FINAL REPORT TO OFFICE OF NAVAL RESEARCH
Contract No. N00014-75-C-0456

SHOCK-BOUNDARY LAYER INTERACTION
EFFECTS IN TRANSONIC FLOW FIELDS

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January 1980

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1. INTRODUCTION

Shock-boundary layer interaction can significantly influence the transonic flow and aerodynamics of missiles, wings and turbine blades. This influence is not only local but can also extend significantly downstream within the boundary layer and thereby alter the global properties of lift and drag. Some of the important effects that these interactions may exert even in the non-separating case are: (a) the interactive-thickening slightly alters the large-scale local inviscid pressure and both the shock location and obliquity; (b) the interaction zone itself does not scale with the local boundary layer thickness, thereby introducing a kind of "unit Reynolds number" effect; (c) possible incipient local separation at the shock foot if the local shock strength is strong enough (and/or Reynolds number is low), which then drastically changes the entire nature and extent of the interaction to a larger scale one involving a bifurcated shock interaction pattern; (d) an overall increase of the boundary layer displacement and momentum thicknesses downstream; (e) additional downstream distortion for some considerable distance of the more detailed boundary layer properties such as the shape factor and skin friction.

In view of these effects, it is important that shock-boundary layer interactions and their Reynolds and Mach number scaling be fundamentally understood and appropriate theoretical tools be developed for their prediction in engineering applications. Accordingly, in 1972 the author and his colleagues embarked on a basic research program toward these goals; this report summarizes the results achieved by this effort for the case of non-separating 2-D turbulent flow, and their implications as regards further research and applications.

2. LOCAL INTERACTION THEORY

2.1) Basic Interactive Flow Model

It is well-known experimentally that when separation occurs, the disturbance
flow pattern associated with normal shock-boundary layer interaction is a very complicated one involving a bifurcated shock pattern\textsuperscript{1,2}, whereas the un-separated case pertaining to turbulent boundary layers up to $M_{\infty} \leq 1.3$ has instead a much simpler type of interaction pattern which is more amenable to analytical treatment (see Fig. 1). With some judicious simplifications, it is possible to construct a fundamentally-based approximate analytical theory of the problem in this latter case. For the sake of orientation and completeness, a brief summary of this theory will now be given (full details can be found in Refs. 3 & 4).

The flow consists of a known incoming isobaric turbulent boundary layer profile $M_0(y)$ subjected to small transonic perturbations due to an impinging weak normal shock. In the practical Reynolds number range of interest here $[Re_L = 10^6$ to $10^8]$ we purposely employ a non-asymptotic disturbance flow model in the turbulent boundary layer patterned after the Lighthill-Stratford-Honda double-deck approach\textsuperscript{5-8} that has proven highly successful in treating a variety of other problems involving turbulent boundary layer response to strong rapid adverse pressure gradients,\textsuperscript{5-12} and which is supported by a large body of transonic and supersonic interaction data plus a general theoretical study\textsuperscript{13}. The resulting flow model (Fig. 2) consists of an inviscid boundary value problem surrounding a shock discontinuity and underlaid by a thin viscous disturbance sublayer that contains the upstream influence and skin friction perturbation. An approximate analytic solution is further achieved by assuming small linearized disturbances ahead of and behind the nonlinear shock jump plus neglect of the detailed shock structure within the boundary layer, which give accurate predictions for all the properties of engineering interest when $M_{\infty} \leq 1.05$. The resulting equations can be solved by operational methods yielding the interactive pressure rise and displacement thickness growth plus a recently-extended\textsuperscript{4,13}
skin friction solution downstream as well as upstream of the shock foot containing non-linear incipient-separation effects. The solution contains all the essential global features of the mixed transonic viscous interaction flow, and detailed comparisons with experiment have shown that it gives a very good account of all the important engineering features of the interaction over a wide range of Mach-Reynolds number conditions. Consequently, it is believed that this theory provides a good account of the interaction region for the purposes of practical transonic flow field analyses on wings or projectiles.

