PARALLEL \( \Sigma \)-ERASING ARRAY ACCEPTORS

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ABSTRACT

A parallel \( \Sigma \)-erasing array acceptor (P\( \Sigma \)-EAA) is introduced. It is proved that the class accepted by P\( \Sigma \)-EAA's is exactly the context-free array languages.

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1. Introduction

In [1], Rosenfeld introduced isotonic array grammars, and in [2] Cook and Wang presented a Chomsky hierarchy of isotonic array grammars and languages. Acceptors for type 0 and type 1 array languages were given in Milgram and Rosenfeld [3] and an acceptor for type 3 array languages was given in [2]. But an acceptor for type 2 or context-free languages remains as an open problem.

Cook and Wang's finite state array acceptors (FSAA's) for type 3 array languages are considered as erasing array acceptors in the following sense: At each step, an FSAA M is in some state q scanning an input symbol a. In a given move, M marks the scanned array symbol with ✰ (the input symbol is erased by changing it to a symbol with ✰), changes state and moves one square left (L), right (R), up (U), or down (D). An array A is accepted iff M, beginning in its initial state scanning some array cell, scans and marks each non-# symbol ✰ of the A with ✰ and ends in one of its final states. In this definition of acceptability, there is the problem that M itself never knows whether every non-# cell is marked with ✰.

In this note, we introduce a kind of erasing array acceptor similar to the above mentioned one, which acts in parallel and erases input symbols according to given rules. Further, the acceptor is able to traverse the area of input symbols after
a cell falls into a special state. This ability is provided in order to avoid the above-mentioned problem. This automaton can be considered as a restricted cellular array acceptor; it is called a parallel ε-erasing array acceptor (PΕ-EAA).

The purpose of this note is to solve the open problem concerning the context-free array languages proposed in [2]. We prove that the class accepted by PΕ-EAA's is exactly the context-free array languages. A key idea in the proof is based on a definition of a nondeterministic PΕ-EAA such that it is able to retrace backward derivations in an isotonic context-free array grammar.

In this note, we assume that the reader is familiar with isotonic array grammars, languages, and their hierarchy given in [1], [2] and also with the fundamental definitions of cellular array acceptors as in [4].
2. Definitions

We first give some notation and definitions about PE-EAA's. In this note, we assume that input arrays are surrounded by the edge symbol #.

**Definition 2.1** A parallel ε-erasing array acceptor (PE-EAA) is a two-dimensional cellular acceptor satisfying the following (i)-(vi):

(i) For a set \( S \) of input states, a set \( Q \) of non-input states, and a set \( T \) of traversal states, \( S \cap Q = \emptyset, S \cap T = \emptyset, Q \cap T = \emptyset \).

(ii) For each \( A \in Q \), \( A^L, A^R, A^U, \) and \( A^D \) are also in \( Q \).

(iii) \( S^t \in Q \) is a special "dead" blank state.

(iv) \( S^t, S \in Q \) are the quasi-accepting state and the accepting state, respectively.

(v) \( \bar{S} \in Q \) is the "broken" state.

(vi) \( \delta \) is the transition function, i.e. \( \delta: (\Sigma \cup Q \cup T)^5 \rightarrow (\Sigma \cup Q \cup T) \).

In this note, we consider exclusively nondeterministic acceptors. In Definition 2.1, \( \delta(X,Y,Z,U,V) \exists W \) means that one of the values of \( \delta \) over the first 4 arguments is \( * \). That is, the first 4 arguments of \( \delta \) are occupied by the 4 neighbors in the indicated positions of the last argument. For intuitive understanding, \( \delta(X,Y,Z,U,V) \) is sometimes denoted by \( \delta(X \ Y \ Z) \).
δ is defined as a mapping satisfying the following (1)-(10):

1. δ(X,Y,Z,U,δ) is always δ, except in the case of traversal mentioned in (9) below. This means that δ is the special dead blank state.

2. δ(X,Y,Z,U,A) ∈ AH, where H ∈ {L,R,U,D}, in cases not contradicting (5) and (6) below.

3. For each H ∈ {L,R,U,D} and A ∈ Q, δ(X,Y,Z,U,AH) is always δ.

Conditions (1) and (3) correspond to "erasing".

4. For each input state a, the first 4 variables X,Y,Z,U of δ(X,Y,Z,U,a) are dummy.

5. δ(A E C) depends on E and D only. In the other cases, B
   the situation is the same, i.e., for example δ(A E C)
   depends on E and C only, δ(A R E C) depends on E and A only,
   D
   and so on.

