A SURFACE DUCT LEAKAGE TERM APPLICABLE TO SHALLOW DUCTS AT LOW FREQUENCIES

G. GARTRELL
A new term is proposed for inclusion in a formula in common use for the calculation of acoustic propagation loss to extend its range of applicability to a shallow duct at low frequencies.

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1. INTRODUCTION

Baker (ref. 1) has systematically tested the validity of a formula for calculation of acoustic propagation loss given by Saxton (ref. 2), and as a result has suggested a revised coefficient for describing leakage from a surface duct. The formula is

\[
\text{Loss (dB)} = 20 \log R + 59.3 + (\alpha + \beta)R \\
(\text{for } R \leq 0.221(H)^{0.5})
\]

\[
\text{Loss (dB)} = 10 \log R + 5 \log H + 53 + (\alpha + \beta)R \\
(\text{for } R > 0.221(H)^{0.5})
\]

where \( R \) is range in kilometres and \( H \) is duct depth in metres.

\( \alpha \) is the volume absorption coefficient (discussed in Section 4) and \( \beta \) is Baker's high frequency duct leakage coefficient

\[
\beta \text{ (dB/km)} = \frac{29.1 f}{((4762.6 + 11.5t)H)^{0.5}} (1.4)^n
\]

where \( t \) is temperature (degrees C), \( f \) is frequency (kHz), \( H \) is duct depth (m), and \( n \) is sea state.

Baker's term is empirically derived and is only applicable with confidence to frequencies in the 3.25 to 7.5 kHz range and ducts generally deeper than 25 m.

For the frequencies and duct depths involved, a number of modes will be strongly trapped, and scattering due to duct roughness, particularly at and near the sea surface, will be the dominant leakage mechanism. As the frequency is decreased the commonly encountered range of duct depths, from 10 m to 40 m, traps less and less modes. Whereas the scale of duct roughness may be less significant as wavelength increases, a new and more fundamental duct leakage phenomenon becomes increasingly significant.

A surface duct still enhances near surface propagation even when it supports only one mode. As the duct depth decreases, or the frequency decreases, the trapping of the mode will become weaker, and leakage of the mode energy more significant. The important point is that ducted propagation is not a simple stop or go state. Between the extremes of no duct at all and a strongly trapping duct there is a full spectrum of partial trapping which may still be a major factor in propagation between two points.

Any acoustic propagation prediction model which uses a wave theory method can adequately demonstrate this. In the present study the parabolic equation propagation loss model (ref. 3) has been used to predict propagation loss as a function of range for frequencies in the range 200 Hz to 2.0 kHz, and for constant duct depths from 10 m to 100 m.

A comparison of the predictions enables the functional form of the partial trapping leakage coefficient (\( \gamma \)) to be established (see Section 3). Since the leakage coefficient \( \beta \) determined by Baker becomes insignificant in the region in which the \( \gamma \) becomes significant, and vice versa, the two terms can be simply added to provide a useful extension of the general formula down to 200 Hz.
2. PROPAGATION LOSS PREDICTIONS

Propagation loss predictions for simple isothermal ducts over a typical thermocline gradient to a totally absorbing bottom were made for a range of duct depths from 10 m to 100 m and frequencies from 200 Hz to 2 000 Hz.

Figure 1 shows the resultant predictions at 1 400 Hz for a source at 20 m and a receiver at 30 m for ducts ranging from 20 m to 60 m. The 20 m duct curve is almost identical to one with the receiver at 10 m even though in one case the receiver is 10 m above the duct bottom and in the other case it is 10 m below. The source in this example is of course only on the edge of the duct at 20 m. The 25 m and 30 m ducts show significantly better propagation, with the source now properly in the duct, although the receivers are respectively just below, and on the edge of the duct. Curves for receivers at 10 m depth for the latter two cases show identical slopes but about 5 dB less loss. This clearly illustrates that the mode amplitude distribution associated with a duct extends well beyond what in ray theory would be regarded as the bottom of the duct. The 40 m and 60 m ducts strongly enhance the propagation between 20 m source and 30 m receiver. Comparison of the propagation loss curves for the 60 m duct and the 100 m duct shows little average difference in range, but a significant change in structure. In the 60 m duct case we can see a periodic interference structure at short range which dies out to leave a smooth curve after about 20 km. This represents interference between the first and second modes. The second mode is only weakly trapped and dies out quickly, leaving the first mode. Extending the duct depth to 100 m preserves the second mode with little apparent leakage out to the 50 km maximum range of the model study, since the interference pattern in this instance is well developed over the entire range.

Although the presence of the second mode periodically reduces the propagation loss by about 2 to 3 dB, the range averaged loss does not show any significant improvement over the 60 m case. The relative amplitudes of the different modes excited depend on the depth of the source within the duct.

Figure 2 shows propagation loss from a 20 m source to a 150 m receiver. The curves for 20 m, 25 m, 30 m and 40 m ducts are self explanatory. The 60 m duct curve shows some interference structure, but very high loss due to the strong trapping of the first mode and partial trapping of the second. The 100 m duct curve shows a more complicated behaviour. In this case because of the depth of the duct, the mode tail extending below the duct is still significant at 150 m. The interference pattern below the duct differs markedly from that at the 30 m depth within the duct and probably involves the leaky third mode. Figure 3 shows field intensity diagrams for ducts with depths ranging from 30 m to 100 m for 1 400 Hz, and close inspection of these diagrams clearly explains all the curves of figures 1 and 2. The intensity diagrams vary from less than 70 dB loss for pure white, through 9 grey shades each representing a five dB step, to greater than 110 dB loss represented by the darkest tone. Range in the diagram runs from 0 to 50 km, and depth in water from 0 to 300 m. Below 300 m the model is given a highly absorbing non-reflective bottom to ensure that only the original surface duct energy contributes to the field.

