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LEVEL II

ESTIMATION OF VELOCITY DISTRIBUTION OF FRAGMENTING WARHEADS USING A MODIFIED GURNEY METHOD

THESIS

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Yves J. Charron
Capt CAF

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(6) ESTIMATION OF VELOCITY DISTRIBUTION

OF FRAGMENTING WARHEADS USING A

MODIFIED GURNEY METHOD.

THESIS

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(10) Yves JO/Charron, CD, B Eng

Capt Canadian Armed Forces

Graduate Aeronautical Engineering

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Acknowledgements

Having spent most of my military career in armament and weapon related work, I welcomed an opportunity to broaden my knowledge in the more theoretical side of explosives and weapons. The project I undertook was quite interesting, even though it only scratches the surface of the vast domain of explosives, explosive reactions, damage done by explosives, etc. Since this material was new to me, I have learned a great deal. This study is a good starting point from which a more detailed study of gas dynamics, warhead design, explosive reactions, experimental testing, etc... could be pursued. It has made me aware that there is much to learn.

I would like to thank Dr D.W. Breuer, my faculty advisor, for introducing me to the topic. His suggestions and comments kept me in the right direction and made the study a more instructive experience. I would also like to thank Capt J. Stinson for his help in obtaining an error-free computer program.

Lastly, I wish to thank my wife and son who have spent many lonely hours while I studied. Their support and affection eased the burden considerably.

Y. Charron

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List of Symbols

Symbol	Definition	units
C	Mass of Explosive	g
C_D	Coefficient of Drag	
C/M	Explosive Mass to Metal Mass Ratio	
D	Detonation Velocity of Explosive	mm/μs
E	Explosive Energy	kcal/g
F(x)	Modification Factor	
L/D	Length to Diameter Ratio	
M	Mass of Metal	g
N	Mass of Tamper	g
N/M	Tamper Mass to Metal Mass Ratio	
R	Radius of Charge	cm
V	Velocity	mm/μs
V_R	Residual Velocity	mm/μs
V_{50}	Ballistic Limit	mm/μs
r	Radial Distance	cm
v	Velocity	mm/μs
x	Axial Distance	cm
$\alpha/2$	Angle Between Surface Normal and Fragment Velocity Vector	degree
ρ	Density	g/cc

Subscripts

M	Pertains to Metal Fragments
N	Pertains to Tamper Mass
f1	Pertains to First Layer of Fragments
f2	Pertains to Second Layer of Fragments
p	Pertains to Plastic
pl	Pertains to Plywood
s	Pertains to Steel
0	Initial Conditions

Abstract

The Gurney energy method, an analytical method with which to compute fragment velocity distributions for continuous casing warheads, is reviewed and a method to model velocity losses due to end effects on cylindrical charges and end projector type warheads is presented. The Taylor formula, which estimates the angle of projection of the fragments, and a modified Taylor formula, which gives better estimates, are also presented. The method is extended for use with casings made of pre-formed fragments. Finally, four conceptual preliminary designs, based on information acquired in the project, are investigated.

The modifications made to the equations provide improved results and the examples confirm that the modified Gurney method is a quick, inexpensive tool for use in preliminary warhead design.

ESTIMATION OF VELOCITY DISTRIBUTION
OF FRAGMENTING WARHEADS USING A
MODIFIED GURNEY METHOD

I Introduction

Warheads form an important part of a weapon system and proper design is necessary to ensure a high probability of kill. When a weapon system is designed, the warhead is generally not an "off-the-shelf" item and it is designed with the intent of defeating a particular class of targets. The damage mechanisms of conventional warheads are blast and fragments. In some cases, such as air-to-air missiles, it is the fragments generated from the explosion of the warhead which will incapacitate or destroy the target by piercing or passing through it. The destructive power of the fragments is actually derived from their kinetic energy and, in order to estimate that energy, the mass and velocity of the fragments must be known. The velocity and mass distribution of fragments from exploding warheads is a complex function. This is exemplified in Fig 1 where a 20-mm round is loaded with tetryl and is detonated. The flash radiographies, which have been touched up to ease reproduction, show that the round expands circumferentially and

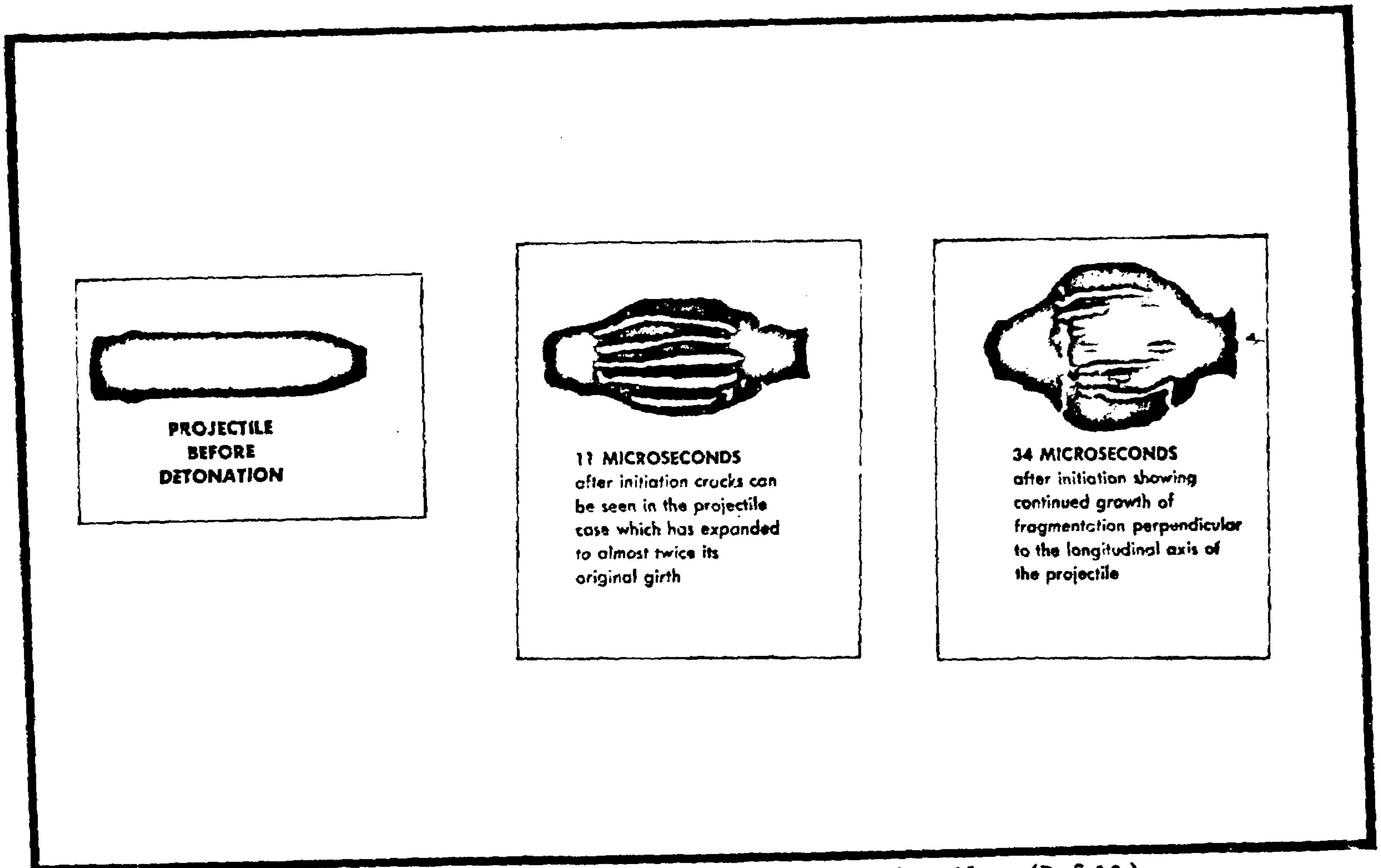


Fig 1. Detonation of Tetryl Filled 20 mm Projectile (Ref 18)

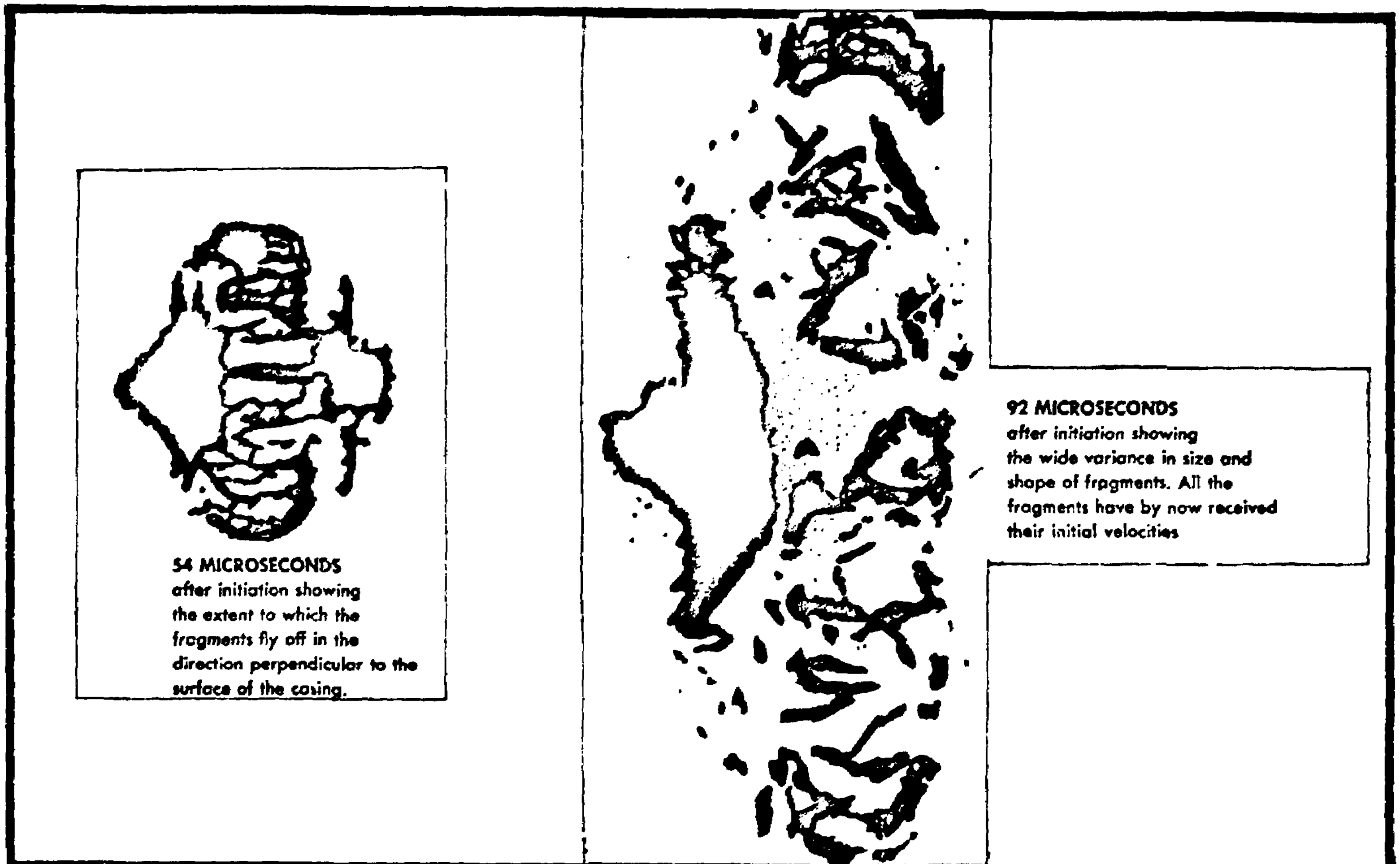


Fig 1 (Cont'd). Detonation of Tetryl Filled 20 mm Projectile (Ref 18)

eventually breaks into many fragments of differing shapes, and masses. This is the typical behavior of an exploding warhead.

Of prime importance to the designer is the velocity distribution, which he must be able to estimate to a reasonable degree of accuracy to determine the damage that the warhead can inflict. There are essentially three methods which can be used for warhead design. The first is the experimental method where a warhead of a certain design is tested and modified until the desired characteristics are attained. A second method is to use computer-aided design. Several computer programs exist which estimate velocity distributions quite well; an example is the HEMP code (Ref 9). Unfortunately, both methods are time consuming and can be very expensive. The third method is a theoretical method which is relatively simple, accurate for cases where $L/D>2$, and is well suited to preliminary design of warheads. It is known as the Gurney energy method. Because of the simplistic approach taken in deriving the relationship between fragment velocities, Gurney energy and C/M, the Gurney energy method holds but for $.1 \leq C/M \leq 5$ (Ref 6,13). For values of C/M outside these limits, gas dynamic equations must be used to compute reasonable velocity estimates. However, most warheads used in modern weapons fall within that range of C/M's and the Gurney method can be used to predict fragment velocity distributions, which, in most cases, are within ten percent of experimental data. However, experimental data are difficult to obtain accurately because of the nature of the process. The fragments move so rapidly that it is nearly impossible to measure exact velocities. Also in cases where length to diameter (L/D) of the warhead is less or equal to two, estimates of the velocity distributions calculated by Gurney's method are in error by as much as 30 percent at the free ends.

Purpose

The purpose of this study is to collect and present currently available information on the Gurney method for estimating velocity distributions for warheads with continuous casings and to extend it for use on warheads made of pre-formed, or discrete, fragments. Also, the equations will be modified to account for variations in velocities due to end effects, which are especially important in warheads where $L/D \leq 2$. Finally, the material developed and presented will be used in the conceptual preliminary design of three example warheads to illustrate its application to the design of different warhead geometries. This improved method should be used for preliminary design because it is quicker, easier to use and cheaper than the other methods.

In addition to the Gurney method, Taylor's formula, which estimates the angle of departure of the fragments, will be reviewed. A more accurate equation which estimates the angle of departure will be presented.

Modification to Gurney and Taylor Equations

The modifications to the Gurney method are derived from available experimental data. From that data, the influence of end effects on the velocity distribution of the fragments from cylindrical and end projector type warheads can be observed. Terms in the Gurney equation for a particular simple metal/explosive geometry will be modified such that the computed velocity distribution approximates the test data for that particular geometry. These data are contained in Ref 2, 9, 10, 11 and 19. Although there is not an abundance of experimental data, it is sufficient to determine appropriate modifications to the Gurney equations. The modified equations are intended for use in preliminary design, after which experimental testing or computer simulation would be used to finalize the design. Some conceptual preliminary designs of end projector type warheads will be investigated. This type of warhead is particularly suited to air-to-air missiles using proportional navigation. Being a directional warhead, its fragments would travel approximately along the line of sight to the target and, if detonated close enough to the target, should give higher probabilities of kill than a similar missile using an isotropic warhead.

II Review of the Derivation of Gurney and Taylor Formulae

Gurney Equations (Ref 6, Ref 13)

The following developments may be found in the cited references and are presented here for the sake of completeness and convenience of the reader.

The Gurney method is straightforward and is based on energy and momentum balances. It is accurate over a wide range of charge to mass ratios (C/M) and works best for one-dimensional translation of a metal surface (Ref 10). The method makes a number of assumptions which simplify the problem. First, in a metal/explosive system, the chemical energy of the explosive is assumed to be completely converted to kinetic energy upon detonation of the explosive. This energy imparts a velocity to the product gases and the metal casing. Secondly, the velocity profile of the product gases is linear and it is constant throughout the metal thickness. Representation of this is shown in Fig 2. Thirdly, charge gases after detonation are assumed to be equally dense everywhere and are expanding uniformly. Lastly, rarefaction waves which are created behind the reaction zone are neglected.

