This document is disseminated under the sponsorship of the Department of Transportation in the interest of information exchange. The United States Government assumes no liability for its contents or use thereof. The contents of this report do not necessarily reflect the official view or policy of the Coast Guard, and they do not constitute a standard, specification, or regulation. This report, or portions thereof, may not be used for advertising or sales promotion purposes. Citation of trade names and manufacturers does not constitute endorsement or approval of such products.
### METRIC CONVERSION FACTORS

**Approximate Conversions to Metric Measures**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>When You Know</th>
<th>Multiply by</th>
<th>To Find</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LENGTH</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td></td>
<td>1.6</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>cm</td>
<td></td>
<td>0.394</td>
<td>meters</td>
<td></td>
</tr>
<tr>
<td>mm</td>
<td></td>
<td>0.0394</td>
<td>kilometers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>AREA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.362069</td>
<td>square inches</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.111111</td>
<td>square meters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.000022</td>
<td>square kilometers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.000022</td>
<td>hectares (10,000 m²)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MASS (weight)</td>
<td></td>
</tr>
<tr>
<td>kg</td>
<td></td>
<td>2.20462</td>
<td>pounds</td>
<td></td>
</tr>
<tr>
<td>lb</td>
<td></td>
<td>0.453592</td>
<td>kilograms</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>VOLUME</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
<td>milliliters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
<td>liters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
<td>cubic meters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TEMPERATURE (exact)</td>
<td></td>
</tr>
<tr>
<td>°C</td>
<td></td>
<td>9/5(when temperature Fahrenheit) - 32</td>
<td>°F</td>
<td></td>
</tr>
</tbody>
</table>

**Approximate Conversions from Metric Measures**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>When You Know</th>
<th>Multiply by</th>
<th>To Find</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LENGTH</td>
<td></td>
</tr>
<tr>
<td>in</td>
<td></td>
<td>2.54</td>
<td>meters</td>
<td></td>
</tr>
<tr>
<td>ft</td>
<td></td>
<td>0.3048</td>
<td>meters</td>
<td></td>
</tr>
<tr>
<td>km</td>
<td></td>
<td>0.000621</td>
<td>miles</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>AREA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.59039</td>
<td>square inches</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.10000</td>
<td>square meters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.000247</td>
<td>square kilometers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.29030</td>
<td>acres</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MASS (weight)</td>
<td></td>
</tr>
<tr>
<td>gr</td>
<td></td>
<td>0.00007</td>
<td>grains</td>
<td></td>
</tr>
<tr>
<td>kg</td>
<td></td>
<td>1.00000</td>
<td>grams</td>
<td></td>
</tr>
<tr>
<td>lb</td>
<td></td>
<td>0.453592</td>
<td>pounds</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>VOLUME</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
<td>liters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
<td>cubic meters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TEMPERATURE (exact)</td>
<td></td>
</tr>
<tr>
<td>°F</td>
<td></td>
<td>5(when temperature Fahrenheit) + 32</td>
<td>°C</td>
<td></td>
</tr>
</tbody>
</table>

The Towboat Maneuvering Simulator is based on the integration in time of the differential equations which describe the motions of the towboat and barge string in three degrees of freedom, i.e., yaw, sway and surge.

This volume of the simulator documentation provides a description of the basic equations of motion included in the simulator and the relationships used to determine external forces and moments due to a bow thruster and wind. These equations are completely general in nature and could be used, with the proper hydrodynamic coefficients, to describe the maneuvering of vessels other than towboats.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1</td>
</tr>
<tr>
<td>2.0</td>
<td>2</td>
</tr>
<tr>
<td>3.0</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>13</td>
</tr>
<tr>
<td>3.2</td>
<td>14</td>
</tr>
<tr>
<td>3.3</td>
<td>17</td>
</tr>
<tr>
<td>4.0</td>
<td>18</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>19</td>
</tr>
</tbody>
</table>
### NOTATION

