PERFORMANCE OF OPTICAL RECEIVERS IN DETECTION OF VARIABLE OUTPUT
PERFORMANCE OF OPTICAL RECEIVERS IN DETECTION OF VARIABLE DUTY CYCLE MANCHESTER CODED DATA

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PERFORMANCE OF OPTICAL RECEIVERS IN DETECTION OF VARIABLE DUTY CYCLE MANCHESTER CODED DATA

THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air Training Command in Partial Fulfillment of the Requirements for the Degree of Master of Science

by

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Preface

This study was undertaken at the request of the Systems Avionics Division System Development Group of the Air Force Avionics Laboratory at Wright-Patterson Air Force Base. The System Development Group had received conflicting recommendations from contractors who had independently studied the prospects of variable duty cycle Manchester coding. This report provides an additional independent recommendation arrived at by basic receiver modeling and sound statistical theory. I hope the report is understandable, sensible, and valuable in resolving the point in question.

Special thanks go to Captain Stanley R. Robinson, my advisor, for his consistent motivation and encouragement, to Captain Danny E. Eklund for his careful reading, and to Miss Sheri Vogel for her fine typing job. This thesis is dedicated to the future of Corinne Nicole Cilley.

ROBERT A. CILLEY
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Abstract

The goal of this paper is to determine whether the probability of error in bit detection by optical receivers can be improved through the use of a variable duty cycle Manchester modulation format and proper receiver design. It has been speculated that a short source duty cycle would improve receiver performance, particularly when the source average power is constant. The parameters assumed to affect the probability of error include optical power, temperature, pulse width, amplifier gain, signal shot noise, thermal noise, and the circuit parameters of resistance and capacitance.

Performance predictions are made for the optimum receiver of signals in additive white noise, the matched-filter receiver, and two suboptimum receivers. The first suboptimum receiver is nothing more than a detector and an FET amplifier. The second consists of the same components with an equalizing filter added. Graphical data showing performance under extreme variations of the parameters listed above indicate that the matched-filter receiver is far superior to the suboptimum receivers. The equalized receiver performs better than the basic detector/amplifier receiver only under the most favorable operating conditions. The performance variation with source duty cycle varies with operating conditions. Under worst conditions the maximum duty cycle is recommended.
PERFORMANCE OF OPTICAL RECEIVERS IN DETECTION OF VARIABLE DUTY CYCLE MANCHESTER CODED DATA

I. Introduction

The System Avionics Division System Development Group, of the Air Force Avionics Laboratory, is currently sponsoring an effort to determine fiber-optic data bus receiver requirements for optimal performance in military avionic applications. A fiber-optic data bus is desirable because of its high bandwidth, low loss, small size and weight, ruggedness and flexibility, low cost, electrical isolation, and immunity from nuclear radiation and other electromagnetic and radio frequency interference [Ref. 1]. These features are invaluable when applied in military aircraft, where the amount and complexity of the avionics are greater than ever.

Requirements for military avionic data buses are set forth in MIL-STD-1553B (USAF) [Ref. 2]. The standard specifies a Manchester modulation format. Figure 1 illustrates this format as adapted for use with optical (unipolar) pulses. A bit period, which is $2T_p$ seconds long, is divided into two time slots. A unipolar pulse is transmitted in one of the two time slots depending on whether the source bit is a ONE or a ZERO. A ONE is represented by a pulse in the first time slot. A pulse in the second time slot indicates a data ZERO.

The study done in this paper is motivated by the possibility that the receiver performance can be improved by shortening the duty cycle of the optical source. Figure 2 illustrates the variable duty cycle Manchester
Figure 1. Unipolar Manchester Modulation Format.

Figure 2. Variable Duty Cycle Manchester Modulation Format.
format, in which the duration of the pulse is a fraction, x, of the full duty cycle Manchester pulse. If the source maintains a constant average power output, a decreased duty cycle means increased peak power at the sampling instant. The shorter pulse also means that the receiver bandwidth must be increased to prevent distortion. A study of the noise power at the output of the receiver is required to determine whether the noise penalty incurred by such an increase in bandwidth is too great for an improvement in performance to be realized.

The goal of this paper is to determine whether the probability of error in bit detection can be improved through use of the variable duty cycle format and proper receiver design. The parameters which affect the probability of error include available optical power, length of optical pulse, amplifier gain, dark current, signal shot noise, thermal noise, and the receiver circuit parameters of resistance and capacitance.

Assumptions

Several assumptions must be made to simplify the analysis and limit the scope of the study. Since the chief goal is to minimize performance degradation as pulses are detected by the receiver, it is assumed that the pulses arrive in "perfect" condition. That is, they are taken to have whatever form is convenient (square pulses in this case) so that the effect of propagation through the receiver may be easily shown. The implications of square optical pulses at the detector are that the optical source has infinitesimal switching times and that the optical fiber and couplers contribute no amplitude or phase distortion. The source is further assumed to be either peak or average power limited. In the former case the amplitude of each optical pulse is constant regardless of the duty cycle. In the
latter case the pulse amplitude varies with duty cycle so as to maintain a constant average power. Modern sources are capable of producing very short pulses (subnanoseconds) with very high peak amplitudes (tens of watts). Heating of such sources and fundamental limitations on their bandwidths render the average power limitation assumption invalid for very small source duty cycles. The laser source, for example, has an emission efficiency and a laser threshold which vary exponentially with temperature [Ref. 3]. In addition, the width of the pulse in the time domain is inversely proportional to the (finite) oscillation width of the laser [Ref. 4].