The assumptions made enable us to reduce what would be complex equations to simple ones which can be solved to yield reasonable results for velocity distributions. Although the chemical energy is actually transformed to kinetic and thermal energy and light, the kinetic energy

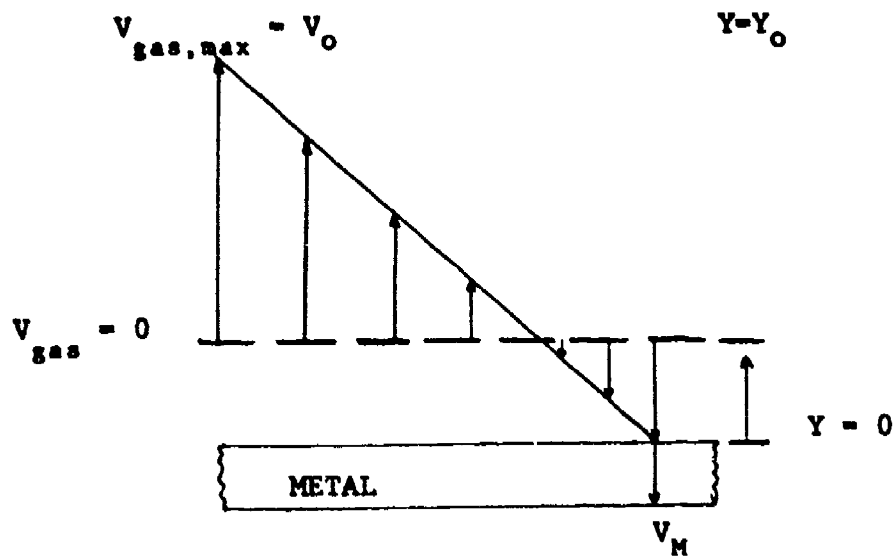


Fig 2. Linear Gas Velocity Profile in Metal/Explosive System

term greatly overpowers all other terms, except for nuclear explosions, which will not be considered. The assumption about constant density is far from reality because the gases near the reaction zone will be denser. Henry (Ref 6) compared the Gurney method with one based on a parabolic distribution of charge gas density. Results showed that the curves for characteristic velocity of the fragments coincide for values of C/M up to 1.0, whereas at C/M of 5.0, the difference is approximately 12 percent. However, experimental data are not available to support or contradict Henry's results. The added complexity of assuming a parabolic distribution is not worthwhile and a constant density assumption permits an easy derivation of a simple relationship between velocity, Gurney energy and C/M, which gives a reasonably accurate fragment velocity distribution. Finally, the last assumption about rarefaction waves is also misleading. Upon initiation of the explosive by some adequate means, a detonation wave travels into the explosive, and rarefaction waves travel in the opposite direction (Ref 22). The effect of the rarefaction waves is to decrease the pressure which builds up to accelerate and rupture the casing. Although the last two assumptions do not truly represent reality, they constitute an important part of Gurney's method by introducing two cancelling errors; first, assuming constant density causes the velocity estimate to be low; secondly, neglecting the existence of rarefaction waves causes the velocity estimate to be high. This results in the method being accurate over a wide range of C/M ratios (0.1 to 5.0)(Ref 6,13).

The Gurney method determines the initial velocity of the metal casing as a function of C/M and the Gurney specific energy, $\sqrt{2E}$. The functions differ for various warhead geometries, but only involve C/M and $\sqrt{2E}$. The quantity $\sqrt{2E}$ is determined empirically and is a characteristic of each explosive. It has been measured for several commonly used explosives. The values for the Gurney energy quoted in this report were obtained from Ref 4 and 13. For some explosives, two or more different values are given, illustrating the difficulty encountered in obtaining exact data.

However, the difference between the values is slight. Whenever Gurney energy values were required for computations, the smallest values found in the references were used because the Gurney method overestimates the velocity in most instances and by using the smallest value of $\sqrt{2E}$, agreement between experimental data and the Gurney estimate was better. In analyzing the experimental data used later in this report, it was necessary to estimate the value of the Gurney energy of explosives for which no data were found. Since the energy appears to vary linearly with the detonation velocity of the explosive, a linear least squares fit was used to obtain the energy values. The graph is shown in Appendix A. The experimental data were available for cylindrical charges and sandwich-type configurations; consequently, derivation of the Gurney equations for those geometries will be reviewed. Equations for other geometries can be found in Ref 6 and 13.

A cylindrical charge is illustrated in Fig 3. The cylinder is assumed infinite in length so that end effects can be neglected.

From the conservation of energy principle, the total chemical energy of the explosive before detonation, CE, is approximately equal to the kinetic energy after detonation (thermal and light energy are neglected).

$$\begin{matrix}\text{Chemical Energy} \\ \text{of Explosive}\end{matrix} = \begin{matrix}\text{Kinetic Energy} \\ \text{of Metal}\end{matrix} + \begin{matrix}\text{Kinetic Energy} \\ \text{of Product Gases}\end{matrix} \qquad (1)$$

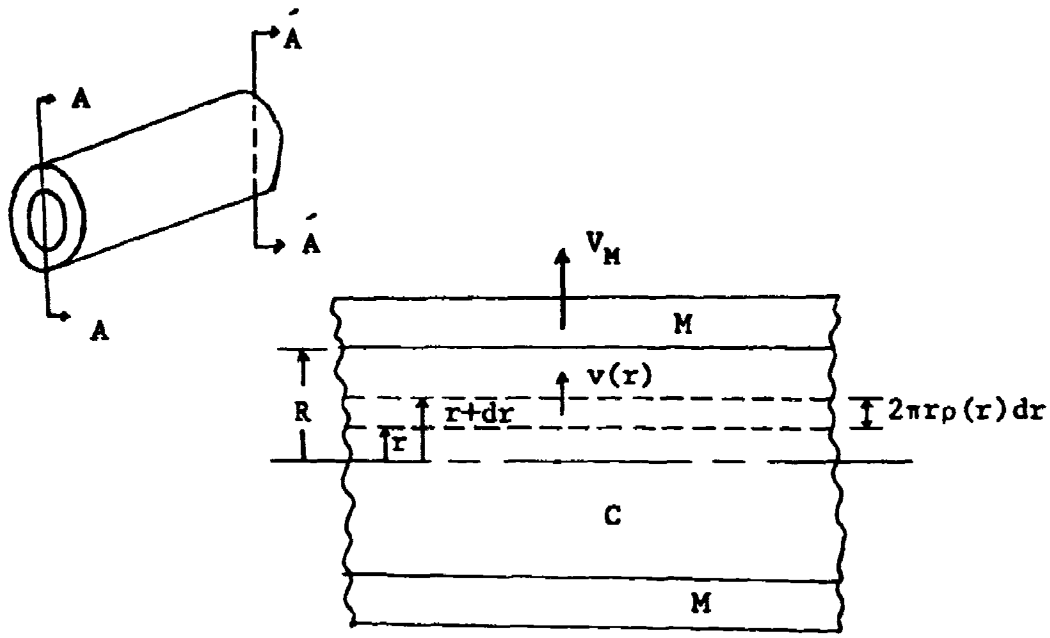


Fig 3. Cylindrical Casing

$$CE = \tfrac{1}{2}MV^2 + \tfrac{1}{2}\int_0^R v^2(r)\,2\pi r\rho(r)\,dr \qquad (2)$$

where

C = mass of explosive per unit area

E = energy per unit mass of explosive

M = mass of metal per unit area

V = initial velocity of metal imparted by the explosion

$\rho(r)$ = density of the charge gas at some point r

R = displacement of the casing at some time after detonation

r = distance from centerline

Hence, for a cylinder of unit length, the mass of the charge gas between r and r+dr is $2\pi r\rho(r)dr$. The density, assumed constant, at r is given by

$$\rho(r) = C/\pi R^2 \tag{3}$$

and the velocity at r is

$$v(r) = \left(\frac{r}{R}\right)V_M \tag{4}$$

because of the assumptions of constant density and linear velocity profile.

Substituting for $\rho(r)$ and $v(r)$ into Eq (2) and integrating, we obtain

$$CE = \tfrac{1}{2}MV_M^2 + \frac{CV_M^2}{4} \tag{5}$$

from which

$$V_M = \sqrt{2E}\left(\frac{M}{C} + \frac{1}{2}\right)^{-\frac{1}{2}} \tag{6}$$

or

$$V_M = \sqrt{2E}\left(\frac{C/M}{1 + .5\ C/M}\right)^{\frac{1}{2}} \tag{7}$$

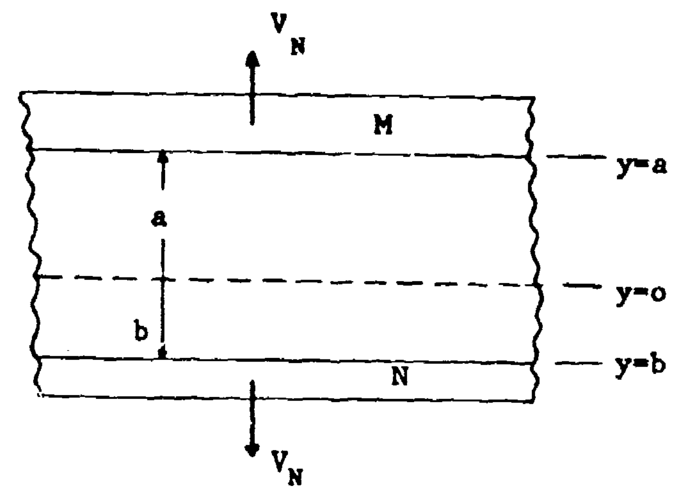


Fig 4. Asymmetric Sandwich

The equation for an asymmetric sandwich is developed as follows:

Figure 4 represents an asymmetric sandwich. The sandwich is assumed infinite in length so that end effects can be neglected.

After detonation, there will exist a plane where the gas is stationary; let this plane be at a distance a from the surface of the plate of mass M per unit area and at a distance b from the surface of the plate of mass N per unit area. Applying the energy conservation principle to before and after the detonation,

$$\begin{array}{l}\text{Chemical Energy} \\ \text{of Explosive}\end{array} = \begin{array}{l}\text{Kinetic Energies} \\ \text{of Plates M and N}\end{array} + \begin{array}{l}\text{Kinetic Energies of} \\ \text{Charge Gases Between} \\ \text{Plates M and N}\end{array} \qquad (8)$$

that is,

$$CE = \tfrac{1}{2}MV_M^2 + \tfrac{1}{2}\int_0^a v^2(y)\rho(y)dy + \tfrac{1}{2}NV_N^2 + \tfrac{1}{2}\int_0^b v^2(y')\rho(y')dy' \qquad (9)$$

where

V_M = initial velocity of plate of mass M

V_N = initial velocity of plate of mass N

The remaining terms are defined as in Eq 2 and primes indicate the negative y direction.

Now from the conservation of momentum,

$$\begin{array}{l}\text{Momentum of} \\ \text{Plate M}\end{array} + \begin{array}{l}\text{Momentum of gases} \\ \text{between o and a}\end{array} = \begin{array}{l}\text{Momentum of} \\ \text{Plate N}\end{array} + \begin{array}{l}\text{Momentum of} \\ \text{gases between} \\ \text{o and b}\end{array} \qquad (10)$$

that is,

$$MV_M + \int_0^a \rho(y)v(y)dy = NV_N + \int_0^b \rho(y')v(y')dy' \qquad (11)$$

but since the gases are assumed to expand uniformly

$$V_M = \left(\frac{a}{b}\right) V_N \qquad (12)$$

which implies that

$$v(y) = \left(\frac{y}{a}\right) V_M \qquad (13)$$

$$v(\acute{y}) = \left(\frac{\acute{y}}{b}\right) V_N \qquad (14)$$

Also, the density was assumed to be constant, implying that,

$$\rho(y) = \rho(\acute{y}) = \frac{C}{(a+b)} \qquad (15)$$

Substituting Eqs (12), (13), (14) and (15) into (11) and integrating yields

$$MV_M + \frac{C}{a+b} \frac{a}{2} V_M = NV_M \left(\frac{b}{a}\right) + \frac{C}{a+b} \frac{b^2}{2} \frac{V_M}{a} \qquad (16)$$

solving for b/a gives

$$\frac{b}{a} = \frac{C/M + 2}{C/M + 2N/M} \qquad (17)$$

Substituting Eqs (13), (14) and (15) into Eq (9) and solving gives

$$\frac{2CE}{V_M^2} = M + \frac{C}{a+b}\frac{a}{3} + N\frac{b^2}{a^2} + \frac{C}{a+b}\frac{b^3}{3a^2} \tag{18}$$

Solving for V_M gives

$$V_M = \sqrt{2E}\left[\frac{1}{3}\frac{1+(b/a)^3}{1+(b/a)} + \frac{N}{C}\left(\frac{b}{a}\right)^2 + \frac{M}{C}\right]^{-1/2} \tag{19}$$

The value of V_N is determined from Eq (12).

If Eq (17) is substituted into Eq (19), we obtain

$$V_M = \sqrt{2E}\left[\frac{1 + A^3}{3(1+A)} + \frac{N}{C}A^2 + \frac{M}{C}\right]^{-1/2} \tag{20}$$

where

$$A = \frac{C/M + 2}{C/M + 2N/M} = \frac{b}{a} \tag{21}$$

The equation for an open-faced sandwich can be easily obtained by letting $N=0$. This gives

$$V_M = \sqrt{2E} \cdot \frac{\sqrt{3}\ (C/M)}{((C/M)^2 + 5(C/M) + 4)^{1/2}} \tag{22}$$

We have thus derived the Gurney equations which will be necessary to analyze the experimental data.

Taylor's Formula (Ref 13)

As was stated earlier, Taylor's formula predicts the angle of departure of the fragments. This formula is derived as follows. Figure 5 shows the detonation of a charge against a metal plate. The detonation velocity, D, is parallel to the surface of the plate. It is assumed that acceleration of the plate to its final velocity is instantaneous and steady state is achieved. The plate is assumed to undergo no net shear flow and therefore there is no change in length or thickness. Although the plate accelerates gradually to its final velocity, V_M, the time lapse is so short that for practical purposes it can be

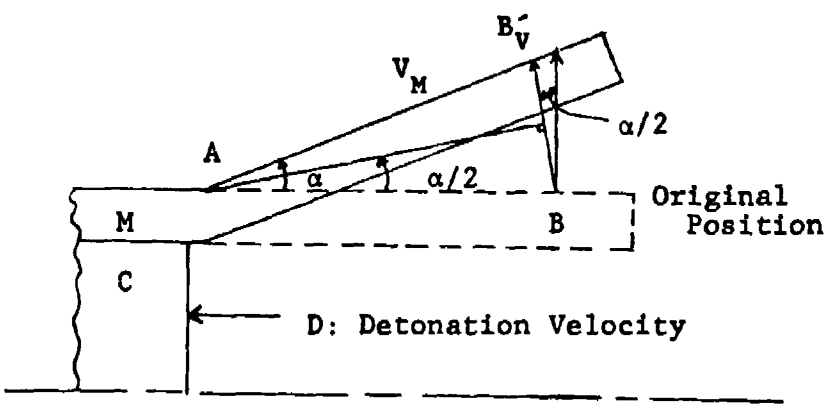


Fig 5. Projection of Metal Plate by Detonation of Explosive

assumed to be instantaneous. At some time after detonation, the deflection of the plate from its original position is α degrees. The point B therefore goes to point $\acute{B}$ and

$$AB = A\acute{B} \tag{23}$$

The line AC is drawn perpendicular to $B\acute{B}$. Therefore, angle CAB = angle $CA\acute{B}$ = $\alpha/2$, since $AB\acute{B}$ is an isosceles triangle. The velocity vector D is along AB and the velocity vector V_F is along $B\acute{B}$. From trigonometric and geometric relations, we obtain

$$\text{Sin } \alpha/2 = \frac{V_M/2}{D} \tag{24}$$

which is Taylor's formula.