It is noted that the foregoing solution may be used with an incoming turbulent boundary layer profile model input from either experimental data or any theoretical prediction method. In our earlier studies we employed an accurate and especially convenient composite Law of the Wall - Law of the Wake profile model for equilibrium turbulent flows; more recently we have generalized it to include a nonequilibrium upstream flow history characterized by the three arbitrary parameters preshock Mach number, boundary layer displacement thickness Reynolds number and the value of the incompressible shape factor $H_{11}$.

Typical results of the theory are shown in Figs. 3-6. The predicted influence of Reynolds number on the pressure field is shown in Fig. 3; the extent of the interaction upstream and downstream decreases with increasing $Re_L$, tending toward a simple step pressure rise in agreement with both experimental observations and Navier-Stokes numerical simulation of turbulent interactions. The upstream influence distance $x_{up}$ ahead of the shock (where the interactive pressure rise is only 5%) at various shock strengths as a function of Reynolds number is shown in Fig. 4 plotted in ratio to $\delta_0$. These results agree with several detailed correlation studies of upstream influence data on interacting turbulent boundary layers that directly verify the present non-asymptotic triple deck flow model. The corresponding displacement thickness growth (Fig. 5)
is also of practical interest since this often has a significant back-effect on the inviscid flow and shock position on airfoils or in channel flows. It is seen that the predicted displacement growth decreases significantly with increasing Reynolds number. Note also that the overall streamwise extent of the interaction does not scale proportionally to the boundary layer thickness $\delta_0$ even in the non-separating case. The interactive skin friction distribution (Fig. 6) shows the typical decrease toward the shock owing to the adverse pressure gradient disturbance induced by the interaction; increasing shock Mach number enhances this owing to the stronger local interaction pressure gradient involved. When the interaction is strong enough, the present theory predicts vanishing skin friction and a very short separation bubble slightly behind the shock foot, as confirmed by detailed studies of transonic turbulent boundary layer interactions. The relative effect of the interaction at a given $M_1$ decreases at higher $Re_L$, incipient separation occurring more readily at lower Reynolds number as observed experimentally.

Direct comparisons with available data on two unseparated airfoil flows are shown in Fig. 7. The theory predicts the upstream influence well whereas it overestimates the pressure recovery downstream. This is typical of such airfoil tests and is caused by the fact that the actual shock occurring in airfoil experiments is usually oblique (albeit still with subsonic post-shock flow) owing to the interactive displacement thickness back-effect on the surrounding inviscid flow; this lowers the actual pressure rise 20-30% below the normal shock value. As illustrated by the good comparison with some DFVLR-Göttingen interaction data on a supercritical wing section shown in Fig. 8, when this obliquity is incorporated the present theory gives a satisfactory account of the interaction downstream as well as upstream of the shock. Finally, we show here (Fig. 9) some additional favorable comparisons of our theory with data from
the classical interaction experiments of Ackeret, Feldmann and Rott.\(^1\)

2.2) Further Refinements and Extensions of the Theory

The fundamental soundness and adaptibility of the foregoing flow model has permitted several useful extensions. These include consideration of pressure gradient affects in the background (non-interacted) flow\(^30\), the influence of wall curvature\(^31\), and allowance for moderate blowing or suction effects normal to the wall.\(^4\) Moreover, the presence of channel walls was studied\(^32\) and a method developed\(^14\) for applying the above theory to the case of channel or tube flows; this proves very important in the interpretation of interaction experiments carried out in such flows.

Some additional interesting extensions have also been made in response to questions about transonic flow behavior and simulation under the unusual conditions pertaining to cryogenic wind tunnels. Thus the influence of both non-adiabatic walls (heat transfer)\(^33,34\) and low temperature real gas effects\(^34,35\) on various features of shock-turbulent boundary layer interactions was studied by appropriate generalization of our basic interaction model.

