ERRORS IN THE MEASUREMENT OF ANGLE WITH A LASER INTERFEROMETER.

APR 79  L M RAMSAY  D B PROWSE

UNCLASSIFIED  MRL-TN-420
ERRORS IN THE MEASUREMENT OF ANGLE WITH A LASER INTERFEROMETER

Lilian M. Ramsay and David B. Prowse*

Approved for Public Release

* Previously at Materials Research Laboratories. Now at the National Measurement Laboratory.
ABSTRACT

It is shown that when a laser interferometer is used for general angle measurement, including the calibration of angle measuring instruments such as rotary tables, errors of 30" or more can occur unless great care is taken in the optical alignment. On the other hand, if the instrument is carefully aligned, an accuracy of ± 1" may be obtained over a range of ± 14°. The errors that occur are independent of the sensitivity of the interferometer system.

Approved for Public Release

* Previously at Materials Research Laboratories. Now at the National Measurement Laboratory.

© COMMONWEALTH OF AUSTRALIA 1979
It is shown that when a laser interferometer is used for general angle measurement, including the calibration of angle measuring instruments such as rotary tables, errors of 30" or more can occur unless great care is taken in the optical alignment. On the other hand, if the instrument is carefully aligned, an accuracy of ± 1" may be obtained over a range of ± 14°. The errors that occur are independent of the sensitivity of the interferometer system.
CONTENTS

1. INTRODUCTION 1
2. ANALYSIS AND DISCUSSION 2
3. CONCLUSION 5

REFERENCE 5

* * *
1. INTRODUCTION

The advent of the laser interferometer has revolutionised the measurement of length and has made possible measurements to an accuracy of 1 part in $10^6$ on a routine basis, these measurements being limited significantly only by a knowledge of the temperature of the work piece. By use of two retroreflectors with a fixed separation it is possible to convert distance on a laser interferometer to angle. This has been done commercially by Hewlett-Packard who market an angle accessory for their laser interferometer which can measure angles over a range of $\pm 10^\circ$ with a resolution of 0.1" and which has an accuracy specification of $\pm 1$ second up to 1000 seconds and $\pm 4$ seconds per degree up to 10 degrees. Schatz (1) describes a similar instrument which has a range of $\pm 2$ degrees and an accuracy within 0.03 second.

The principle of operation of these instruments can be seen by an examination of Figure 1 where it can be seen that as the reflector mount rotates then the path length between the two arms changes, and this change is related to the angle of rotation. The Hewlett-Packard accessory consists of two components, a right-angle beam bender which is both wrung and bolted on to the remote interferometer (a beam splitter mounted in a precision metal cube), and a reflector mount in which two retroreflectors (or corner cubes) are mounted a fixed distance $L = 52.388 \text{ mm (2.0625 inch)}$ apart. When the retroreflectors are separated by this distance, then, for small angles, 25.4 nm (1 \mu in) on the interferometer display is equal to 0.1" of rotation of the reflector mount relative to the remote-interferometer-and-beam bender assembly.

This accessory has been found to work well for measurement of pitch and yaw on machine tools and for straightness and flatness measurements. However, because this type of instrument does not measure angles directly, but sines of angles, then errors can arise in general angle measurement and in the calibration of angle measuring instruments, such as rotary tables, if the reflector mount is not set correctly to zero.
2. ANALYSIS AND DISCUSSION

Schatz (1) has shown that

\[ D = L \sin \epsilon + (b' - b) \left[ \frac{n - \sin^2 \epsilon}{(n^2 - \sin^2 \epsilon)^{\frac{1}{2}}} - 1 \right] \]  

(1)

where

- \( D \) is the interferometer reading
- \( L \) is the separation of the retroreflectors
- \( \epsilon \) is the angle turned through
- \( n \) is the refractive index of the glass in the retroreflectors
- \( b, b' \) are the heights from the face to the apex of the retroreflectors.

For \( \epsilon = 10^\circ \), \( b' - b = 10 \mu m \) and \( n = 1.5 \) then the second term in equation (1) produces a correction of 0.5". For matched corner cubes \( (b' - b) \) should be less than 10 \( \mu m \). Hence in the remainder of the paper this correction term is ignored and equation (1) is written as

\[ D = L \sin \epsilon \]  

(2)

Consider the reflector mount set on a rotary table as in Figure 1, with the retroreflectors normal to the laser beam, or what is described in this paper as the "zero position". Then if the mount is rotated by a small angle \( \alpha \) radian from the "zero position", giving a reading \( x_1 \), then

\[ \sin \alpha = \frac{x_1}{L} \]

If the reading displayed on the interferometer is zeroed and the mount then rotated through a further angle \( \phi \), to make up a total angle \( \theta \), then \( \phi \) is given by

\[ \sin \phi = \frac{x_2}{L} \]

where \( x_2 \) is the new reading. This equation and equation (4) below should actually be of the form

\[ \sin \phi = \frac{x_2 \cos \alpha}{L} \]
However this produces a correction of only 0.17" in $\delta \theta$ (defined after equation (4) below) for $\theta = 14^\circ$ and $\alpha = 40'$. So except for work of the highest accuracy it can be ignored, although the values given in Table 1 include this correction.

