BATHYMETRIC DATA REDUCTION SUBSYSTEM

Synectics Corporation

P. Bell
R. Howarth

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED
This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-79-212 has been reviewed and is approved for publication.

APPROVED:

Amanda L. Hasemeier
AMANDA N. HASMEIER
Project Engineer

APPROVED:

ROSS H. ROGERS, Colonel, USAF
Chief, Intelligence and Reconnaissance Division

FOR THE COMMANDER:

JOHN P. Huss
Acting Chief, Plans Office

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (IRM) Griffiss AFB NY 13441. This will assist us in maintaining a current mailing list.

Do not return this copy. Retain or destroy.
UNCLASSIFIED

The subject final technical report covers Phase I - the Basic Operating Capability of the Bathymetric Data Reduction Subsystem. The report discusses and evaluates problems encountered during Phase I and the solutions which Synectics Corporation provided to these problems.

There are four (4) parts to this report. First, a discussion of problems which is basically mathematical. Secondly, discussion of software accomplishments pertain to subsystem development. Part III, method of inaccuracy occurring -
in data input and manipulation are discussed. Finally, the fourth part evaluates the scope and limitations of the hardware configuration along with system software provided by Data General Corporation.
<table>
<thead>
<tr>
<th>SECTION</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1-1</td>
</tr>
<tr>
<td>1.1</td>
<td>Purpose</td>
<td>1-1</td>
</tr>
<tr>
<td>1.2</td>
<td>Subjects to be discussed</td>
<td>1-1</td>
</tr>
<tr>
<td>2</td>
<td>MATHEMATICAL APPLICATIONS</td>
<td>2-1</td>
</tr>
<tr>
<td>2.1</td>
<td>Purpose</td>
<td>2-1</td>
</tr>
<tr>
<td>2.2</td>
<td>Registration</td>
<td>2-1</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Method of Solution</td>
<td>2-1</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Source and History of Approach</td>
<td>2-6</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Empirical Results and Evaluation</td>
<td>2-6</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Future Topics of Interest</td>
<td>2-6</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Coordinate Transformations</td>
<td>2-7</td>
</tr>
<tr>
<td>2.3</td>
<td>Coordinate Transformations</td>
<td></td>
</tr>
<tr>
<td>2.3.1</td>
<td>Problem to be Solved</td>
<td>2-7</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Method of Solution</td>
<td>2-7</td>
</tr>
<tr>
<td>2.3.2.1</td>
<td>Mercator Mapping Equation</td>
<td>2-8</td>
</tr>
<tr>
<td>2.3.2.2</td>
<td>Inverse Mercator Mapping Equation</td>
<td>2-8</td>
</tr>
<tr>
<td>2.3.2.3</td>
<td>Transverse Mercator Mapping Equation</td>
<td>2-11</td>
</tr>
<tr>
<td>2.3.2.4</td>
<td>Inverse Transverse Mercator Equations</td>
<td>2-11</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Source and History of Approach</td>
<td>2-14</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Empirical Results and Evaluations</td>
<td>2-14</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Future Topics of Interest</td>
<td>2-14</td>
</tr>
<tr>
<td>2.4</td>
<td>Geographic Sectioning Algorithms</td>
<td>2-14</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Problem to be Solved</td>
<td>2-14</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Method of Solution</td>
<td>2-14</td>
</tr>
<tr>
<td>2.4.2.1</td>
<td>Circle Search</td>
<td>2-15</td>
</tr>
<tr>
<td>2.4.2.2</td>
<td>Polygon Search</td>
<td>2-16</td>
</tr>
<tr>
<td>2.4.2.3</td>
<td>Path Search</td>
<td>2-21</td>
</tr>
<tr>
<td>2.4.3</td>
<td>Source and History of Approach</td>
<td>2-22</td>
</tr>
<tr>
<td>2.4.4</td>
<td>Empirical Results and Evaluations</td>
<td>2-22</td>
</tr>
<tr>
<td>2.4.5</td>
<td>Future Topics of Interest</td>
<td>2-22</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS, Continued

SECTION  TITLE  PAGE
1  SOFTWARE ACCOMPLISHMENTS  3-1
1.1  Purpose  3-1
1.2  Digitization and Voice Entry Subsystem  3-1
1.2.1  Sounding Data  3-1
1.2.2  Pathograms  3-1
1.2.3  Registration  3-2
1.2.4  Review Mode  3-2
1.3  Batch Processes Subsystem  3-2
1.3.1  Geographic to Table Conversion  3-2
1.3.2  Table to Geographic Conversion  3-3
1.3.3  Plot Functions  3-4
1.4  Data Base Subsystem  3-4
1.4.1  Sectioning  3-4
4  THE PROBLEMS OF ACCURACY  4-1
4.1  Purpose  4-1
4.2  Data  4-1
4.2.1  Accuracy of Source Analog Data  4-1
4.2.1.1  Charts  4-1
4.2.1.2  Data Entry of Analog Source Materials  4-2
4.2.2  Accuracy of Processed Data  4-2
4.2.2.1  Registration  4-2
4.2.2.2  Coordinate Transformations  4.3
4.3  Problem of Validation  4-4
5  SYSTEM CONFIGURATION  5-1
5.1  Purpose  5-1
5.2  Hardware Configuration  5-1
<table>
<thead>
<tr>
<th>SECTION</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3</td>
<td>BDRS Software Configuration</td>
<td>5-5</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Key Areas</td>
<td>5-5</td>
</tr>
<tr>
<td>5.3.1.1</td>
<td>Multitasking</td>
<td>5-5</td>
</tr>
<tr>
<td>5.3.1.2</td>
<td>Overlays</td>
<td>5-5</td>
</tr>
<tr>
<td>5.3.1.3</td>
<td>User Device Implementation</td>
<td>5-6</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Conclusions and Recommendations</td>
<td>5-6</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-1</td>
<td>BASIC OPERATING CAPABILITY</td>
<td>5-2</td>
</tr>
<tr>
<td>5-2</td>
<td>STATION 1</td>
<td>5-3</td>
</tr>
<tr>
<td>5-3</td>
<td>STATION 2</td>
<td>5-4</td>
</tr>
</tbody>
</table>
EVALUATION

The effort provided an initial capability to create and update a bathymetric data library employing voice data entry augmented digitizing and editing functions. This effort significantly enhances the human factors aspect of large volume geographic point data digitizing. The mathematical data manipulations have been structured to significantly reduce error buildup in the manipulated/transformed data. Lastly, the digital data library provides timely query responses and data availability. Future plans are to increase the scope of the mathematical data manipulations to provide for a wider range input and output formats, to introduce a wider range of logical retrievals at the data base for increased responsiveness and to optimize the voice entry human factors aspects.

JOSEPH J. PALERMO
Laboratory Contract Manager
SECTION I
INTRODUCTION

1.1 Purpose

The purpose of this document is twofold. First, a critical account is provided of the important subsystem and technical problems which were encountered in Phase I of BORS. Second, a critical account and evaluation is given of the solutions which were provided by Syntectics Corporation to these problems.

1.2 Subjects to be Discussed

This document is divided into four parts. Part I is dedicated to a discussion of problems whose solutions are mathematical in nature. Part II is comprised of a discussion of the software accomplishments pertaining to subsystem problems. Part III deals with the myriad contexts in which the perennial problems of accuracy occur in data input and manipulation. Finally, Part IV critically evaluates the scope and limitations of the hardware configuration utilized in conjunction with the system software provided by Data General Corporation.
SECTION II
MATHEMATICAL APPLICATIONS

2.1 Purpose

The purpose of this section is to provide a detailed account of the mathematical algorithms and their applications within the BORS problem environment. The three contexts in which algorithms have been developed are registration, coordinate transformations and geographic sectioning.

2.2 Registration

2.2.1 The problem to be solved by n point registration is twofold. First, a function must be constructed whose domain is comprised of physical X-Y locations on a digitizing table and whose range is comprised of X-Y coordinates in an earth rectangular frame, given that n values (3 to 8) of the function are known. The construction of this function is referred to as day '1' n-point registration. Second, a function must be constructed which maps table X-Y locations to table X-Y locations given that n (from 3 to 8) values of the function are known. This construction is entitled day 'n' n-point registration.