**DIRECTIONAL STABILITY AND CONTROL**

The following nomenclature conforms to DNTRI Report 137 and M6TH Report 5695 where applicable. The positive direction of axes, angles, forces, moments, and velocities are shown by Figure 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Dimensional Form</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>( C_{D} )</td>
<td>Propeller diameter</td>
</tr>
<tr>
<td>( D )</td>
<td>( D_{s} = \frac{D}{L} )</td>
<td>Steady-turning diameter</td>
</tr>
<tr>
<td>( I_{x} )</td>
<td>( I_{x} = \frac{I_{x}}{\frac{1}{2}mL^{2}} )</td>
<td>Moment of inertia of ship about x-axis</td>
</tr>
<tr>
<td>( I_{y} )</td>
<td>( I_{y} = \frac{I_{y}}{\frac{1}{2}mL^{2}} )</td>
<td>Moment of inertia of ship about y-axis</td>
</tr>
<tr>
<td>( I_{z} )</td>
<td>( I_{z} = \frac{I_{z}}{\frac{1}{2}mL^{2}} )</td>
<td>Moment of inertia of ship about z-axis</td>
</tr>
<tr>
<td>( J )</td>
<td>( J_{t} = \frac{u}{nD} )</td>
<td>Propeller advance coefficient based on ship speed ( u )</td>
</tr>
<tr>
<td>( J_{o} )</td>
<td>( J_{o} = \frac{u_{o}}{nD} )</td>
<td>Propeller advance coefficient at steady ship command speed ( u_{o} )</td>
</tr>
<tr>
<td>( k_{x} )</td>
<td>( k_{x} = \frac{x}{L} )</td>
<td>Radius of gyration of ship about x-axis</td>
</tr>
<tr>
<td>( k_{y} )</td>
<td>( k_{y} = \frac{x}{L} )</td>
<td>Radius of gyration of ship about y-axis</td>
</tr>
<tr>
<td>( k_{z} )</td>
<td>( k_{z} = \frac{x}{L} )</td>
<td>Radius of gyration of ship about z-axis</td>
</tr>
<tr>
<td>( l )</td>
<td>( l = \frac{1}{L} )</td>
<td>Characteristic length; length between perpendiculars for commercial ships</td>
</tr>
</tbody>
</table>
\[ l_d = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Dynamic stability lever} \]

\[ l_p = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Damping lever} \]

\[ l_v = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Static stability lever} \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Mass of ship} \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Hydrodynamic moment component about x axis (yawing moment)} \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Aerodynamic moment component about x axis} \]

\[ \omega = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Yawing moment when } \beta = 0 \text{ or } 0 \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } r \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } (q-1) \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Coefficient used in representing } N \text{ as a function of } p \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Second order coefficient used in representing } N \text{ as a function of } r \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Coefficient used in representing } N \text{ as a function of } r \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } v \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } (q-1) \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Coefficient used in representing } N \text{ as a function of } \eta \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Coefficient used in representing } N \text{ as a function of } \eta \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Second order coefficient used in representing } N \text{ as a function of } v \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ Second order coefficient used in representing } N \text{ as a function of } v \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } \eta \]

\[ N = \frac{L}{L - \frac{L}{L - \frac{L}{L}}} \text{ First order coefficient used in representing } N \text{ as a function of } \eta \]
Propeller revolution rate

Propeller revolution rate at steady command speed

Ordered propeller revolution rate

Overshoot width of path

Overshoot heading angle; measured from value at second execute

Steady-turning radius

Angular velocity component about z axis relative to fluid

Angular acceleration component about z axis relative to fluid

Tactical diameter

Transfer

Time

Time at i
th execute in an overshoot or zigzag maneuver

Time at initiation of a maneuver

Time to reach 90-degree change of heading in a turn

Time to reach 180-degree change of heading in a turn

Linear velocity of origin of body axis relative to fluid

Component of U in direction of the x axis

Time rate of change of u in direction of the x axis

Command speed: steady value of ahead speed component u for a given propeller rpm for \( \beta = \delta r = 0 \); sign changes with propeller reversal

Linear velocity of current

Component of \( U_C \) in direction of x axis

Linear velocity of origin of body axis relative to fluid

Component of \( U_R \) in direction of x axis

Velocity at rudder due to motion and propeller race

Wind velocity
Component of $U_4$ in direction of $y$ axis

Component of $U_5$ in direction of $y$ axis

Absolute speed in knots

Steady approach speed in knots

Speed in knots at 90-degree heading change in a turn

Speed in knots at 180-degree heading change in a turn

Component of $V$ in direction of the $y$ axis

Time rate of change of $V$ in direction of the $y$ axis

Component of $x$ in direction of $y$ axis

Longitudinal body axis; also the coordinate of a point relative to the origin of body axes