3. GLOBAL INTERACTION EFFECTS

3.1) Downstream Effects of Interactions on Boundary Layers

In addition to the increased displacement thickness on the body, the foregoing discussion shows that the skin friction level following the interaction is significantly reduced; combined with the attendant distortion of the profile shape, these facts suggest that the subsequent downstream boundary layer development may retain a "memory" of the interaction effects for a considerable distance (over and above a simple thickening), particularly as regards possible incipient separation in the adverse pressure gradient region on the aft portion of the body. This "after-effect" question was therefore subjected to detailed
study by the author and one of his students,\textsuperscript{36} using the two-layer turbulent boundary layer program of Moses\textsuperscript{37} as a model of the downstream viscous flow; the program is initialized behind the interaction so as to account either fully, partially (\(\delta^*\) - effect only) or not at all for the preceding interaction. Calculations were then made of the subsequent downstream turbulent boundary layer behavior (\(H\), \(C_f\), \(\delta^*\), \(\varepsilon^*\)) in typical airfoil post-shock adverse pressure gradients for different assumed local interactive shock strengths and positions or Reynolds numbers.

Some typical results are illustrated in Fig. 10. They clearly show that the behavior of the boundary layer and incipient separation in the trailing edge region for a given downstream adverse pressure gradient field depends strongly on the "competition" between this field and the after-effect of the highly-non equilibrium profile distortion due to the interaction. Roughly speaking, this after-effect extends 20 to 30\% chord downstream and increases with shock strength and decreasing Reynolds number. If the trailing edge region lies within this range of the shock, it is thus seen that a simple thickening effect alone is not sufficient to account for the interaction and would result in an inaccurate prediction of the rearward boundary layer shape factor, skin friction and incipient separation properties including their scaling. Especially notable is the interaction - induced hastening of separation downstream (\(C_f \to 0\)). These theoretical predictions are further supported by data obtained in Gottingen\textsuperscript{24} on a supercritical airfoil boundary layer flow through a moderately-strong non-separating shock interaction region; as shown in Fig. 11, comparison with the observed downstream behavior of both \(H\) and \(C_f\) shows poor agreement when only the \(\delta^*\) - effect of the interaction is accounted for but good agreement when additional effect of the interactive skin friction reduction is also included.
The aforementioned downstream effects are deemed of practical importance for two major reasons: (1) in regions of sustained adverse pressure gradient that often follow the short-scale interaction zone, the shape of the velocity profile and streamwise shear stress distribution (as well as thickness) are of considerable importance to the aerodynamic design of an airfoil or wing; (2) the altered boundary layer properties (especially possible incipient separation) near the trailing edge and into the wake can further exert a powerful effect on the overall aerodynamics via their influence on the Kutta condition and on possible buffet onset.

3.2) Global Analysis of Supercritical Airfoil Flow

See Appendix A.

4. FUTURE DIRECTIONS FOR RESEARCH AND APPLICATION

On the basis of the foregoing successful basic research and the construction of a fully-operational computer program version of the resulting interaction theory, it is felt that it is now possible to provide a correctly-modeled account of shock-boundary layer interaction within supercritical airfoil design and analyses codes. In particular, we have seen the imbedding of our interaction solution as a local "interactive module" within a combined inviscid flow - boundary layer computer program so as to enable an improved study of the important trailing edge region in aft-loaded airfoils that now includes the upstream presence of shock interaction effects, as well as the possible onset of incipient separation beneath or downstream of the shock.

As regards recommended further work, there are four areas of great practical interest. (1) Extension and application of the present interaction theory to the unsteady case (examining first the validity of the quasi-steady approximation) in order to study unsteady air loads due to flutter at transonic speeds.
(2) Adaptation of the interaction analysis to three dimensional flow fields on finite-span wings, at least outside wing/fuselage juncture or tip - influence regions; this now appears feasible to study and is clearly of great practical interest. Once again the goal would be to imbed this extended interaction theory in a global flow field analysis program. (3) Given our progress in adapting the interaction theory to the presence of channel walls, one may now study in more detail the effects of shock-boundary layer interactions on transonic internal flows within engine inlets and ducts and turbomachinery blade passages and cascades. The influence of these interactions on the resulting losses and downstream effects, especially with incipient separation, is important to understand and predict in practice. (4) Extension of our basic work to study the interaction of shocks with flows containing significant streamwise vorticity, which occurs in certain types of aerodynamic configurations and/or in connection with the presence of vortex generators upstream.
REFERENCES