Day '1' registration, from an empirical point of view, results in the geodetic significance of points on a map which has been placed on the digitizing table. Day 'n' registration results in mapping back to day 1 the points on the map as it lies on the table translated or rotated relative to the day 1 registration of the map. Since both algorithms are identical from a mathematical point of view, it suffices to give a presentation of day 1 registration.

2.2.2 Method of Solution

The idea of the n-point registration is to convert X-Y table values to a form which can be converted to lat/long values on the earth's surface. A pure table X-Y value is physically meaningless.

The problem is a typical statistical one of finding a function given that you know some of the ordered pairs that the function must satisfy. In registration you know:
To be more precise, let \( X_i, X, \ldots X_n \) be the map scale mil values of the registration points, let \( Y_1, Y_2, \ldots Y_n \) be the map scale mil values of the registration points, let \( X_1, X_2, \ldots X_n \) be the known table \( X \) values, let \( Y_1, \ldots Y_n \) be the table \( Y \) values. You know for each ordered pair of \( X-Y \) table points \( X_i, Y_i \), the map scale mil equivalent \( X_i^1, Y_i^1 \). This constitutes the known relations which must be approximated by the function being sought. What properties should the function have besides fitting the known data? Simplicity requires that the function be linear. Geometrical considerations suggest finding two functions \( f, g \) such that

\[
\begin{align*}
(a) & \quad f(X_i, Y_i) = X_i^1 \\
(b) & \quad g(X_i, Y_i) = Y_i^1
\end{align*}
\]

Given the assumption of linearity we get

\[
\begin{align*}
(a) & \quad f(X_i, Y_i) = AX_i + BY_i + C = X_i^1 \\
(b) & \quad g(X_i, Y_i) = DX_i + EY_i + F = Y_i^1
\end{align*}
\]

Now we must impose some conditions which allow us both to solve for \( A, B, C, D, E \) and \( F \) unambiguously and which lend geodetic meaning to the physical table \( X-Y \) values. That is, given an arbitrary table \( X-Y \) pair, the functions \( f \) and \( g \) must map the pair into an approximately correct map scale mil ordered pair.

The classical approach to a statistical problem in a linear model is the least squares best fit algorithm. The idea is to minimize the sum of the squares of the differences between known data values and the values predicted statistically. In registration we have known map scale mil values. We want the functions \( f \) and \( g \) to yield predicted map scale mil values such that we minimize the sums of the squares of the differences between the predicted map scale mil values (i.e., \( f(X_i, Y_i) \), \( g(X_i, Y_i) \) and the known map scale mil values (i.e., \( X_i^1, Y_i^1 \)).

More explicitly, given that

\[
\begin{align*}
(a) & \quad X_i^1 = AX_i + BY_i + C \\
(b) & \quad Y_i^1 = DX_i + EY_i + F
\end{align*}
\]
minimize
\[(c) \sum_{i=1}^{n} [x_i^2 - (AX_i + BY_i + C)]^2\]
\[(d) \sum_{i=1}^{n} [y_i^2 - (DX_i + EY_i + F)]^2\]
by picking the correct values of A, B, C, D, E, and F

Let \(H(X_i, Y_i) = \sum_{i=1}^{n} [x_i^2 - (AX_i + BY_i + C)]^2\)

and \(G(Y_i, X_i) = \sum_{i=1}^{n} [y_i^2 - (AX_i + BY_i + C)]^2\)

the requirement of minimizing these expressions is logically equivalent to
the following six equations:

\[(1) \frac{\partial H}{\partial A} = 0\]
\[(2) \frac{\partial H}{\partial B} = 0\]
\[(3) \frac{\partial H}{\partial C} = 0\]
\[(4) \frac{\partial G}{\partial D} = 0\]
\[(5) \frac{\partial G}{\partial E} = 0\]
\[(6) \frac{\partial G}{\partial F} = 0\]

Taking the derivatives and simplifying we get the following six equations:

\[(1) A \sum_{i=1}^{n} x_i^2 + B \sum_{i=1}^{n} x_i y_i = C \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i^2 x_i\]
\[(2) A \sum_{i=1}^{n} x_i y_i + B \sum_{i=1}^{n} y_i^2 = C \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i^2 y_i\]
\[(3) A \sum_{i=1}^{n} x_i^2 + B \sum_{i=1}^{n} y_i^2 = C \sum_{i=1}^{n} x_i y_i\]
\[(4) D \sum_{i=1}^{n} x_i^2 + E \sum_{i=1}^{n} x_i y_i + F \sum_{i=1}^{n} y_i^2 = C \sum_{i=1}^{n} x_i y_i\]
\begin{align*}
(5) & \quad D Y_{i} + E Y_{i}^2 + F Y_{i} = Y_{i} Y_{i} \\ 
& \quad 1-1 \quad i-1 \quad 1-1 \quad i-1 \\
(6) & \quad D Y_{i} + E Y_{i} + n - Y_{i} Y_{i} \\ 
& \quad 1-1 \quad i-1 \quad i-1 \\

Note that all expressions are known in the above equations except for \(A, B, C, D, E,\) and \(F.\) To solve for these, treat equations 1, 2, and 3 as 3 equations in 3 unknowns \(A, B,\) and \(C\) and treat equations 4, 5, and 6 as 3 equations in 3 unknowns \(D, E,\) and \(F.\) Then we can write equations 1-3 and 4-6 as:

\[ X Y = Z \]
\[ X W = V \]

where

\[
X = \begin{pmatrix}
\sum_{i=1}^{n} \left( X_{i} \right)^2 & \sum_{i=1}^{n} X_{i} Y_{i} & \sum_{i=1}^{n} X_{i} \\
\sum_{i=1}^{n} X_{i} Y_{i} & \sum_{i=1}^{n} \left( Y_{i} \right)^2 & \sum_{i=1}^{n} Y_{i} \\
\sum_{i=1}^{n} X_{i} & \sum_{i=1}^{n} Y_{i} & N
\end{pmatrix}
\]

\[
Y = \begin{pmatrix}
A \\
B \\
C
\end{pmatrix}
\]

\[
W = \begin{pmatrix}
D \\
E \\
F
\end{pmatrix}
\]

\[
Z = \begin{pmatrix}
\sum_{i=1}^{n} X_{i}^2 X_{i} \\
\sum_{i=1}^{n} X_{i} Y_{i} \\
\sum_{i=1}^{n} X_{i} \\
\sum_{i=1}^{n} Y_{i}
\end{pmatrix}
\]

\[
V = \begin{pmatrix}
\sum_{i=1}^{n} Y_{i}^2 X_{i} \\
\sum_{i=1}^{n} Y_{i} Y_{i} \\
\sum_{i=1}^{n} Y_{i} \\
\sum_{i=1}^{n} Y_{i}^2
\end{pmatrix}
\]

Note that \(X\) is a symmetric matrix.
To solve for $Y - \begin{pmatrix} A \\ B \\ C \end{pmatrix} \cdot \begin{pmatrix} w \\ F \end{pmatrix}$ find the inverse of $X^{-1} X^{-1}$ to get

\[
\begin{align*}
(1) \quad \begin{pmatrix} A \\ B \\ C \end{pmatrix} & \quad X^{-1} Z \\
(2) \quad \begin{pmatrix} D \\ E \\ F \end{pmatrix} & \quad X^{-1} V
\end{align*}
\]

The matrix $X^{-1}$ exists if its determinant is non-zero which seems to work always. If it does not work, new registration points must be chosen.

The software for this approach requires solving for the elements of $X, Z, W$, finding $X^{-1}$, and multiplying $X^{-1} Z$ and $X^{-1} V$. Having found these values we have for an arbitrary table XY pair $X_i, Y_i$ that

\[
\begin{align*}
(1) \quad X^i & = AX_i + BY_i + C \\
(2) \quad Y^i & = DX_i + EY_i + F
\end{align*}
\]

Upon conversion of $X^i, Y^i$ to meters we multiply by the map scale to get $U^i, V^i$, the earth scale meter equivalents of $X^i, Y^i$. This pair $U^i, V^i$ is the input into the inverse map projection. The output is the pair $\theta, \psi$ called the latitude and longitude of the table point $X_i, Y_i$. 

