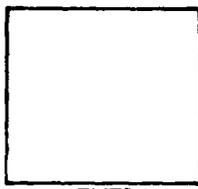




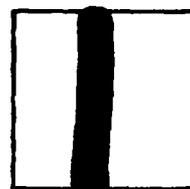
PHOTOGRAPH THIS SHEET

ADA 080822

DTIC ACCESSION NUMBER



LEVEL



INVENTORY

FTD-ID(RS)T-0370-79

DOCUMENT IDENTIFICATION

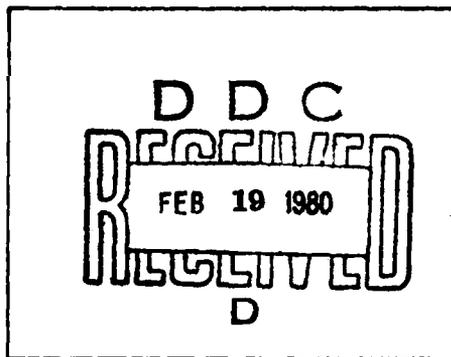
DISTRIBUTION STATEMENT A

Approved for public release;  
Distribution Unlimited

DISTRIBUTION STATEMENT

ACCESSION FOR	
NTIS	GRA&I <input checked="" type="checkbox"/>
DTIC	TAB <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION /	
AVAILABILITY CODES	
DIST	AVAIL AND/OR SPECIAL
A	

DISTRIBUTION STAMP



DATE ACCESSIONED

Empty box for date received in DTIC

DATE RECEIVED IN DTIC

PHOTOGRAPH THIS SHEET AND RETURN TO DTIC-DDA-2

ADA 080822

# FOREIGN TECHNOLOGY DIVISION



MOMENTUM AND ENERGY RELAXATION OF AN ELECTRON IN A CRYSTAL  
I. GENERAL RELATIONS FOR TEST ELECTRON

By

P. A. Kazlauskas, I. B. Levinson



Approved for public release;  
distribution unlimited.

79 08 20 213

## EDITED TRANSLATION

FTD-ID(RS)T-0370-79

21 March 1979

MICROFICHE NR: *FTD-79-C-000422*

MOMENTUM AND ENERGY RELAXATION OF AN ELECTRON  
IN A CRYSTAL I. GENERAL RELATIONS FOR TEST ELECTRON

By: P. A. Kazlauskas, I. B. Levinson

English pages: 25

Source: Litovskiy Fizicheskiiy Sbornik, Vol. 6,  
Nr. 1, 1966, pp. 33-44

Country of Origin: USSR

Translated by: Robert Allen Potts

Requester: AFWL/IN

Approved for public release; distribution unlimited.

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

\*ye initially, after vowels, and after ь, ь; e elsewhere.  
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

0370

MOMENTUM AND ENERGY RELAXATION OF AN ELECTRON IN A CRYSTAL

I. General Relations for Test Electron.

P. A. Kazlauskas, I. B. Levinson.

(Submitted 6 Mar. 1965)

The values characterizing relaxation and fluctuation of energy and momentum of a test electron in a crystal are examined during collisions with phonons and static impurities. These values are studied both for elastic and for inelastic scattering at various lattice temperatures.

### Introduction

The speed of transmission of energy and momentum from electrons to the lattice determines the degree of deviation of distribution of the electric field from equilibrium with the absence of a field. Strictly speaking, by energy we should mean its excess above thermal energy, and by momentum we should mean directed momentum, for example, projection to the direction of the field. It is obvious that losses of energy and momentum by the electron can be described with the aid of some times of relaxation.

The most common method of introduction of relaxation time is based on examination of the test particle, as is usually done in the theory of plasma [1].

At this work a systematic examination is conducted of the values, describing the interaction of the test electron with the lattice (phonons or static defects) for an isotropic model of semiconductor. In detail is studied the degree of elasticity of scattering on acoustic and optical phonons depending on the energy of electron and the temperature of the lattice.

## 1. Laws of dispersion.

The law of dispersion of electrons in isotropic model is determined by the connection between energy  $\epsilon$  and the absolute value of momentum  $p$  in the form  $\epsilon = \epsilon(p)$  or  $p = p(\epsilon)$ . Hence is determined speed

$$v(\epsilon) = \frac{\partial \epsilon}{\partial p} \quad (1.1)$$

and the density of states

$$g(\epsilon) = 4\pi p^2(\epsilon)/v(\epsilon). \quad (1.2)$$

For parabolic zone with effective mass  $m$

$$\begin{aligned} \epsilon(p) &= \frac{1}{2m} p^2, & p(\epsilon) &= (2m\epsilon)^{\frac{1}{2}}, \\ v(\epsilon) &= (2\epsilon/m)^{\frac{1}{2}}, & g(\epsilon) &= 4\pi(2m)^{\frac{3}{2}} \epsilon^{-\frac{1}{2}}. \end{aligned} \quad (1.3)$$

The law of dispersion of phonons is assigned by the dependence of frequency  $\omega$  on the absolute value of momentum  $q$  in the form  $\omega = \omega(q)$ . For acoustic phonons there is taken

$$\hbar\omega(q) = sq, \quad (1.4)$$

where  $s$  - the speed of sound. For optical phonons

$$\hbar\omega(q) = \hbar\omega_0(1 - aq^2), \quad (1.5)$$

where  $\omega_0$  - limiting frequency and  $a$  - coefficient, responsible for dispersion of optical phonons of order  $(mM)^{\frac{1}{2}}$ .

where  $M$  - mass of nucleus.