The velocity V_M can be obtained from Gurney's equation for the appropriate geometry.

Accuracy of the Equations

As stated before, the Gurney equations are reasonably accurate for a large number of configurations, L/D and C/M. In a cylindrical bomb of constant diameter and L/D>2, the initial fragment velocity is essentially the same for all the fragments in the central part of the casing, neglecting the small portion of fragments generated at the ends. However, in many cases, the diameter of the explosive charge is not constant. Many projectiles have ogival noses. It is therefore more appropriate to take a local charge to mass ratio for use in Gurney's equations. These equations were derived using an element of explosive/metal system of unspecified size. For example, to use the total C/M for a cone shaped warhead to find the velocity distribution of the whole casing would be erroneous. The fragments at the apex of the cone would have a much lesser velocity than the ones at the base, but using the total C/M would predict the same velocities for all the fragments. Consequently, care must be taken in applying Gurney's formula to ensure that the correct C/M is used. The accuracy of the method can be observed in Fig 6 where the velocity distribution obtained by Gurney's method, using local C/M along the projectile, is compared to experimental data for a typical high explosive projectile (Ref 10,11). As can be seen, the agreement is quite good. It must be noted that in this case the length to diameter ratio is fairly large.

At either end, where the C/M is small, Gurney's equation predicts a lower value than at the center of the projectile and this is borne out by the data. It is expected that similar agreement should occur for other warheads of similar configurations. Unfortunately, it has been observed that Gurney's method does not work well for configurations where $L/D \lesssim 2$, even if local C/M's are used. The estimates of the fragment velocities at the ends are in error by as much as 28 percent in some cases. Figure 7 compares the estimate from Gurney's equation, Eq (7), with experimental data for a cylindrical charge of L/D=2. Although the C/M is the same for the whole length of the cylinder, which implies that the velocity should be the same for all the fragments, it can be seen that the fragment velocities at the ends are less than at the center. The Gurney equation provides a reasonable estimate for fragments located from about 0.5 to 0.8 of the relative distance from the initiated end, but fails for the rest of the casing. Taylor's formula also shows agreement and large errors within roughly the same areas, which is to be expected, since its estimate depends on the velocity obtained from Gurney's equation.

The equation loses its accuracy at the ends because of the free end effects. These end effects result from rarefaction waves, which are neglected in Gurney's theory, but it appears that they must be taken into account. As the Gurney equations are presented, they are not adequate for evaluating velocity distributions for warheads with L/D≤2. Computer codes are more accurate. The best HEMP code, which assumes elastic-plastic properties for the casing and accounts for gas leakage between the fragments, models the experimental data quite accurately (Ref 9). Unfortunately, it is a complex tool requiring much computer time to obtain the results. A method of modifying the Gurney equation for cylindrical charges so that it computes more accurate results is investigated in the next section. Use of an appropriately modified Gurney equation would save time and money. Preliminary design could be done with this equation and the final design confirmed with one or two runs of the HEMP code.

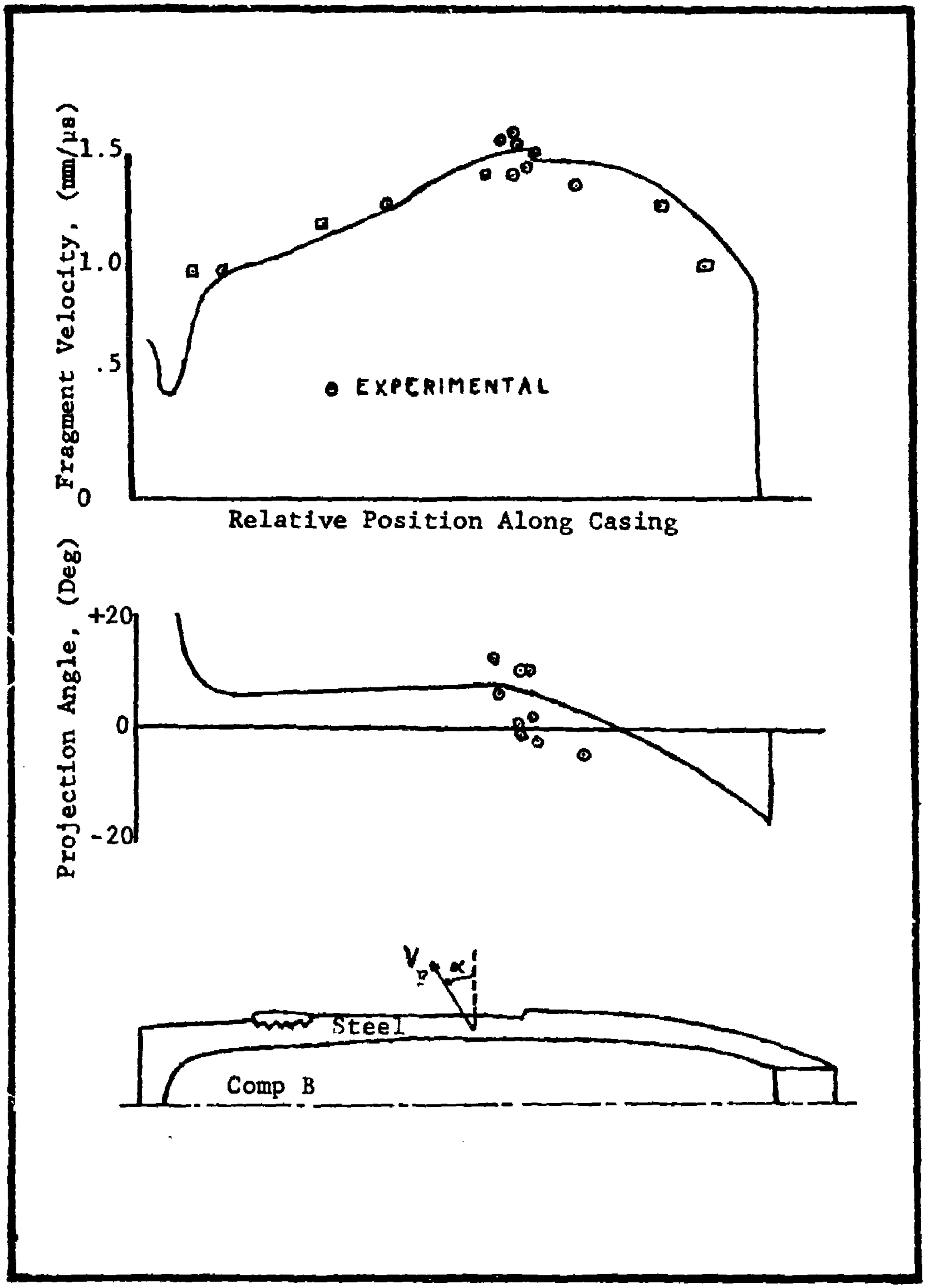


Fig 6. Comparison of Gurney and Taylor Formulae with Experimental Data For H.E. Projectile (Ref 10,11)

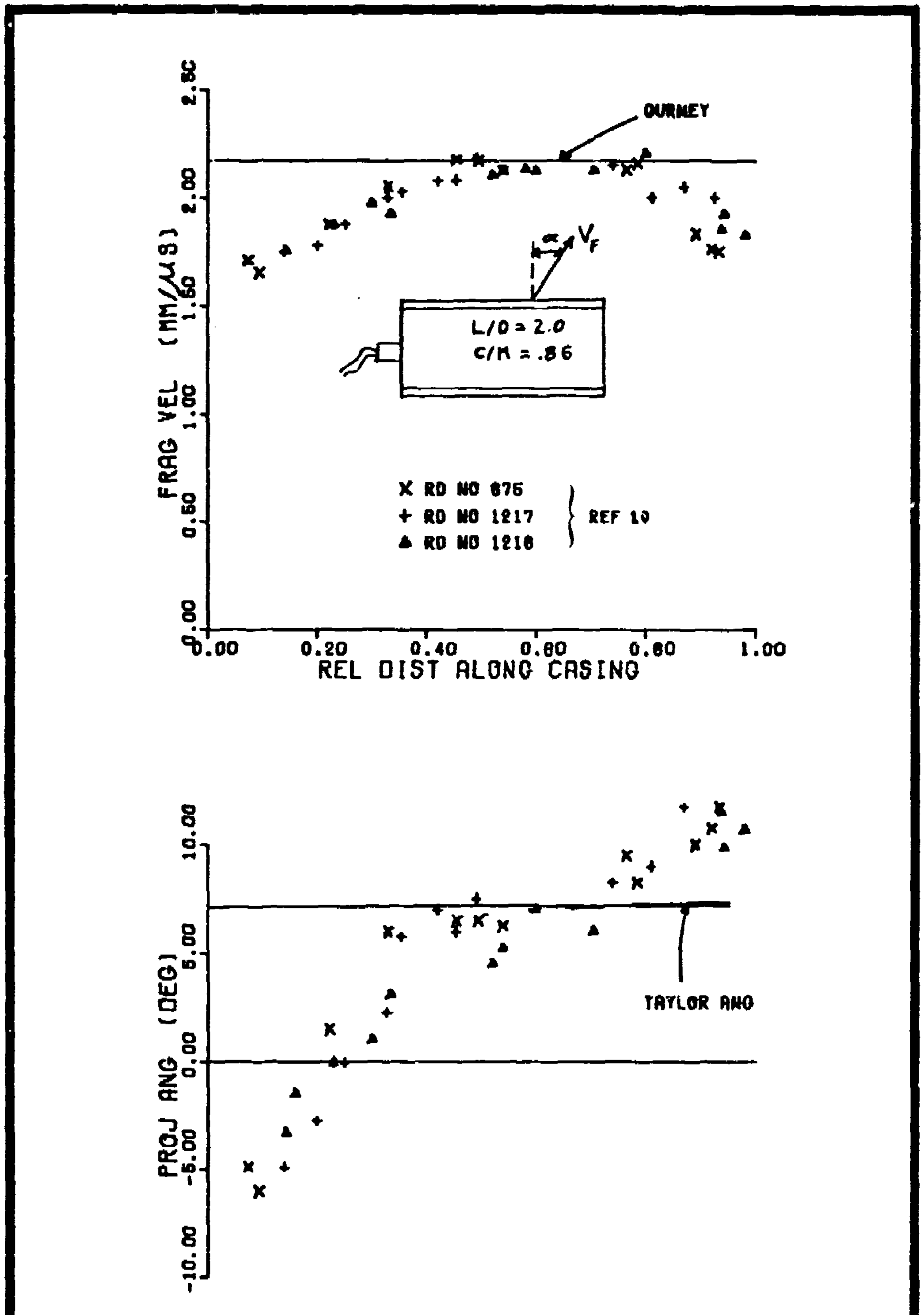


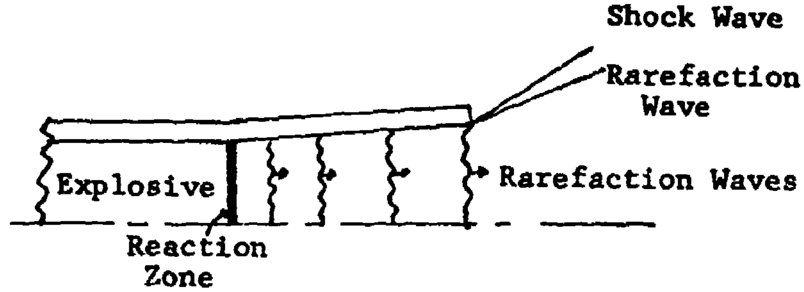
FIG 7. COMPARISON OF GURNEY AND TAYLOR FORMULA WITH EXPERIMENTAL DATA FOR CYLINDER FILLED WITH OCTOL.

III Modification of the Formulae

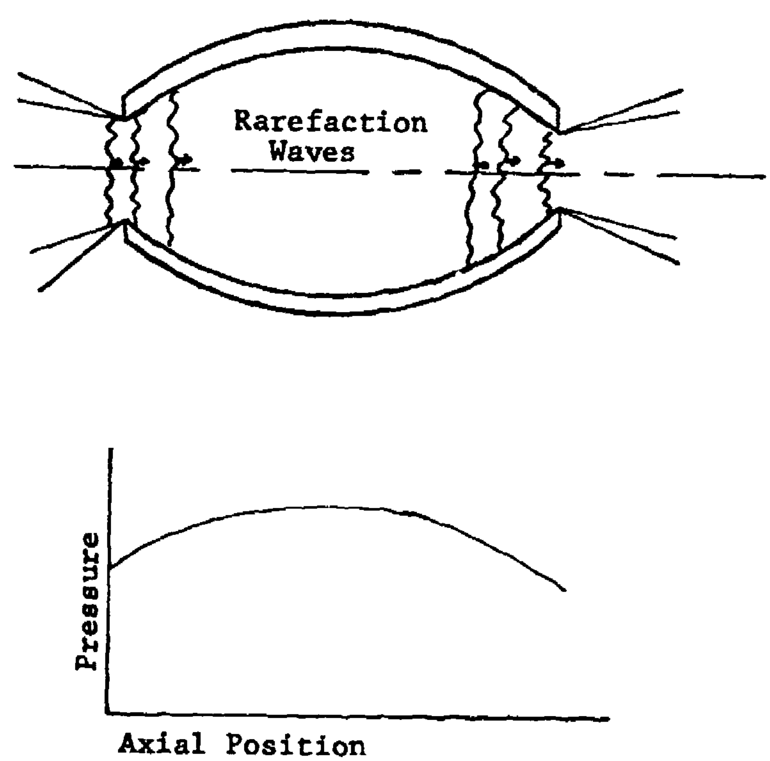
End Effects

End effects are present in all cases but have a varying degree of importance in computing the velocity distribution of the fragments. In cylindrical warheads of $L/D>2$, the end effects could be neglected and the error in computing the velocity distribution would be small. Most of the fragments in such cases would attain the maximum velocity which can be achieved for a given C/M and L/D. The fragments at the ends would have reduced velocities but since L/D is large, the overall percentage of those fragments would be small. Gurney's equation would give a reasonable estimate of the velocity distribution, being in error only at the ends. However, for design purposes, it is desirable to be able to estimate the velocity distribution for all the fragments. In cases where $L/D\leq 2$, the influence of end effects are such that few of the fragments reach velocities predicted by Gurney's equation for a cylindrical charge. Consequently, a method which yields better estimates is required.