THIS PAGE IS BEST QUALITY PRACTICABLE

F. D. P. Furness, 1972

2-5
2.2.4 Source and History of Approach

The adopted approach to registration was developed "in-house" at Synectics Corporation. In particular, the idea of solving for the X and Y dimensions independently resulted in minimizing errors which could result from redigitizing single control points when a registration is unacceptable.

2.2.4 Empirical Results and Evaluation

Since the mathematical model of registration is both statistical and linear, there are empirical situations in which problems can arise. In the ideal situation, the source map is not distorted badly by shrinkage of expansion. We have noted that such distortion is rarely compensated for within the linear model. A distorted map can be registered within the residual tolerance level. This should not be taken as evidence that the statistical fit has adequately compensated for the distortion. The model is linear; therefore, the scaling which occurs is uniform along an axis. Thus, the actual map shrinkage is spread uniformly over the map. This could make the functions constructed in registration both meet the residual test and fail to provide an adequate mapping. Thus, the chart must be known to be in good condition or poor empirical data from the table might be allowed to infiltrate the data base.

It is crucial to pick registration points intelligently. If 8 registration points are picked representative of the map as a whole; that is, in no obvious geometrical pattern, and the document is in good physical condition, then the results of registration can be trusted. All mathematical computations are performed in over-kill double precision.

2.2.4 Future Topics of Interest

Since the mathematical model of registration is linear, the idea obviously arises of utilizing a model in which non-linear terms contribute to statistical prediction. The linear model is quite satisfactory given a chart in good condition and an intelligent choice of registration points. Thus the employment of a non-linear model would be motivated if an attempt were made to register charts in less than optimal condition.
The linear model could also be supplemented with a normally distributed error term whose expectation value is zero and whose standard deviation reflects user input in accuracy in entering a control point. However, it seems arbitrary to assign such a standard deviation for an arbitrary user. Thus, this term does not appear in the functions constructed by registration.

2.1 Coordinate Transformations

2.3.1 Problem to be Solved

The problem to be solved by the coordinate transformations is to convert between earth rectangular and geodetic coordinates utilizing the Mercator and Transverse Mercator projections. The conversion from geodetic to earth rectangular coordinates is effected by the map projection itself. Inverses had to be constructed as well to convert earth rectangular to geodetic coordinates.

2.3.2 Method of Solution

The projection types utilized are conformal projections. As Thomas has shown in "Conformal Projections in Geodesy and Cartography", Coast and Geodetic Survey Special Publication 251, all conformal mappings of the spheroid upon a plane are expressed by the analytic function

\[ X+iy = f(\lambda+i\varphi) \]

where

\[ r = \ln \left( \frac{\tan \left( \frac{\pi}{4} + \frac{\varphi}{2} \right)}{\frac{1-\varepsilon \sin \varphi}{1+\varepsilon \sin \varphi}} \right)^{\frac{\xi}{2}} \]

and \( \varphi \) equals the latitude of the point, \( \lambda \) the longitude and \( \varepsilon \) the eccentricity. The form of the Mercator and Transverse Mercator mapping is then determined by which line or lines in the projection are to be held true to scale and by the necessary geometric form of map elements corresponding to meridians and parallels. Once the form of \( f \) is determined, the real and imaginary parts of \( X+iy = f(\lambda+i\varphi) \) are equated which must in turn satisfy the well known Cauchy Riemann equations:

\[
\begin{align*}
(1) \quad \frac{\partial x}{\partial \lambda} &= \frac{\partial y}{\partial r} \\
(2) \quad \frac{\partial x}{\partial r} &= -\frac{\partial y}{\partial \lambda}
\end{align*}
\]
to finally obtain \( x \) as a function of \( \lambda \) and \( r \) and \( y \) as a function of \( \lambda \) and \( r \). The mapping equation is thus derived.

### 2.3.2.1 Mercator Mapping Equation

The Mercator projection is the simplest and most basic of the conformal map projections. The form of \( f \) is linear and the initial conditions quite simple. As Thomas indicates, the initial condition is that the scale of the projection must be true at the equator. Thus, for a latitude of \( \phi = 0 \), \( r = \ln(1) = 0 \), and \( y = 0 \), and \( x = a\lambda \), the product of the equatorial radius and the longitude \( \lambda \). Thus since \( x + iy = f(\lambda + ir) \) we have \( a\lambda + i(\lambda +\lambda) = f(\lambda + ir) = x + iy \). Equating real and imaginary parts of \( x + iy = a(\lambda + ir) \) we have

1. \( x = a\lambda \)
2. \( y = ar = a \ln \left( \frac{\tan(\pi + 1\phi)}{4} \right) \)
   \[ = a \ln \left( \frac{(\sin \phi + 1)}{\cos \phi} \right) \left( \frac{(1 - \sin^2 \phi)}{(1 + \sin^2 \phi)} \right) \]

The scale or magnification of a point at latitude \( \phi \) given a conformal projection is the Jacobian of \( x \) and \( y \) with respect to \( r \) and \( \lambda \) divided by \( N \cos \phi \) where \( N = \frac{a}{\sqrt{1 - \epsilon^2 \sin^2 \phi}} \), the distance from the point along that line from the minor axis of the spheroid perpendicular to the line tangent to the point. In this case the scale become \( a \cdot \sec \phi \).

### 2.3.2.2 Inverse Mercator Mapping Equation

Given the Mercator mapping function \( Y = f(x) \) which maps a latitude to an earth scale meter value, find a method to calculate the latitude given the earth scale meter value.

The Mercator mapping equation in question takes the form:

\[
Y = \left( \frac{A}{SCALAC} \right) \ln \left[ \frac{\tan \left( \frac{\pi}{4} + 1\phi \right)}{4} \right] \cdot \frac{1 - \text{ECC} \cdot \sin(P)}{\text{ECC} \cdot \sin(P)} \frac{ZEC/2}{1} \]

Where \( Y = \) the earth scale meter value and \( P = \) the latitude value.

Given this form for \( f(x) \), an expression \( 6(x) \) must be derived such that the form \( p = 6(P) \) is obtained. Given an initial approximation \( P_0 \) to \( P \), Weinstein iteration is used to improve the desired latitude value.
To derive the form $P - 6(P)$ we proceed as follows. Since SCALAC is a constant we eliminate it to get

$$Y_1 = A \ln \left\{ \frac{\tan \left( \frac{\pi + 1P}{4} \right) \cdot 1 - \text{ZECC} \cdot \sin(P) \cdot \text{ZECC}/2}{1 + \text{ZECC} \cdot \sin(P)} \right\}$$

$$Y_1 = A \ln \left( \frac{\tan \left( \frac{\pi + 1P}{4} \right)}{2} \right) + Y(P)$$

where $Y(P) = A \ln \left( \frac{1 - \text{ZECC} \cdot \sin(P) \cdot \text{ZECC}/2}{1 + \text{ZECC} \cdot \sin(P)} \right)$

But $Y(P) = A \cdot \text{ZECC} \cdot \text{ZECC}/2 \ln \left( \frac{1 - \text{ZECC} \cdot \sin(P)}{1 + \text{ZECC} \cdot \sin(P)} \right)$

So if we let $B = \text{ZECC} \cdot \sin(P)$ and use the series expansion $1/2 \ln \left( \frac{1 + B}{1 - B} \right) = \frac{1}{3}B + \frac{1}{5}B^3 + \ldots$ with $AE2 = A \cdot \text{ZECC}$, $AE4 = A \cdot \text{ZECC} \cdot \text{ZECC}/3!$ we get

$$Y(P) = \sin(P) \cdot (AE2 + \sin^2(P)) \cdot (AE4 + \sin^2(P) \cdot AE6)$$

Therefore:

$$Y_1 = A \ln \left( \frac{\tan \left( \frac{\pi + 1P}{4} \right)}{2} \right) - Y(P)$$

$$\frac{Y_1 - Y(P)}{A} = \ln \left( \frac{\tan \left( \frac{\pi + 1P}{4} \right)}{2} \right)$$

$$\exp \left\{ \frac{Y_1 - Y(P)}{A} \right\} = \tan \left( \frac{\pi + 1P}{4} \right)$$

$$P = 2 \cdot \left( \arctan \left( \exp \left( \frac{Y_1 - Y(P)}{A} \right) \right) \right) - \frac{\pi}{4}$$

Thus we have derived the correct form $P = 6(P)$ for use by the iteration scheme.