The comparison of laws of dispersion of electrons and phonons determines the characteristic energy  $\epsilon_0$ , at which the electron and phonon have identical pulses to them. It is determined as the solution of equation

$$\epsilon = A\omega(q) \Big|_{q=p(\epsilon)} \quad (1.6)$$

For optical phonons without dispersion  $\epsilon_0 = A\omega_0$ . For acoustic  $\epsilon_0$  depends on the law of dispersion of electrons and is found from equation

$$\epsilon = sp(\epsilon). \quad (1.7)$$

For acoustic phonons there exists another characteristic energy  $\epsilon'_0$ , at which the electron and phonon have identical speeds. It is found from equation

$$v(\epsilon) = s. \quad (1.8)$$

For parabolic zone

$$\epsilon_0 = 2ms^2, \quad \epsilon'_0 = \frac{1}{2} ms^2. \quad (1.9)$$

Subsequently it will be assumed that  $\epsilon(p)$  increases faster than  $p$ . Then, as is easy to see from Fig. 1,  $\epsilon'_0 < \epsilon_0$ .

Fig. 1. Law of dispersion of electrons and characteristic energies of acoustic scattering.

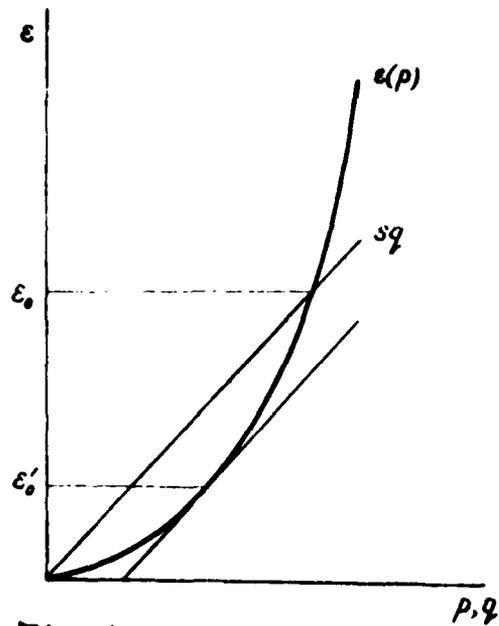


Fig. 1.

Furthermore, with this assumption

from follows  
 из  $\epsilon \gg \epsilon_0$  следует  $sp(\epsilon) \ll \epsilon$ ,  
 " из  $\epsilon \ll \epsilon_0$  следует  $sp(\epsilon) \gg \epsilon$ .  
 (1.10)

For optical phonons it will be convenient to consider that

$\epsilon'_0 = \epsilon_0$ .

2. Probability of scattering in isotropic model.

The probability of scattering of electron from a state with

momentum  $p$  to a state with momentum  $p'$ , connected with radiation (upper sign) or absorption (bottom sign) of phonon with momentum  $q$ , is taken usually in the following form [2]:

$$W^{\pm}(p, p') = \frac{2\pi}{\hbar} \left( N(q) + \frac{1}{2} \pm \frac{1}{2} \right) \left[ \frac{\hbar}{2m\omega(q)} \right]^2 M(p, p') \delta(\epsilon(p') - \epsilon(p) \pm \hbar\omega(q)) \quad (2.1)$$

In this case during computation it is necessary to consider  $q = \pm(p - p')$ . Equilibrium number of phonons enters the last expression

$$N(q) = \left[ \exp \frac{\hbar\omega(q)}{kT} - 1 \right]^{-1} \quad (2.2)$$

and also the square of the electron matrix element  $M(p, p')$ , connecting the beginning and end electron states.

In the isotropic model it is accepted to consider that  $N$  depends only on the value of momentum of the radiated or absorbed phonon and does not depend on the direction of this momentum. The laws of dispersion of electrons and phonons also are considered isotropic. All this, in essence, indicates some averaging with respect to angles, which can be done differently (either in the entire probability of transition, or in separate factors); therefore the values, figuring in the isotropic model, are determined actually with accuracy to a factor on the order of one, depending on the method of averaging. Exceptions are cases, when some value, for example  $\epsilon(p)$ , in actually is isotropic. Since, generally speaking, division into longitudinal and transverse phonons takes place only along certain

directions, then in actuality the mentioned averaging is performed not only by directions of the momentum of phonon, but also by the polarizations of phonon.