The electronic signals produced by the receiver are assumed to be directly proportional to the incident optical power, which is assumed to be noiseless. Signal shot noise and dark current are introduced by the detector, and thermal (Johnson) noise is introduced by biasing resistors and the amplifier.

A simple level detector is assumed to respond to the output of the receiver. The signals received originate from a single source. There is no interference from other sources on the data bus.

Plan of Attack

Study of the roles of the various system parameters is simplified by first examining the detection of a single bit which is unperturbed by intersymbol interference. (Intersymbol interference is the overlapping of pulses which have been spread by filtering in the channel and the receiver.) Comparison of the performances of different types of receiver will be made. When the conditions of best performance are known for these receivers, the study may be expanded to include the effects of intersymbol interference.
A realistic noise equivalent circuit will be developed for the detector/amplifier combination. The equivalent circuit will allow the determination of the characteristics of the noise at the amplifier output.

The probability of error in detection of a Manchester bit is minimized when the peak amplitude of the pulse (in either time slot) is maximum and when the variance is minimum for both pulse present and pulse absent events. Therefore, the receiver performance will be compared in terms of a ratio of the pulse peak amplitude and the sum of the pulse present and pulse absent variances.

The first receiver type to be studied employs no post-amplifier filters, and will be referred to as a "non-filtered" receiver. The threshold detector operates on the low-pass filtered output of the amplifier. The second type uses an equalizer. An equalizer is an additional network having a frequency characteristic such that the total attenuation of the two networks in cascade will be independent of frequency over the bandwidth of interest [Ref. 5:505]. The output pulse shape is improved in such a receiver, but a significant noise penalty is paid. Finally, the receiver which yields a maximum output in response to an input corrupted by additive white noise, the matched-filter receiver, will be studied. Present specifications in MIL-STD-1553B preclude the use of a matched-filter receiver because of synchronization problems. The matched-filter receiver performance will provide a benchmark against which the other receivers can be compared. Should a synchronous fiber-optic system be developed, its performance would be predicted by these results.

The amplitude-variance ratios of each type of receiver can be maximized through the choice of system parameters. Variation of each ratio will be illustrated for dark current and signal shot noise, and thermal noise limited receivers.
Real receivers are designed to detect sequences of pulses, not solitary pulses. If the transfer function of a receiver is such that intersymbol interference occurs, its effect on performance must be taken into account. A discussion of this problem and its solution through the concept of correlative coding will follow the determination of the receiver performance in solitary pulse detection.

In the following sections a noise model for the basic receiver will be developed, and the probability of error in bit detection for non-filtered, equalized, and matched-filter receivers will be expressed. The receiver performances will be compared and recommendations of receiver type, receiver parameters, and optical source duty cycle will be made.
II. Basic Receiver Model

The basic receiver model [Ref. 6] is shown in Figure 3. A photodiode converts the received optical pulses to a signal photocurrent, $i_s$. This photocurrent is corrupted by two noise sources, shown in Figure 3 as one current source and one voltage source (referred to the input of the amplifier). These are assumed to be independent Gaussian sources to simplify the analysis. An amplifier and a threshold detector circuit follow the photodiode.

Detector/Amplifier Noise Equivalent Circuit

The detector/amplifier combination to be studied employs a photodiode and a transimpedance amplifier with a field-effect-transistor (FET) front end. Its circuit diagram was shown in Figure 3. The combination noise equivalent circuit is shown in Figure 4. The photodiode is modeled as a capacitive current source with an output, $P/x$, proportional to the incident optical power. The constant of proportionality, $r$, is the responsivity (in amperes per watt of incident optical power) of the detector. Since responsivities between 0.5 and 1.0 are available, and for convenience, a responsivity of 1.0 will be assumed throughout this paper. A current source, $i_{DN}$, represents the signal shot noise and dark current. The FET amplifier is dominated by thermal noise generated within the FET channel [Ref. 7]. The channel thermal noise and the thermal noise due to the parallel combination of detector, amplifier, and bias resistances are combined in a current source, $i_{TN}$.

In order to evaluate the receivers some typical values, or ranges of values, of the system parameters must be assumed. First, a bit rate of 5 megabits per second is assumed. For a Manchester code this corresponds
Figure 3. Basic Receiver Model.

Figure 4. Detector/Amplifier Noise Equivalent Circuit.
to a sampling rate of 10 MHz, for which $T_p$ is 100 nsec [Ref. 8]. The photodiode is assumed to have a responsivity of 1.0 amp/watt and a dark current between 1 and 100 na (at $T=25^\circ$C) [Ref. 9]. Transconductance in an FET typically ranges from 10 to $10^4$ $\mu S$ (micro-siemanns) at 25°C [Ref. 10]. Typical values of shunt resistance and capacitance are 1 M$\Omega$ and 3pf respectively [Ref. 8]. (A range of resistance from $R=10^4\Omega$ to $R=10^8\Omega$ will be studied.) The assumed range of required optical power at the output of the fiber will be from 1 mwatt to 10 $\mu$watts. This corresponds to a range found to produce bit error rates in the vicinity of $10^{-12}$ in a report by Harris Corporation [Ref. 11:3-37]. Finally, any receiver that is recommended must be superior over the entire range of temperatures likely to be found in avionic circuit board environments. Present standards [Ref. 12] specify a range (for Class III equipment) of -55°C to +125°C. The range studied here will be from +25°C to +125°C with the assumption that the performance is only improved as the temperature is lowered.

