. The estimated white noise matrix $W(k)$ is derived from the variance figures $VAR(RANGE, AZ, EL)$ adjusted for range and elevation of observation, and decomposed into the geocentric coordinate system.

Typical range error variance values for AN/FPQ-11 and FPQ-6 radars for a rocket range rate of 1.5 km/sec at 20 to 30 secs. from launch, and a range velocity lag coefficient of 0.2 kyds/kyd/sec. are approximated by

$$(0.2 \cdot 1.5)^2 \approx 0.1 \text{ km}^2$$

Corresponding azimuth and elevation error values for base angular variance ranging from 10.0 to 0.01 mil² result in variance figures from 10⁻⁵ to 10⁻⁸. At 30 km range and 60° elevation these are equivalent to variances of 0.005 km² and below. $[10^{-5} \cdot 30^2 \cdot \cos(60^\circ)]$.

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