End effects result from the rarefaction waves generated in the system. When detonation occurs, a shock wave passes through the explosive, transforming the chemical energy of the explosive to kinetic energy and a reaction zone of finite thickness forms behind the shock wave. The pressure and temperature rise sharply behind the shock, but since there is no interface to maintain this rise, they begin to fall off. The explosive process continues through the reaction zone and beyond into the burnt gases so that rarefaction waves are created (Ref 21). The process is sketched in Fig 8. Because of the high detonation velocities involved in the process, the explosive matter is completely detonated before the casing breaks apart. The rarefaction waves formed at the initiated end of the explosive travel in a direction opposite that of the detonation shock. The center part of the detonation products is still confined and the pressure loss is less there than at the initiated end. When the detonation shock reaches the opposite end, the gases are no longer confined and the rarefaction waves cause the loss of pressure. In cases where L/D is small, the rarefaction waves reach farther inside the casing towards the middle and the pressure is decreased over a wider axial distance. Because the pressure is decreased, the velocity of the fragments will be less. This loss in velocity is similar to having a smaller C/M at the ends, that is, the velocity is proportional to the C/M. Therefore, end effects can be modelled by decreasing the local C/M along the casing by an appropriate amount. The difficulty lies in finding the amount of the decrease. Whatever the modification required is, it must give reasonable results and be of a simple form so that it is easy to apply. Returning to Fig 8, it can be seen that the pressure loss, consequently the loss



a) Progression of Detonation



b) Complete Detonation of Explosive.
Casing has not burst yet.

Fig 8. Explosive Process

in velocity, varies with the position along the casing and is greater at the ends. End-initiated cylinders are investigated, since experimental data are available for these.

Modification to the Gurney Formula (Cylinders)

Reduction of the local C/M can be achieved by assuming that, for computational purposes, an amount of explosive is removed from the ends. Because simplicity is a factor, simple geometric shapes, such as cones or hemispheres, were investigated as likely amounts to be removed. The method was to assume the removal of a cone or hemisphere of certain dimensions from either end, compute the velocity distribution using Gurney's formula for a cylindrical case, with the appropriate local C/M and compare the results obtained to the experimental data for that particular combination of explosive type, L/D, and C/M. Best agreement was obtained when two cones, one of height 2R and of base equal to the diameter, and one of height R and of base equal to the diameter, were removed from the initiating end and the opposite end, respectively (Ref 20). Figure 9 illustrates this. The cylinder is divided into three regions. In region B, no modification is required and the correction factor is 1.0. In region A and C, the factor is determined as follows.

In region A,

$$\frac{r}{R} = \frac{2R-x}{2R} = 1 - \frac{x}{2R} \qquad (25)$$

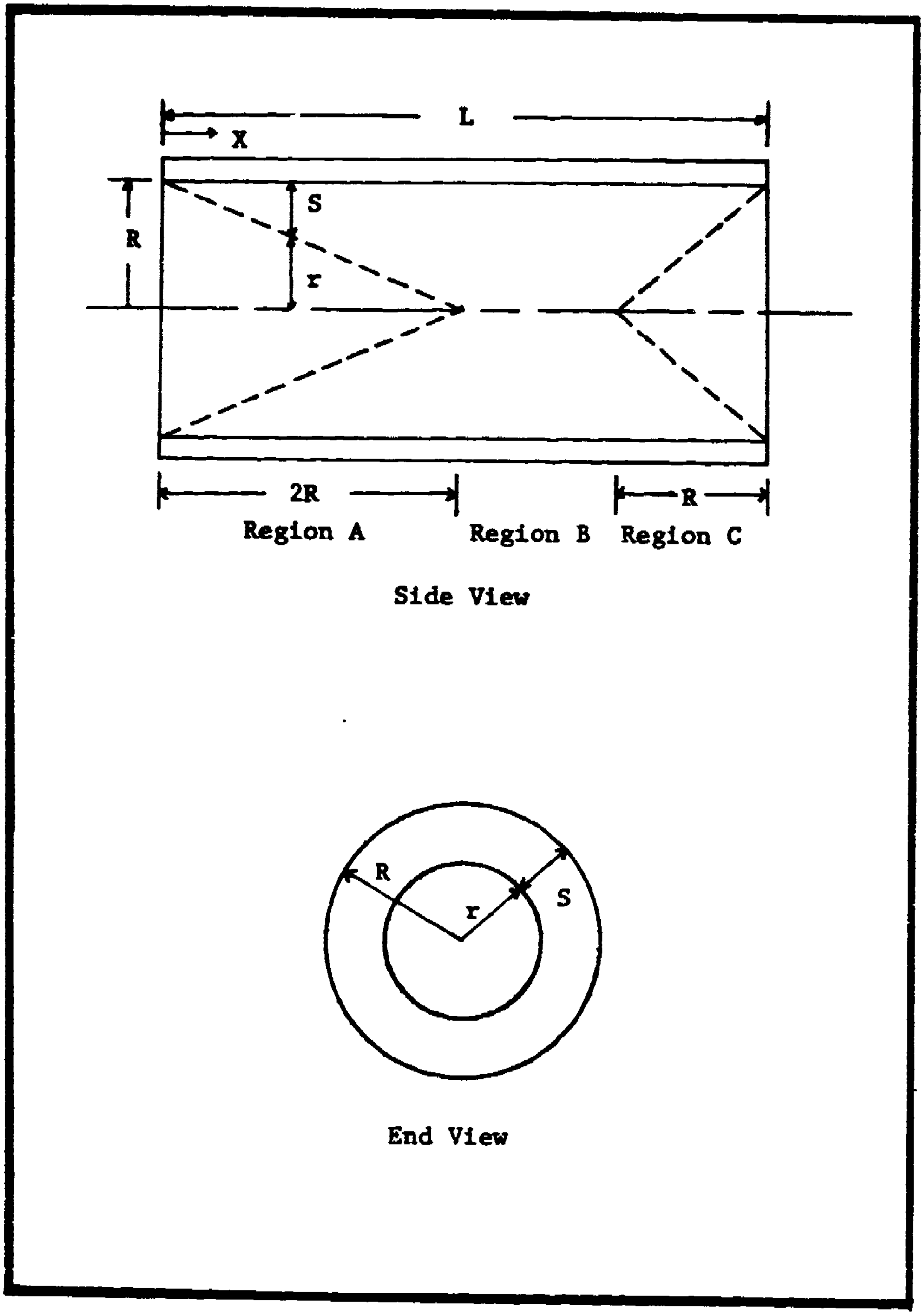


Fig 9. Modification for Cylinder Filled with Explosive

If the cylinder is viewed from the end, the proportional amount of explosive at that location is

$$F(x) = \frac{\text{area of sector S}}{\text{total area}} = \frac{\pi R^2 - \pi r^2}{\pi R^2} = 1 - \left(\frac{r}{R}\right)^2$$

$$= 1 - \left(1 - \frac{x}{2R}\right)^2 \qquad (26)$$

Similarly in region C, the amount of explosive at any one point would be

$$F(x) = 1 - \left(1 - \frac{L - x}{R}\right)^2 \qquad (27)$$

Therefore, the correction factor for the whole cylinder can be written in a compact form, suitable for insertion in a computer program, as

$$F(x) = 1 - \left(1 - \min\left(\frac{x}{2R}\ ,\ 1.0,\ \frac{L - x}{R}\right)\right)^2 \qquad (28)$$

Multiplying C/M by F(x) will give the appropriate local C/M to use in Gurney's equation. The cone at the initiating end is greater because it allows for the build-up of the explosion at that end.

The Gurney equation to be used for cylindrical cases when $L/D \leq 2$, filled with any explosive, and with $.1 \leq C/M \leq 5$ becomes

$$V_M(x) = \sqrt{2E}\left[\frac{F(x)C/M}{1 + .5F(x)C/M}\right]^{\frac{1}{2}} \tag{29}$$

<u>Modification to the Taylor Formula (Cylinders)</u> (Ref 20)

In Taylor's derivation for deflection angle, steady state conditions were assumed to exist. In many cases, this is incorrect because there exists a velocity gradient in the metal. It is therefore necessary to use a different formula to obtain the projection angles. The equation proposed by Randers-Pherson gives reasonable agreement between calculated values and experimental data. In his derivation, he considered two closely spaced points along the metal surface. Each point was assumed to accelerate according to the equation

$$V = V_o\left[1 - e^{-\left[\frac{t-T}{\tau}\right]}\right] \tag{30}$$

where V is the velocity at any instant, V_o is the ultimate velocity which the element would attain, T is the time at which the detonation front reaches the element, and τ is the time constant of acceleration, i.e. the time required for the element to reach $V_o(1-1/e)$. V_o, T, and τ all vary with initial location. If there is no velocity gradient, due to the acceleration of the metal, V_o and τ are independent of initial location and the two elements follow identical trajectories. That equation is

$$\text{Sin } \alpha = \frac{V_M}{2D} - \frac{V_M' \tau}{2} - \frac{(V_M' \tau)^2}{5} \qquad (31)$$

where

α = angle from vertical (see Fig 6)

V_M = fragment velocity, obtained from the modified Gurney equation

D = detonation velocity of the explosive

V_M' = velocity gradient in the metal

τ = time constant of acceleration

The equation was derived empirically. The velocity gradient V_M', is the difference in velocity between two adjacent points, divided by the distance separating them, and the value of τ, defined as the time for the element to reach $V_o(1-1/e)$, is estimated from computing the velocity of the fragment at different times using experimental data for radial expansion of cylindrical charges in the standard cylinder test. It may be noticed that if there is no velocity gradient, the equation becomes Taylor's formula.

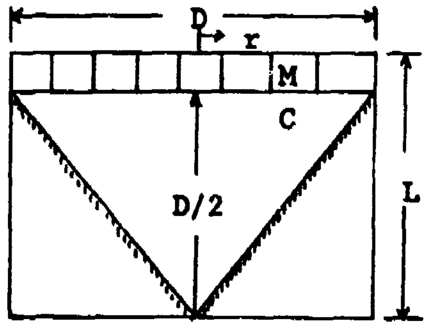
Modification for Pre-Formed Fragments (Cylinders)

From explosive tests with steel cylinders with continuous casings, the first appearance of gas leakage is at expansion ratios of 1.6 to 2.1, depending on the type of explosive filling, i.e. its power (Ref 10). In similar tests with cylinders made of pre-formed fragments, gas leakage occurs at expansion ratios of 1.18 to 1.26 (Ref 19). This implies that much of the energy escapes between the fragments and is not available to accelerate the fragments. This can be represented by reducing the overall C/M for the cylinder. Therefore, the correction was obtained by reducing the total C/M until acceptable agreement between computations and experimental data was obtained.

Modification to the Equation for End Projectors

One of the purposes of this study is to apply a modified Gurney method to the design of end projectors. End projectors of interest are of circular shape (to fit inside an air-to-air missile) and are made of pre-formed fragments. Such warheads are also subject to end effects; however, the modification will be different since the configuration is different from that of the cylindrical case. In an end projector, the detonation wave is parallel to the layer of fragments to be projected, instead of being perpendicular as it is in a cylinder. Figure 10 illustrates a cross section of a sample end projector. Since end effects were modelled by reducing the local C/M in the case of cylinders, it is appropriate to do the same for this geometry, because the presence of rarefaction waves will reduce the fragment velocities at the edges of the projector like they did at the ends of cylinders. In this case, the Gurney equation to be used is that for an open-faced sandwich (see Fig 10). Since the

$$V_M(r) = \sqrt{2E}\left[\frac{\sqrt{3}F(r)C/M}{\left[(F(r)C/M)^2 + 5F(r)C/M + 4\right]^{1/2}}\right]$$



where

$$F(r) = \left[1 - \frac{r(I)}{R}\right]^2 \quad (31)$$

Fig 10. Correction for End Projector Warhead

geometry involved is different, the correction for end effects must be slightly different. Again the reduction of local C/M is to be modelled using a simple geometric shape It was found that best agreement with experimental data (Ref 11) occurred when a cone of explosive of height D/2 and base D was assumed to be the amount of explosive present (See Fig 10). Also, since the end projector is made of pre-formed fragments, the total C/M is reduced to account for gas leakage between the fragments. The reduction is assumed to be the same amount as that for cylindrical case.

To recapitulate, the influence of end effects on the velocity distribution of fragments is modelled by modifying the Gurney equations as follows:

i) for cylinder charges, continuous casing, it is assumed, for computational purposes, that an amount of explosive is removed from both ends of the cylinder. That amount is in the form of a cone, which enables computation of local C/M to be done easily.

ii) for end projector type warheads, continuous casing, it is assumed, for computational purposes, that an amount of explosive is removed from the edges. It is assumed that only a cone of explosive remains. The local C/M can then be computed easily.

iii) for warheads made of pre-formed fragments, it is assumed that the actual C/M for the warhead is reduced to some effective C/M which reflects the greater energy loss.

The Taylor equation is modified in accordance with Ref 20. Using the modified formulae, a computer program was written to calculate the necessary parameters, velocity distribution and projection angles, and plot them so the results could be compared with experimental data available. The computer program is contained in Appendix C.

IV Results of Application of Modified Formulae

Experimental Data

The modified equations were used to calculate velocity distribution and projection angles for end-initiated cylinders (L/D=2.0) filled with various explosives, and two different C/M's for each explosive. Cylinders made of preformed fragments, C/M=.93 and two different L/D's were also investigated. The results were then compared to the experimental data for those configurations. Experimental data are difficult to obtain for exploding warheads. Velocities can be measured by different methods, such as X-Ray photography, or electronic screens, but velocities obtained from different methods are slightly different for the same explosive/metal geometry. Hence, some methods are more accurate than others and it is impossible to give an exact value for the velocity distribution for a particular test. However, it is assumed that the data used, to which the equations are compared, were obtained by using the same measuring techniques and apparatus for all the tests, since they were obtained by the same organization. Therefore, although the velocities measured may not be the exact velocities, the data has the same error sources due to the method of measurement used. The data plotted in Fig 11 to Fig 16 are for tests in which the geometries were kept the same for three different explosives (Octol, Comp B, and TNT) and for two different C/M's for each explosive. For Octol and TNT, the tests with cylinders of L/D=2.0 were repeated three times each for the same C/M and for TNT, they were done six times in one instance and five in the other for the same C/M. Further insight can be gained into the accuracy of the data as follows; as can be seen in Fig 11 to Fig 16 different results are obtained in successive tests using the same C/M, L/D and explosive. For example for TNT, L/D=2, C/M=.77, the difference is approximately 17 percent between results of RD899 and RD1208; for Comp B, C/M=.39, it is as much as 13% at some positions for RD1211 and RD1212. Also, for any one test, the data points do not form a smooth curve. The more tests of any particular configuration (e.g.TNT, L/D=2, C/M=.77), the more the data are spread. The variations in the data are due to a number of factors. First, there will be interaction in the fragment cloud so that some fragments will collide with each other and thus, the velocity distribution will not be a smooth curve and, depending on the number of interactions, since the metal casing will fragment differently in each test, there will be a difference in the velocities from test to test. Second, although the external geometry has been kept the same, the properties of the explosive may differ. The rate of detonation of an explosive is influenced by the temperature, pressure, density of packing and humidity. Consequently, unless all external conditions are the same for each test, different results will be obtained and when all these differences are added together, the velocities obtained differ between tests of same L/D, C/M, and explosive. Although the velocity distribution has a certain amount of error, most of the tests (except for some tests with TNT) show the influence of end effects.

Cylinders with Continuous Casings

Figures 11 to 16 show the results from the Gurney, modified Gurney and modified Taylor equations plotted with the experimental data for explosive cylinder tests, for each case, L/D=2.0. Because the experimental data are different for tests of the same explosive, L/D and C/M, the estimates from the Gurney, modified Gurney, and modified Taylor equations should be compared to an average of the test results for each explosive and C/M for a more meaningful comparison.