Fig. 1 Effect of Separation on Interaction Flow Pattern

Fig. 2 Theoretical Model of Non-Separating Interaction (Schematic)

Fig. 3 Reynolds Number Influence on Interaction Pressure Field

Fig. 4 Upstream Influence vs. Mach and Reynolds Number

Fig. 5 Scale Effect on Interactive Displacement Thickness Growth

Fig. 6 Reynolds Number Influence on Interactive Skin Friction Distribution
A. Comparison of Predicted Local Interaction Pressures with NAE Experiments for a Supercritical NACA 64A410 Airfoil: $M_a = .70, \text{Re}_c = 8 \times 10^6$

**Fig. 7**
Comparison of Theory with NAE Supercritical Airfoil Data

B. Comparison of Predicted and Experimental Pressures for the NACA 64A410 Airfoil: $M_a = .751, \text{Re}_c = 35 \times 10^6$

**Fig. 8**
Comparison of Present Theory with DFVLR-Göttingen Flow Measurements on Supercritical Wing Section
Fig. 9. Comparison with Wind Tunnel Data of Ackeret, Feldmann and Rott
Fig. 10. Typical Results for Interaction After-Effects on Downstream Turbulent Boundary Development (Shock at .7c; \( M_l = 1.2 \) ahead of shock, \( Re_c = 10^5 \).)
A COMPUTATIONAL PROCEDURE FOR
TRANSONIC AIRFOIL FLOW INCLUDING A SPECIAL SOLUTION
FOR SHOCK BOUNDARY LAYER INTERACTION

by

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The influence of a shock-boundary layer interaction on a supercritical airfoil flow field is significant, because it governs the way the boundary layer responds to the subsequent adverse pressure gradients and hence influences the flow conditions at the trailing edge. It is, therefore, important to incorporate a detailed treatment of the interaction region in the overall flow field analysis.

Existing treatments of shock-boundary layer interaction regions within such analysis codes have, unfortunately, relied on such simplistic treatments as artificial smearing of the pressure gradient for an ordinary boundary layer code or the use of a viscous ramp model; neither of the treatments can correctly account for the effects of the shock on the boundary layer properties needed downstream. The present work gives the results of incorporating a correct detailed accounting for the shock boundary layer interaction within a state-of-the-art viscous-inviscid computation method. The soundness of the approach is demonstrated by comparison with experimental data for the pressure distribution, the displacement thickness and the skin friction coefficient.

The present approach consists of imbedding a solution for the local interaction as a module within the boundary layer-inviscid flow computation code. The method is comprised of the following components:

**Inviscid flow theory:** The solution of the inviscid equations are obtained with the relaxation technique of Jameson [1] for the full potential equation which provides a fully conservative rotated scheme as well as a standard non-conservative formulation. The calculations are carried out in a computational plane obtained by conformally mapping the airfoil to a circle.
The solution is obtained in a sequence of decreasing mesh size. Jameson's accelerated iterative method is used to speed up convergence.

Boundary layer theory: Here Rotta's integral method [2] is used. This method is based on simultaneously integrating the van Kármán momentum equation and the energy equation to obtain the displacement thickness. In order to solve the above equations additional relations for the shape factor, the skin friction coefficient and the dissipation coefficients are employed. The computation starts at a point $x_1$ close to the stagnation point, using for the laminar boundary layer between the stagnation point and $x_1$ similar solutions based on the Falkner-Skan equation.