The initial approximation $P_0$ to $P$ is obtained by considering $\text{ZECC} = 0$; that is, a spherical earth model is employed. Proceeding as before we have:

$$Y_1 = A \ln \left( \frac{\tan \left( \frac{\pi + 1P}{4} \right)}{2} \right)$$

$$P_0 = 2 \cdot \left( \arctan \left( \exp \left( \frac{Y_1}{A} \right) \right) \right) - \frac{\pi}{4}$$

Thus $P_0$ is calculated exactly.

Having obtained $P_0$ and $P = 6(P)$ Wegstein iteration is used as follows:
(1) Initialization

\[ P_0 = P_0 \]

\[ P_1 = G(P_0) \]

\[ P_1 = G(P_1) + G(P_1) - P_1 \]

\[ \frac{P_1 - P_0}{G(P_1) - P_1} - 1 \]

(2) At iteration \( n+1 \)

\[ P_{n+1} = G(P_n) \]

\[ P_{n+1} = G(P_{n+1}) + G(P_{n+1}) - P_{n+1} \]

\[ \frac{P_{n+1} - P_n}{G(P_{n+1}) - P_{n+1}} - 1 \]

(3) Convergence at tolerance \( E \)

\[ \frac{P_{n+1} - P_n}{P_{n+1} - E} \]

This will occur unless \( G(P) = 1 \) at \( P \). If convergence does not occur, the initial approximation \( P_0 \) is output as the latitude.
2.3.3.3 Transverse Mercator Mapping Equation

As Thomas points out, the initial condition of the Transverse Mercator projection is that the scale is true along the central meridian of the map. Thus, for a point at longitude zero and latitude \( \phi \) we have \( x=0 \). Given that \( x+iy=f(\lambda+ir) \) we have \( iy=f(ir) \), hence \( \sqrt{\sec^2 \phi \cdot 15^\circ} \) the distance along the meridian from the equator at longitude \( \lambda \) to the point at latitude \( \phi \) with \( R=\) the radius of curvature \( -(1-\varepsilon^2)\cdot a \cdot (1-\ell^2 \sin^2 \phi)^{-3/2} \). But \( r=R \int_0^\phi \sec \phi \, d\phi \) by the condition on \( R \) for all conformal projections. Thus \( dr=R \sec \phi \, d\phi \) and \( Rd\phi=N \cos \phi \, dr \). Therefore \( S=\int_0^\phi N \cos \phi \, dr = f(r) \) since \( \lambda \) is zero. Thus we have \( x+iy=f(r) \). Now we want to subject \( x \) and \( y \) to the Cauchy Riemann equations so that equations for \( x \) and \( y \) in terms of \( \lambda \) and \( r \) are needed. Thomas proceeds to expand \( x+iy=f(\lambda+ir) \) about the point \( z=ir \) in a Taylor Series. Then real and imaginary components are equated yielding the desired forms for \( x \) and \( y \) in terms of \( \lambda \) and \( r \). Once all terms are computed and the Cauchy Riemann equations applied, Thomas reaches his desired mapping equations.

The scale \( K \) for the projection at a point \( \lambda, \phi \) is

\[
K = \left[ \left( \frac{\partial x}{\partial \lambda} \right)^2 + \left( \frac{\partial y}{\partial \lambda} \right)^2 \right]^{1/2} N \cos \phi
\]

\[
= \frac{1}{N \cos \phi} \frac{\lambda x}{\partial \lambda} \cdot (1 + \tan^2 \phi)^{1/2}
\]

2.3.2.4 Inverse Transverse Mercator Equations

The following algorithm, taken from Rapp and Sprinsky, Page 30 is employed.

\[
\Delta \lambda = \sec \phi \cdot \frac{1}{N} \left( \frac{1}{6} - \frac{1}{2} \frac{1}{N} \frac{1}{N} \right) \left( 1 + 2t_1^2 + \eta_1^2 \right) + \frac{1}{2} \frac{1}{N} \left( 5 + 28t_1^2 + 6\eta_1^2 + 6t_1^2 \eta_1^2 \right) + \frac{24}{1} + \frac{6}{1} + \frac{6}{1} + \frac{1}{1} \frac{1}{N} \left( 61 + 90t_1^2 + 45t_1^4 + 107\eta_1^2 \right)
\]

\[
\phi = \phi' + \frac{t}{2} \left( \frac{1}{1} + \eta_1^2 \right) \frac{1}{N} \frac{1}{2} \frac{1}{4} \left( \frac{1}{1} + \eta_1^2 \right) \left( 5 + 3t_1^2 + \eta_1^2 - 4\eta_1^4 \right)
\]

\[
+ \frac{9}{1} \frac{1}{N} \frac{1}{4} \frac{1}{2} \frac{1}{4} \left( 61 + 90t_1^2 + 45t_1^4 + 107\eta_1^2 \right) - \frac{16}{1} \frac{1}{2} \frac{1}{4} \frac{1}{2} \frac{1}{4} \left( 61 + 90t_1^2 + 45t_1^4 + 107\eta_1^2 \right)
\]

\[
- 162t_1^2 \sin^2 \phi' + 45t_1^2 \sin^2 \phi' \frac{x}{n} \]

2-11
where:  
\[ t_1 = \tan \phi' \]
\[ \eta_1^2 = e'^2 \cos \phi' \]
\[ e'^2 = \text{second eccentricity squared} \]
\[ \phi' = \text{foot point latitude} \]
\[ X = \text{UTM Easting} \]
\[ N = a'/\left(1 - e'^2 \sin^2\phi\right)^{1/2} \]
\[ a' = 0.99964 \]
\[ a = \text{semi-major axis of the ellipsoid} \]
\[ \Delta \lambda = \text{longitude difference from central meridian of point} \]
\[ \phi = \text{latitude of point} \]
\[ e'^2 = \text{eccentricity squared} \]

Having computed \( \Delta \lambda \), and \( \phi \), the longitude itself must be computed as follows:

\[ \lambda = - \left( \Delta \lambda - (6(30 - n) - 3) \rho \right) \]

where:  
\[ \lambda = \text{longitude} \]
\[ \rho = 0.017453279252 \]
\[ n = \text{zone number} \]

The first negative converts longitude positive eastward to positive westward.

A first approximation to foot point latitude is computed from:

\[ \phi' = S(l + s^2(\text{AA} + s^2(\text{BB} + s^2(\text{CC} + s^2\text{DD})))) \]

where:
\[ \text{AA} = -3e^2/6 \]
\[ \text{BB} = (12e^2 + 45e^4)/120 \]
\[ \text{CC} = -(48e^2 + 1023e^4 + 1170e^6)/5040 \]
\[ \text{DD} = (192e^2 + 18304e^4 + 75099e^6 + 6048e^8)/362880 \]
\[ Y - \frac{Y}{(1.9996a(1-c^2))} \]

\[ Y - \text{UTM Northing} \]

Improved approximations of \( \psi' \) are obtained by Newton iteration, that is:

\[ \psi'_{(2+1)} - \psi'_{(1)} = \frac{F}{F'} \]

where:

\[ F = S - S_e \]

and:

\[ S = A\psi' - \left( b/2 \right) \sin \left( 2\phi' \right) + \left( C/4 \right) \sin \left( 4\phi' \right) \]

\[ - \left( D/b \right) \sin \left( b\phi' \right) \]

\[ A = 1 + \frac{1}{4}v^2 + \frac{15}{64}v^4 + \frac{175}{256}v^6 \]

\[ B = \frac{1}{4}v^2 + \frac{15}{16}v^4 + \frac{525}{512}v^6 \]

\[ C = \frac{15}{64}v^4 + \frac{105}{256}v^6 \]

\[ D = \frac{15}{512}v^6 \]

and:

\[ F' = \left( 1 - e^2 \sin^2 \phi' \right)^{\frac{1}{2}} \]
2.1.3 Source and History of Approach

The Thomsen derived projections have been rigorously tested and approved in many cartographic projects. The inverse to the Mercator mapping was developed at Syntaxis Corporation. The inverse to the Transverse Mercator mapping has been obtained from Kapp and Sprinsky.