**Output Variance**

The noise at the input of the receiver is nonstationary. That is, it depends on whether a pulse is present or absent. This section presents the variances at the output of the amplifier for both cases.

With no pulse present, the input noise is a composite of dark current and thermal noises. The power spectral density of this component is [Ref. 6]

$$S_n(f) = qI_D + \frac{4kT}{R} + \frac{2.8kT}{g_m} \left[ \frac{1}{R^2} + (2\pi f)^2 \right]$$

(1)

where $R$ = total combined shunt resistances of the detector, amplifier, and bias resistors (ohms)
$C =$ total combined shunt capacitance of the detector and amplifier (farads)
$q =$ charge of an electron ($1.6\times10^{-19}$ coulombs)
$k =$ Boltzmann's constant ($1.38\times10^{-23}$ joules per degree Kelvin)
$T =$ temperature (degrees Kelvin)
$g_m =$ FET transconductance (siemens)
$I_D =$ dark current (amperes)

When the pulse is absent, dark current and thermal noise are the only components of the input noise. The variance of the amplifier output, which will be denoted by $\sigma_n^2$, is the integral over a bandwidth of at least $1/xT_p$ Hz of the product of the noise power spectral density above and the square of the receiver amplitude response. (The assumed significant bandwidth extends to the first zero-crossing of the pulse spectral density function.)

When the pulse is present the input noise also contains a signal shot noise component. In this case the amplifier output variance, which will be denoted by $\sigma^2$, is the same as that described above for pulse absent with an additional component of $qP/x\int h^2(t-t)dt$ [Ref. 13:113]. The function $h(t)$ is the output response of the detector/amplifier to a unit impulse of optical power at the input.

The amplifier output noise is "colored" and varies as the third power of the desired bandwidth. It is not generally desirable, then, to increase the amplifier bandwidth in hopes of improving the quality of the signal. The following chapter will use these output noise variances in an expression of the receiver probability or error in bit detection. Later sections will consider which system parameters can be varied in order to improve the probability of error.
III. Probability of Error in Bit Detection

The performance of the receiver can be measured by its probability of error in bit detection \( (P_E) \). For unipolar Manchester coded data, the probability that one bit is detected in error is the probability that one or both of the voltages received during the period \( 2T_p \) is in error. In symbols,

\[
P_E = P(O)[P(v_1>\beta)+P(v_2<\beta)+P(v_1>\beta)P(v_2<\beta)\text{ZERO sent}] \\
+P(1)[P(v_1<\beta)+P(v_2>\beta)+P(v_1<\beta)P(v_2>\beta)\text{ONE sent}]
\]

(2)

where \( P(O) \) and \( P(1) \) are the respective probabilities that ONEs and ZEROs are sent, \( v_1 \) and \( v_2 \) are the voltages received during the first and second halves of the bit period, and \( \beta \) is the decision threshold. If ONEs and ZEROs are equally likely \( (P(O)=P(1)=\frac{1}{2}) \) and \( \beta \) is chosen so that \( [P(v_1>\beta)\text{ZERO sent}] = [P(v_2<\beta)\text{ZERO sent}] = [P(v_1<\beta)\text{ONE sent}] = [P(v_2>\beta)\text{ONE sent}] \), then the overall \( P_E \) is (see Appendix A.)

\[
P_E = \sqrt{\pi} \text{ erfc} \left( \frac{s(xT_p)}{\sqrt{2}\sigma_0} \right) + \frac{\pi}{4} \text{ erfc} \left( \frac{s(xT_p)}{\sqrt{2}(\sigma_1+\sigma_0)} \right)^2
\]

(3)

where \( s(xT_p) \) is the output sample given that a pulse is present, \( \sigma_0^2 \) and \( \sigma_1^2 \) are the variances as described in the previous section, and

\[
\text{erfc}(w) = \frac{2}{\pi} \int_w^\infty \exp(-z^2)dz.
\]

The probability of error will be minimum for the maximum of the ratio in the argument of the complementary error function above. This argument will be referred to simply as ARG. In the next section expressions for ARG will be developed for two suboptimum receiver types, the non-filtered and equalized receivers, and an analogous expression will be developed for the matched filter receiver.
Non-Filtered Receiver

The response, $h(t)$, at the output of the receiver without post-amplifier filters, to a unit impulse of optical power at the input is

$$h_N(t) = \frac{1}{C} \exp(-t/RC)U(t) \quad (4)$$

where $U(t)$ is the unit step function. The corresponding transfer function is

$$H_N(f) = \frac{R}{1+j2\pi fRC} \quad (5)$$

Note that there is a pole at the cutoff frequency, $1/RC$ Hz.

The response to a current pulse of amplitude $P/x(rP_0/x)$ and duration $xT_p$ seconds is

$$v(t) = \begin{cases} (PR/x)[1-\exp(-t/RC)] & 0 < t < xT_p \\ (PR/x)[\exp(xT_p/RC)-1] \exp(-t/RC) & t > xT_p \end{cases} \quad (6)$$

With no pulse present the output variance is

$$\sigma_0^2 = \int_{-1/xT_p}^{1/xT_p} S_n(f)|H_N(f)|^2 df$$

$$= \left(4kT+qI_DR\right) \tan^{-1} \left[ \frac{2\pi RC}{xT_p} \right] + \frac{5.6kT}{g_mxT_p} \quad (7)$$

When a pulse is present the output variance is

$$\sigma_1^2 = \sigma_0^2 + \frac{q(PR/x)}{2C} \left[1-\exp(-2xT_p/RC)\right] \quad (8)$$
The response of this receiver may be improved by equalizing (correcting the RC characteristic). However, equalizing introduces more noise. Pulse amplitude and output variance expressions will be developed for the equalized receiver in the next section.

