A Shear Stabilized Cylindrically Symmetric Mirror Plasma

W. M. MANHEIMER

Plasma Theory Branch
Plasma Physics Division

December 21, 1979

NAVAL RESEARCH LABORATORY
Washington, D.C.

Approved for public release; distribution unlimited.

80 1 28 088
This memorandum report examines the possibility of shear stabilization of a cylindrically symmetric mirror plasma.
A SHEAR STABILIZED CYLINDRICALLY SYMMETRIC MIRROR PLASMA

In order to avoid flute instabilities, magnetic mirror plasmas are almost always confined in minimum B configurations.\(^1\),\(^2\) While these configurations are MHD stable, they are not cylindrically symmetric. However, cylindrically symmetric configurations have certain advantages in that the coil structures are much simpler. Also it might be possible to compress a cylindrically symmetric mirror confined plasma with an imploding liner. This note proposes such a cylindrically symmetric MHD stable mirror plasma.

The idea is to use shear stabilization rather than minimum B. This essentially means running a current through the center of the mirror field.

To analyze this, model the flute instability by a simple g mode. Let us assume a 2:1 mirror ratio so the radius of the plasma at the midplane is larger by \(\sqrt{2}\) than at the mirror neck, as shown in Fig. 1. Model the field line by

\[
y = r_o + 0.15 r_o \cos \left( \frac{2\pi x}{L} \right). \tag{1}
\]

Then the minimum radius of curvature \(R\) (at midplane) is \(1/R = 0.15 r_o \left( \frac{2\pi}{L} \right)^2\). This corresponds to the most unstable gravity.

Note: Manuscript submitted October 30, 1979.
Then say

\[ g = \frac{v_i^2}{R} = 0.15 \frac{v_i^2}{r_o} \left(\frac{2\pi}{L}\right)^2 \]  

(2)

Now the Suydam style criterion\(^3,4\) for stabilizing g modes is

\[ 4\pi g \frac{\partial \rho}{\partial x} \frac{L^2}{B^2} \approx \frac{4\pi g \rho L^2}{\frac{\partial \rho}{\partial x} B^2} < \frac{1}{4} \]  

(3)

where \(L_s\) is the shear length

\[ L_s^{-1} \approx \frac{B_\theta}{B_z} \frac{1}{q} \frac{dq}{dr} \]  

(4)

and

\[ q = \frac{2\pi B_z r}{B_\theta L} \]  

(5)

If the current all flows within a radius \(a\), \(B_\theta = B_\theta(a) \frac{a}{r}\) so

\[ L_s^{-1} \approx \frac{2 B_\theta}{B_z r} \]  

(6)

Taking \(r = r_o\) the shear stabilization condition becomes

\[ 8\pi \rho v_i^2 (0.15) \left(\frac{2\pi r_o}{L}\right) < B_\theta^2 \]  

(7)

If \(n = 10^{13}\), \(T_i = 1\) KeV, \(R_o = 2\) cm, \(L = 40\) cm like the plasma in Ref. 2, we find,

\[ B_\theta(r_o) = 200 \text{ G} \]  

(8)
since $B_0 = I/5r_o$ that this means a current down the center
\[ I = 2 \text{K Amps.} \] (9)
which is a relatively modest current.

This current could come from a wire or discharge down the middle. If the latter, the discharge itself must be MHD stable also. If the electrode radius (assumed to be at mirror neck) is 0.7 cm and current radius is $a = 1$ cm at midplane, then if $I = 2$ kA amps and $B_0 = 10$ kG,

\[ q(a) = \frac{aB_0}{L} \frac{10\pi a^2 B_0}{I L} \approx 7.5 \] (10)
so the discharge should be stable if it has a diffuse profile.

The appealing thing about this theory is that it utilizes only a single concept, shear stabilization of a mirror plasma. This could be tested on a relatively small low cost experiment like that of Ref. 2. If it works, one then has the option of using a minimum $B$ configuration if cylindrical symmetry is unimportant, or a shear stabilized configuration if cylindrical symmetry is important.

**Acknowledgment**

The author would like to thank Dr. A. E. Robson for very useful discussions. This work was supported by the U. S. Department of Energy.
References


Fig. 1. Schematic of a shear stabilized mirror plasma.
Fig. 1 - Schematic of a shear stabilized mirror plasma