After such averaging the probability of scattering in the isotropic model is convenient to write out in the following manner, dividing the emission and absorption terms:

$$W(\epsilon, \epsilon', \chi) = W^+(\epsilon, \epsilon', \chi) + W^-(\epsilon, \epsilon', \chi), \quad (2.3)$$

where

$$W^{\pm}(\epsilon, \epsilon', \chi) = \frac{2\pi}{\hbar} B(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \delta(\epsilon' - \epsilon \pm \hbar\omega(q)). \quad (2.4)$$

Here  $\chi$  - angle of scattering, and  $\epsilon$  and  $\epsilon'$  - initial and final energy of electron. In this case it is necessary to consider

$$q^2 = p^2(\epsilon) + p^2(\epsilon') - 2p(\epsilon)p(\epsilon')\cos\chi \quad (2.5)$$

The dimensions of the utilized values are the following:  $W$  -  $(s)^{-1} \times (\text{momentum})^{-3}$ ,  $B$  -  $(\text{energy})^2 \times (\text{momentum})^{-3}$ .

Factor  $(\text{momentum})^3$  is connected with the density of states in the space of momenta, factor  $(s)^{-1}$  - with the frequency of transitions, and  $(\text{energy})^2$  - with the value of square of the matrix element of electron-phonon interaction.

Subsequently we will use the moments of probability of transition

$$W_i(\epsilon, \epsilon') = \frac{1}{4\pi} \int d\Omega W(\epsilon, \epsilon', \chi) P_i(\cos\chi), \quad d\Omega = d\alpha \sin\chi d\chi, \quad (2.6)$$

with the aid of which is written out expansion

$$W(\mathbf{r}, \mathbf{r}', \chi) = \sum_{l=0}^{\infty} W_l(\mathbf{r}, \mathbf{r}') (2l+1) P_l(\cos \chi) \quad (2.7)$$

The most generally used are  $W_0$  (average with respect to angles),  $W_1$ ,  $W_2$ , and also transport moments

$$W_1(\mathbf{r}, \mathbf{r}') = W_0(\mathbf{r}, \mathbf{r}') - W_2(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \int d\Omega W(\mathbf{r}, \mathbf{r}', \chi) (1 - \cos \chi), \quad (2.8a)$$

$$W_2(\mathbf{r}, \mathbf{r}') = \frac{2}{3} (W_0(\mathbf{r}, \mathbf{r}') - W_1(\mathbf{r}, \mathbf{r}')) = \frac{1}{12\pi} \int d\Omega W(\mathbf{r}, \mathbf{r}', \chi) \sin^2 \chi \quad (2.8b)$$

All these moments can be represented as the sums of emission and absorption parts. Let us write out their expressions through  $B(\mathbf{q})$ . With the aid of (2.5) there is replaced variable of integration  $\chi$  by  $q$ , and we obtain

$$W_l(\mathbf{r}, \mathbf{r}') = \frac{2\pi}{h} \frac{1}{2p(\mathbf{s})p(\mathbf{s}')} \int_{p(\mathbf{s})-p(\mathbf{s}')}^{p(\mathbf{s})+p(\mathbf{s}')} dq q B(q) P_l(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \times \\ \times \delta(\mathbf{r}' - \mathbf{r} \pm h\mathbf{w}(q)), \quad (2.9)$$

where

$$P_l(q) = P_l(\cos \chi)$$

with

$$\cos \chi = \frac{p^2(\mathbf{s}) + p^2(\mathbf{s}') - q^2}{2p(\mathbf{s})p(\mathbf{s}')} \quad (2.10)$$

In the case of scattering on static defects (impurities, dislocations) in Born approximation the probability of scattering depends only on the transmitted momentum  $q$ . Therefore in isotropic model for scattering on static defects

$$W(\epsilon, \epsilon', \chi) = \frac{2\pi}{\hbar} B(q) \delta(\epsilon - \epsilon'). \quad (2.11)$$

### 3. Characteristics of scattering of test particle.

The behavior of the test particle is described by different times of fluctuation or relaxation, which in view of the isotropicity of the model depend only on energy  $\epsilon$ .

Life time

$$\frac{1}{\tau_0(\epsilon)} = \int (dp') W(p, p'). \quad (3.1)$$

Relaxation time of momentum of its longitudinal component

$$\frac{p(\epsilon)}{\tau(\epsilon)} = \int (dp') W(p, p') (p - p')_{||}. \quad (3.2)$$

Transverse component of momentum does not relax in view of isotropicity.

The time of fluctuation of transverse component of momentum, or the time of deflection of momentum

$$\frac{\rho^2(\alpha)}{\tau_1(\alpha)} = \int (d\mathbf{p}') W(\mathbf{p}, \mathbf{p}') (\mathbf{p} - \mathbf{p}')^2 \quad (3.3)$$

Time of fluctuation of longitudinal component of pulse

$$\frac{\rho^2(\alpha)}{\tau_2(\alpha)} = \int (d\mathbf{p}') W(\mathbf{p}, \mathbf{p}') (\mathbf{p} - \mathbf{p}')^2_{||} \quad (3.4)$$

Signs  $\perp$  and  $||$  pertain to the direction of initial momentum  $\mathbf{p}$ . By carrying out integration with respect to angles in (3.1) - (3.4), it is possible to express the characteristic times through spherical moments  $W_l$ . In this case it is convenient to introduce time  $\tau_m$  according to

$$\frac{\rho^2(\alpha)}{\tau_m(\alpha)} = \int_0^\pi d\epsilon' g(\epsilon') \rho^2(\epsilon') W_l(\alpha, \epsilon') \quad (3.5)$$