2.1.4 Empirical Results and Evaluation

The transformations from earth rectangular to geographic coordinates, when combined with the transformations derived in registration, are accurate to within a second of arc within the well-known bounds of application of the map projections themselves. They are accurate enough to satisfy any requirement which stays within this limitation. The inverse Transverse Mercator mapping is confined to one zone at a time. The inverse Mercator mapping equation handles arbitrary earth rectangular values.

2.1.5 Future Topics of Interest

The inverse transverse Mercator mapping is restricted to transforming earth rectangular coordinates within a single zone to geographics. It might prove beneficial to generalize the mapping to allow zone overlap if this is possible.

2.4 Geographic Sectioning Algorithms

2.4.1 Problem to be Solved

The sectioning algorithms provide the capabilities to perform circle, path, and polygon searches within areas generated on the surface of the earth. The earth is imagined to be a sphere of unit radius.

2.4.2 Method of Solution

To perform a given search, the routine must be called twice: the first call generates the appropriate parameters to describe the region in question, and the second call accesses points and calculates whether they fall inside the test region.
(A) First Call

On first call the input lat-long values of the center and radial point are converted to spherical coordinates on a unit sphere.

(B) Second Call

First the input test point is converted to spherical coordinates on a unit sphere. Then the great circle distances from the center point to the radial point and from the center point to the test point are calculated. If the latter distance is less than the former, the test point is within the region defined by the center and radial points.

The distances are calculated as follows:

Let \( \mathbf{R} \) and \( \mathbf{S} \) be vectors emanating from the origin of the sphere to the 2 points in question. The dot product of \( \mathbf{R} \) and \( \mathbf{S} \) is the cosine of the angle between the two vectors. The angle is the great circle distance in question. Formally we have

\[
\mathbf{R} \cdot \mathbf{S} = X_1 X_2 + Y_1 Y_2 + Z_1 Z_2
\]

but

\[
X_1 = \sin \theta_1 \cdot \cos \phi_1
\]

\[
Y_1 = \sin \theta_1 \cdot \sin \phi_1
\]

\[
Z_1 = \cos \theta_1
\]

\[
X_2 = \sin \theta_2 \cdot \cos \phi_2
\]

\[
Y_2 = \sin \theta_2 \cdot \cos \phi_2
\]

\[
Z_2 = \cos \theta_2
\]

where \( \theta \) is latitude and \( \phi \) is longitude. Thus \( \mathbf{R} \cdot \mathbf{S} = \sin \theta_1 \sin \theta_2 (\cos(\phi_1 - \phi_2)) + \cos \theta_1 \cos \theta_2 \), letting \( W = \) distance sought we have

\[
W = \sqrt{X_1^2 + Y_1^2 + Z_1^2 - 2 \mathbf{R} \cdot \mathbf{S}}
\]
\[ W = \arccos \left( \sin \theta_1 \sin \theta_2 \left( \cos (\phi_1 - \phi_2) \right) + \cos \theta_1 \cos \theta_2 \right) \]

Q.E.D.

### 2.4.2.2 Polygon Search

**(A) First Call**

On first call, the input lat-long values of the polygonal vertices are converted to Cartesian X-Y-Z coordinates on the surface of a unit sphere centered at the origin of the coordinate system.

**(B) Second Call**

On second call the input lat-long values of the test points are converted to Cartesian X-Y-Z coordinates on the surface of a unit sphere centered at the origin of the coordinate system.

To calculate whether a given test point lies within the polygonal test region, the following insight is utilized. Suppose the \( n \) vertices of the polygon have coordinates \( <X_1, Y_1, Z_1>, <X_2, Y_2, Z_2>, \ldots, <X_n, Y_n, Z_n> \) and the test point has coordinates \( <X_t, Y_t, Z_t> \). Consider the planes defined by the triangles whose vertices are \( \{<X_1, Y_1, Z_1>, <X_2, Y_2, Z_2>, <X_3, Y_3, Z_3>\}, \{<X_1, Y_1, Z_1>, <X_3, Y_3, Z_3>, <X_4, Y_4, Z_4>\}, \ldots, \{<X_1, Y_1, Z_1>, <X_{n-1}, Y_{n-1}, Z_{n-1}>\}, <X_n, Y_n, Z_n> \) and the line defined as that line connecting the origin to the coordinates \( <X_t, Y_t, Z_t> \). Let \( W_1, W_2, \ldots, W_{n-2} \) be the points of intersection between the line and each of the planes. Let \( \theta_1, \theta_2, \theta_3 \) be for a given triangle and a given plane, the angles described from the point of intersection \( W_k \) to the triangular vertices \( \{<X_1, Y_1, Z_1>, <X_{k-1}, Y_{k-1}, Z_{k-1}>\}, <X_k, Y_k, Z_k>\} \). If \( \theta_1 + \theta_2 + \theta_3 = 2\pi \) then the point of intersection lies within the triangular region. If this occurs for at least one triangle, the point is obviously in the polygonal region.
The following is a formal derivation for a test point and a given triangle. First the equation of the plane of the triangle is calculated, then the point of intersection is calculated, then the origin of the coordinate system is translated to this point of intersection, and finally the angles are calculated and summed.

1. Calculate equation of plane of triangle

The general equation of a plane is given by the formula \(A \cdot X + B \cdot Y + C \cdot Z = 0\). To determine the planar equation given 3 points \(<X_1, Y_1, Z_1>\), \(<X_2, Y_2, Z_2>\), \(<X_3, Y_3, Z_3>\) we have:

\[
A = X_1(Y_2Z_3 - Z_2Y_3) - Z_1(Y_2 - Y_3) + (Y_2Z_3 - Z_2Y_3)
\]

\[
B = Y_1(Z_2 - Z_3) - Z_1(Y_2 - Y_3) + (X_2Z_3 - X_2Y_3)
\]

\[
C = Z_1(Y_2 - Y_3) - Y_1(X_2 - X_3) + (X_2Y_3 - Y_2X_3)
\]

\[
D = -[X_1(Y_2Z_3 - Z_2Y_3) - Y_1(X_2Z_3 - Z_2X_3) + Z_1(X_2Y_3 - Y_2X_3)]
\]

Given that the three points are triangular vertices, the above coefficients determine the plane of the triangle.

2. Point of Intersection

Suppose that \(RP(I)\), \(I=1,2,3\) is the XYZ coordinate of the test point. Then the equation of the line in space through the origin and the test point is

\[
\frac{X}{RP(1)} = \frac{Y}{RP(2)} = \frac{Z}{RP(3)}
\]

Suppose \(RP(I) \neq 0\). Then we proceed as follows:

\[
Y = \frac{RP(2) \cdot X}{RP(1)} \quad Z = \frac{RP(3) \cdot X}{RP(1)}
\]

\[
A \cdot X + B \cdot \left[\frac{RP(2) \cdot X}{RP(1)}\right] + C \cdot \left[\frac{RP(3) \cdot X}{RP(1)}\right] + D = 0
\]

\[
X \cdot \left[\frac{A + B \cdot RP(2)}{RP(1)}\right] + C \cdot \left[\frac{RP(3)}{RP(1)}\right] = -D
\]
\[ X = \frac{D}{A + B \cdot \text{RP}(2)} + C \cdot \text{RP}(3) / \text{RP}(1) \]

Let \( E = \text{RP}(2) / \text{RP}(1) \)
\( F = \text{RP}(3) / \text{RP}(1) \)
\( H = A \cdot E \cdot F \)

then we have
\[ X = -D / H \]

Let \( \text{RP}(I), 1 \leq I \leq 3 \) be the coordinate of the point of intersection.