Therefore

$$\theta = \phi + \alpha = \sin^{-1}(x_2/L) \pm \sin^{-1}(x_1/L)$$  \hspace{1cm} (3)

However when the rotation is carried out in one operation then

$$\theta = \phi + \alpha = \sin^{-1}(x_2 \pm x_1)/L$$  \hspace{1cm} (4)

The magnitude of the error $\delta \theta$, given by the difference (equation (3)) - (equation (4)), is shown in Figure 2 for various values of $\theta$ and $\alpha$. There is a small asymmetry between the curves for positive and negative values of $\alpha$ which amounts to 7.0" for $\theta = 14^\circ$ and $\alpha = 40'$. Since $\theta$ is a function only of the ratio $x/L$, $\delta \theta$ is independent of $L$ and so the values obtained apply to any instrument, regardless of the separation between the retroreflectors. For example, Schatz (1) used $L = 206.2648$ mm (four times the value for the Hewlett-Packard instrument) to obtain high sensitivity, but the same $\delta \theta$ values will apply to his and the Hewlett-Packard instrument. Thus, by studying Table 1 it can be seen that to calibrate a rotary table over an interval of $-10^\circ$ to $+10^\circ$ to an accuracy of 1" it is necessary to find the "zero position" to within 1'. This difficulty does not arise with the use of such instruments for machine tool calibration or straightness measurements, as deviations of more than 1" very rarely occur, and even at this angle, for $\alpha = 10'$, $\delta \theta$ is only of the order of 0.1". Since 10' is equivalent to 2.9 mm/m this alignment accuracy is relatively easy to achieve for translation of the retroreflector mount by 0.5 m or more.

The angle accessory was used for the calibration of a precision rotary table and Figure 3 is a diagram of the errors over the range $-14^\circ$ to $+14^\circ$ obtained for various values of $\alpha$. The errors were given by the difference between the rotary table reading and the angle calculated by means of equation (4). The initial autoreflection and angular measurement system alignment procedures necessary for setting up the laser interferometer components were carried out but it was found that ultimately a "zero position" had to be established by trial and error. This was done by calculating the sum of the squares of these errors for each of a number of runs. The initial position of the rotary table was altered by a small increment between each run and the curve for which the sum of squares was the least, best approximated the abscissa axis, and gave the "zero position" of the rotary table. This gives a quantitative means of selecting the "zero position", as this is not obvious merely from inspection of the calibration values. It is, however, just possible for an angular movement to have a "mechanical" error in its rotation that exactly compensates the error for a particular value of $\alpha$. In this case the sum of squares method does not work. However, if another segment is measured, as in the calibration of a rotary table, then the discontinuity in the measured errors would immediately show up the problem.
For tables with errors of more than a few seconds finding the "zero position" could be a very time consuming process, especially as it would have to be repeated for each segment of the table calibrated. In this case an autocollimator could be used to either establish the "zero position" by aligning the retroreflectors and the beam splitter,* or as a null indicator of the "zero position" for each new segment.

If the theoretical values shown in Figure 2 are used to correct the respective curves of Figure 3, then for the 11 curves this produces a maximum scatter of 2" at any of the calibration points over the range ± 14°, which is well within the specification for the Hewlett-Packard instrument given in Section 1. Thus, with care the instrument can be used over a range of ± 14° to an accuracy of ± 1".

For specification of ± 4 seconds per degree Table 1 shows that it is possible for this to be exceeded unless reasonable care is exercised. Thus the specification allows Δθ = 40" for θ = 10° and this occurs for α = 40°.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALUES OF Δθ IN SECONDS CALCULATED FOR THE WORST CASE OF (φ + α)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ</th>
<th>-1'</th>
<th>-5'</th>
<th>-10'</th>
<th>-20'</th>
<th>-30'</th>
<th>-40'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1°</td>
<td>0.01</td>
<td>0.05</td>
<td>0.10</td>
<td>0.22</td>
<td>0.34</td>
<td>0.45</td>
</tr>
<tr>
<td>2°</td>
<td>0.04</td>
<td>0.19</td>
<td>0.39</td>
<td>0.83</td>
<td>1.30</td>
<td>1.79</td>
</tr>
<tr>
<td>5°</td>
<td>0.23</td>
<td>1.16</td>
<td>2.37</td>
<td>4.87</td>
<td>7.49</td>
<td>10.23</td>
</tr>
<tr>
<td>10°</td>
<td>0.93</td>
<td>4.67</td>
<td>9.41</td>
<td>19.11</td>
<td>29.08</td>
<td>39.32</td>
</tr>
<tr>
<td>14°</td>
<td>1.84</td>
<td>9.24</td>
<td>18.58</td>
<td>37.58</td>
<td>56.99</td>
<td>76.78</td>
</tr>
</tbody>
</table>