6. In case of a collision in (5), the acceptor goes into the broken state S, i.e., for example, δ(A E C) = S,
   B
   δ(A R E C) = S, and so on.

7. For the broken state S, δ(X,Y,Z,U,S) = S.

8. δ(X E Z) takes a value different from E only when X = AR,
   U
   Y = BD, Z = CL, or U = DU for some A,B,C,D. This corresponds to the "context-free" property.

9. When a cell falls into the quasi-accepting state S, δ is defined to begin traversal of the input. The states of the set T are used for this traversal.

10. In the process of traversal, a cell changes to the broken state when two states of E U Q are in the input area;
otherwise it falls into the accepting state at the starting cell of $S^t$ after finishing the traversal. The detailed definition of $\delta$ to satisfy (10) is complicated and will not be given here.
3. Main theorem

We prove that the class accepted by PE-EAA's is exactly the isotonic context-free array languages. Let G be an isotonic context-free array grammar. L(G) represents the language generated by this grammar. Also, let M be a PE-EAA. T(M) means the set accepted by this acceptor M.

Lemma 3.1 Any context-free array language is accepted by a PE-EAA.

Proof:

Let G be a type 2 grammar G=(N,T,#,P,S) and let M be a PE-EAA. We prove that \( W \subseteq M(L(G) = T(M)) \).

Any isotonic context-free array language (ICFAL) is generated by the following normal form of rewriting ([2]):

\[
\#A \rightarrow BC, \quad A\# \rightarrow BC, \quad \# \rightarrow B, \quad A \rightarrow B, \quad A \rightarrow a.
\]

Therefore, an array generated by the normal form is accepted by a PE-EAA which retraces backward rewriting rules. That is, \( \delta \) of this PE-EAA is defined as follows:

\[
\delta(\ldots, a) \ni A \quad \text{if } A \rightarrow a \text{ is in } P,
\]

\[
\delta(\cdot B \cdot C^L) \ni A \quad \text{if } A\# \rightarrow BC \text{ is in } P,
\]

\[
\delta(\cdot B \cdot C) \ni A \quad \text{if } \#A \rightarrow BC \text{ is in } P,
\]

\[
\delta(\cdot B \cdot C^U \cdot B^D) \ni A \quad \text{if } A \rightarrow B \text{ is in } P,
\]

\[
\delta(\cdot C \cdot) \ni A \quad \text{if } \#B \text{ is in } P.
\]
In this definition, if a symbol $A$ in the production is $S$, then the corresponding $A$ for $\delta$ is $S^t$. It is easily seen that an input array ends in the accepting state $S$ at some square and each non-$\#$ cell of the array goes into $\#$ if the input array is generated by $G$. //

**Lemma 3.2** The set accepted by a $P_\#$-EAA is a context-free array language.

**Proof:**

The proof is similar to Lemma 3.1. We prove that $\forall M \exists G(L(G)=T(M))$. We define an isotonic context-free array grammar $G=(N,T,\#,P,S)$ from a given $P_\#$-EAA $M$ as follows:

$\begin{align*}
A & \rightarrow a \quad \text{if } \delta(.,.,.,.,a) \ni A \\
A# & \rightarrow BC \quad \text{if } \delta(\cdot B C^L) \ni A \\
\#A & \rightarrow BC \quad \text{if } \delta(B^L C \cdot) \ni A \\
A & \rightarrow B \quad \text{if } \delta(\cdot B \cdot C^U) \ni A \\
\# & \rightarrow C \quad \\
\# & \rightarrow B \quad \text{if } \delta(\cdot B \cdot C) \ni A \\
A & \rightarrow C \quad \text{if } \delta(\cdot C \cdot) \ni A \\
S & \rightarrow S^t
\end{align*}$

For the other cases of $\delta$, the corresponding rewriting rules are not given. It can be seen without difficulty that an
input array is generated by this isotonic context-free array grammar $G$ iff it is accepted by $M$.

**Theorem 3.3** The class accepted by PE-EAA's is exactly the isotonic context-free array languages.

**Proof:**

This is obtained from Lemmas 3.1 and 3.2.
4. Remark

In this note, we have considered parallel $\Sigma$-erasing array acceptors for the isotonic context-free array languages. We could also define sequential acceptors which simulate $P\Sigma$-EAA's; but it seems that these acceptors are somewhat more complicated.
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References


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