Then we have
\[
\begin{align*}
\text{RP}(1) &= X = -D / H \\
\text{RP}(2) &= \text{RP}(2) \cdot \text{RP}(1) / \text{RP}(1) \\
\text{RP}(3) &= \text{RP}(3) \cdot \text{RP}(1) / \text{RP}(1)
\end{align*}
\]

(3) Translate origin to point of intersection

Suppose \( \langle X_j, Y_j, Z_j \rangle \) is an arbitrary point in X-Y-Z space. Then \( \text{SJK}(I), 1 \leq I \leq 3 \) is defined as follows
\[
\begin{align*}
\text{SJI}(1) &= X_j - \text{RP}(1) \\
\text{SJI}(2) &= Y_j - \text{RP}(2) \\
\text{SJI}(3) &= Z_j - \text{RP}(3)
\end{align*}
\]

Then \( \text{SJI}(I) \) is the translated point
\[ A = \begin{pmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{pmatrix} \]

\[ CB = A - RPI \]

Note the CB is the array A translated to a new origin at RPI. Thus each column of CB is a coordinate. To find \( \theta_1, \theta_2, \) and \( \theta_3 \) proceed as follows. Take the following vector dot products

1. \( \langle CB(I,1), CB(I,2) \rangle \)
2. \( \langle CB(I,1), CB(I,3) \rangle \)
3. \( \langle CB(I,2), CB(I,3) \rangle \)

according to the formula

\[
A \cdot B = \sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2} \cdot \cos \theta
\]

where

\[
A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}
\]

\[
B = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}
\]

Given that the following assignments are made

1. \( RMAG1 = (CB(1,1))^2 + (CB(2,1))^2 + (CB(3,1))^2 \)^{1/2} \]
2. \( RMAG2 = (CB(1,2))^2 + (CB(2,2))^2 + (CB(3,2))^2 \)^{1/2} \]
3. \( RMAG3 = (CB(1,2))^2 + (CB(2,3))^2 + (CB(3,3))^2 \)^{1/2} \]

we have

1. \( CB(I,1) \cdot CB(I,2) = RMAG1 \cdot RMAG2 \cdot \cos \theta_2 = DOT1 \)
2. \( CB(I,1) \cdot CB(I,3) = RMAG1 \cdot RMAG3 \cdot \cos \theta_1 = DOT2 \)
3. \( CB(I,2) \cdot CB(I,3) = RMAG2 \cdot RMAG3 \cdot \cos \theta_3 = DOT3 \)
Therefore

(1) \[ \theta_2 = \frac{\text{DOT1}}{(\text{RMAG1} \cdot \text{RMAG2})} \]

(2) \[ \theta_1 = \frac{\text{DOT2}}{(\text{RMAG1} \cdot \text{RMAG3})} \]

(3) \[ \theta_3 = \frac{\text{DOT3}}{(\text{RMAG2} \cdot \text{RMAG3})} \]

Therefore

(1) \[ \theta_2 = \text{ARCCOS} (\theta_2) \]

(2) \[ \theta_1 = \text{ARCCOS} (\theta_1) \]

(3) \[ \theta_3 = \text{ARCCOS} (\theta_3) \]

2.4.2.3 Path Search

(A) First Call

First the 3 points input which define the path are converted to
XYZ coordinates on the surface of a unit sphere yielding \( <X_1, Y_1, Z_1> \), \( <X_2, Y_2, Z_2> \), \( <X_3, Y_3, Z_3> \). The vertices of the quadrangle defined by the path are found as follows. Let \( \Delta X = X_3 - X_2 \), \( \Delta Y = Y_3 - Y_2 \) and \( Z = Z_3 - Z_2 \). The four vertices are

(1) \( <X_3, Y_3, Z_3> \)

(2) \( <X_2 - \Delta X, Y_2 - \Delta Y, Z_2 - \Delta Z> \)

(3) \( <X_1 - \Delta X, Y_1 - \Delta Y, Z_1 - \Delta Z> \)

(4) \( <X_1 + \Delta X, Y_1 + \Delta Y, Z_1 + \Delta Z> \)

This follows by the obvious symmetry of the sphere

(B) Second Call

This is just a 4 sided polygon search.
2.4.3 Source and History of Approach

The sectioning algorithms were completely developed in-house at Synectics Corporation.

2.4.4 Empirical Results and Evaluations

The sectioning algorithms have been tested thoroughly and are accurate in typical application environments. The only source of approximation consists in assuming that the earth is circular. Thus geodetic paths on the ellipsoid are considered to be great circle paths on the surface of the earth. For large earth test areas, this could result in some inaccuracy in polygon searches. The extent of the inaccuracy has not been rigorously analyzed.

2.4.5 Future Topics of Interest

The polygon sectioning routines are employed within the context of determining whether a given quadrangular region overlaps a given polygonal region. An algorithm needs to be developed which determines whether two such regions overlap. At present, a serial search method is employed to solve this problem and this seems to be a needless consumption of time.

It might also be of interest to develop an algorithm which would approximate the region of overlap in question with a many sided polygon. In this way, a third region could be introduced so that a determination could be made whether three regions overlap. Thus, in general, an iterative procedure would exist to determine when n regions overlapped and an approximate polygonal representation of the overlap would be obtained.

Finally, the polygon search might be generalized to more complex polygonal regions; that is, regions where angles could be obtuse between adjacent sides. This would probably be required if the iterative procedure described above were developed.
SECTION III
SOFTWARE ACMPLOISHMENTS

1.1 Purpose

The purpose of this section is to discuss some of the more interesting developments which have occurred during the creation of the Phase 1 BDRS software. The discussions included are not intended to exhaust all facets of the system. The focus of each separate discussion will be from the key novel capabilities provided by the software in satisfying the requirements of the three BDRS subsystems: digitization and voice entry, batch and data base.

3.2 Digitization and Voice Entry Subsystem

3.2.1 Sounding Data

The key accomplishment of this BDRS Subsystem is that of providing the capability to digitize sounding data from nautical charts and store this sounding information in a compact and conveniently accessible form. Sounding data may be entered manually with a keyboard and vocally via communication between the Eclipse C300 and the Threshold 500 Voice Recognition terminal/display device. As the voice data is entered, a visual display is provided on the Tektronix 4010 display unit.

The sounding data may also be edited by the user in a very simple way using either the special keyboard or the 'voice box'. Thus, a complete capability is provided to create and edit features of sounding data which makes maximum use of system resources to ease the task of the user in data entry and edit.

3.2.2 Fathograms

An important capability of this BDRS subsystem is that of digitizing fathograms; that is, graphs of depth/time coordinates supplemented with such parameters as geographic fixes, loxodrome bearings, and ship velocities. Once a digitized fathogram file has been created, a file of geographic-depth positions can in principle be constructed and entered into a data base. Thus, the analog information of the fathogram is integratable into the general BDRS data base framework.
3.2.3 Registration

The algorithm of registration has been thoroughly discussed in Section 2.2 of this document. It should, however, be emphasized here that the development and implementation of this algorithm within this subsystem provides the subsystem user a very efficient and easily employed method of registering charts so that information in a feature digitized from that chart is easily accessed and accurately edited. In the Lineal Input System (LIS) no such method was employed, and the errors inherent in both the LIS day 'l' and day 'n' registration procedures have been eliminated. Within the conceptual structure of the registration scheme developed by Synectics Corporation, it becomes an easy matter to edit table files even when they are created by the batch program which converts arbitrary geographic files to table files. Thus, an arbitrary geographic file constructed from information in the database can in principle be converted to table form, edited, and reconverted to geographic coordinates. A more thorough discussion of this will be found in the discussion of the batch program which converts geographic files to table files, in Section 3.3.1 of this document.