The effect of relative movement of the various components was studied and it was found that rotation of the laser about an axis parallel to the rotary table axis does not change the interferometer reading but causes a corresponding change in the "zero position". However, rotation of the remote interferometer assembly about a like axis causes a corresponding change in reading, but no change in the "zero position". Thus the laser

* Because of the difficulty in using the front face of a retroreflector for autocollimation without attaching a reflector to it, this method will only work if the optical axis of the retroreflectors coincides with the mechanical axis of their mount - to which the necessary reflector may be attached.
may be mounted remotely from any machine tool being tested without the 
results being significantly affected, but it must be rigidly fixed in 
relation to any angular measuring device, such as a rotary table, that the 
interferometer may be used to calibrate.

3. CONCLUSION

The results given above were obtained by use of the Hewlett-Packard 
angle accessory but they apply to any similar instrument, since $\theta_0$ is 
independent of $L$. Because of the errors described in this paper care should 
be exercised when these instruments are used for general purpose measurement 
of angles greater than $1^\circ$.

REFERENCE

Bulletin d'information du Bureau National Metrologie, 2, (31), 
28-32.
FIG. 1 - Use of the Hewlett-Packard laser interferometer angle accessory to calibrate a rotary table.
FIG. 2 - Values of $\delta \theta$ for various values of $\theta$ and $\alpha$. 
FIG. 3 - Errors in a rotary table calibrated with the angle accessory. The different curves are for the values of $\alpha$ given in Figure 2.
DISTRIBUTION LIST

MATERIALS RESEARCH LABORATORIES

Chief Superintendent
Superintendent, Physics Division
Mr. E.R. Harrison
Miss L.M. Ramsay
Dr. D.B. Prowse
Library
Librarian, N.S.W. Branch (Through Officer-in-Charge)
Officer-in-Charge, Joint Tropical Trials and Research Establishment

DEPARTMENT OF DEFENCE

Chief Defence Scientist
Deputy Chief Defence Scientist
Controller, Projects and Analytical Studies
Superintendent, Military Advisers Branch
Head, Laboratory Programs Branch
Army Scientific Adviser
Air Force Scientific Adviser
Naval Scientific Adviser
Chief Superintendent, Aeronautical Research Laboratories
Director, Defence Research Centre
Senior Librarian, Defence Research Centre
Librarian, R.A.N. Research Laboratory
Document Exchange Centre (16 copies)
Principal Librarian, Campbell Park Library ADSATIS Annex
Central Office, Directorate of Quality Assurance - Air Force
Director, Joint Intelligence Organisation
Head, Engineering Development Establishment
Librarian, Bridges Library, Royal Military College

DEPARTMENT OF PRODUCTIVITY

NASA Canberra Office
Head, B.D.R.S.S. (Aust.)
Manager, Ordnance Factory, Bendigo
Manager, Ordnance Factory, Maribyrnong
Manager, Ammunition Factory, Footscray
Manager, Government Aircraft Factory

OTHER FEDERAL AND STATE DEPARTMENTS AND INSTRUMENTALITIES

The Chief Librarian, Central Library, C.S.I.R.O.
Australian Atomic Energy Commission Research Establishment
MISCELLANEOUS - AUSTRALIA

Hibbert Machine Tool Service Co., East St. Kilda
Technisearch Ltd., Metrology Laboratory, Production Engineering Department, Melbourne

MISCELLANEOUS - OVERSEAS

Defence Scientific and Technical Representative, Australian High Commission, London
Assistant Director/Armour and Materials, Military Vehicles and Engineering Establishment, England
Reports Centre, Directorate of Materials Aviation, England
U.S. Army Standardization Group, Office of the Scientific Standardization Representative, Canberra, A.C.T.
Chief, Research and Development, Defence Scientific Information Service, Ontario, Canada (2 copies)
The Director, Defence Scientific Information and Documentation Centre, Delhi, India
Colonel B.C. Joshi, Military, Naval and Air Adviser, High Commission of India, Red Hill, A.C.T.
Director, Defence Research Centre, Malaysia
Exchange Section, British Library, Yorkshire, England
Periodicals Recording Section, Science Reference Library, British Library, Holborn Branch, London
INSPEC: Acquisition Section, Institution of Electrical Engineers, England
Overseas Reports Section, Defence Research Information Centre, England
Library, Chemical Abstracts Service, Columbus, Ohio, U.S.A.