The $x$ coordinate of $C_B$

The $x$ coordinate of $C_G$

Aerodynamic force component along $X$ axis

A coordinate of the displacement of $C_G$ relative to the origin of a set of fixed axes

Hydrodynamic force component along $x$ axis (longitudinal, or axial force)

Second order coefficient used in representing $X$ as a function of $r$. First order coefficient is zero

Coefficient used in representing $X$ as a function of $u$

Second order coefficient used in representing $X$ as a function of $u$ in the non-propelled case. First order coefficient is zero

Coefficient used in representing $X$ as a function of the product $v r$

Second order coefficient used in representing $X$ as a function of $v$. First order coefficient is zero

First order coefficient used in representing $X_{v r}$ as a function of $(q-1)$

Second order coefficient used in representing $X$ as a function of $r$ at $q = 0$. First order coefficient is zero
Second order coefficient used in representing $x_{0r}$ as a function of $\eta$

$y$: $y' = \frac{y}{L}$
- Lateral body axis; also the coordinate of a point relative to the origin of body axes.

$y_0$: $y_0' = \frac{y_0}{L}$
- The $y$ coordinate of CG.

$y_0'$: $y_0' = \frac{y_0}{L}$
- A coordinate of the displacement of CG relative to the origin of a set of fixed axes.

$y$: $y' = \frac{y}{\frac{1}{2}pl^{2}U}$
- Hydrodynamic force component along $y$ axis (lateral force).

$y_A$: $y_A' = \frac{y_A}{\frac{1}{2}pl^{2}U_A}$
- Aerodynamic force component along $Y$ axis.

$y_s$: $y_s' = \frac{y_s}{\frac{1}{2}pl^{2}U}$
- Lateral force when $\beta = 0, \gamma = 0$.

$Y_p$: Distance of port propeller from centerline.

$Y_s$: Distance of starboard propeller from centerline.

$Y_p$: $Y_p' = \frac{Y_p}{\frac{1}{2}pl^{2}U}$
- First order coefficient used in representing $Y$ as a function of $r$.

$Y_{qq}$: $Y_{qq}' = \frac{Y_{qq}}{\frac{1}{2}pl^{2}U}$
- First order coefficient used in representing $Y_q$ as a function of $(\eta - 1)$.

$Y_p$: $Y_p' = \frac{Y_p}{\frac{1}{2}pl^{s}}$
- Coefficient used in representing $Y$ as a function of $r$.

$Y_p[r]$: $Y_p[r]' = \frac{Y_p[r]}{\frac{1}{2}pl^{s}}$
- Second order coefficient in representing $Y_p$ as a function of $r$.

$Y_p[r]_r$: $Y_p[r]_r' = \frac{Y_p[r]_r}{\frac{1}{2}pl^{s}}$
- Coefficient used in representing $Y_p[r]$ as a function of $r$.

$Y_v$: $Y_v' = \frac{Y_v}{\frac{1}{2}pl^{2}U}$
- First order coefficient used in representing $Y$ as a function of $v$.

$Y_{vq}$: $Y_{vq}' = \frac{Y_{vq}}{\frac{1}{2}pl^{2}U}$
- First order coefficient used in representing $Y_v$ as a function of $(\eta - 1)$.

$Y_s$: $Y_s' = \frac{Y_s}{\frac{1}{2}pl^{s}}$
- Coefficient used in representing $Y$ as a function of $\gamma$.

$Y_v[r]$: $Y_v[r]' = \frac{Y_v[r]}{\frac{1}{2}pl^{s}}$
- Coefficient used in representing $Y_v$ as a function of $r$.

$Y_v[v]$: $Y_v[v]' = \frac{Y_v[v]}{\frac{1}{2}pl^{s}}$
- Second order coefficient used in representing $Y$ as a function of $v$.
\[ Y_{v|v|q} \quad Y_{v|v|q} = \frac{Y_{v|v|a}}{\rho u^2} \]  
First order coefficient used in representing \( Y_{v|v|} \) as a function of \((q-1)\)

\[ Y_{0r} \quad Y_{0r} = \frac{Y_{0r}}{\rho u^2} \]  
First order coefficient used in representing \( Y \) as a function of \( r \)

\[ Y_{0mp} \quad Y_{0mp} = \frac{Y_{0mp}}{\rho u^2} \]  
First order coefficient used in representing \( Y_{0r} \) as a function of \((q-1)\)