3.2.4 Review Mode

The capability exists to display the data of a table file on a Tektronix 4010 display given a user selected window. Each feature, whether consisting of trace data, discrete points, or soundings, is searched and data which falls within the window is displayed. This provides the user with all he needs to discover where he desires to make edit changes. The scope of such an editing capability in relation to the Data Base is further discussed in Section 3.4 of this document.

3.3 Batch Processes Subsystem

3.3.1 Geographic to Table Conversion

The novel feature provided by this capability is that the resultant table file can be immediately edited, in principle, by employing a day 'n' registration. This can be seen as follows. When the file is converted, the registration points of the file are also converted. The resultant file has been day
I registered in the sense that a perfect fit is obtained with least square best fit coefficients of \( \phi \). This means that any table point in the file need only be scaled to earth scale meters followed by a translation from the chart earth scale meter lower left hand corner to a true origin to obtain the true earth scale meter value of the table point. The plotted chart, when placed on the table, is registered as a day n registration of the perfect fit file. Therefore, given that a set of registration points are in the geographic file, an arbitrary geographic file can be brought to table, edited, and reconverted to geographic form.

The mathematical simulation of this is as follows. Suppose \((\phi, \lambda)\) are the latitude and longitude of an arbitrary coordinate in the geographic file. Let \(X_L, Y_L\) be the map scale meter value of the lower left hand corner of the chart. Let \(X_\phi, Y_\lambda\) be the map scale meter value of the coordinate \((\phi, \lambda)\). The table \(X, Y\) of the point is

1. \(X = (X_\phi - X_L) + 3\) inches, conversion to miles
2. \(Y = (Y_\lambda - Y_L) + 3\) inches, conversion to miles

Let \(X_n, Y_n\) be the map point \(X, Y (\phi, \lambda)\) when the map is placed on the table which is produced by a plot of the table file. Then registration will map \(X_n, Y_n\) to \(X, Y\) and \(X_\phi, Y_\lambda\) need only be calculated from equations 1 and 2 to give the map scale meter values of \((\phi, \lambda)\). When scaled to earth scale meters by multiplying by the map scale, the earth scale meter value is obtained for \((\phi, \lambda)\).

3.3.2 Table to Geographic Conversion

A novel feature provided by this capability is that two distinct kinds of table file can, in principle, be converted to geographic form; digitized table files, and table files created by geographic to table conversion. The difference between the conversion is that the latter table files are transformed to earth scale meter coordinates as described in Section 3.3.1, whereas the former table files are converted to earth scale meter coordinates by accessing the day 1 least square best fit coefficients created when the table file is registered as described in Section 2.2.2.
A second novel feature of this conversion consists in the way in which point to point data is treated in trace features. Point to point data consists of two absolute embedded points occurring successively in the feature. A straight line is implicitly assumed to exist between the two points. If the data is converted to geographics and then back to table using a different map projection, the straight line will cover different map scale meter points. The solution to this problem consists in generating X-Y points on the line connecting the two points in the original table file at increments of 127 miles in either X or Y. As an incremental X-Y position on the line is obtained, it is converted to geographics. Thus, the implicit line as a whole is representative converted to geographics in the table to geographic conversion function.

3.3 Plot Functions

The plot software has several features worthy of especial notice. The development of this software is such that, in principle, any other type of plotter can be added to the BDRS hardware configuration simply by accessing an intermediate plot file. Second, the Xynetics interface routines are written in simple Fortran and are in principle, usable by other computer systems. Finally, the development of further plot functions is readily integratable into the current software due to the simple calling sequence of any plot function. For example, functions existing on other computer systems interfacing with a Calcomp plotter may be integrated into the BDRS system to produce the same type of output.

3.4 Data Base Subsystem

3.4.1 Sectioning

The Data Base subsystem provides the capability of creating BDRS geographic files on the basis of search criteria which are thoroughly discussed in Section 2.4.2 of this document. In principle, the following sequence of steps could be taken, utilizing this capability, to quickly edit the files included in the source sectioned files. First, the created geographic file is converted to table and plotted. The plot is then registered and the table file reviewed.
until the data to be edited is found. The edit mode is then entered to edit the file. Each feature edited identifies the source ID and document number of the source sectioned file, as well as the number of the feature relative to this source sectioned file. The edited file is then reconverted to geographics with a flag set at each feature which has been edited. Finally, the source sectioned files are updated to reflect the edits that were made on the table file. The following diagram reflects the flow of this sequence.

SOURCE SECTIONED FILES

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

GEOGRAPHIC FILE

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

TABLE FILE

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

EDITED TABLE FILE

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

GEOGRAPHIC FILE

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

EDITED SOURCE SECTIONED FILES
SECTION IV
THE PROBLEMS OF ACCURACY

4.1 Purpose

The purpose of this section is to analyze the critical contexts within which considerations of accuracy are relevant. In each such context, the intent is to identify both where considerations of accuracy have been analyzed and where further analysis is required.

4.2 Data

4.2.1 Accuracy of Source Analog Data

4.2.1.1 Charts

The accuracy of data portrayed on man-made navigational charts and maps is clearly suspect in many instances. Charts become distorted because of shrinkage and expansion, and the distortion is clearly a highly complex nonlinear phenomena from a mathematical point of view. The algorithm of registration does little, if anything, constructive to compensate for this distortion as registration is represented within the classical linear model of maximum likelihood which assumes uniform stretching or shrinking in an arbitrary direction.

When such charts are digitized and converted to geographic coordinates, information becomes available to the data base which seriously threatens its integrity. Thus, the ability to edit the data base becomes of paramount importance. A serious look should be taken at the model, provided by Synectics Corporation, in Section 3.4.1 of this document, which purports to encompass such an editing capability.

The accuracy of data portrayed on charts produced mechanically by plotters on such systems as the Lineal Input System, presents another problem as the material out of which the charts are made is much more resistant to distortion. These problems will be discussed momentarily in Section 4.3 of this document.
4.2.1.2 Data Entry of Analog Source Materials

Data is entered by the user within the digitization subsystem of BDRS. When graphical feature data is entered with a cursor, the data is accepted as is by the system; that is, there is no error term employed by the software to smooth out the errors which occur in data entry itself. In a linear model, the typical assumption of statisticians is that such error can be modeled by a probability distribution with a zero expectation and a variance characteristic of the particular user. Whether the inclusion of such an error term would be fruitful within BDRS itself has not been analyzed empirically.

A natural question to ask is how to check whether the data entered by the user is representative of the data supplied on the chart. The proof plot provides the check required. If the proof plot is a perfect overlay of the original source chart as it is taped on the table when digitized, then each digitized point is within an epsilon of its correct position. That is, depending upon the resolution of the data when digitized (from 1 mil to 10 mils), and accuracy of the plotting device. It may appear (to the human eye) that two chart points overlap each other, but in reality may actually be a few mils apart.

The accuracy of the sounding data entered by the user can be checked by the user as he enters the sounding value. This value is displayed on the Tektronix 4010 display. If the user is employing the Threshold 500 Voice unit, his entry is further displayed on the Threshold display unit.

4.2.2 Accuracy of Processed Data

4.2.2.1 Registration

The algorithm of registration is thoroughly discussed in Section 2.2.2 of this document. In this section several corollaries pertaining to the problem of accuracy will be deduced from the mathematical properties of the algorithm.

It was noted in Section 2.2.4 of this document that registration points should be picked judiciously; that is, representative of the chart as a whole and not in any obvious geometrical pattern. The reason for this latter condition finds its justification in the mathematical fact that an earth scale
meter rectangular or arbitrary vertices can result from a distorted rectangle with acceptable residuals. For example, in registering a UTM chart, if the vertices of a rectangle are used for control points and the user enters the wrong northings and eastings for the points, then if his incorrect entries also constitute the vertices of a rectangle, the registration algorithm will produce a mapping which has acceptable residuals. Other obvious geometric shapes have this same characteristic.

A second property of the algorithm, which is actually quite unfortunate, is that a seriously distorted chart may be registered with acceptable residuals. Thus, it is the users responsibility to ensure that he is not allowing poor data into the BDPS system. There is a check which can be employed for this case. It is discussed in Section 4.2.2.2 of this document.