The upper graph of Fig 11 shows the velocity distribution obtained by the Gurney equation (dashed line) and the modified Gurney equation (solid line) plotted with the experimental data for cylinders filled with Octol, C/M=.86. It can be observed that the Gurney equation estimate reasonably well fragment velocities between $.45 \leq x/L \leq .85$. However, the estimate is progressively larger in error as $.45 \geq x/L \geq 0$ and $.85 \leq x/L \leq 1.0$. On the other hand, the estimates from the modified Gurney equation are reasonably accurate for $.15 \leq \frac{x}{L} \leq .95$. The modified Gurney equation is reasonably accurate for approximately 80 percent of the relative distance, whereas, the Gurney equation is only accurate for 40 percent of the relative distance. The modified equation reproduces fairly closely tests RD875 and RD1217 between $.1 \leq x/L \leq .95$.

The bottom graph compares the results of the modified Taylor equation with the experimental data for the same cylinder tests. As can be observed, the modified Taylor equation gives a reasonably accurate estimate compared to the estimate from the normal Taylor equation (see Fig 7). The superiority of the modified Taylor equation over the normal Taylor equation is clearly evident in this case.

In the upper graph of Fig 12, the velocity distribution obtained by the Gurney and modified Gurney equations are plotted with the experimental data for cylinders filled with Comp B, $C/M = .79$. It can be observed that the modified Gurney equation gives a reasonably accurate estimate of the velocity distribution for $.1 \leq x/L \leq .95$, whereas the Gurney equation is only accurate for $.4 \leq x/L \leq .85$. Furthermore the modified equation reproduces fairly closely test RD588 between $.1 \leq x/L \leq .95$.

The bottom graph compares the results of the modified Taylor equation with the experimental data for the same cylinder tests. As can be observed, the modified Taylor equation gives a reasonably accurate estimate compared to the estimate from the normal Taylor equation (see Fig 7). The superiority of the modified Taylor equation over the normal Taylor equation is clearly evident in this case.

The same information is contained in Fig 13 for cylinders filled with TNT, C/M=.77. In both graphs, the data show a wide spread and the agreement of the equations with the data is not as good as in the previous cases. However, the modified equation reproduces fairly well test RD1208. Most of the tests show that end effects are present and, for this reason, the modified Gurney equation is assumed to give better estimates than the Gurney equation. The estimates of the projected angles are clearly too high. However, the modified Taylor equation is superior to the normal Taylor equation.

Figure 14 shows the velocity distribution obtained from the modified Gurney and Gurney equations in the upper graph for cylinders filled with Octol, C/M=.43. Again, it can be observed that the modified Gurney equation gives better estimates than the normal Gurney equation. The bottom graph shows the estimates of the projected angles from the modified Taylor equation and experimental data for the same tests. The agreement of the modified Taylor equation with the experimental data is better than that with the normal Taylor equation.

Figures 15 and 16 contain the same information as the other figures but for cylinders filled with Comp B, C/M=.39, and cylinders filled with TNT, C/M=.38, respectively. In Fig 15, agreement of the equation with the experimental data is not as good as the agreement in most of the previous figures. However, the modified equations are superior to the normal equations for obtaining estimates of velocity and projected angle. The experimental data in Fig 16 shows a very large spread. Both the Gurney and the modified Gurney equations overestimate the velocity distributions. Neither equation gives the good agreement obtained in most of the other cases. The modified Gurney equation should be superior to the normal Gurney equation because it accounts for end effects, which although not evident from the data, are present.

The results of the modified Gurney equation are not plotted for $x/L<.15$ and $x/L>.95$ because the velocity decreases rapidly to zero in those regions and this contradicts reality since the end fragments will have a finite velocity. A better representation would be to continue the fragment velocity curve with the same constant slope obtained for the last points in the plot. However, since there are few experimental data points in those regions, it is difficult to ascertain whether this procedure should be followed. Since the modified equation gives reasonable agreement in the region $.15<x/L<.95$, it is deemed appropriate to represent end effects.

Cylinders with Pre-Formed Fragments

The equations are valid for cylinders made with preformed fragments. However, since the fragments separate earlier than cylinders with continuous casings and thus let the energy dissipate faster, it is expected that the velocity should be lower. This is confirmed by the experimental data plotted in Fig 17. Although the C/M is .93, the maximum fragment velocity attained is approximately seven percent lower than tests using the same explosive and L/D, but where the C/M is only .86 (see Fig 11). If no modification for pre-formed fragments is made, the results of the Gurney equation are in error by more than 10 percent, as shown in the upper graph of Fig 17. The lower graph shows the results of the modified Taylor equation. Since there is less energy to accelerate the fragments, this is equivalent to using a smaller effective C/M. Consequently, starting with the results of the Gurney equation, modified for end effects, the C/M of the charge was reduced until good agreement with experimental data was obtained. This occurs when the effective C/M is taken to be 80 percent of the actual C/M. Using this effective value for C/M in the modified Gurney equation yields the results shown in the upper graph of Fig 18. The C/M used in the normal Gurney equation has also been reduced by 20 percent. The difference between the modified equation and experimental data is on the order of one percent for the velocity distribution for $.1<x/L<.95$. The plot in the bottom graph gives a fairly good representation of the projected angles. Tables VII and VIII give the numerical results of the standard and modified equations.

When cylinders with $L/D<2$ are investigated, the modified equations give slightly less satisfactory results. Experimental data for tests with cylinders of $L/D=1.0$ are compared to the results of the modified equations in Fig 19. The same corrections as those for cylinders with $L/D=2.0$ are used. As can be seen, the equation slightly overestimates the velocities at $x/L \doteq .7$ and underestimates them towards the ends. The maximum fragment velocities attained are more than 20 percent lower than that for L/D=2.0, even though the C/M is .93 for both cases. This is because the rarefaction waves have a greater effect on the cylinder and cause a greater loss in velocity. Using the same modifications as that for cylinders with L/D=2.0 give a larger error (approximately 9 percent at $x/L \doteq .2$) but the equation gives acceptable results for $.2 < x/L < .9$ which are clearly superior to the estimates given by the normal Gurney equation corrected for preformed fragments only. A different correction could be used to obtain better results, but, since the results are fairly good, the same correction factors should be used for all cylinders of different L/D's to retain a simple, easy to use method. Table IX gives the numerical results of the standard and modified equations.

From the results obtained, it can be seen that the Gurney equation for cylinders made with continuous casings, modified for end effects as shown in Fig 9, will give reasonably accurate estimates of the fragment velocity distribution. Also, for cylinders made with pre-formed fragments, the results show that the C/M should be reduced by 20 percent, in addition to the modification for end effects.

End Projectors

The results of the corrected Gurney equation for end projectors are tabulated in Table X. Since pre-formed fragments are used, the effective C/M used in making the

Table I

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with Octol, C/M=.86

L/D=2.0

Detonation Velocity=8.20 mm/μs   Gurney Constant=2.80 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	2.171	1.088	.746	-26.460
.100	2.171	1.450	1.882	- 7.443
.150	2.171	1.679	3.020	- 2.899
.200	2.171	1.840	4.018	- .089
.250	2.171	1.955	4.837	1.897
.300	2.171	2.040	5.483	3.381
.350	2.171	2.100	5.981	4.528
.400	2.171	2.140	6.355	5.439
.450	2.171	2.164	6.623	6.181
.500	2.171	2.171	6.802	6.693
.550	2.171	2.171	6.923	6.923
.600	2.171	2.171	7.021	7.021
.650	2.171	2.171	7.099	7.099
.700	2.171	2.171	7.163	7.163
.750	2.171	2.171	7.216	7.654
.800	.171	2.140	7.157	8.982
.850	2.171	2.040	6.854	10.894
.900	2.171	1.840	6.205	13.675
.950	2.171	1.450	4.905	21.978

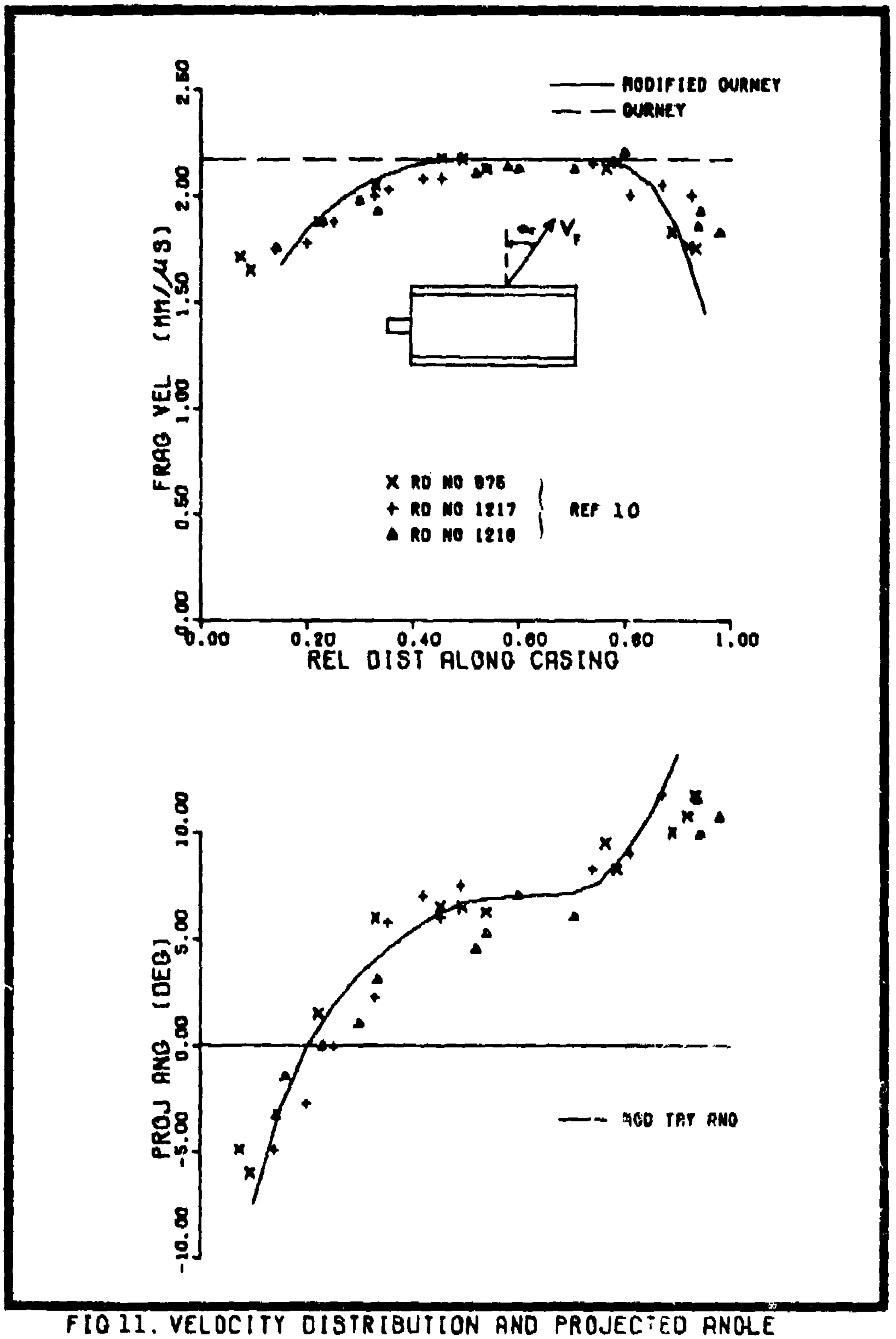


FIG 11. VELOCITY DISTRIBUTION AND PROJECTED ANGLE FOR CYLINDER FILLED WITH OCTOL. L/D=2.0, C/M=.86.

Table II

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with Comp B, C/M=.79

L/D=2.0

Detonation Velocity=7.75 mm/µs   Gurney Constant=2.68 mm/µs

Rel Length	Init Vel mm/µs	Mod Init Vel mm/µs	Tay Ang Deg	Mod Tay Ang Deg
.050	2.017	1.001	.726	-25.041
.100	2.017	1.337	1.836	- 7.166
.150	2.017	1.552	2.953	- 2.805
.200	2.017	1.703	3.935	- .083
.250	2.017	1.812	4.742	1.854
.300	2.017	1.892	5.380	3.309
.350	2.017	1.949	5.873	4.438
.400	2.017	1.987	6.242	5.337
.450	2.017	2.010	6.508	6.071
.500	2.017	2.017	6.684	6.576
.550	2.017	2.017	6.803	6.803
.600	2.017	2.017	6.899	6.899
.650	2.017	2.017	6.976	6.976
.700	2.017	2.017	7.039	7.039
.750	2.017	2.017	7.091	7.524
.800	2.017	1.987	7.030	8.834
.850	2.017	1.892	6.725	10.704
.900	2.017	1.703	6.076	13.393
.950	2.017	1.337	4.786	21.549

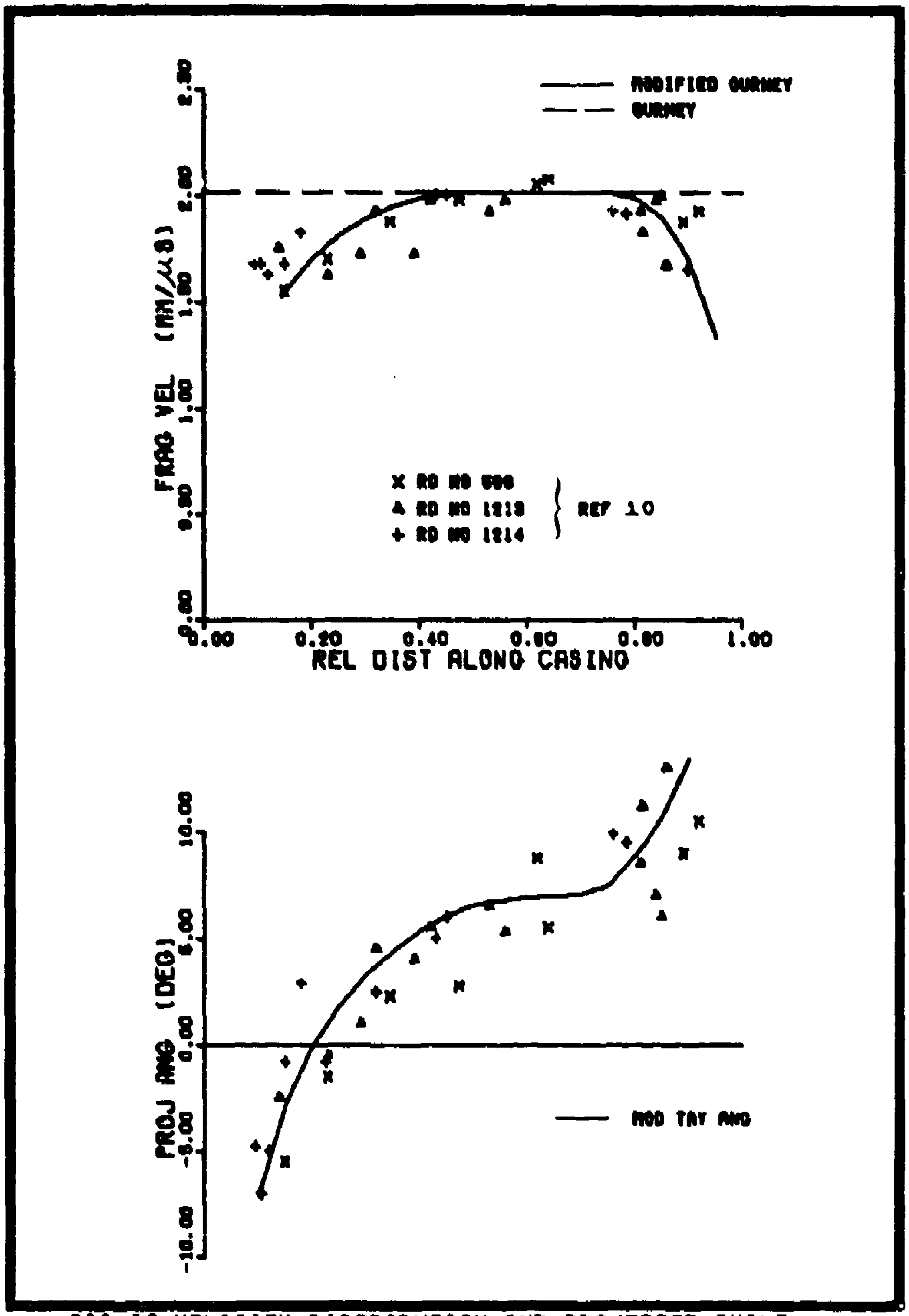


FIG 12. VELOCITY DISTRIBUTION AND PROJECTED ANGLE
FOR CYLINDER FILLED WITH COMP B. L/D=2.0. C/M=.79

Table III

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with TNT, C/M=.77

L/D=2.0

Detonation Velocity=6.95 mm/μs   Gurney Constant=2.37 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	1.767	.875	.707	-22.603
.100	1.767	1.169	1.791	- 6.494
.150	1.767	1.358	2.881	- 2.443
.200	1.767	1.490	3.841	.116
.250	1.767	1.586	4.630	1.947
.300	1.767	1.657	5.254	3.328
.350	1.767	1.707	5.736	4.400
.400	1.767	1.741	6.098	5.255
.450	1.767	1.761	6.358	5.950
.500	1.767	1.767	6.530	6.430
.550	1.767	1.767	6.647	6.647
.600	1.767	1.767	6.740	6.740
.650	1.767	1.767	6.815	6.815
.700	1.767	1.767	6.877	6.877
.750	1.767	1.767	6.928	7.332
.800	1.767	1.741	6.867	8.552
.850	1.767	1.657	6.567	10.288
.900	1.767	1.490	5.930	12.787
.950	1.767	1.169	4.667	20.768

