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A brief survey of the historical development of the field of thermal stresses, with some identification of areas fruitful for future research.

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THERMAL STRESSES: A SURVEY

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I want to thank the organizers of this Congress for asking me to present the opening lecture. I thought it would be appropriate if I were to start with a rapid review of the field of thermal stresses and give some thoughts regarding an assessment (although necessarily a very personal one) of the present state of the field, and finally include guesses as to what the future might bring.

The history of the development of the field of thermal stresses forms an interesting study of a growth of a scientific discipline. It may thus be useful not only in judging what has happened and is likely to happen in the field itself, but may provide some broad observations in a general way upon progress in a scientific field.

A glance at the number of publications in this field (Tablel, [1]) shows that a remarkable growth has taken place in the field of thermal stresses, but that it has taken place relatively recently. For example, one may note that only 17 papers were published in the first 65 years of the subject (i.e., from 1835-1900), and an equal number in the next 20 years. But an admittedly incomplete listing for the two years 1972 and 1973 alone [2] contains roughly ten times that number of papers, and there is no doubt that the rate of publications has continued to increase ever since.

Rather than to dwell on the number of papers published, however, it is probably more instructive to consider a few of the landmarks in the long history of the subject, and thus to see whether the <u>scientific</u> advance followed a similarly increasing rate of growth.

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TABLE 1
Thermal Stress Publications [1]

	Number of Publications During the five-year period	Cumulative
before 1836	4	4
1836-1840	2	6
1841-1845	1	7
1846-1850	0	7
1851-1855	0	7
1856-1860	0	7
1861-1865	0	7
1866-1870	1	8
1871-1875	1	9
1876-1880	1	10
1881-1885	2	12
1886-1890	0	12
1891-1895	2	14
1896-1900	3	17
1901-1905	5	22
1906-1910	5	27
1911-1915	2	29
1916-1920	5	34
1921-1925	15	49
1926-1930	15	64
1931-1935	21	85
1936-1940	30	115
1941-1945	30	145
1946-1950	83	228
1951-1955	204	432

Aside from some preliminary experimental investigations on the effect of temperature and deformations of solids, the first paper on thermoelasticity is that of Duhamel [3]. With this paper, published only fifteen years after the appearance of Fourier's treatise on heat conduction [4] and thirteen years after the publication of Navier's classical paper on the foundations of the theory of elasticity [5], the subject seems to have emerged full grown. In this paper the various boundary-value problems in linear thermoelasticity are formulated (including the effect of thermomechanical coupling), they are simplified and solved for a number of special cases. Little advance occurred for almost 60 years or so, that is until the work of the Italian elasticians (notably Almansi [6] and Tedone [7]), in the decade from about 1897 to 1907. Their researches paved the way for Muskhelishvili's later studies of two dimensional problems, including the introduction of the dislocation analogy [8] and the use of complex variables [9]; foreshadowed by this early work was also Goodier's method, published in 1937 [10].

In addition to the above rather basic advances, other papers were published in the first century of thermoelasticity, dealing with a variety of solutions to specific problems. Among these might be mentioned those relating to stresses, in spheres and cylinders (the first ones being respectively those of Hopkinson in 1879 [11] and of Leon in 1904 and 1905 [12 and 13]), Timoshenko's analysis of bimetallic strips [14], and some work on thermoelastic plates.

The foundations of thermoelasticity were thus well laid, and led to the solution in the 30's and 40's of a large number of problems of practical interest. Particular emphasis was given to the study of problems with common practical geometries, such as cylinders, spheres and plates, so that the practical solution of problems related to many important engineering applications was then well understood and within the reach of the engineering community. We might perhaps mention two papers of a somewhat different cast, published during this period, namely Signorini's [15] on finite thermoelastic deformations, and Biot's [16] study of the stresses arising under steady-state temperature distributions.

This activity can be said to culminate in the appearance of the first text on the subject, namely that of Melan and Parkus [17] in 1953; although restricted to steady-state temperature distributions, the appearance of this book may be said to mark the first recognition that thermoelasticity was indeed a field in its own right, distinct from elasticity (or inelasticity) on the one hand, and heat conduction on the other.

At the same time, substantial work on the inelastic behavior of solids under thermal effects was begun, with consequent appearance of important generalizations of the theories of viscoelasticity (e.g. [18], where the viscoelastic-elastic analogy was first given for incompressible materials, later generalized [19,20],and plasticity (e.g.[21,22]).Simultaneously, advances were being made in the extension of the domain of thermoelasticity: important topics of interest were the establishment of thermoelastic energy theorems [23,24] and the analysis of thermally-induced waves in solid bodies [25] and of thermally-induced vibration of beams [26]. Coupled to these developments was the establishment of theorems on overall thermoelastic deformations [27] and the introduction into thermoelasticity the well known structural use of the principle of virtual work [28]. These were accompanied by considerable effort at noting both the similarities and the differences between elastic and thermoelastic behaviors and theories. Perhaps the feeling at the time could best be summarized by noting that the similarities were useful in providing a direct extension of known procedures into the new field, while the differences had to be kept in mind so as not to be led astray. Examples of this sort of work are given by [28,29,30].