Equalized Receiver

Recall that to correct frequency distortion in a transmission network, an additional network, an "equalizer", must be added so that the total attenuation of the two networks in cascade is independent of frequency over the bandwidth of interest. A network which will adequately equalize the output of the amplifier will cancel the effect of the pole in the network transfer function at $1/RC$ Hz by introducing a zero at the same frequency. The transfer function of any such realizable filter will have poles at higher frequencies. The filter could be designed so that these poles occur at $1/\times T_p$ Hz, thus passing the most important frequency information of the pulse with no distortion. The equalized version of the non-filtered network of the previous section, then, has a frequency response of

$$H_E(f) = H_N(f) \times H_{EQ}(f) = 1 \quad (|f| < 1/\times T_p \text{ Hz})$$

(9)

where the unit amplitude is chosen arbitrarily. The amplitude of the voltage pulse at the output of the receiver is then $P/x$, and the variances are (assuming that any resistance in the equalizer can be chosen so that the additional thermal noise is negligible)

$$\sigma_0^2 = \left[ \frac{2.8kT}{g_m R^2} + \frac{4kT}{R} + qI_D \right] \times \frac{2}{\times T_p} + \frac{2.8kT}{g_m} \left(2\pi C\right)^2 \times \frac{2}{3(\times T_p)^3}$$

(10)

and

$$\sigma_1^2 = \sigma_0^2 + qP/x$$

(11)
Enough information is now available for comparison of the complementary error function arguments, ARG, for the two suboptimum receivers. The comparison will be done numerically following the development of an analoguous expression for the matched-filter receiver.

**Matched-Filter Receiver**

The receiver which yields a maximum output in the presence of additive white noise is the matched-filter receiver [Ref. 14:234-245]. The noise which is under study, however, is non-white. In order to take advantage of the matched-filter concept a "whitening" filter may be introduced to the non-filtered receiver. This filter distorts the signal, as well as the noise, so the matched filters must be matched to the distorted signal.

Analysis of the matched-filter receiver is greatly simplified if Fourier techniques can be used. The analysis requires the use of the noise power spectral density. Because the noise is non-stationary (it depends on whether the signal is present or absent), however, the noise power spectral density technically does not exist [Ref. 14:183].

The noise may be taken as approximately stationary by assuming that the signal shot noise exists during the entire Manchester bit period whether a pulse is present or not. The actual noise will never be any worse than this, so there is no possibility of over-optimistic results.

The probability of error in detection of pulses in stationary non-white noise is [Ref. 15:317]

\[
(P_e)_M = \sqrt{\frac{\pi}{2\pi}} \text{erfc} \left( \frac{1}{2\sqrt{\pi}} \left[ \int_{-\infty}^{\infty} \frac{|S(f)|^2}{S_{nw}(f)} \, df \right]^{1/2} \right)
\]  

(12)
where \( S(f) \) is the Fourier transform of the \( xT_p \) second-long square pulse, and \( S_{nw}(f) \) is the non-white noise power spectral density defined by

\[
S_{nw}(f) = S_n(f) + qI_s
\]  

(13)

The signal shot noise component, \( qI_s \), is now assumed to be constant. It is defined by

\[
I_s = q \frac{P}{x}
\]  

(14)

Evaluation of the argument of the complementary error function above yields

\[
\text{ARG}_M = \frac{P/x}{2\pi^{3/2}\sqrt{2}} \left[ -\frac{\pi x T_p}{A^2} - \frac{B}{2A^{3/2}} \left[ 1 - \text{Bexp}(-2\pi x T_p \sqrt{A/B}) \right] \right]^{1/2}
\]  

(15)

where

\[
A = \frac{4kT}{R} + \frac{2.8kT}{g_m R^2} + q(I_D + \frac{P}{x})
\]

and

\[
B = 2.8kT(2\pi C)^2/g_m
\]

The exponential term has a maximum value of one, and its factor, \( B \), is negligible in comparison to one for the ranges of temperature and transconductance that are likely to be encountered. The task, then, is to maximize the remainder of the argument above. An approximation of \( \text{ARG}_M \) is

\[
\text{ARG}_M \approx \frac{P/x}{2\pi^{3/2}\sqrt{2}} \left( -\frac{\pi x T_p}{A^2} - \frac{\sqrt{B/A}}{2A} \right)^{1/2}
\]

(16)

The probability of error in detection of a full Manchester bit is the probability that one or both of the parts of the bit are detected incorrectly.

In symbols, for a matched-filter receiver
\[
(P_e)_M = 2\sqrt{\pi} \text{erfc}(\text{ARG}_M) + \pi[\text{erfc}(\text{ARG}_M)]^2 \tag{17}
\]

Because of synchronization and delay problems in military avionic communication systems, the designers of such systems have standardized the use of asynchronous signaling. If these problems could be overcome, there would be no need to study the suboptimum receivers. In the following chapter these receivers will be compared to each other, as well as to the optimum receiver, and conclusions about their relative qualities will be made.
IV. Observations

To this point a model of the fiber-optic system, expressions for the performance of three particular types of optical receiver, and a set of typical values of the system operating parameters have been presented. These developments may now be used to numerically evaluate and compare the receivers.