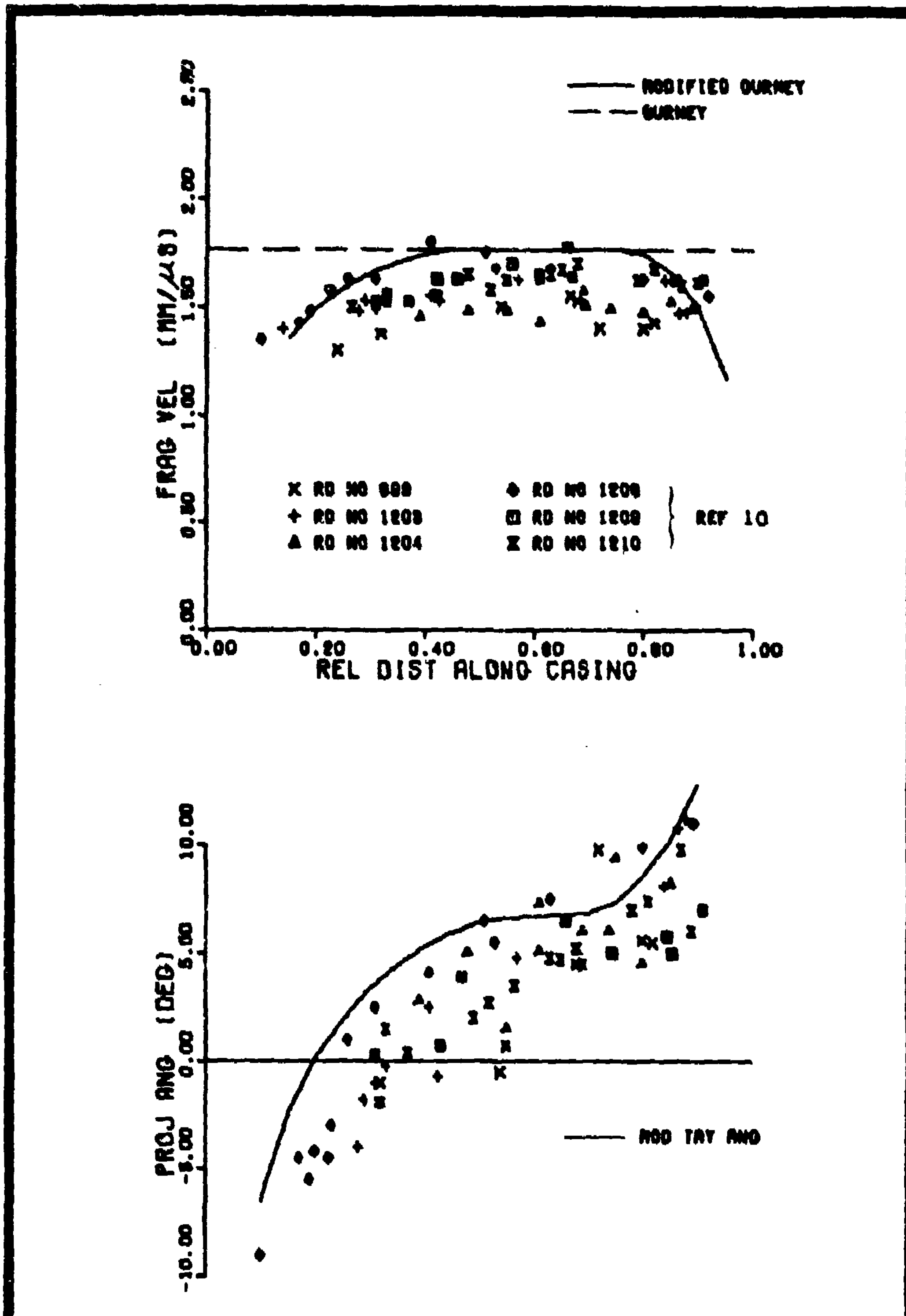


FIG 13. VELOCITY DISTRIBUTION AND PROJECTED ANGLE
FOR CYLINDER FILLED WITH TNT. L/D=2.0, C/M=.77

Table IV

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with Octol, C/M=.43

L/D=2.0

Detonation Velocity=8.20 mm/μs   Gurney Constant=2.80 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	1.666	.784	.538	-17.840
.100	1.666	1.061	1.377	- 5.710
.150	1.666	1.245	2.238	- 2.493
.200	1.666	1.377	3.007	- .396
.250	1.666	1.476	3.648	1.145
.300	1.666	1.549	4.161	2.333
.350	1.666	1.602	4.559	3.277
.400	1.666	1.638	4.859	4.043
.450	1.666	1.659	5.073	4.677
.500	1.666	1.666	5.213	5.115
.550	1.666	1.666	5.306	5.306
.600	1.666	1.666	5.380	5.380
.650	1.666	1.666	5.440	5.440
.700	1.666	1.666	5.489	5.489
.750	1.666	1.666	5.530	5.922
.800	1.666	1.638	5.470	7.092
.850	1.666	1.549	5.193	8.712
.900	1.666	1.377	4.641	10.915
.950	1.666	1.061	3.588	18.036

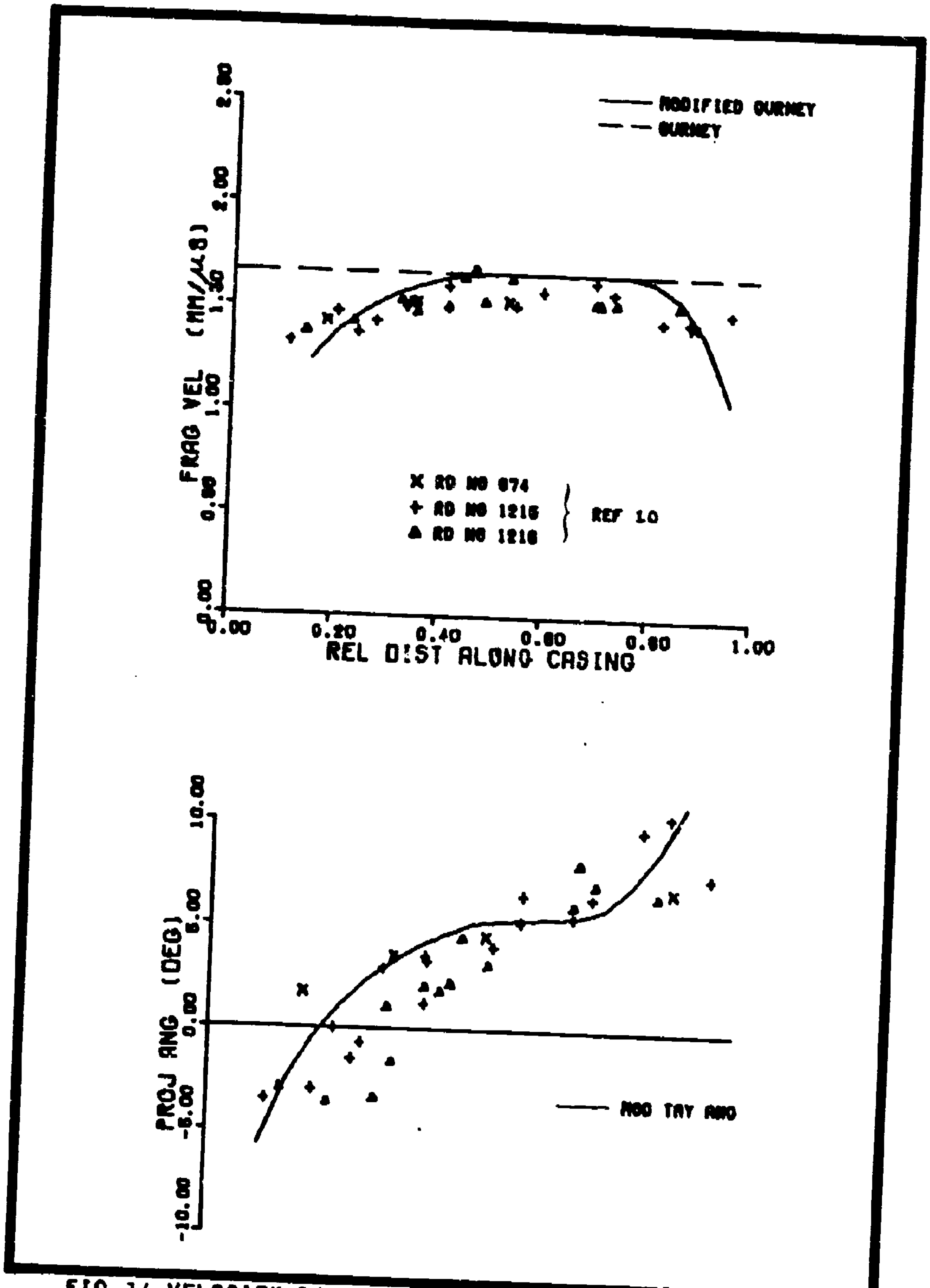


FIG 14. VELOCITY DISTRIBUTION AND PROJECTED ANGLE FOR CYLINDER FILLED WITH OCTOL. L/D=2.0. C/M=.43

Table V

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with Comp B, C/M=.39

L/D=2.0

Detonation Velocity=7.75 mm/μs   Gurney Constant=2.77 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	1.582	.740	.537	-17.466
.100	1.582	1.003	1.378	- 5.625
.150	1.582	1.178	2.241	- 2.455
.200	1.582	1.305	3.015	- .375
.250	1.582	1.399	3.660	1.160
.300	1.582	1.470	4.177	2.349
.350	1.582	1.521	4.580	3.295
.400	1.582	1.556	4.882	4.046
.450	1.582	1.576	5.099	4.701
.500	1.582	1.582	5.240	5.142
.550	1.582	1.582	5.333	5.333
.600	1.582	1.582	5.408	5.408
.650	1.582	1.582	5.468	5.468
.700	1.582	1.582	5.518	5.518
.750	1.582	1.582	5.558	5.953
.800	1.582	1.556	5.497	7.123
.850	1.582	1.470	5.219	8.733
.900	1.582	1.305	4.653	10.899
.950	1.582	1.003	3.589	17.899

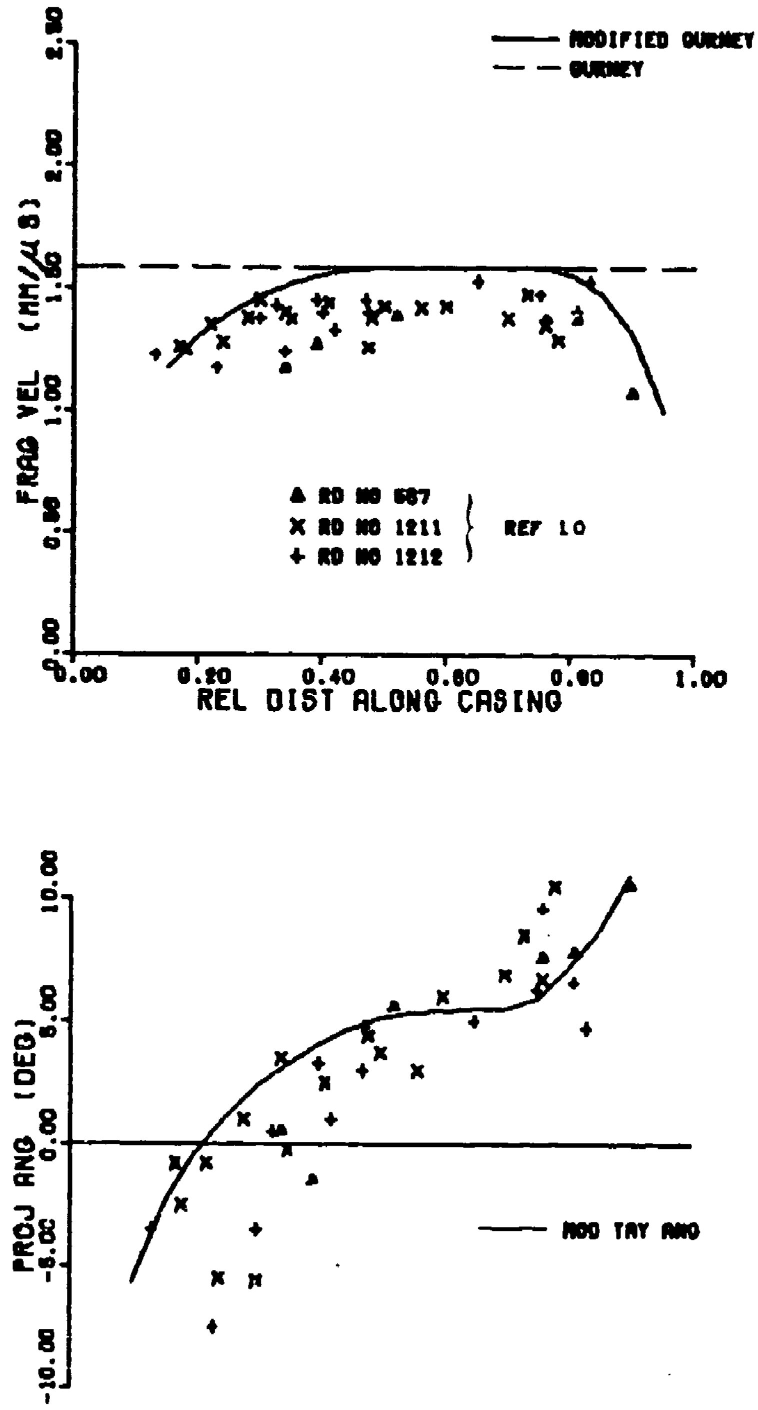


FIG 15. VELOCITY DISTRIBUTION AND PROJECTED ANGLE FOR CYLINDER FILLED WITH COMP B. L/D=2.0. C/M=.39

Table VI

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with TNT, C/M=.38

L/D=2.0

Detonation Velocity=6.95 mm/μs   Gurney Constant=2.44 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	1.379	.644	.521	-15.779
.100	1.379	.873	1.337	- 5.076
.150	1.379	1.026	2.176	- 2.138
.200	1.379	1.136	2.927	- .193
.250	1.379	1.219	3.555	1.250
.300	1.379	1.280	4.057	2.370
.350	1.379	1.325	4.449	3.262
.400	1.379	1.355	4.743	3.987
.450	1.379	1.373	4.954	4.586
.500	1.379	1.379	5.091	5.000
.550	1.379	1.379	5.182	5.182
.600	1.379	1.379	5.254	5.254
.650	1.379	1.379	5.313	5.313
.700	1.379	1.379	5.361	5.361
.750	1.379	1.379	5.400	5.765
.800	1.379	1.355	5.340	6.846
.850	1.379	1.280	5.069	8.329
.900	1.379	1.136	4.518	10.325
.950	1.379	.873	3.483	17.016