Then we have

$$\frac{1}{\tau_0} = \frac{1}{\tau_m} \quad (3.6)$$

$$\frac{1}{\tau_1} = \frac{1}{\tau_m} + \frac{1}{\tau_{11}} \quad (3.7)$$

$$\frac{1}{\tau_2} = \frac{2}{3} \frac{1}{\tau_m} - \frac{2}{3} \frac{1}{\tau_{11}} \quad (3.8)$$

$$\frac{1}{\tau_3} = \frac{1}{\tau_m} - 2 \frac{1}{\tau_{11}} + \frac{1}{3} \frac{1}{\tau_m} + \frac{2}{3} \frac{1}{\tau_{11}} \quad (3.9)$$

For symmetric scattering, when the probability of scattering does not depend on  $\chi$ , i.e.,  $W_l=0$  with  $l \neq 0$ , we have  $1/\tau_m=0$  with  $l \neq 0$ . In this case from the four times  $\tau_0, \tau_1, \tau_2$  and  $\tau_3$  only two are independent, because they all are expressed through  $\tau_m$  and  $\tau_{11}$ . For elastic scattering times  $\tau_m=\tau$ , do not depend on  $\alpha$ . Among times  $\tau_0, \tau_1, \tau_2$  and  $\tau_3$  only three are independent, because they all are

expressed through  $\tau_0$ ,  $\tau_1$  and  $\tau_2$ .

Energy relaxation is determined by lost power

$$Q(\mathbf{p}) = \int (d\mathbf{p}') [\epsilon(\mathbf{p}) - \epsilon(\mathbf{p}')] W(\mathbf{p}, \mathbf{p}'). \quad (3.10)$$

In isotropic model  $Q$  does not depend on the direction  $\mathbf{p}$  and after integration with respect to angles takes the form

$$Q(\epsilon) = \int_0^{\infty} d\epsilon' g(\epsilon') (\epsilon - \epsilon') W_0(\epsilon, \epsilon') \quad (3.11)$$

From physical considerations it is clear that  $Q(\epsilon)$  should change sign with some  $\epsilon^*$  of order  $kT$ ; with  $\epsilon < \epsilon^*$  the electron acquires energy from the scattering system and  $Q < 0$ , and with  $\epsilon > \epsilon^*$  the electron gives out energy to this system and  $Q > 0$ . Therefore it is possible to write

$$Q(\epsilon) = \frac{\epsilon - \epsilon^*}{\tau(\epsilon)}, \quad (3.12)$$

where  $\tau$  is the relaxation time of energy for the test particle.

Another characteristic of the energetic interaction between test particle and the scattering system is

$$D(\mathbf{p}) = \frac{1}{2} \int (d\mathbf{p}') [\epsilon(\mathbf{p}) - \epsilon(\mathbf{p}')]^2 W(\mathbf{p}, \mathbf{p}'). \quad (3.13)$$

In contrast to  $Q$ , this value identically considers both the loss and the acquisition of energy of test particle. Therefore  $D$  describes not the relaxation of energy, but its fluctuation, i.e., exchange of

energy between the test particle and the scattering system. In the isotropic model

$$D(\epsilon) = \frac{1}{2} \int_0^{\infty} dt' g(\epsilon') (\epsilon - \epsilon')^2 W_0(\epsilon, \epsilon'). \quad (3.14)$$

By analogy with the relaxation time it is possible to introduce the time of fluctuation of energy  $\tau$

$$\frac{1}{\tau(\epsilon)} = \frac{D(\epsilon)}{\epsilon^2}. \quad (3.15)$$

#### 4. Qualitative examination of scattering.

Let us investigate qualitatively the character of scattering of electron with energy  $\epsilon$  using laws of conservation, which will be conveniently written in the following form:

$$q = [p^2(\epsilon) + p^2(\epsilon') - 2p(\epsilon)p(\epsilon') \cos \chi]^{\frac{1}{2}}, \quad (4.1)$$

$$\epsilon' - \epsilon = \pm \hbar \omega(q). \quad (4.2)$$

These relationships are depicted graphically in Fig. 2 for acoustic scattering and in Fig. 3 for optical scattering.

Fig. 2. Region of integration during acoustic scattering.