Shock wave-boundary layer interaction theory: For non-separating interactions (local Mach number $M_1 \leq 1.3$ in the Reynolds number range $Re \sim 10^6 - 10^8$) a non-asymptotic triple deck distribution flow model of normal shock turbulent boundary layer interaction is employed [3, 4]. The model consists of an inviscid region surrounding a shock discontinuity and an underlying thin viscous disturbance sublayer that contains the upstream influence and skin friction perturbation. An approximate analytic solution is achieved by assuming small linearized-disturbance ahead of and behind the non-linear shock-jump, with a simplified treatment of the detailed shock structure within the boundary layer down to the sonic level.

Coupling procedure: A coupling procedure has been carefully worked out. A representation is given in Fig. 1. (It should be noted that the experimental pressure distribution in Regions 1 and 3 is in general given by the inviscid computation.)

The boundary layer theory is used in Regions 1 and 3, the shock wave boundary layer interaction theory in region 2.

The input required for Region 2, viz. the shock upstream Mach number, the Reynolds number based on the displacement thickness and the shape factor is directly obtained from Region 1. The interacting module then computes the pressure distribution and the distribution of boundary layer parameters within the inter-
action region. To initiate the boundary layer computation in Region 3 the required input, viz. the momentum thickness and the energy thickness, are computed from the parameters supplied by Region 2.

The inviscid computation is iteratively carried out for the airfoil plus displacement thickness. The computation is started with an assumed displacement thickness, which is updated in subsequent steps by the method described above.

The inclusion of a special solution for the shock boundary layer interaction into a viscous/inviscid computation method and its application to transonic airfoil flow analysis is considered a contribution to the state-of-the-art in this field.

Experimental Study: Boundary layer and flow field measurements were carried out at the DFVLR on two supercritical airfoils having different characteristics in the pressure distribution. The free stream conditions were such that the local shock-upstream Mach Number varied between 1.2 and 1.4 for Reynolds Numbers between $2 \times 10^6$ and $4 \times 10^6$. In addition, the initial boundary layer condition was varied by changing the tripping device location. On one of the airfoils additional pressure distribution and wake measurements were carried out in the Lockheed CFWT at Reynolds Numbers between $4 \times 10^6$ and $30 \times 10^6$. Results of these experiments are compared with results of the aforementioned theory.

Representative results: Figure 1 shows a comparison between experimental results and results from the boundary layer / shock boundary layer interaction theory for a given experimental pressure distribution. The agreement in displacement thickness upstream and downstream as well as in the interacting region is excellent. It will be shown that the good agreement in the downstream region is due to the proper representation of the shock boundary layer interaction. Agreement in skin friction
coefficient is not as good; however, the interaction theory seems to predict the minimum quite accurately. Note, that the skin friction does not enter the overall computation but can be used to predict separation onset.

A comparison of the results of the complete method with experiment is given in Figure 2. Particularly good agreement is obtained in the shock location and the pressure rise across the shock. Additional such comparative examples for various parametric conditions as well as a critical assessment of the method will be given in the full paper.

Concluding remarks: The present investigation shows that it is important to include a correct treatment of shock boundary layer interaction into a viscous/inviscid transonic airfoil flow computation in order to obtain the correct boundary layer parameters immediately downstream of the interaction as input for the boundary layer computation downstream. It is also shown that details of the interaction, e.g., the local shape of the displacement surface, can be ignored in the inviscid computation.

A coupling of the interaction theory with a viscous/inviscid method for transonic projectile flow is described in [5].

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Figure 1: Comparison of Boundary Layer/Shock Boundary Layer Interaction Theory with Experiment
Airfoil CASI 10 - 2 / Uo A2

$M_{ao} = 0.764$

Re = $29.8 \times 10^6$; Transition at 8% c

Theory: $\alpha = 0.7^\circ$  $C_L = 0.520$

Experiment: $\alpha = 0.8^\circ$  $C_L = 0.521$

Fig. 2: Comparison of Results of Complete Method with Experiment
APPENDIX B

List of Publications Generated by Contract Research

During the contractual period 1972 - 1979, the following technical publications have been generated, or are under preparation.