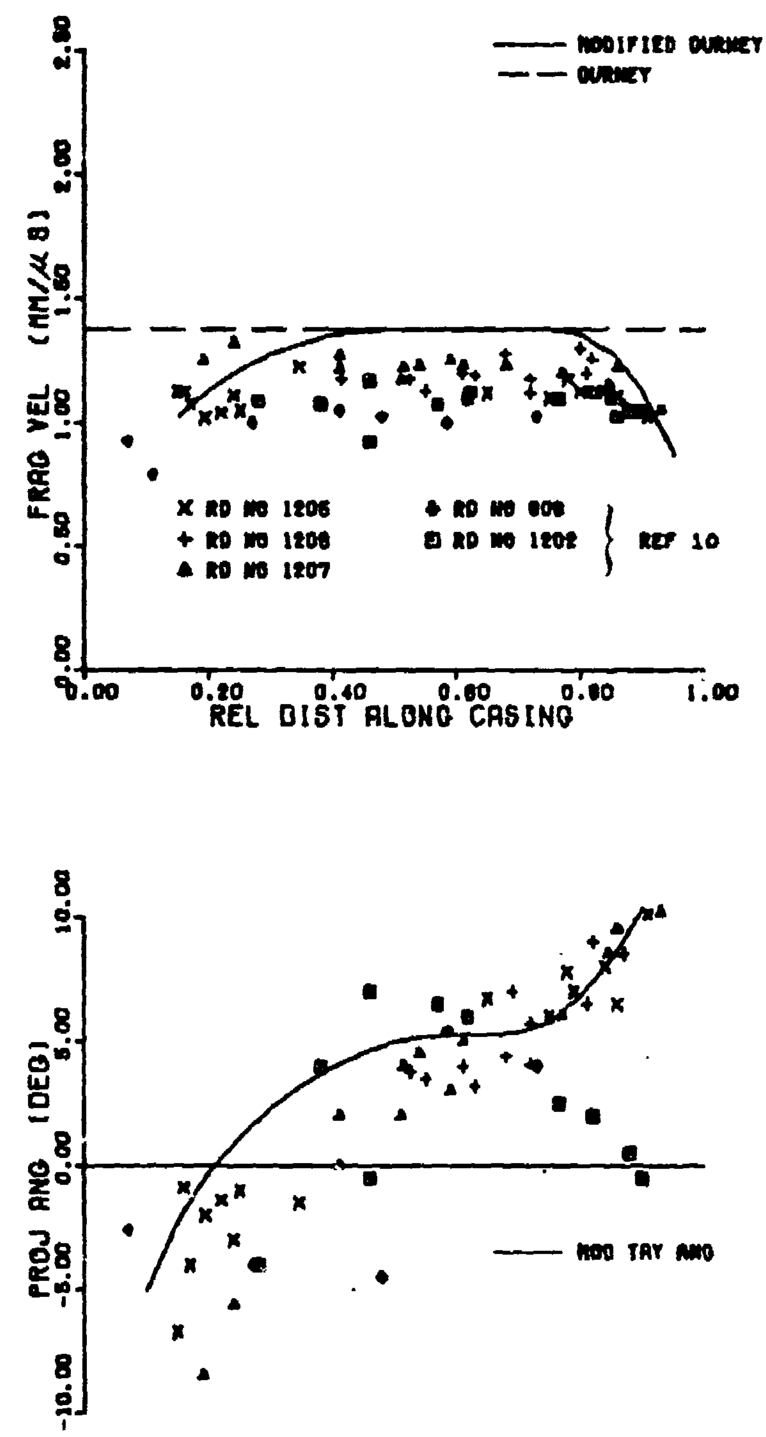


FIG 16. VELOCITY DISTRIBUTION AND PROJECTED ANGLE FOR CYLINDER FILLED WITH TNT. L/D=2.0, C/M=.38

Table VII

Comparison of Modified and Standard Gurney and Taylor Formulae for Cylinder Filled with Octol, C/M=.93
Pre-Formed Fragments - No Correction

L/D=2.0

Detonation Velocity=8.20 mm/μs   Gurney Constant=2.80 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	2.062	.983	.626	-17.810
.100	2.062	1.320	1.603	- 5.281
.150	2.062	1.539	2.614	- 1.910
.200	2.062	1.696	3.531	.298
.250	2.062	1.812	4.307	1.922
.300	2.062	1.899	4.939	3.170
.350	2.062	1.964	5.441	4.152
.400	2.062	2.010	5.832	4.938
.450	2.062	2.041	6.127	5.576
.500	2.062	2.058	6.340	6.099
.550	2.062	2.062	6.481	6.435
.600	2.062	2.062	6.585	6.585
.650	2.062	2.062	6.669	6.669
.700	2.062	2.062	6.737	6.784
.750	2.062	2.058	6.781	7.371
.800	2.062	2.010	6.669	8.446
.850	2.062	1.899	6.336	9.762
.900	2.062	1.696	5.685	11.729
.950	2.062	1.320	4.442	18.864

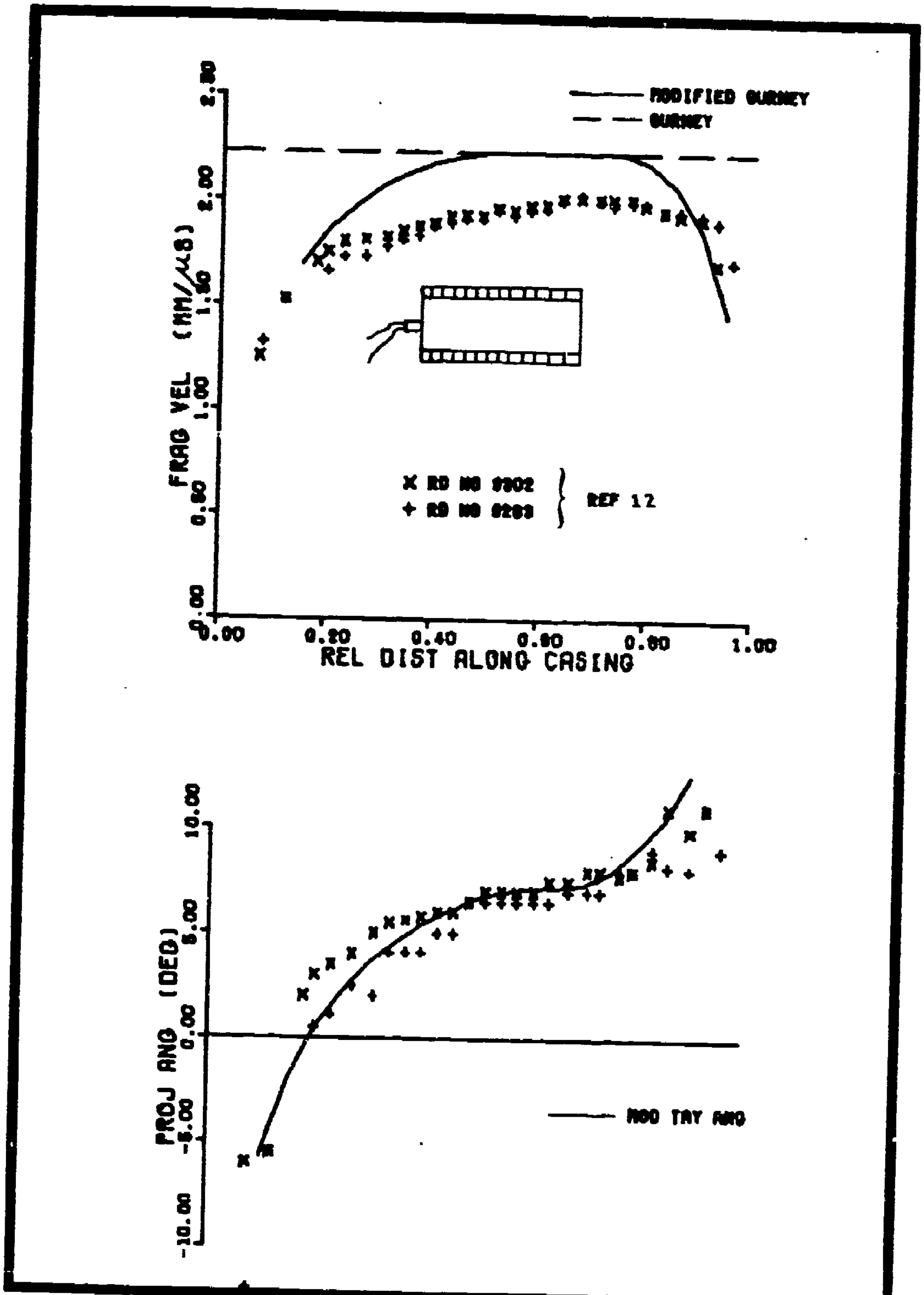


FIG 17. VELOCITY DISTRIBUTION AND PROJECTED ANGLE FOR CYLINDER FILLED WITH OCTOL PRE-FORMED FRAGMENTS. L/D=2.0, C/M=.93

Table VIII

Comparison of Modified and Standard Gurney and Taylor Formulae
for Cylinder Filled with Octol
Corrected for Pre-Formed Fragments (CM=.8* cm)

L/D=2.0   C/M=8*.93=.744

Detonation Velocity=8.20 mm/μs   Gurney Constant=2.80 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	2.062	.983	.626	-17.810
.100	2.062	1.320	1.603	- 5.281
.150	2.062	1.539	2.614	- 1.910
.200	2.062	1.696	3.531	.298
.250	2.062	1.812	4.307	1.922
.300	2.062	1.899	4.939	3.170
.350	2.062	1.964	5.441	4.152
.400	2.062	2.010	5.832	4.938
.450	2.062	2.041	6.127	5.576
.500	2.062	2.058	6.340	6.099
.550	2.062	2.062	6.481	6.435
.600	2.062	2.062	6.585	6.585
.650	2.062	2.062	6.669	6.669
.700	2.062	2.062	6.737	6.784
.750	2.062	2.058	6.781	7.371
.800	2.062	2.010	6.669	8.446
.850	2.062	1.899	6.336	9.762
.900	2.062	1.696	5.685	11.729
.950	2.062	1.320	4.442	18.864

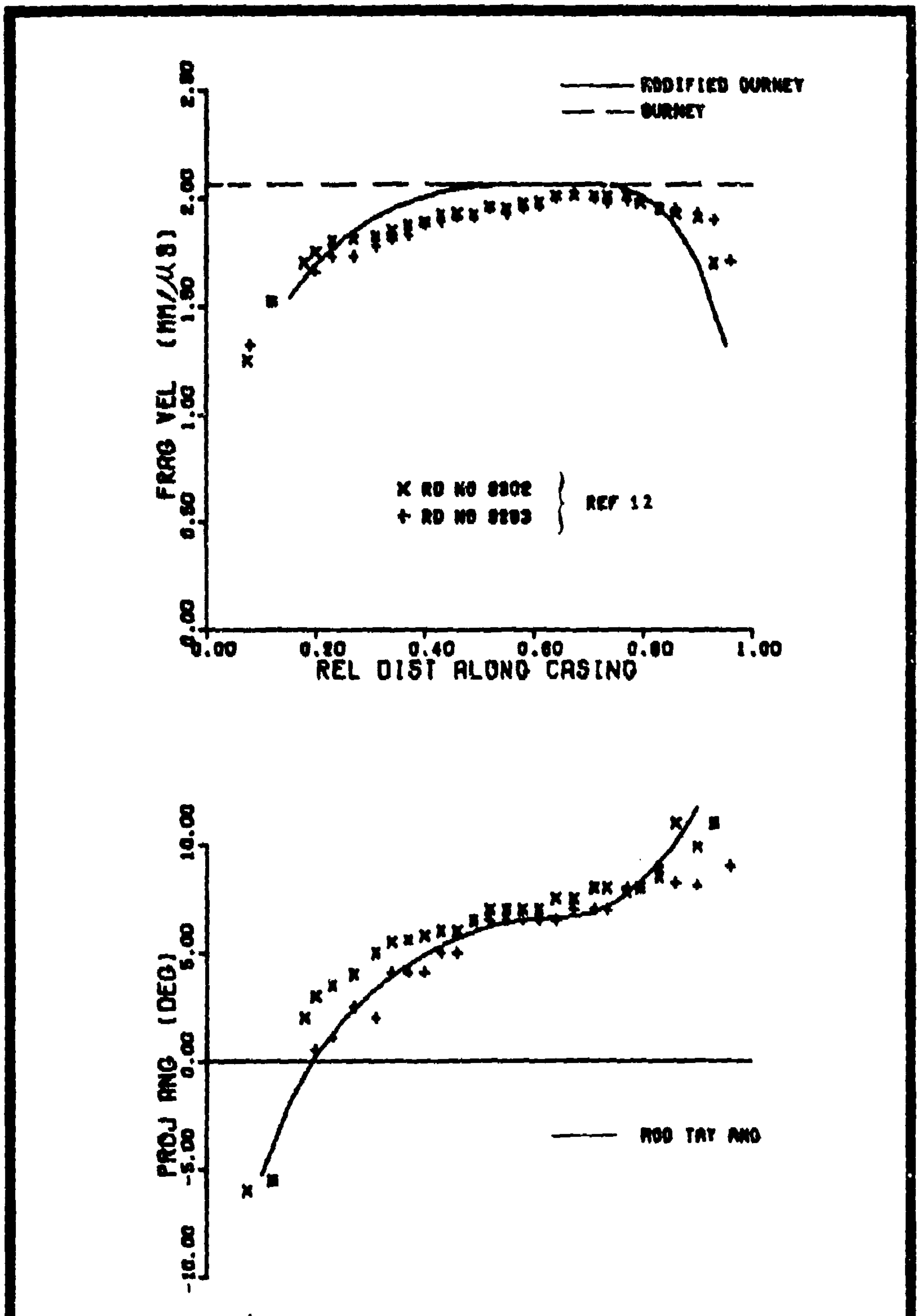


FIG 18. VELOCITY DISTRIBUTION AND PROJECTED ANGLE
FOR CYLINDER FILLED WITH OCTOL
CORRECTED FOR PRE-FORMED FRAGMENTS, L/D=2.0, C/M=.744