Signal shot noise is most influential in all three receiver types when the optical power is high, the temperature is low, and the FET transconductance is high. Figure 5 shows plots of ARG in this case for all three of the receiver types for peak as well as average power limited sources. In no case is it seen that the signal shot noise causes the average power limited system to perform worse than the peak power limited system (for the range of duty cycle shown). Further efforts, therefore, may be concentrated on the average-limited case.

As observed in Figure 5, the matched filter receiver is by far the most desirable of the types being studied here. When the received power is high, Figure 5 indicates that a matched-filter receiver and a source with a large duty cycle produce the lowest probability of error for both peak and average power limited sources. Figure 6 shows that when the received power is low, however, the average power limited source with a small duty cycle produces better results. As seen in Figure 6, $\text{ARG}_M$ for the average power limited source peaks as the duty cycle is brought down to about $10^{-2}$.
Figure 5. Variation of ARG when Signal Shot Noise is Most Significant (high source power).
Figure 6. Variation of ARG when Signal Shot Noise is Most Significant (low source power).
If the matched-filter receiver cannot be used because of synchronization problems, another receiver must be chosen. The remainder of this section is devoted to the comparison of the performances of the non-filtered and equalized receivers.

Variation with Signal Shot Noise

The first question to be asked about these suboptimum receivers is whether the signal shot noise is strong enough to significantly affect the probability of error at high power. We again consider the operating conditions of Figure 5. Figure 7 depicts the same situation for average power limited sources with three values of resistance. The available power is assumed to be low (1 mwatt). Figure 8 shows the same situation for high power (10 μwatts). Apparently, the non-filtered receiver is more affected by signal shot noise, but even with an increase in power by 40 db, the maximum difference between the changes in ARG is only about 3 db. Below about 10 μwatts, then, signal shot noise is negligible. The analysis may then be continued for systems in which signal shot noise is not a factor (low received power).

Variations with Dark Current and Channel Thermal Noise

The next question concerns the sensitivity of the suboptimum receivers to variations in photodiode dark current and amplifier transconductance. These parameters are functions of temperature and vary as

\[ I_D(T) = I_{D0} \times 2^{\frac{T-T_0}{T_0}} \]  

\[ g_m(T) = g_{m0}(\frac{T}{T_0})^{-3/2} \] (18) (19)
Figure 7. Variation of ARG when Signal Shot Noise is Most Significant (low source power, suboptimum receivers only).
Figure 8. Variation of ARG when Signal Shot Noise is Most Significant (high source power, suboptimum receivers only).
Because dark current and channel thermal noise are larger at high temperatures, their effects will be studied at the upper temperature limit, 125°C.

The plots in Figures 9 and 10 show the variation in ARG for conditions under which the dark current is most significant. (The receivers are dark current limited.) These conditions are high transconductance and high temperature. Figure 9 is done for $I_{D0} = 1$na. Figure 10 is done for $I_{D0} = 100$na. The non-filtered receiver is affected more than the equalized receiver by increasing dark current; especially at high resistances. The non-filtered receiver with $R = 1M\Omega$ and a source duty cycle of 0.1 maintains its performance better, however, and would be the choice when the dark current is high. When the dark current is low this same receiver performs only slightly worse than the best equalized receiver, where $x=1$.

The effect of variations in $g_{mo}$ are shown in Figures 11 and 12. Transconductance is most significant when the dark current is low and the temperature is high. In Figure 11, $g_{mo}$ is $10^4\mu S$. In Figure 12 it is reduced to $10\mu S$. It is expected that, due to the $x^3$ term in $ARG_E$, the performance of an equalized receiver would improve faster with duty cycle than that of the non-filtered receiver. Figure 11 confirms that for large resistances and duty cycles the equalized receiver performs better than the non-filtered receiver when $g_{mo}$ is high. The best equalized receiver performance, at $x=1$, is only slightly better than the best non-filtered receiver performance, at $x=0.1$.

It is important to observe in Figure 12 that the non-filtered receiver argument is about 5 db higher, for $R=10^6$ and $x=1$, than the equalized receiver when the transconductance is low. When the transconductance is high the non-filtered receiver performance with $x=0.1$ is only slightly worse than that of the equalized receiver with $x=1$. 

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Figure 9. Variation of ARG when Dark Current is Most Significant (low \(I_D\)).
Figure 10. Variation of ARG when Dark Current is Most Significant (high $I_D$).

\[ P = 10^{-9} \text{ W} \quad T = 398^\circ \text{K} \]
\[ g_m = 10^{-2} \text{ S} \quad I_D = 10^{-7} \text{ A} \]
Figure 11. Variation of ARG when Transconductance is Most Significant (high $g_m$).

$P = 10^{-9}$ W  $T = 398^\circ$K
$g_m = 10^{-2}$ S  $I_D = 10^{-9}$ A

LETTERs - N-FILTERED
NUMBERS - EQUALIZED
A. $1 + R = 1E4$  B. $2 + R = 1E6$
C. $3 + R = 1E8$
Figure 12. Variation of ARG when Transconductance is Most Significant (low $g_{mo}$).
Conclusions Concerning the Suboptimum Receivers

The preceding numerical analysis has shown the variations in performance to be expected when varying the system parameters. Conclusions may now be drawn about the better suboptimum receiver of single pulses.

It is logical to choose detectors with very low dark currents and amplifiers with very high transconductances. The monetary cost of such choices may be prohibitive, however. For this reason the recommendations to be made must be conservative.