A final corollary of this algorithm is that the proper registration of a good source chart generates an extremely accurate mapping from the coordinate frame of the table to the earth rectangular frame in day 1 registration. The simplicity of the algorithm conjoined with the partitioning of symmetric matrices to find inverses results in a very accurate mapping.

4.2.2.2 Coordinate Transformations

When geographic coordinates are mapped to the earth scale meter frame and then back to the geographic frame, the resultant geographic coordinates are within a second of arc of the original geographic coordinates. The case of iterating this procedure to check for an accumulation of harmful error has not been checked rigorously.

Since the coordinate transformations and the registration transformation are so accurate, the following sequence constitutes an excellent test of whether the source analog chart is distorted badly.

(1) Register the chart and build a table file.
(2) Convert table file to geographic file.
(3) Convert geographic file to table file.
(4) Plot table file.
If the plot overlays cleanly with the source data, then the chart is in good condition. Otherwise, the accuracy of the chart is seriously suspect.

4.3 Problem of Validation

In order to evaluate the accuracy of the map projections, charts must be utilized which represent data to the degree of accuracy being tested. The Hydrographic Center has provided such a chart for the Mercator projection test. Charts of such high caliber will also be provided for each mapping demanded.

To evaluate the accuracy of the polygon search algorithm, a high precision Gnomic projection chart is required. The great circle vertices of the polygon map to straight lines on such a chart. Thus, one may construct with precision arbitrary earth polygonal regions on the chart and test arbitrary points for inclusion within the defined chart polygonal region.

If the accuracy of the map projections is precisely known, the accuracy of the functions constructed in registration can be tested by simply comparing their output with the known output of geographic points either identical to or different from the registration points.

The overall problem of all of these validation procedures is that they demand charts which are 'known' to be accurate within the degree of precision for which a test is to be made. If the charts are machine produced from the Lineal Input System (LIS), then the projections must conform to those employed in LIS itself. Whether this is desirable or not has not been rigorously analyzed within the BDRS research.

The procedure to this point has been to implement the LIS geographic to table projections for the Mercator and Transverse Mercator mappings. The equations have been checked and validated within BDRS. Whether such a procedure will be effective for dealing with the Polyconic and Lambert Conformal projections is not at present known.
SECTION V
SYSTEM CONFIGURATION

5.1 Purpose

The purpose of this section is three-fold. First, to provide a detailed description of the hardware configuration employed in Phase I of the BDRS. Secondly, to identify and discuss key technical areas which pertain to the development of the BDRS functional capabilities. Thirdly, to present conclusions and recommendations pertaining to the BDRS.

5.2 Hardware Configuration

Figure No. 5-1 illustrates the BOC hardware configuration which consists of the following:

- Data General ECLIPSE C300 Processor - 128K core memory
- Data General Magnetic Tape Units, 9 Track (2)
- Data General 6012 CRT
- Centronics Line Printer
- Data General, 92MB Disk Drive, dual portable
- Data Automation Digitizing Tables (2)
- Tektronix Graphic Terminal, 4010, (2)

Station One (Figure No. 5-2)

- 42" X 60" active Area Data Automation (X/Y Digitizer Table)
- Standard Cursor (five push buttons)
- Tektronix 4010 CRT
- 16 Key Keyboard

Station Two (Figure No. 5-3)

- 42" X 60" active area Data Automation (X/Y Digitizer Table)
- Special cursor (LED display/five push buttons)
- Tektronix 4010 CRT
- Threshold Technology, Inc., Model 500 voice data entry terminal
- 16 Key Keyboard

5-1
5.3 BDRS Software Configuration

5.3.1 Key Areas

The key areas which pertain to the utilization of vendor supplied system software (MRDOS/INFOS) within the BDRS are as follows:

- **Multitasking** - A multiple task environment is one in which logically distinct tasks compete simultaneously for the use of system resources.

- **Overlays** - Overlaying is a technique used for loading routines into main memory from some type of mass storage during the execution of a program.

- **User Device** - User devices are any device that is not part of Data General's equipment identification during a system generation (SYSGEN). These devices are identified to the system at run time via operating system subprogram calls. (Reference Data General Application Note: "User Device Driver Implementation In The Real Time Disk Operating System", 017000002-03).

5.3.1.1 Multitasking

The BDRS subsystem software utilizes the multitasking capability provided by Data General's Mapped Real Time Disc Operating System (MRDOS/INFOS). The digitization software consists of three (3) tasks which can run in the foreground of the C300 Data General processor. The first task functions merely to wake up the two remaining tasks: table 1 and table 2. Thus, two digitizing tables can be operated concurrently in one ground of the system. In the remaining ground, either Data Base or Batch can be executed.

5.3.1.2 Overlays

The table 1 and table 2 software modules are very similar. The execution of each is divided into overlays. Each task has one overlay segment assigned to it. A detailed examination of the actual overlay structure by
overlay name and routine name is depicted in Volume I Bathymetric Data Reduction Subsystem Software Documentation BOC Phase, Section 3.2.2.

Each of the three (3) data base related processing modes (On-Line, Batch and Master) utilize the overlaying technique in performing their respective processes. Consistent throughout the above mentioned processes is the fact that each processing function is defined as an overlay and is loaded into core only as the result of a user selecting the particular function. The implementation of such a philosophy was made simple by the fact that the three processing modes were developed in a modular fashion, thus allowing for straight forward overlay identification and structuring.

5.3.1.3 User Device Implementation

The digitizing tables, voice recognition unit, and each Tektronix 4010 CRT associated with a table are introduced to the system software in the same way. The method of introduction is described in detail in Data General's User Manual entitled "User Device Implementation". It suffices here to say that each device is assigned a number and the interrupt service routine address for each device is stored in a system vector table.

When an interrupt occurs in a multitasking environment, the multitasking activity is suspended. When the interrupt has been serviced, it is the responsibility of the interrupt service routine to reawaken the environment. This is accomplished by sending the appropriate message to the task interrupted which has issued a receive message request prior to the interrupt. The system then activates this task and the multitasking environment is rescheduled.

5.3.2 Conclusions and Recommendations

Utilizing the INPOS/MRDOS operating system (normally called INPOS) both foreground and background processing may be operated concurrently. As stated in paragraph 5.3.1.1, two digitizing tables can be operated in one ground, namely the foreground and either the Data Base or Batch functions can be operated in the background. The operating system (INPOS), digitizing
functions and data base functions, are highly overlayed processes, that is, they rely on disk resident overlays for loading functional subprograms related to their specific functions. This, coupled with the facts that the digitizing processes is a time critical activity, requiring system resources on "demand" (i.e., CPU) while data collection is taking place, and the BDRS data base INPOS files are large randomly organized disk files which need be accessed an indeterminate number of times depending on the complexity of the user requests for data, supports Sylectic's beliefs that the disk is over burdened due to the frequency and location of disk access which caused excess head movement otherwise known as thrashing.

To alleviate the above mentioned problems, two possible solutions are recommended. The first solution would be to dedicate a disk (preferably a quick fixed head disk) to be used exclusively for storage/retrieval of the following data: MDMOS/INPOS overlays, BDRS subsystem's overlays, intermediate working files and any non-data base related disk activities. The second possible solution would be to dedicate a processor exclusively for the data base functions sharing a master disk for infrequent bulk data exchange between the two processors.

It should also be pointed out that in the production environment, the present line printer output capability may be inadequate. The Centronics Line Printer Model 101, currently part of the BDRS hardware configuration, outputs 165 characters per second (cps). When volume hardcopy reports are necessary, the line printer and disk line printer spooler are tied up for long periods of time. When this occurs, no other function may use the printer or spooler for producing any hardcopy output. With this fact, serious consideration should be given to acquiring a faster line printer for the production environment.
MISSION
of
Rome Air Development Center

RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control Communications and Intelligence (C3I) activities. Technical and engineering support within areas of technical competence is provided to ESD Program Offices (POS) and other ESD elements. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.