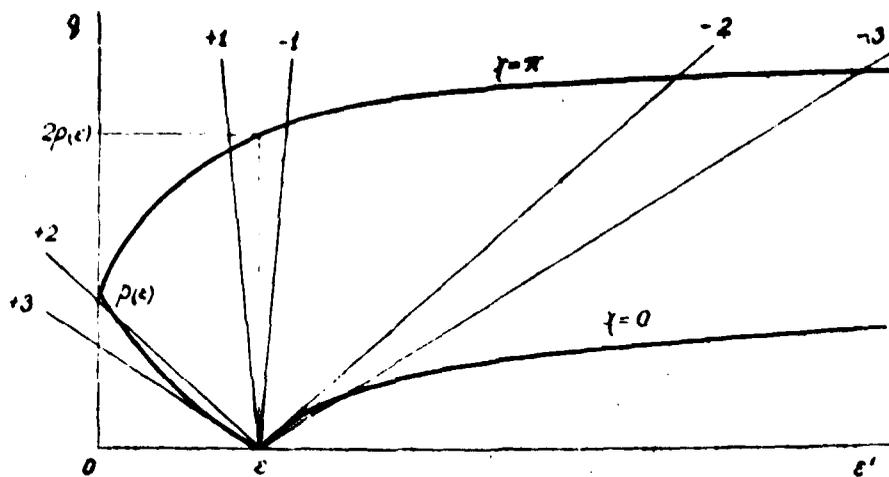
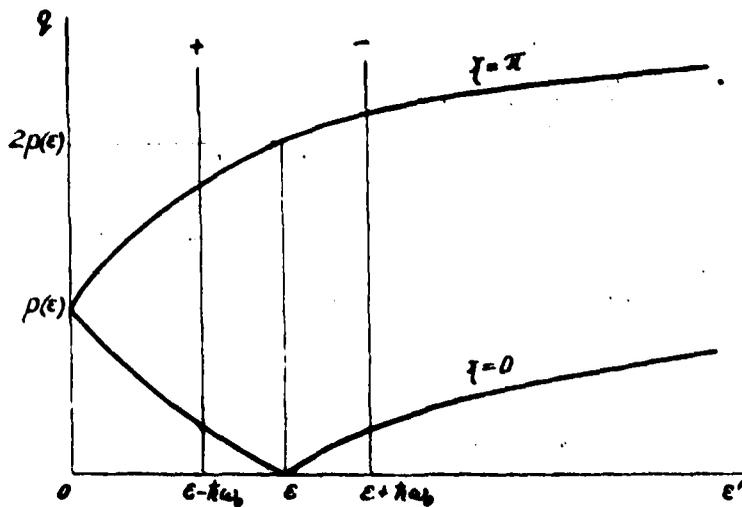


Fig. 2.

Fig. 3. Region of integration during optical scattering.



The curves, marked by values  $\chi$ , correspond to the right side of equation (4.1); pairs of lines, marked (+) and (-), correspond to equation (4.2), emission and absorption respectively. For acoustic scattering there are three pairs, marked 1, 2, 3; they illustrate the

different types of solutions of system (4.1)-(4.2) depending on the initial energy  $\mathcal{E}$ , respectively, in cases:

1) - with  $\epsilon_0 < \epsilon$ , emission and absorption occur with scattering to all angles;

2) - with  $\epsilon'_0 < \epsilon < \epsilon_0$ , emission is hampered and is possible only with scattering forward  $\chi < \pi/2$ ;

3) - with  $\epsilon < \epsilon'_0$ , emission generally is impossible.

For optical scattering there are only two cases, which are obtained with  $\epsilon'_0 = \epsilon_0$ .

Let us investigate now the degree of elasticity of acoustic scattering depending on  $\mathcal{E}$ . In order to use Fig. 2, instead of change of  $\mathcal{E}$  it is more convenient to consider the change of  $s$ . If  $\epsilon > \epsilon_0$  (or  $s \rightarrow 0$ ), then  $q \approx 2p(\epsilon)$  and change of energy  $\Delta \epsilon \approx 2p(\epsilon) \Delta \epsilon$ , i.e., the scattering is elastic. With decrease of the energy up to values  $\epsilon \sim \epsilon_0$ , the scattering is more inelastic, and  $\Delta \epsilon \sim \epsilon \sim \epsilon_0$ . If  $\epsilon < \epsilon_0$  (or  $s \rightarrow \pi$ ), then, as is easy to see, for determination of the final energy we have  $\epsilon' \approx sp(\epsilon)$ , whence  $\epsilon' \approx \epsilon_0$ . Thus,  $\Delta \epsilon \approx \epsilon' > \epsilon$ , i.e., with  $\epsilon < \epsilon_0$ , scattering is highly inelastic.

It is obvious that these conclusions about the degree of elasticity of scattering depending on the comparison of  $\epsilon$  and  $\epsilon_0$  are valid also for optical scattering.

Let us discuss now the questions connected with temperature. Let us examine the elastic scattering. First of all, it will be universally elastic only with  $kT \gg \epsilon_0$ . Secondly, the character of distribution of phonons, on which scattering occurs, is of interest. It is determined by the comparison of  $\hbar\omega = \Delta\epsilon$  and  $kT$ , where, as seen from the above-presented discussion,  $\Delta\epsilon = \hbar\omega_0$  for optical phonons, and  $\Delta\epsilon = sp(\epsilon) = (2ms^2\epsilon)^{\frac{1}{2}}$

for acoustic phonons. For high temperatures  $kT \gg \Delta\epsilon$  the scattering occurs on classically equally distributed phonons, the number of which  $N \sim kT/\Delta\epsilon \gg 1$ . For low temperatures  $kT \ll \Delta\epsilon$  the number of significant phonons  $N \sim \exp(-\Delta\epsilon/kT) \ll 1$  and scattering occurs mainly due to the spontaneous emission of phonons. Let us emphasize again that for acoustic scattering the character of the temperature, high or low, depends on the energy of electron  $\epsilon$ .