\( \beta \)  
Angle of drift

\( \beta_f \)  
Angle of drift relative to fluid

\( \delta_r \)  
Deflection of flanking rudder

\( \delta_g \)  
Deflection of steering rudder

\( \delta_{rp} \)  
Steady rudder angle at \( i \)th execute in an overshoot or zigzag maneuver; \( i = 1, 2, 3, \ldots \)

\( \dot{\delta}_r \)  
Flanking rudder deflection rate

\( \dot{\delta}_g \)  
Steering rudder deflection rate

\( \xi_{1h} \quad \xi_{1h}' = \frac{\xi_{1h}}{U} \)  
Roots of characteristic stability equation for horizontal plane motions, \( i = 1 \) or 2

\( \phi \)  
Heading or yaw angle

\( \phi_i \)  
Heading angle at \( i \)th execute in an overshoot or zigzag maneuver, measured from value at first execute; \( i = 2, 3, \ldots \)

\( \dot{\phi} \)  
Rate of change of heading

\( \bar{v}_h \quad \bar{v}_h = \frac{v_h}{U} \)  
Height of loop at neutral rudder angle from spiral maneuver

\( \bar{v}_1 \quad \bar{v}_1 = \frac{v_1}{U} \)  
Rate of change of heading at \( i \)th execute in an overshoot or zigzag maneuver; \( i = 2, 3, \ldots \)

\( \omega' \)  
Frequency of oscillation
1.0 INTRODUCTION

The towboat maneuvering simulator is based on the integration in time of the differential equations which describe the motions of the towboat and barge string in three degrees of freedom, i.e., yaw, sway and surge. The theoretical background for these equations are presented in Reference 1, "The Prediction of River Tow Maneuvering Performance," U.S. Coast Guard Report No. CG-D-32-78. This reference presents the basic equations and a complete set of hydrodynamic coefficients for a representative towboat and barge train. These coefficients were obtained by model tests.

This section of the simulator documentation provides a description of the basic equations of motion included in the simulator and the relationships used to determine external forces and moments due to a bow thruster and wind. These equations are completely general in nature and could be used, with the proper hydrodynamic coefficients, to describe the maneuvering of vessels other than towboats.
The mathematical model for the maneuvering of a river tow consists of the coupled differential equations in three degrees of freedom (yaw, sway and surge) which describe the motions in the \(X, Y\) plane and the complete set of hydrodynamic coefficients and external forces which are required in order to numerically integrate these equations. There are also auxiliary equations which describe the response of the steering and propulsion system to external inputs.

A complete set of three coupled differential equations with all of the terms necessary to simulate normal maneuvers of surface ships are presented in Reference 2. These equations have been used by HYDRONAUTICS, Incorporated to calculate the maneuver trajectories for a wide range of surface ship types in deep and shallow water. These equations are based on a more complete set of equations developed by the U.S. Navy for the simulation of submarine motions in six degrees of freedom. The equations in Reference 2 differ from other sets of equations, such as those of Reference 3 used to describe surface ship maneuvers primarily in the way higher order terms are introduced. The equations of Reference 3 use a Taylor expansion which results in odd functions being represented by linear and cubic terms. The equations of Reference 2 are a square absolute representation for higher order terms so that odd functions are represented by linear and square terms. At large drift angles, which is a likely operating condition for a river tow, the forces and moments are dominated by cross flow drag which is proportional to velocity squared. Thus, it is better to use equations in which forces and moments are proportional to velocity squared rather than cubed. As a result, the equations presented in Reference 2 were selected.
to form the basis for the equations which describe the maneuvering of a river tow. There are a number of modifications which must be made to these equations. These modifications and the resulting set of equations are described in the following paragraphs.

The following equations in three degrees of freedom are referred to a right-hand orthogonal system of moving axes, fixed in the body, with its origin normally located at the center of mass of the body. The positive direction of the axes, angles, linear velocity components, angular velocity components, forces and moments are given in Figure 1. Unless otherwise indicated in the Notation, the numerical values for the hydrodynamic coefficients used with the equations are for the ship propulsion point ($n = 1.0$). The equations are written in terms of the complete barge flotilla towboat configuration. Thus the values of the coefficients embrace the interaction effects between rudder and hull, propeller and hull, and propeller and rudder as determined from the model tests of the complete configuration.