The most ideal situation is depicted in Figures 13 and 14. In this case the transconductance is high and the dark current is low. At high duty cycles, and for both high and low temperatures, the best equalized receiver performs better than the best non-filtered receiver. At 25°C, and for high resistance, $\text{ARG}_E$ at $x=1$ is about 6 db higher than $\text{ARG}_N$ at $x=0.1$.

Without the luxuries of large transconductance and small dark current, the story is different. Figures 15 and 16 show this case at both extremes of the temperature range. The transconductance is not enough to overcome the noise penalty incurred by the wider equalized frequency response. Under these conditions the non-filtered receiver performs better than the equalized receiver. The best non-filtered receiver performance occurs at high duty cycles.

Under the most ideal conditions, and particularly when the amplifier transconductance is high, it has been shown that the equalized receiver comes closer than the non-filtered receiver to the optimum matched-filter receiver performance in the detection of Manchester data. There are two reasons why the equalized filter should not be recommended, however.
Figure 13. Variation of ARG under Most Ideal Conditions (low temperature).
Figure 14. Variation of ARG under Most Ideal Conditions (high temperature).
Figure 15. Variation of ARG under Worst Conditions (high temperature).
Figure 16. Variation of ARG under Worst Conditions (low temperature).
First, the unusually large transconductance of $10^4$ μS used here is difficult to achieve, and increases the cost of the receiver. Second, the equalized receiver will tend to oscillate. Feedback impedances between the gate and drain of the FET, for example, will allow the closed loop gain of the system to reach one at high frequencies. [Ref. 16:175]. Because the non-filtered receiver performances observed are not significantly worse than those of the equalized receiver, and because of the inherent problems with equalization listed above, the non-filtered receiver should be used in an asynchronous system. Furthermore, in a properly designed system with a non-filtered receiver, maximum duty cycle pulses produce the best performance.
V. A Solution of the Intersymbol Interference Problem in the Non-Filtered Receiver

The previous discussion has been limited to consideration of detection of a single pulse, free from interference from all preceding pulses. Interference will always exist, however, in the non-filtered receiver. Its capacitance causes a "residual" voltage to exist at the times that subsequent pulses are due to be sampled. This residual voltage is what is called intersymbol interference.

A reasonable criterion for the significance of the residual voltage is that it exceeds an arbitrary percentage of the peak signal value. We may avoid all interference by constraining the residual level due to an immediately preceding pulse. The ratio, $p$, of the residual voltage level due to a pulse in the $N$th previous time slot to the maximum value of the present pulse is

$$p = \frac{\exp(x T_p / RC) - 1}{1 - \exp(-x T_p / RC)} \exp[-T_p(x+N)/RC]$$

(20)

Using the assumed values of $T_p$ and $C$, and taking $R=1M\Omega$ and $x=1$ for best non-filtered receiver performance, the value of $p$ for $N=1$ is found to be 0.967. The intersymbol interference is certainly excessive!

A technique for improving the performance of digital communication systems by using intersymbol interference in a controlled way was advanced by Adam Lender in the 1960's [Ref. 17]. The technique, called correlative coding, uses the knowledge of the amount of interference due to previous pulses to determine the nature of the present pulse.
When the binary signals to be transmitted consist of positive or zero amplitude pulses, a possible correlative coding scheme is described as follows: Adjust the transmitted amplitude of the present pulse so that the sum of the received pulse amplitude and the amplitudes of the residual voltages due to all significant previous pulses is one constant if the present pulse amplitude is positive, or another constant if the present pulse amplitude is zero. In mathematical symbols,

\[ b_{0}(a_{0}+1)[1-exp(a)] + \sum_{n=1}^{N} b_{n}(a_{n}+1)exp(-\alpha)exp(n\alpha)=v_{0} \quad \text{or} \quad v_{1} \]  \hspace{1cm} (21)

where the \( a_{i} \) are ZEROS or ONES, the \( b_{i} \) are the amplitudes actually sent, the \( s_{i} \) are the total received levels, and \( \alpha=-xT_p/TC \). Solving for \( b_{0} \),

\[ b_{0}|\text{ZERO sent} = (v_{0}-r)/(1-e\alpha) \]  \hspace{1cm} (22)

and \( b_{0}|\text{ONE sent} = (v_{1}-r)/2(1-e\alpha) \)  \hspace{1cm} (23)

Each \( b_{i} \), of course, should be positive for an optical system.

The probability of error when correlative coding is used is then exactly as found in Appendix A with appropriate provisions made for the use of the two new signal levels. The numerator of the complementary error function argument becomes \((s_{1}-s_{0})\). Since \( s_{1} \) cannot be greater than \( s(xT_p) \) and \( s_{0} \) must be greater than zero, the probability of error for this coding scheme cannot be less than that predicted for solitary pulse detection.

The number of registers required to store the amplitudes of the \( N \) significant previous pulses is found by again using Equation 20. The values of \( N \) required to store residual voltages of at least 0.1, 0.01, and 0.001 times the peak signal value are 70, 139, and 208 respectively. The numbers of Manchester bits which must be stored are found by dividing these two numbers by two.
The success of this correlative coding scheme depends on whether the optical source can be switched with minimum probability of error. The advantage of the scheme is that the randomness of the pulse train is compensated for by the transmitter, which eliminates variance due to intersymbol interference at the receiver output.
VI. Summary

The performance of three types of optical receiver in the detection of variable duty cycle Manchester coded data has been studied. The optimum receiver of single pulses in Gaussian noise, the matched-filter receiver, was shown to be far superior to the others, but could not be used due to the asynchronous requirement of the signaling. The performances of two alternative receivers, the non-filtered and equalized receivers, were compared. It was shown numerically that, despite the better frequency response of the equalized receiver, the non-filtered receiver performed nearly as well for most operating conditions. The cost of components and risk of oscillations in the equalized receiver has motivated the recommendation of the non-filtered receiver with source duty cycle equal to one and an intermediate combined shunt resistance.