For inelastic scattering, as was shown, outside the dependence on the energy of electron always  $\Delta\epsilon \sim \epsilon_0$ . Therefore the character of distribution of phonons depends on the comparison of  $kT$  and  $\epsilon_0$ . At high temperatures  $kT \gg \epsilon_0$ , the significant phonons are distributed

classically and their number is great:  $N \sim kT/\epsilon_0 \gg 1$ . In this case for electron of energy  $\epsilon > \epsilon_0$ , the probabilities of absorption and emission have identical order of value. At low temperatures  $kT < \epsilon_0$ , the number of significant phonons is small:  $N \sim \exp(-\epsilon_0/kT) \ll 1$ . In this case for electron with energy  $\epsilon > \epsilon_0$ , the probability of spontaneous emission is much greater than the probability of absorption.

Let us examine now the peculiarities of scattering of electrons with energy  $\epsilon < \epsilon_0$  at low temperatures. The emission of phonon for such an electron is impossible; the probability of absorption is small. After absorption occurs, the energy of electron is made of order  $\epsilon_0$ . Now for the electron the probability of spontaneous emission is much greater than the probability of absorption and much greater than the probability of the first absorption. Therefore actually one should consider that the primary absorption is accompanied by instantaneous reemission, and one should consider the process of scattering as combined [3, 4].

The resulting two-stage scattering on optical phonons is elastic thanks to their small dispersion, since the reemitted quantum with respect to energy almost coincides with the initial absorbed. For two-stage scattering on acoustic phonons there is no such elasticity.

5. General formulas for computation of the characteristics of scattering.

Let us compute now the values, characterizing the test particle, having expressed them in the form of integrals by  $q$ , containing function  $B(q)$ . All these values will be represented in the form of the sum of two terms, corresponding to the radiation (top sign) and absorption (bottom sign).

Life time

$$\frac{1}{\tau_0(\epsilon)} = \frac{1}{\tau_0^+(\epsilon)} + \frac{1}{\tau_0^-(\epsilon)}, \quad (5.1)$$

where

$$\frac{1}{\tau_0^\pm(\epsilon)} = \frac{2\pi}{\hbar} \frac{2\pi}{p(\epsilon)} \int_{\pm} dq q B(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))}. \quad (5.2)$$

Relaxation time of momentum

$$\frac{1}{\tau(\epsilon)} = \frac{1}{\tau^+(\epsilon)} + \frac{1}{\tau^-(\epsilon)}, \quad (5.3)$$

where

$$\frac{1}{\tau^\pm(\epsilon)} = \frac{2\pi}{\hbar} \frac{\pi}{p^2(\epsilon)} \int_{\pm} dq q B(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))} \times \\ \times \left[ p^2(\epsilon) - p^2(\epsilon \mp \hbar\omega(q)) + q^2 \right]. \quad (5.4)$$

The deviation time of momentum

$$\frac{1}{\tau_1(\epsilon)} = \frac{1}{\tau_1^+(\epsilon)} + \frac{1}{\tau_1^-(\epsilon)}, \quad (5.5a)$$

where

$$\frac{1}{\tau_{\pm}^{\pm}(\epsilon)} = \frac{2\pi}{\hbar} \frac{\pi}{p^{\pm}(\epsilon)} \int_{\pm} dq q B(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))} \times$$

$$\times \left[ q^{\pm} \left( p^{\pm}(\epsilon) + p^{\pm}(\epsilon \mp \hbar\omega(q)) \right) - \frac{1}{2} \left( p^{\pm}(\epsilon) - p^{\pm}(\epsilon \mp \hbar\omega(q)) \right)^2 - \frac{1}{2} q^{\pm} \right]. \quad (5.6a)$$

Time of longitudinal fluctuation of momentum

$$\frac{1}{\tau_{||}(\epsilon)} = \frac{1}{\tau_{||}^{+}(\epsilon)} + \frac{1}{\tau_{||}^{-}(\epsilon)}, \quad (5.56)$$

where

$$\frac{1}{\tau_{||}^{\pm}(\epsilon)} = \frac{2\pi}{\hbar} \frac{2\pi}{p^{\pm}(\epsilon)} \int_{\pm} dq q^3 B(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))} - \frac{1}{\tau_{\pm}^{\pm}(\epsilon)}. \quad (5.66)$$

Power of energy relaxation

$$Q(\epsilon) = Q^{+}(\epsilon) - Q^{-}(\epsilon), \quad (5.7)$$

where  $Q^{+}$  - power, given off during radiation of phonons, and  $Q^{-}$  - power, obtained during absorption of phonons:

$$Q^{\pm}(\epsilon) = \frac{2\pi}{\hbar} \frac{2\pi}{p^{\pm}(\epsilon)} \int_{\pm} dq q B(q) \hbar\omega(q) \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))}. \quad (5.8)$$

The characteristic of energy fluctuation

$$D(\epsilon) = D^{+}(\epsilon) + D^{-}(\epsilon), \quad (5.9)$$

where

$$D^{\pm}(\epsilon) = \frac{2\pi}{\hbar} \frac{\pi}{p^{\pm}(\epsilon)} \int_{\pm} dq q B(q) [\hbar\omega(q)]^2 \left[ N(q) + \frac{1}{2} \pm \frac{1}{2} \right] \frac{p(\epsilon \mp \hbar\omega(q))}{v(\epsilon \mp \hbar\omega(q))}. \quad (5.10)$$

The integration limits, bottom  $q_1^{\pm}(\epsilon)$  and top  $q_2^{\pm}(\epsilon)$ , are determined by laws of conservation.