It is well to note that the great interest in thermal effects in structures, illustrated by the works which have been mentioned, did not spring alone from a spontaneous burst of intellectual curiosity, but was greatly spurred by problems arising in engineering practice, primarily in the then rapidly developing fields of rocket-powered high-speed flight and by the nascent exploitation of nuclear sources of energy. It seemed an appropriate time to collect the diverse strands of the subject in a single book, which would hopefully make available to researchers, practicing engineers, and students the physical and mathematical foundations of the field, as well as provide a compendium of previous work, and thus a sound basis for future advances, all in a single source and from a unified point of view. Indeed, this is what J.H. Weiner and I attempted to do at just about that time [31], and I must say that it has always been a source of comfort to us that the appearance of the book did not immediately result in an immediate regression of all work in the field. Indeed, the study of thermal effects in solids and structures continued to grow unabated in a number of directions. Among these we might mention investigations of many specific and practical problems on the basis of the theory that was by then well understood, but which nevertheless presented considerable difficulty when actual solutions were sought, because of such factors as complicated geometries, complicated heating or cooling conditions, and complicated elastic material behavior in extreme temperature regimes. It goes without saying that developments in these directions were accompanied by extensive application and adaptation of numerical methods.

At the same time, other problems were arising which lay at the boundary of validity of what may be called standard thermoelastic theory. These therefore required more careful assessment of the basis of that theory, and particularly the validity of some of the usual simplifying assumptions common in thermal stress analysis. Thus research shedding new insight into the effect of thermoelastic coupling was undertaken, including such fundamental work as a proof of uniqueness [32], analysis of thermoelastic waves [25,33,34,35], the solution of boundary value problems in coupled thermoelasticity [36], and considerations of electromagnetic coupling [37]. A summary of the results of these researches in the general field of dynamic thermoelasticity appeared in Nowacki's book published in 1966 [38]. Examination of the phenomenon of the so called second speed of sound, necessitating the inclusion of hyperbolic terms in the heat conduction equation, emerged shortly thereafter [39].

Another subject which began to attain prominence in the early sixties dealt with the study of mechanical behavior accompanying changes of phase [40], including the determination of transient and residual stresses and deformations. Application of these were important in traditional areas, such as the solidification of castings, but it was not long before the possibility of accurate thermomechanical analyses of ablating bodies [41] and of nuclear reactor meltdown accidents [42] began to be seriously examined. Allied to these works is the analysis of thermoelastic waves during change of phase contained in [43].

Another emerging technology of the sixties was concerned with composite materials; here too the behavior under non-isothermal conditions needed to be examined [44]. Also of growing importance was the theory of anisotropic thermoelasticity [45]. In parallel with the work which has been mentioned, more fundamental efforts at simplifying the practical solution of thermoelastic problems were continuing. Some of these were directed as the development of approximate analytical procedures (and, of course, at establishing the underlying theory; e.g., [45]),at constructing general bounds on the thermal stresses due to arbitrary temperature distributions [46], and at estimating errors in approximate calculations [47]. Not to be overlooked is the considerable body of analytical results pertaining to particular problems, especially in Japan, for example [48]).

The preceding overview up to about a decade or so ago has attempted to give a rapid chronological summary of the subject of thermal stresses. The references listed are those which appear to be the earliest in the particular topic mentioned, with no effort being made at providing a comprehensive bibliography in any one topic. The latter would be a tremendous task, particularly because of the great volume of publications which, as has been mentioned, continues to appear in print. It is nevertheless hoped that the above discussion has helped in putting in some historical perspective the origins of much of today's research. One word of demurral must however be added, because of the restriction here solely to advances of an analytical or theoretical nature. It must therefore be remembered that in many cases considerable experimental research had to be carried out in order to identify either the physical phenomena (for example, the second speed of sound) or the material properties in the specific thermal environment, and of course to verify theoretical results. No discussion of these aspects is presented here, but their essential role must not go unnoticed. The reader will also note that coverage of numerical methods has likewise been omitted here.

I think it is now at any rate clear that, by any reasonable definition, the field of thermal stresses can be considered to have reached full maturity. It might be of interest, in fact, to note what characteristics may be thought of as typical of a mature field, and to see whether a framework appropriate for examining present research and possible future directions might not thus emerge. Perhaps the following four closely related features may be identified as pertaining to a field that has reached maturity:

1. A tendency to take for granted some of the fundamental assumptions underlying the commonly used theory, and not to examine too carefully the conditions in which particular equations might be valid. Another way of stating this might be to say that the field is ready for "handbook engineering" exploitation. Many instances of this can be found: in addition to examples previously mentioned, it appears that some of the early concerns with the validity of thermoelastic beam theory [24] and thermoelastic theory [48,49] are now virtually unnoticed, as are the caveats which have been issued concerning the use of conventional thermoelastic theory and formulas in the analysis of beams of arbitrary cross-section [50] and rings [51]. Similarly, the careful assessment of criteria for uncoupling [48] is no longer a cause of great concern. The reasons for this are evident: experience has taught that reliance on the accepted formulas is generally justified, and it would be both unhealthy and inefficient to indulge in protracted worry. Nevertheless, the earlier studies point to special, but by no means pathological, cases, in which analyses more accurate than the conventional ones are needed, and it would be well if the former found their place in the courses on thermal stresses and thus at least in the subconscious of the practicing engineers.