The performance of the non-filtered receiver is destroyed by intersymbol interference. This problem can be overcome, it was shown, through the use of correlative coding at the transmitter. If the cost and other inherent problems of implementing such coding is less than that of equalization, the non-filtered receiver design can be adopted with satisfactory results.


APPENDIX A

Derivation of the Probability of Error in Manchester Bit Detection

The probability that a Manchester bit is detected incorrectly is

\[
P_E = P(0)[P(v_1 > \beta) + P(v_2 < \beta) + P(v_1 > \beta)P(v_2 < \beta)] \text{ZERO sent]} + P(1)[P(v_1 < \beta) + P(v_2 > \beta) + P(v_1 < \beta)P(v_2 > \beta)] \text{ONE sent]}
\]  

where \( P(0) \) and \( P(1) \) are the respective probabilities that a ZERO is sent and a ONE is sent, \( v_1 \) and \( v_2 \) are the voltages received during the first and second halves of the period, \( 2T_p \), and \( \beta \) is the decision threshold. If ONEs and ZEROs are equally likely (\( P(0)=P(1)=\frac{1}{2} \)) and \( \beta \) is chosen for minimum probability of error (the two signal levels have equal conditional probabilities of error in detection) [Ref. 14:81], then the two terms in the right hand side of Equation A.1 are equal and it follows that

\[
P_E = 2[P(v_1 > \beta) \text{ZERO sent}] + [P(v_1 > \beta) \text{ZERO sent}]^2
\]  

When a ZERO is sent \( v_1 \) is the signal level with the pulse absent. This is a Gaussian noise level. Thus, \( v_1 \) is a random variable with zero mean and variance \( \sigma_0^2 \). Its probability density function is

\[
f_{v_1}(v) = \frac{\exp[-v^2/2\sigma_0^2]}{\sqrt{2\pi\sigma_0^2}}
\]  

The probability described above is then

\[
P_E = 2 \int_{\beta}^{\infty} f_{v_1}(v)dv + \left[ \int_{\beta}^{\infty} f_{v_1}(v)dv \right]^2
\]
and, upon substitution for the argument in the exponent,

\[ P_E = \sqrt{\pi} \text{erfc}\left[ \frac{B}{\sqrt{2}\sigma_0^2} \right] + \frac{\pi}{4} \left[ \text{erfc}\left[ \frac{B}{\sqrt{2}\sigma_0^2} \right] \right]^2 \]  

(A.5)

where \( \text{erfc}(z) \) is the complementary error function, defined as

\[ \text{erfc}(z) = \frac{2}{\pi} \int_{z}^{\infty} \exp(z^2)dz \]  

(A.6)

When a ZERO is sent, \( v_2 \) is the signal level with the pulse present. This is just the sum of the pulse amplitude and the Gaussian noise level with the pulse present (different from the Gaussian noise level with the pulse absent because of the added signal shot noise).

The random variable, \( v_2 \), then, has a mean of \( s(xT_p) \), the amplitude of the pulse, and a variance of \( \sigma_1^2 \). Its probability density function is

\[ f_{v_2}(v) = \frac{\exp\left(-\frac{(v-s(xT_p))^2}{2\sigma_1^2}\right)}{\sqrt{2\pi\sigma_1^2}} \]  

(A.7)

The two density functions, Equations A.3 and A.7 are shown in Figure A.1. From the figure,

\[ [P_E | a \text{ ZERO is sent}] = P(v_1 > B) + P(v_2 < B) + P(v_1 > B)P(v_2 < B) \]

\[ = \int_{B}^{\infty} \frac{\exp\left(-v^2/2\sigma_0^2\right)}{\sqrt{2\pi\sigma_0^2}} dv \]

\[ = \int_{B}^{\infty} \frac{\exp\left[-(v-s(xT_p))^2/s\sigma_1^2\right]}{\sqrt{2\pi\sigma_1^2}} dv \]

(Equation continued on next page.)
Figure A.1 Probability Density Functions of the Receiver Output for Pulse Absent \( f_{v_1}(v) \) and Pulse Present \( f_{v_2}(v) \) Events.

\[
\begin{align*}
&+ \int_{\beta}^{\infty} \frac{\exp(-v^2/2\sigma_0^2)}{\sqrt{2\pi\sigma_0^2}} \, dv \\
&+ \int_{-\infty}^{\beta} \frac{\exp(-(v-s(xT_p))^2/2\sigma_1^2)}{\sqrt{2\pi\sigma_1^2}} \, dv 
\end{align*}
\]

(A.8)

The second term of the right hand side is equivalent to

\[
\int_{2s(xT_p)-\beta}^{\infty} \frac{\exp(-(v-s(xT_p))^2/2\sigma_1^2)}{\sqrt{2\pi\sigma_1^2}} \, dv 
\]

(A.9)

After substituting for the argument of the exponent, this becomes

\[
\frac{1}{\sqrt{\pi}} \int_{s(xT_p)-\beta}^{\infty} \frac{\exp(-w^2)}{\sqrt{2\sigma_1^2}} \, dw 
\]
\[ = \sqrt{\frac{\pi}{2}} \text{erfc} \left[ \frac{s(x_T p) - \beta}{\sqrt{2\pi\sigma_1^2}} \right] \quad (A.10) \]

The first term can also be expressed in terms of the complementary error function. After substituting for the argument of the exponential we find that this term is equivalent to

\[ \int_{\frac{\beta}{\sqrt{2\sigma_0^2}}}^{\infty} e^{-w^2} dw = \frac{\sqrt{\pi}}{2} \text{erfc} \left( \frac{\beta}{\sqrt{2\sigma_0^2}} \right) \quad (A.11) \]

The value of \( \beta \) has been chosen such that \( P(v_1 > \beta) = P(v_2 < \beta) \).