Their values, or equations, from which they should be found, are

indicated in Table 1.

Table 1. Integration limits for acoustic and optical scatterings with arbitrary law of electron dispersion.

		(a) Emission		(b) Absorption	
		$\omega_1$	$\omega_2$	$\omega_1$	$\omega_2$
(c) Asymmetric	$\omega < \omega_0$	-	-	$q - p(\epsilon + \omega) - p(\epsilon)$ ( $q \neq 0$ )	-
	$\omega_0 < \omega < \omega_1$	-	$q - p(\epsilon) - p(\epsilon - \omega)$ ( $q \neq 0$ )	-	$q - p(\epsilon) + p(\epsilon + \omega)$
	$\omega_1 < \omega < \omega_2$	$q - 0$	$q - p(\epsilon) + p(\epsilon - \omega)$	$q - 0$	-
	$\omega_2 < \omega$	-	-	-	-
(d) Symmetric	$\omega < \omega_0$	-	-	$q - p(\epsilon + \hbar\omega) - p(\epsilon)$	$q - p(\epsilon + \hbar\omega) + p(\epsilon)$
	$\omega_0 < \omega$	$q - p(\epsilon) - p(\epsilon - \hbar\omega)$	$q - p(\epsilon) + p(\epsilon - \hbar\omega)$	-	-

Key: (a) Emission. (b) Absorption. (c) Optical. (d) Acoustic.

Dashes indicate that emission or phonons is forbidden by the laws of conservation.

The above-provided formulas are obtained if in the determination of the appropriate value we place the needed moment  $W$ , in the form (2.9). The obtained double integral with respect to  $q$  and  $\epsilon$  is prevalent in the region, limited by curves  $\chi=0, \pi$  in Figs. 2 or 3. Using  $\delta$ -function, it can be reduced to the integral with respect to

curve  $\epsilon' = \epsilon_1 + A\omega^2(q)$  with parameter  $q$ .

Let us note that formulas (5.1) - (5.10) can be highly simplified for parabolic zone with any law of dispersion of phonons and for optical phonons without dispersion with any law of dispersion of electrons.

Institute of Physics and Mathematics of the Academy of Sciences of  
Lithuanian SSR. Vilnius State Pedagogical Institute.

## Appendix

Matrix elements  $B(q)$  for different mechanisms of scattering.

The majority of semiconductors have two atoms each in the elementary cell. Therefore in the isotropic model, where there is actually conducted neutralization with respect to polarizations of phonons, it is necessary to examine two types of phonons: acoustic and optical. They will be distinguished by designations A and O. The phonons of each type scatter electrons, generally speaking, by two means. Spread of oscillations in the lattice, first of all, creates distortion of electric microfield in the elementary cell, which leads to change of the law of dispersion of electron, and, secondly, in the elementary cell creates dipole moment, which leads to the appearance of electric macrofield. Scattering due to the first effect is accepted to call deformation, and due to the second effect - polarization. These two types of scattering will be distinguished by designations D and P. Thus, four mechanisms of scattering exist: DA (on deformation potential of acoustic phonons), PA (piezo-acoustic),

DO (on deformation potential or optical phonons), PO (polarization). The mechanisms of PA and PO are possible only in crystals with different atoms in elementary cell.

Scatterings D and P on phonons of one type do not interfere, since the corresponding matrix elements are phase shifted  $\pi/2$ . Therefore it is possible to consider D and P as independent scattering mechanisms.

Below are provided expressions  $B(q)$  for all four scattering mechanisms.

Mechanism DA [5]:

$$B(q) = B_0 q, \quad B_0 = \frac{1}{2} \frac{1}{(2\pi\hbar)^3} \frac{C_1^2}{\rho s}$$

Here  $\rho$  - density of crystal,  $C_1$  - constant of deformation potential (dimensionality of energy). The speed of sound  $s$  is usually considered some average longitudinal velocity.

Sometimes other constants are used:

$$C[6] = C_1, \\ C[7, 8] = \frac{3}{2} C_1$$

Mechanism PA [9]:

$$B(q) = B_0 q^{-1}, \quad B_0 = \frac{1}{(2\pi\hbar)^3} \frac{(eE_0)^2}{\rho s} N^2$$

Here we introduced characteristic field  $E_0$ , determining the

piezoconnection of electrons with acoustic oscillations. There is also used dimensionless coefficient of electromechanical connection  $K^2$  [10]. All these values in essence are some averaged with respect to angles

$$E_0 = \left\langle \frac{e}{\epsilon_0} \right\rangle,$$

$$K^2 = \left\langle \frac{e^2}{\epsilon_0 c} \right\rangle.$$

where  $e$  - piezomodulus,  $\epsilon_0$  - static dielectric constant and  $c$  - elastic constant. The speed of sound  $s$  entering  $B_0$  is also a certain average, in which enters the speed of both longitudinal and transverse waves.