An important consideration in the maneuvering of a river tow is the effect of current which can vary significantly along the length of the tow. As a result, it is necessary to introduce the effect of the current velocity into the mathematical model. The approach adopted was to define the hydrodynamic terms in the equations based on the relative velocities and yaw rate between the hull and the fluid rather than the inertial velocities and yaw rate. The relative velocities and relative yaw rate can be calculated by the vector addition of the inertial velocity and inertial yaw rate and the current velocities and current yaw rate. In the numerical integration the procedure is to define a matrix of current speeds ($U_{ci}$) and directions ($\gamma_{ci}$) at points on the $X, Y$ plane. Based on the location of the bow ($X_B, Y_B$) midships
(X_a, Y_a) and stern (X_s, Y_s) of the tow, an interpolation in the current speed and direction matrix is carried out to obtain the current speed and direction at the bow (U_{CB}, \psi_{CB}), midships (U_{CM}, \psi_{CM}) and stern (U_{CS}, \psi_{CS}). The following relationships then apply:

\[
\begin{align*}
U_{CB} &= U_{CB} \cos (\psi_{CB} - \psi) \\
U_{CM} &= U_{CM} \cos (\psi_{CB} - \psi) \\
U_{CS} &= U_{CS} \cos (\psi_{CS} - \psi) \\
U_C &= \frac{U_{CB} + U_{CM} + U_{CS}}{3} \\
V_C &= \frac{V_{CB} + V_{CM} + V_{CS}}{3}
\end{align*}
\]  

\[
r_C = \frac{V_{CB} - V_{CS}}{L}
\]

\[
\begin{align*}
U_R &= u - U_C \\
V_R &= v - V_C \\
r_R &= r - r_C
\end{align*}
\]

In this procedure the mean longitudinal and lateral current velocity in the body axis system is obtained from the average of the values at the bow, midships and stern. The variation of the lateral velocity along the tow is accounted for by the apparent current yaw rate defined by the difference in the lateral velocities at the bow and stern divided by length. This assumes the lateral velocity varies linearly from bow to stern. If this is not the case a more complex relationship would have to be introduced.

The equations of motion formulated for a river tow are as follows:
\[ U_R = \sqrt{u_R^2 + v_R^2} \quad v_R = -U_R \sin \beta_R \quad u_R = U_R \cos \beta_R \]

\[ \beta_R = \arctan \left( -\frac{v_R}{u_R} \right) \]
AXIAL FORCE

\[ m(\ddot{u} - v_r - x_G \dot{x}^2) = \frac{\rho}{2} \dot{\ell}^2 (X_{\dot{u}u} \dot{u}_R + X_{\dot{v}r} \dot{v}_R \dot{r}_R) \]

\[ + \frac{\rho}{2} \ell^2 \left( X_{\dot{u}u} u_R^2 + \frac{\rho}{2} \ell^2 u_R^2 \left( a_1 + \frac{b_1}{2} \eta_s + \frac{c_1}{2} \eta_p + \frac{b_1}{2} \eta_s^2 + \frac{c_1}{2} \eta_p^2 \right) \right) \]

\[ + \frac{\rho}{2} \ell^2 \left[ \frac{X_{\dot{v}v} \eta_1}{2} (\eta_s - 1) + \frac{X_{\dot{v}v} \eta_2}{2} (\eta_p - 1) \right] v_R^2 \]

\[ + \frac{\rho A}{2} \ell^2 X_A X_A u_A^2 + \frac{\rho}{2} \ell^2 \left[ \frac{X_{\dot{u}u}}{2} \left( \frac{X_{\delta_S \delta_S} \delta_s^2 + X_{\delta_F \delta_F} \delta_f^2}{2} \right) + \frac{X_{\dot{v}v}}{2} \left( \frac{X_{\delta_S \delta_S} \delta_s^2 + X_{\delta_F \delta_F} \delta_f^2}{2} \right) \right] \]

\[ (2) \]
LATERAL FORCE

\[ m(\dot{v}t + ur + x_G^\tau) = \frac{\rho}{2} \ell^2 \left( Y_{xv} \ddot{v} + Y_{v|z} \dot{r}_R + Y_{r|z} \dot{r}_R \right) + \frac{\rho}{2} \ell^2 (Y_{xv} \ddot{v}) \]