Equating Equations A.5 and A.6, then, yields

\[ \frac{s(x_T p) - \beta}{\sqrt{2\sigma_1^2}} = \frac{\beta}{\sqrt{2\sigma_0^2}} \quad (A.12) \]

Solving for \( \beta \),

\[ \beta = \frac{\sigma_0 s(x_T p)}{\sigma_1 + \sigma_0} \quad (A.13) \]

Substitution of Equation A.13 into Equation A.5 yields

\[ P_E = \sqrt{\pi} \text{erfc} \left[ \frac{s(x_T p)}{\sqrt{2 (\sigma_1 + \sigma_0)}} \right] \]

\[ + \frac{\pi}{4} \left| \text{erfc} \left[ \frac{s(x_T p)}{\sqrt{2 (\sigma_1 + \sigma_0)}} \right] \right|^2 \quad (A.14) \]
APPENDIX B

Performance Graphs for Optimum and Suboptimum Receivers and Peak and Average Power Limited Sources

The graphs included in this appendix illustrate receiver performance under operating conditions not covered by the text. Variation of ARG is shown for peak and average limited sources. The operating receiver resistance is one megohm.
$P = 1E-5 \, W \quad T = 398 \, K \quad GM = 1E-2 \, S \quad IO = 1E-9 \, A$
\[ P = 1 \times 10^{-5} \text{ W} \quad T = 298 \text{ K} \quad QM = 1 \times 10^{-5} \quad S = 1 \times 10^{-9} \text{ A} \]
$P = 1E-9 \text{ W}$  $T = 398 \text{ K}$  $Q = 1E-5$  $S = 1E-9$  $A$
1.2 MATCH PK.AVE
3.4 IN-FIL PK.AVE
5.6 EQ PK.AVE

ARG (DB)

LOG X

P=1E-9 W T=298 K GM=1E-2 S IO=1E-7 A
$P = 1 \times 10^{-6}$ W, $T = 298$ K, $GM = 1 \times 10^{-5}$ S, $ID = 1 \times 10^{-7}$ A
APPENDIX C

Performance Graphs for Suboptimum Receivers and Average Power Limited Sources

These graphs provide additional receiver performance information for the suboptimum receivers and average power limited sources only. Variation of ARG is shown for three values of receiver resistance, 10 kΩ, 1 MΩ and 100 MΩ, for both types of receiver.
LETTERS - N-FILTERED
NUMBERS - EQUALIZED
A. $1 \cdot R = 1 \cdot 10^4$
B. $2 \cdot R = 1 \cdot 10^6$
C. $3 \cdot R = 1 \cdot 10^8$

P = $1 \cdot 10^{-5}$ W  T = 298 K  GM = $1 \cdot 10^{-2}$ S  ID = $1 \cdot 10^{-7}$ A
LETTERS- N-FILTERED
NUMBERS- EQUALIZED
A. 1 x 10^4  B. 2 x 10^4  C. 3 x 10^4

P = 1E-6 W  T = 398 K  GM = 1E-5 S  ID = 1E-7 A
LETTERS - N-FILTERED
NUMBERS - EQUALIZED
A. 1: R=1E4  B. 2: R=1E6
C. 3: R=1E8

\[ P=1E-5 \quad T=398 \ K \quad QM=1E-2 \quad S \quad I0=1E-9 \ A \]
Vita

Robert Andrew Cilley was born on 12 April 1954 in Aurora, Colorado. He graduated from Novato High School, Novato, California in 1972. He studied electrical engineering at San Jose State University while working as a technician and engineer trainee in data communication at Ames Research Center, Moffett Field, California. Upon graduation he was commissioned as a second lieutenant in the United States Air Force and entered active duty as a graduate electrical engineering student at the Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio. He is a member of Eta Kappa Nu and Tau Beta Pi.

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The goal of this paper is to determine whether the probability of error in bit detection by optical receivers can be improved through the use of a variable duty cycle Manchester modulation format and proper receiver design. It has been speculated that a short source duty cycle would improve receiver performance, particularly when the source average power is constant. The parameters assumed to affect the probability of error include optical power, temperature, pulse width, amplifier gain, signal shot noise, thermal noise, and the circuit parameters of resistance and capacitance.
Performance predictions are made for the optimum receiver of signals in additive white noise, the matched-filter receiver, and two suboptimum receivers. The first suboptimum receiver is nothing more than a detector and an FET amplifier. The second consists of the same components with an equalizing filter added. Graphical data showing performance under extreme variations of the parameters listed above indicate that the matched-filter receiver is far superior to the suboptimum receivers. The equalized receiver performs better than the basic detector/amplifier receiver only under the most favorable operating conditions. The performance variation with source duty cycle varies with operating conditions. Under worst conditions the maximum duty cycle is recommended.