Mechanism PO [3, 4]:

$$B(\varphi) = B_0 \varphi^{-1},$$

$$B_0 = \frac{e^2}{h} \frac{h\nu_0}{(2\pi)^2} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right).$$

Here  $\epsilon_\infty$  and  $\epsilon_0$  - high-frequency and low-frequency dielectric constants. As the parameters, describing the interaction of electrons with polarized oscillations, there are also used:

dimensionless constant of connection

$$\alpha = \frac{e^2}{h\nu_0} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right),$$

characteristic electric field [11]  $eE_0 = 2\pi\nu_0 P_0$  and effective charge, giving moment of cell [12]

$$(e^*)^2 = \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \omega_0^2 \frac{V_0 M}{4\pi}.$$

In the last expressions is designated:  $v$  and  $k$  - speed and momentum of electron with energy  $\hbar\omega$ ;  $V$  - volume of cell and  $M$  - reduced mass of ions.

Let us note that in formula (6.11b) [11] there is no factor 2 in the denominator.

Mechanism DO [7]:

$$n(\omega) = n_0 \left( 1 - \frac{1}{2\omega^2} \frac{D^2}{\hbar^2 \omega_0^2} \right)$$

Here  $D$  - constant of deformation potential for optical phonons, determined as shift of bottom of zone with single relative displacement of sublattices [14],  $\rho^*$  - density of reduced mass of ions; if both ions in the cell are identical, then  $\rho^* = 1/4 \rho$ . Sometimes another constant is used

$$(DK)(\omega) = 2 \frac{1}{\omega} D.$$

where  $D$  on the left - constant dimension of energy and  $K$  - vector of reverse lattice.

#### REFERENCES

1. С. Мантрассер, Статистические проблемы в физике и астрономии, ИЛ, Москва, 1947. См. также Вопросы теории плазмы, Атомиздат, Москва, 1, 90 (1963), 4, 81 (1964).
2. Л. В. Лайман, Электроны и фононы, ИЛ, Москва, 1969.
3. Л. И. Давыдов, И. М. Шмидкевич, УФН, 24, 21 (1940).
4. H. Fröhlich, N. Mott, Proc Roy. Soc., A171, 496 (1939).
5. W. Shockley, J. Bardeen, Phys. Rev., 77, 407 (1950); 80, 72 (1950).
6. А. В. Манасон, Квантовая теория неустойчивости, М. 31, 1941.

7. F. Seitz, *Phys. Rev.*, **73**, 549 (1948).
8. А. Зоммерфельд, Г. Бете, *Электронная теория металлов*, М.—Л., 1938
9. M. J. G. Meijer, D. Polder, *Physica*, **19**, 255 (1953).
10. A. R. Hutson, *A.J. Appl. Phys. Suppl.*, **32**, 2287 (1961)
11. R. Stratton, *Proc. Roy. Soc.*, **A246**, 406 (1958).
12. H. V. Callen, *Phys. Rev.*, **76**, 1394 (1949)
13. А. И. Ансельм, *Введение в теорию полупроводников*, Физматгиз, М.—Л., 1962

#### Summary

Energy and momentum relaxation determines energy and drift velocity distribution of hot electrons.

Following quantities, describing test electron interaction with lattice vibrations and at static imperfections, are considered: life time  $\tau_0(\epsilon)$  (3.1), longitudinal momentum relaxation time  $\tau(\epsilon)$  (3.2), transversal and longitudinal momentum deflection times  $\tau_{\perp}(\epsilon)$  (3.3) and  $\tau_{\parallel}(\epsilon)$  (3.4), energy loss power  $Q(\epsilon)$  (3.11) and energy fluctuation "power"  $D(\epsilon)$  (3.13). Scattering probability is assumed only energy and scattering angle dependent (2.4) (2.7). Electron and phonon dispersion  $\epsilon(p)$  and  $\omega(q)$  is assumed isotropic, but arbitrary. The characteristic energy  $\epsilon_0$  for electron-phonon scattering is determined by  $\epsilon(p)$  and  $\omega(q)$  comparison from equation (1.6).

The scattering elasticity and effective phonon distribution (equipartition or zero point vibrations) for various electron energy  $\epsilon$  and lattice temperature  $T_k$  is analysed. These scattering properties are determined by the comparison of  $\epsilon$ ,  $kT$  and  $\epsilon_0$ .

General formulae for  $\tau_0$ ,  $\tau$ ,  $\tau_{\perp}$ ,  $\tau_{\parallel}$ ,  $Q$  and  $D$  calculation are given by (5.1)–(5.10), where the limits of integration are listed in table 1.

State Pedagogical Institute Vilnius

Institute of Physics and Mathematics  
of the Academy of Sciences of the Lithuanian SSR

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>	<u>ORGANIZATION</u>	<u>MICROFICHE</u>
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	9	E403 AFSC/INA	1
C043 USAMIIA	1	E404 AEDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D LAB/FIO	1	E410 ADTC	1
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NIIS	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NIIS	2
H300 USAICE (USAREUR)	1		
P005 DOE	1		
P050 CIA/CRB/ADD/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		
L.L.L./Code L-389	1		
NSA/1213/TDL	2		