3. DETERMINATION OF THE DUCT LEAKAGE COEFFICIENT

For each in-duct propagation loss curve, such as those for the 40, 60 and 100 m ducts of figure 1, there is a region near the source where the loss approximates spherical spreading, and at a greater range where the average loss would be determined by cylindrical spreading, apart from volume absorption and duct leakage. By subtracting the cylindrical spreading and volume absorption loss factors which have been included in the prediction curves, we are left with a residual loss which is attributable to duct leakage. Figure 4 plots curves of constant duct leakage so determined against frequency, with a commonly used formula for calculation of duct cut-off depth plotted for comparison:
where $f$ is frequency in Hz, and $H$ is duct depth in metres.

This formula is derived from one given in English units by Urick (ref. 4) and assumes a nominal value for sound speed in water of $1500 \text{ m s}^{-1}$.

Above and to the right of a family of curves denoted by $\beta = \gamma$ for sea states 0 to 5 on the same figure, Baker's duct leakage coefficient ($\beta$) becomes dominant. Conversely below the appropriate curve the trapping duct leakage term ($\gamma$) dominates. Since Baker's term has not been verified in this region it is not clear whether the best course of action is simple addition of the two terms, or just choosing the maximum. Since the two terms seem to represent different effects, and choosing to err on the side of caution it is suggested that the terms are simply added until empirical evidence indicates otherwise.

Figure 5 gives a number of curves of constant $\beta$ for sea states 0 to 5, from which the $\beta = \gamma$ curves of figure 4 were calculated.

Figure 6 shows the curves of figure 4, but this time with a logarithmic frequency scale, demonstrating that the curves may be approximated by the set of straight lines of figure 7.

The relationship between the lines of figure 7 is then fitted by a polynomial. The lowest order polynomial providing a reasonable fit over the range is of order 3, yielding the formula for duct leakage coefficient

$$\gamma (\text{dB/km}) = a + bx + cx^2 + dx^3$$

where

- $a = 0.076$
- $b = 0.00284$
- $c = 0.000256$
- $d = 0.00000162$

and

$$x = \left(110 + \frac{1.1767(D - 14.5)}{\log_{10} f - 0.4777}\right)$$

where

- $D$ is duct depth (m)
- $f$ is frequency (kHz)

This expression is easily coded for computing, and may simply be added to the coefficients for absorption ($a$) and empirical high frequency leakage ($\beta$) to extend the applicable range of the surface duct propagation formula usefully down to 200 Hz and to shallow ducts.

4. THE EXTENDED PROPAGATION LOSS FORMULA

While Baker adopts an expression for volume absorption coefficient determined by Hall (ref. 5), particularly at lower frequencies there has been found to be a regional variation in absorption coefficient (ref. 6, 7), and the user may be better served by an expression which takes into account the intended region of application.

For propagation at low frequency the modified Thorp formula of reference 7, is far simpler than the Hall formula of reference 5, and in addition takes into account regional variation.
For the Pacific Ocean this formula is

\[
\sigma (\text{dB/km}) = \left( \frac{0.05 f^2}{1 + \frac{1}{f_0^2}} + 0.01 f^2 \right) \times 1.09
\]

(7)

where \( f \) is frequency in kilohertz.

The overall surface duct propagation formula, following reference 1, may then be expressed as

\[
\text{Loss (dB)} = 20 \log R + 59.3 + (\alpha + \beta + \gamma) R
\]

(for short ranges \( R < 0.221(H)^{0.5} \))

(8)

or

\[
\text{Loss (dB)} = 10 \log R + 5 \log H + 53 + (\alpha + \beta + \gamma) R
\]

(for long ranges \( R > 0.221(H)^{0.5} \))

(9)

where \( R \) is range in kilometres

\( H \) is duct depth in metres

\( \alpha, \beta, \) and \( \gamma \) are defined by equations (3), (5), (6) and (7).

5. CONCLUSION

Simple acoustic prediction formulae can be of great assistance in estimating sonar system performance. Some terms in such formulae, such as Baker's high frequency duct leakage term, are best determined empirically. Baker's term is mainly concerned with near-surface phenomena. However, empirical extension of the formula to the lowest frequencies of significance for ducted propagation is not an easy task. At these frequencies the bottom of the duct becomes increasingly important. The bottom of the duct will frequently be irregular in depth on a large scale, both spatially and temporally, and propagation loss measurements will consequently be quite variable.

For this reason the low frequency leakage coefficient is most simply determined by computer modelling. In modelling the term this way, however, we should remember that the ducts encountered in practice are likely to be variable in depth, and that the strongest leakage will be associated with the shallowest duct region. A relatively small proportion of shallow duct may effectively dominate the leakage over a path consisting otherwise of mainly well established surface duct.

Provided such practical realities are borne in mind, the inclusion of the low frequency duct leakage term provides a useful extension of the Baker formula down to the lowest frequencies of significance for normal surface duct propagation.
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Figure 2.

Propagation loss vs range for various duct depths - deep receiver.
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Figure 7. A family of straight lines closely approximating the curves of figure 6.
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