\[ + \frac{\rho}{2} \ell^2 \left( Y_{xv} u_R r_R + Y_{v|v} \dot{v}_R \right) + \frac{\rho_A}{2} \ell^2 v_A^2 y_A \]

\[ + \frac{\rho}{2} \ell^2 \left( Y_{xv} u_R v_R + Y_{v|v} \dot{v}_R \right) + Y_{v\eta} u_R \dot{r}_R \left( \frac{\eta_s + \eta_p}{2} - 1 \right) \]

\[ + \frac{\rho}{2} \ell^2 \left[ Y_{v|v} \eta \cdot \dot{v}_R \right] + \frac{\rho}{2} \ell^2 Y_{r\eta} u_R r_R \left( \frac{\eta_s + \eta_p}{2} - 1 \right) \]

\[ + \frac{\rho}{2} \ell^2 \left[ u_p^2 \left( \frac{Y_{xv} + Y_{\delta_s} \delta_s + Y_{\delta_F} \delta_F}{2} \right) \right] + u_p^2 \left( \frac{Y_{xv} + Y_{\delta_s} \delta_s + Y_{\delta_F} \delta_F}{2} \right) \]
YAWING MOMENT

\[
I_z \ddot{\theta} + mg_x (\ddot{\theta} - u) = \frac{\rho}{2} I_s \left( N_x \ddot{\theta} + N_v \dot{\theta} + N_r |r| |r_R| r_R \right) + \frac{\rho}{2} I_s N_v \dot{\theta} + \dot{\theta} \nabla \left( \frac{n_s + n_p}{2} - 1 \right) + \frac{\rho}{2} I_s \left( N_v N_v \dot{\theta} + N_v |v| v_R |v_R| \right) + \frac{\rho_a}{2} I_s N_A u_A^2 + \rho \ddot{u}_R^2 u_R^2 \left[ -Y_s \left( \frac{b_1}{2} \eta_s + c_1 \eta_p^2 \right) + Y_p \left( \frac{b_1}{2} \eta_p + c_1 \eta_p^2 \right) \right]
\]

\[u_p^2 = (du_R^2 + eDn_u_R + fD^2 n^2) \text{ where } n = \text{Prop RPM}\]

\[n = \text{Propulsion Ratio} = (n/u_R)/(n_c/u_c)\]
The foregoing equations of motion, as noted previously, are patterned after the quasi-steady state equations of Reference 2. The non-dimensional hydrodynamic coefficients which comprise the basic equations are considered to be independent of speed (Froude number). This assumption is valid since river tows always operate at low Froude number.

These equations differ from the equations of Reference 2 in the following details:

a. They are written in terms of the relative velocities and yaw rate to allow the introduction of varying current as discussed above.

b. Terms are included for steering rudders (steering rudder angle = $\delta_s$) and flanking rudder (flanking rudder angle = $\delta_f$).

c. Terms are included for twin propellers which may operate at different RPM’s and different directions of rotation. The turning moment due to differential thrust is included in the yawing moment, Equation (4).

d. The forces and moments generated by the rudders are based on a velocity defined by

$$U_p^2 = (du_R^2 + eDn_R + fD^2n^2)$$

which is a function of the relative axial velocity, $u_R$, and propeller RPM, $n$. The constants $d$, $e$ and $f$ depend on the sign of $u_R$ and $n$. This allows a proper representation of the rudder forces and moments at zero speed and finite propeller RPM.

In realistic maneuvers, river tows operate both ahead and astern and in some cases at large drift angles. In order to properly represent the hydrodynamic forces and moments which act in such conditions, different sets of hydrodynamic coefficients are
used depending on the relative drift angle $\beta_R$. As will be noted in Reference 1 in which the hydrodynamic coefficients from the model tests are presented, most coefficients depend on the direction of motion, i.e., ahead $270^\circ \leq \beta_R \leq 90^\circ$ or astern $90^\circ \leq \beta_R \leq 270^\circ$. In order to obtain a better representation of the steady sideforce and yaw moment at drift angles near 90 and 270 degrees certain coefficients have an additional value when $30^\circ \leq \beta_R \leq 150^\circ$ or $210^\circ \leq \beta_R \leq 330^\circ$.