2. An increased separation between applied and theoretical work. There is of course considerable interaction between practicing engineers and researchers, but, perhaps as a partial corollary of item 1 above, fundamental work is often regarded as irrelevant to practical solutions, while the need for the latter is often of little interest to the more mathematically inclined researcher. There is, in other words, a tendency to dismiss mathematical theorems by practicing engineers as mere curiosities, while simultaneously they may be quite correctly regarded by the mathematicisns as providing deeper insights theoretically. Attempts to bring the exponents of the two extremes together have been made, and continue to be made, such as, for example, two recent symposia [52,53] in the field of problems involving moving boundaries or change of phase (depending on whether one is mathematically or physically inclined), but much more must be done before a

real dialogue can be said to be taking place. The present Conference appears to be in fact an excellent move in this direction. One reason for the separation is easy to understand: researchers are increasingly drawn to more and more to unconventional problems, which though of course still physically meaningful and important in certain advanced applications, do not fall within the usual experience of most practitioners. A plea can be entered here for more understanding of the problems experiences by both camps.

3. Research on more esoteric problems. This has already been referred to as a partial cause of the increased separation of the practical from the theoretical, but it must be understood, in a mature but not stagnant field, as a case of continuous interchange between the two: as more extreme conditions become analytically tractable, applications for them are found. New applications in turn give rise to new needs, and then further, and more advanced or complex work must take place. This is an unending interaction, and there are numerous indications that in the field of thermal stresses they are indeed occurring in such diverse areas as the effect of extremely rapid intense heating or in change-of-phase problems, including the melting of reactor elements.

4. The appearance of a journal devoted to the field. A need for this is perceived, not necessarily to accommodate the very advanced work, but to provide an outlet for the increased number of workers concerned with a variety of problems of increasing complexity.

I conclude from an examination of the above characteristics that the field of thermal stresses is indeed mature and lively, and certainly the present symposium gives evidence of both. One more question might nevertheless be asked, namely what direction in the field would I deem most interesting and fruitful for future research. The answer to that question must of necessity be a subjective one, since the outlook of any one person is bound to be affected by his own previous work and interests. It might nevertheless be of some value to attempt to give my own views, with the idea that they might provide a useful starting point for discussion and further research.

The topics covered by the present Conference can certainly be taken to form a reasonable overview of current interests, and in fact each paper presented could be taken as representative of a problem that will receive additional future attention. In fact, almost everyone of the broad areas identified below as future research topics finds an illustrative counterpart on the Conference's program.

The following topics are, in my opinion, worthy of the reader's attention:

1. The propagation of thermoelastic waves. Studies should include all coupling effects, consideration of finite propagation speeds, anisotropic and inelastic effects, and identification of the various discontinuities and wave-front characteristics (e.g. [54]).

2. Further development of general theorems in thermoelasticity.

This area is not only an intellectually satisfying one, but also still can disclose useful and important basic information. As examples, one might note Dundurs' work [55] on the a priori prediction of thermoelastic distorsions in steady-state problems, and some useful extensions and applications of thermoelastic energy and virtual work theorems in [56]. Some general theorems in thermopiezoelectricity should also be noted [57].

3. Thermomechanical behavior accompanying changes of phase, and generally in moving-boundary problems. In part no doubt because of the complexity of problems of this type, and of the necessity to incorporate consideration of inelastic behavior of materials in a regime where little accurate information is available, comparatively little has as yet been done in this field.

4. Much more work will no doubt be needed in all aspects of viscoelasticity, plasticity, and combined inelastic effects, ranging from the establishment of accurate constitutive equations, to the solution of practical problems. Similarly, the analysis of anisotropic media and composite materials will need further attention.

5. Thermal effects in fracture are obviously part of a still developing subject.

6. To all of the above must of course be added the developments of numerical and experimental methods, essential for the solution of actual complicated problems. I would add to these, as a useful comparison the establishment of approximate rapid analytical methods of calculation of stresses and deflection, which are extremely useful in the verification of numerical results and in preliminary design.

I believe the above categories contain most of the important topics which require consideration, and are likely to continue to do so for some time to come. What is clear is that there is a great deal of work to be done, and I can probably be most useful at this point by cutting my presentation short and, with renewed thanks to the organizers of this Conference, get out of the way so that work can proceed.

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