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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research done under Air Force Grant No. AFOSR 76-3046 during the period 6.1.78 - 5.31.79 consists mainly of generalizing previous work to obtain new, easily checkable stability criteria for general, dissipative or nondissipative, explicit or implicit difference approximations to hyperbolic mixed initial-boundary value problems. The criteria obtained are independent of the basic scheme and are given solely in terms of the boundary conditions; thus they are

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20. Abstract (Continued)

significantly more convenient than traditional criteria. The results imply that many well known boundary conditions, when used in combination with arbitrary dissipative or nondissipative schemes, always maintain stability.

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PROBLEMS IN STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS

Interim Report

1.6.78 - 5.31.79

by

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79

During the period 6.1.78 - 5.31.79 I was mainly engaged with E. Tadmor in extending previous work [1,2] to obtain new stability criteria for a wide family of difference approximations to hyperbolic initial-boundary value problems in the quarter plane $x \geq 0$, $t \geq 0$.

In our work, [3], the approximated differential system is of the form

$$\partial u(x,t)/\partial t = A\partial u(x,t)/\partial x,$$

where A is a Hermitian nonsingular matrix, and the inflow and outflow unknowns interact at the boundary. For the difference approximation we consider arbitrary stable basic schemes --dissipative or nondissipative, explicit or implicit--together with general boundary conditions which determine the boundary values in terms of outflow values of internal grid points.

Using the stability theory of Gustafsson, Kreiss and Sundström [4] we show that the entire approximation is stable if and only if the scalar component of its outflow part are stable. We thus reduce the overall stability question to that of a scalar outflow problem, and from this point on our purpose becomes to obtain easily checkable, sufficient stability criteria for the reduced outflow problems.

Our results in this vein are essentially independent of the basic scheme and are given entirely in terms of the boundary conditions. Hence, these results are much more convenient than traditional criteria which involve the basic difference scheme as well as the boundary conditions.

As in [2], the main results are for the case where the outflow boundary conditions are translatory, i.e., determined at all boundary points by the same procedure. Roughly speaking, our results assure overall stability if the boundary conditions satisfy certain properties such as solvability and the von Neumann condition, plus a single inequality in one unknown which is usually verified without much effort.

Using the new results, numerous examples were worked out. For example, we show that if the basic scheme is dissipative (explicit or implicit) and two time leveled, and if the outflow boundary values are determined by horizontal extrapolation, then the overall approximation is stable (compare [5, 1, 2].) Surprisingly, it is shown that this result is false if the basic scheme is of more than two time levels; namely, outflow dissipative multi-step basic schemes extrapolated horizontally at the boundary, are not always stable.

For multi-level dissipative basic schemes we show that if the boundary conditions are generated for example by oblique extrapolation (compare [4, 2]), by the Box-Scheme (compare [4,7]), or by the right-sided weighted Euler scheme (compare [8]), then overall stability is assured. For general (dissipative or nondissipative) basic schemes we prove that if the boundary conditions are defined by the right-sided explicit or implicit Euler schemes (compare [4, 7, 2]), then the entire approximation is stable.

At present Tadmore and I are extending the above results to the case where the differential system is of the general form

$$\partial u(x,t)/\partial t = A\partial u(x,t)/\partial x + Bu(x,t) + F(x,t),$$

and where A is Hermitian, B an arbitrary matrix, and F a given inhomogenity vector. This is the most general case studied by Gustafsson, Kreiss and Sundström in [4]. We expect to complete this work by the summer of 1980.

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