As noted in the introduction to this section, a complete set of hydrodynamic coefficients for Equations 2, 3 and 4 for a towboat and barge string are presented in Reference 1. The simulator as presently configured has the hydrodynamic coefficients as constants independent of water depth. Thus, if operations in shallow water are to be simulated, hydrodynamic coefficients applicable to the appropriate depth should be used. As a general rule, the water can be considered as deep if it exceeds the draft of the tow by a factor of 2.5 or 3.

The equations of motion (i.e., equations 2, 3 and 4) are solved stepwise in time in the computer program. In the program the time step, DT is set equal to 1.0 seconds. For a long, relatively slow-moving river tow, a longer time step (2 to 4 seconds) could be used without significantly affecting the results of most maneuvers. The 1.0 second time step was chosen more on the basis of the updating rate for the control display. It was considered desirable to minimize the time delay between a control input and the display response. A 1.0 second time step was considered reasonable on this basis.

At each time step in the solution of the equations of motion, the current velocity at the low, midship and stern of the tow is determined. This calculation is carried out by subroutine CURT.
In the input to the program, the current speed and direction is specified at up to 30 lines or stations across the river. These may be arbitrarily spaced along the river. The river bank is defined in the visual display by lines joining the end points of consecutive stations. As a result, stations should be concentrated in the area of bends or rapid changes in river width. The current speed and direction are also likely to change rapidly in such areas.

At each station, the current speed and direction is specified at 8 evenly spaced points. (see Figure 13 of Volume I). Subroutine CURT determines the current speed and direction at bow, midship and stern by searching the stations in sequence. At each station, the smallest distance between the vessel point and one of the 8 station points is determined. Initially this process is repeated until the minimum distance point is found. The current defined at the closest point is then used in calculations. After the initial time step the search is carried out locally around the station point found to have the minimum distance during the previous time step. This local search technique saves significant computer time.
3.0 EXTERNAL FORCES

3.1 Bow Thruster

The towboat maneuvering simulator contains the provision for a bow thruster. The forces and moments generated by the bow thruster are represented by the following relationships:

Thruster Lateral Force -

\[ F_{Yo} = (T_{MAX}) \times (\text{Percent Output}) = \text{THMAX} \times \text{TRPM} \]

\[ F_{Yo} = \text{Y force at zero forward speed} \]

Percent output ranges from -1.0 to 1.0

The lateral forces of a tunnel type bow thruster decreases with forward speed. Thus, the lateral force introduced into the lateral force Equation (3) is given by:

\[ F_{XP(3)} = F_Y = F_{Yo} (\text{TRC}(1) + \text{TRC}(2) \times \cos (2\pi \frac{u}{V_j})) \] \hspace{1cm} (5)

for \( u/V_j \leq 0.5 \)

\[ = F_{Yo} ((\text{TRC}(1) - \text{TRC}(2)) + \text{TRC}(3) \times (u/V_j - 0.5)) \]

for \( u/V_j > 0.5 \)

where

\[ u = \text{axial speed of vessel} \]

\[ V_j = \text{thruster jet or exit velocity} \]

\[ V_j = (\frac{F_{Yo}}{\rho A})^{\frac{1}{2}} \]

\[ \rho = \text{mean density of water} \]

\[ A = \text{thruster Disc Area} \]
Typical constants for a tunnel bow thruster based on Reference 5 would be:

\[ \text{THMAX} = 10,000 \text{ lbs} \]
\[ \text{TRC(1)} = 0.65 \]
\[ \text{TRC(3)} = 0.18 \]
\[ \text{CTHV} = 1/\rho A = 0.025 \]

**Thruster Yawing Moment**

Because of interaction forces with the hull the yawing moment due to the thruster does not change with forward speed in the same way as the lateral force. The yawing moment due to the thruster which is introduced into the yawing moment equation is given by:

\[
N = F_Y X_c \left( 1 - (1 - 0.67 \frac{u}{VJ}) \left( 1 - \frac{F_Y}{F_Y^o} \right) \right)
\]  (6)

\[ N = \text{Yawing Moment due to thruster} \]
\[ X_c = \text{Longitudinal location of thruster} = YCSP(3) \]

### 3.2 Wind Forces and Moments

The forces and moments acting on the towboat and barges due to wind were calculated using methods and data given in Reference 4. The forces and moment at a given time are calculated using the instantaneous relative wind velocity.

The axial and lateral forces and yawing moment due to wind are given by:

\[
X_w = -\rho C_{xw} V^2 A_p \cos^2 \theta \beta = FX(2)
\]  (7)
\[
Y_w = \rho C_{yw} V^2 A_p \sin^2 \theta \beta = FY(2)
\]  (8)
\[
N_w = Y_w X_{LCP} = FN(2)
\]  (9)
where

\[ X_w \text{ and } Y_w \] are longitudinal and lateral forces (in ship coordinates) due to wind

\[ N_w \] is yawing moment about the c.g. due to wind

\[ \rho_a \] is air mass density

\[ C_{xw} \text{ and } C_{yw} \] are longitudinal and lateral drag coefficients

\[ A_{px} \text{ and } A_{py} \] are longitudinal and lateral projections of above water hull and deckhouses

\[ V_{we} \] is effective wind velocity = \( V_w R \)

\[ \beta_{we} \] is angle measured from the ship x-axis to the effective wind velocity vector = \( \beta_w \)

\[ \lambda_{cp} \] is longitudinal position of the center of pressure of the wind force

The effective wind velocity, including the effect of the boundary layer at the water surface, deduced empirically in Reference 4, is:

\[ V_{we} = \left( \frac{z_{ca}}{32.81} \right)^{0.15} \left( V_{wx}^2 + V_{wy}^2 \right)^{1/2} \]

or

\[ = \left( \frac{z_{ca}}{32.81} \right)^{0.15} \left[ (u + V_w \cos \beta_w)^2 + (v + V_w \sin \beta_w)^2 \right]^{1/2} \] (10)

where

\[ z_{ca} \] is vertical center of area of the ship

\[ V_{wx} \text{ and } V_{wy} \] are longitudinal and lateral components of the effective wind velocity

\[ u \text{ and } v \] are components of ship velocity

\[ V_w \] is actual wind velocity

\[ \beta_w \] is wind heading angle measured from ship x-axis to actual wind velocity vector
The angle $\beta_{we}$ is given by

$$\beta_{we} = \tan^{-1} \left( \frac{u + V_w \cos \beta_w}{v + V_w \sin \beta_w} \right)$$

The center of pressure is determined from the longitudinal center of area, using empirical methods and data from Reference 4 and is:

$$x_{Lcp} = L_{BP} \left[ 3 \frac{x_{ca}}{L_{BP}} - (0.001 + 0.01 \ AR) \left( |\beta_{w}'| - 90 \right) \right]$$

where

- $L_{BP}$ is ship LBP
- $x_{ca}$ is longitudinal position of the center of lateral area $= XLCA$
- $\beta_{w}'$ is the value of $\beta_{we}$ in degrees $= BW$
- $AR$ is hull aspect ratio, $AR = 2A_{py}/L_{BP}^2 = AS$

Based on the data given in Reference 4 and estimated above water hull and deckhouse shapes, the following wind force coefficients would apply to the towboat and barges configuration for which hydrodynamic data are reported in Reference 1.

$$L_{BP} = 745 \text{ ft}$$
$$x_{ca} = 58.8 \text{ ft}$$
$$AR = 0.0167$$
$$C_{xw} = 1.0$$
$$C_{yw} = 1.0$$
$$A_{px} = 1520 \text{ ft}^2$$
$$A_{py} = 4635 \text{ ft}^2$$
$$Z_{ca} = 6.2 \text{ ft}$$
3.3 Other External Forces

At this time no other external forces are included in the simulator. In the future, other external forces such as bank suction, interaction with passing vessels and the effects of mooring lines could be included.
4.0 CONTROL SYSTEMS

The control systems included in the towboat maneuvering simulator include steering control for the steering and flanking rudders, RPM control for the port and starboard propellers and the bow thruster output control. The command inputs for all of these controls are provided thru the computer A to D input from the control station. In the simulator, the response to the control inputs is at a constant rate. The rates used based on Reference 6 are as follows:

- Steering and Flanking Rudders: 5 deg/sec
- Propeller RPM: 20 RPM/sec
- Thruster Output: 10 percent thrust change/sec

The response to the control system inputs is considered representative of a typical towboat. If required in the future, a more complex representation of the response to control inputs could be modeled in the simulator.
REFERENCES


5. Chislett, M.S., and Bjorhkorn, O., "Influence of Ship Speed on the Effectiveness of Lateral-Thrust Unit", Report No. HY-8; HYA, Lyngby, Denmark, April 1900.