Table IX

Comparison of Modified and Standard Gurney and Taylor Formulae
for Cylinder Filled with Octol
Corrected for Pre-Formed Fragments (CM=.8* cm)

L/D=1.0 C/M=.8*.93=.744

Detonation Velocity=8.20 mm/μs Gurney Constant=2.80 mm/μs

Rel Length	Init Vel mm/μs	Mod Init Vel mm/μs	Tay Ang Deg	Mod Tay Ang Deg
.050	2.062	.715	.230	-12.797
.100	2.062	.983	.626	- 4.970
.150	2.062	1.173	1.098	- 2.937
.200	2.062	1.320	1.603	- 1.557
.250	2.062	1.440	2.115	- .462
.300	2.062	1.539	2.614	.462
.350	2.062	1.624	3.088	1.265
.400	2.062	1.696	3.531	1.969
.450	2.062	1.758	3.937	2.592
.500	2.062	1.812	4.307	3.145
.550	2.062	1.859	4.640	3.635
.600	2.062	1.899	4.939	4.072
.650	2.062	1.934	5.205	5.205
.700	2.062	1.899	5.262	6.626
.750	2.062	1.812	5.146	7.391
.800	2.062	1.696	4.918	7.900
.850	2.062	1.539	4.546	8.587
.900	2.062	1.320	3.960	9.765
.950	2.062	.983	2.989	15.021

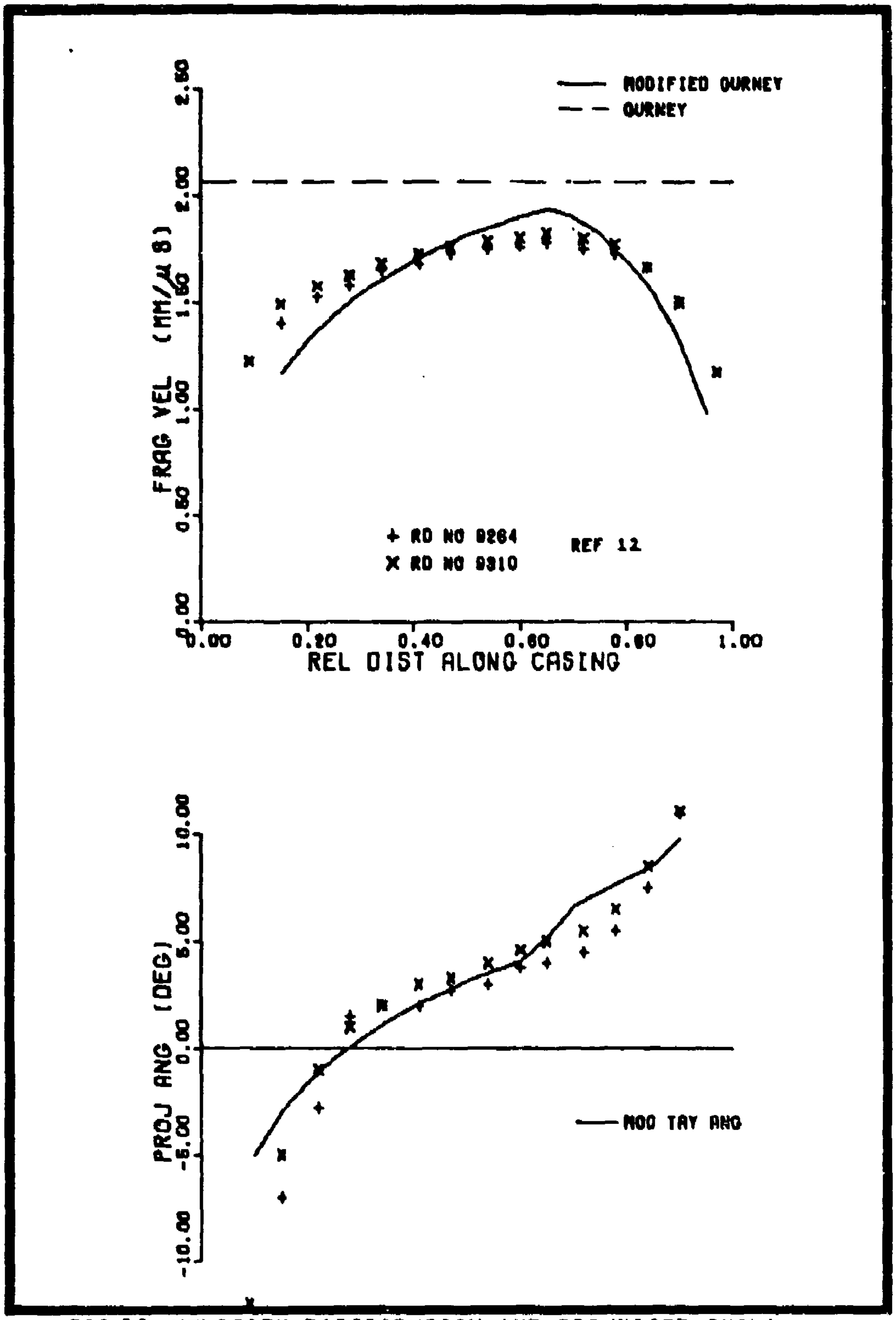


FIG 19 . VELOCITY DISTRIBUTION AND PROJECTED ANGLE
FOR CYLINDER FILLED WITH OCTOL
CORRECTED FOR PRE-FORMED FRAGMENTS, L/D=1.0, C/M=.744

calculations was reduced by 20 percent. Figure 20 shows the geometry of the end projector with a comparison of the results obtained from the Gurney equation (Eq 22) and experimental data (Ref 9). If no corrections are made, the Gurney equation gives a maximum velocity of approximately 2.7 mm/μs. The experimental data, although it may not give the exact velocity for reasons discussed previously, give a maximum velocity of approximately 2.1 mm/μs, a difference of 22 percent from that of Eq 22. However, when the modified Gurney equation (Eq 32) is used, the difference is only 11 percent. Unfortunately, there is only one set of experimental data to test the modified equation. With more experimental data, it might be possible to derive a simple, more precise modification factor. Nevertheless, the present modification is simple and gives reasonable results. It will give results of the same form for any other L/D and C/M, since changing any one or the other ratio changes the size of the cone of the explosive accordingly. The modified equation should therefore give results which are comparable to the experimental data.

A slightly different modification factor for the Gurney equation for an open-faced sandwich was found in Ref 14. The factor proposed is the removal of a cone of explosives with an angle of 30 degrees from the normal at the edges of the charge (see Fig 20).

The results obtained if that factor is used are shown as the dotted line in Fig 21 and in Table X. Unfortunately, only one set of experimental data was found

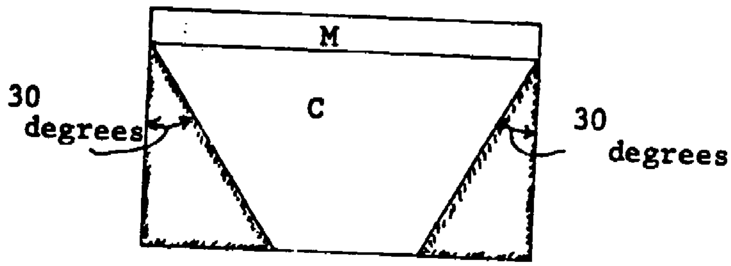


Fig 20. Correction for End Effects for Open-Faced Sandwich

to compare the two factors. As can be seen, the results obtained are not as good as those given by the modification factor given in Eq 32. On the basis of that experimental data, the modification proposed in Section III of this study is superior to the one proposed in Ref 14.

In general, given a cylindrical warhead or an end projector type warhead of a certain L/D and C/M, a reasonably accurate estimate of the velocity distribution for the fragments can be obtained by using the Gurney equations with appropriate modifications for the particular geometry involved.

The angle of departure of the fragments was not found because it would be a very complex undertaking. The Taylor formula applies to a system where the detonation wave is perpendicular to the surface to be projected. If the charge in an end projector is point-initiated, a spherical detonation wave will be formed. Reflections will occur at free

Table X

Velocity Distribution for End Projector

L/D=1/2   C/M=2.34

Detonation Velocity=7.75 mm/μs   Gurney Constant=2.68 mm/μs

Rel Length	Init Vel mm/μs	Init Vel (using 30 deg cone) (Ref 14) mm/μs
.050	.818	1.212
.100	1.314	1.777
.150	1.646	2.089
.200	1.881	2.264
.250	2.051	2.366
.300	2.175	2.366
.350	2.262	2.366
.400	2.321	2.366
.450	2.355	2.366
.500	2.366	2.366
.550	2.355	2.366
.600	2.321	2.366
.650	2.263	2.366
.700	2.175	2.366
.750	2.052	2.354
.800	1.882	2.276
.850	1.647	2.111
.900	1.315	1.816
.950	.819	1.286

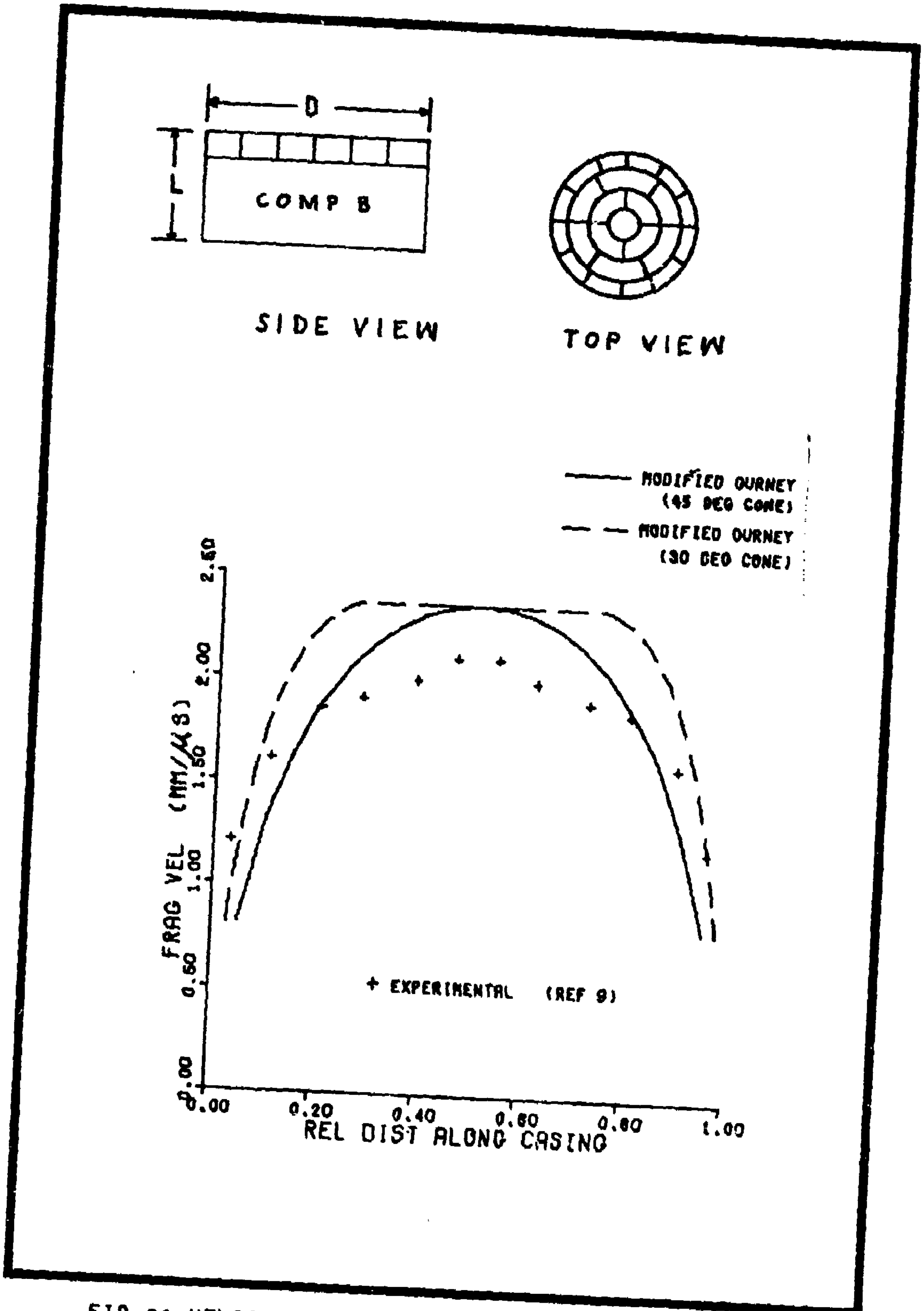


FIG 21. VELOCITY DISTRIBUTION FOR END PROJECTOR
L/D=1/2, C/M=2.35

surfaces and the pattern of the detonation wave will be quite difficult to obtain. An analysis similar to Taylor's could be done for this situation but since the total dispersion of the fragments is of greater interest than that of individual fragments, no attempt was made to obtain the projection angle of the fragments. The total dispersion can be controlled by imbeding the fragments in a plastic ring as was done in the experiments (Ref 2) which will be discussed next.

An attempt was made to use the method on a different type of end projector (see Fig 22). As can be seen, the projector consists of five layers. The uppermost is comprised of 32 aluminum cubes surrounded by plastic. The other layers are C-4, steel, 1/16 inch thick Detasheet, and plywood, in that order. The projector has a 12.7 cm diameter. The Gurney equation for an asymmetrical sandwich was used for this analysis. Experimental results are contained in Ref 2. As is explained in the reference, measurement of the fragment velocities was not easy. Different methods were used and, then, only the speed of the fastest fragment could be obtained for reasons outlined in Ref 2. Although the tests were with essentially the same L/D's, C/M's, and explosive fillings, the measured velocities differed greatly. Consequently, calculated results are best compared to the average measured velocity for any one series of tests. Since only the fastest fragment speed was recorded, which should be the one in the middle of the circular disk, the results of the Gurney equation, with only a correction for using pre-formed fragments, are compared to the experimental data. Detailed calculations for one specific test (Shot No 43) are shown in Appendix B.

The calculated velocity from Gurney's equation for Shot 43 is 2200 m/s, and the average measured speed of the fastest fragment for that series of tests is 2430 m/s. Thus, the calculated value underestimates the velocity by approximately ten percent. The calculations were repeated for Shot No 66 in which almost twice as much C-4 was used. The velocity was calculated to be 2690 m/s, and the average measured velocity for that series of tests is 3410 m/s. In this instance, the method underestimates the velocity by about 21 percent. At this point, it seems that the modified method should not be applied.

The method does not work well because these end projectors are not end initiated. The method of initiation is important in what occurs during the detonation process. Since the C-4 is initiated simultaneously around the perimeter by the Detasheet, a circular shock wave will form and converge towards the center of the charge. The shock wave interacts with itself to produce an amplified wave. The results are similar to a cylinder which is initiated at both ends. Such a case is considered in Ref 18. The velocity of the fragments at the middle is approximately 15 percent greater in such instances than when the cylinder is initiated from one end only. This implies that no reduction should be made in the C/M because of pre-formed fragments, or that the modification can be made and the results increased by 15 percent. Since no additional information is known, the value is approximate because different configurations might yield different percent increases. However, this value should be close to any value obtained for a different configuration. Consequently, if the total effective C/M's are not reduced in Gurney's formula, the results obtained are velocities of 2375 m/s and 2870 m/s for Shots 43 and 66 respectively. This reduces the error to approximately two percent for Shot 43 and 16 percent for Shot 66. It can be concluded that, when initiation causes amplification of the detonation wave, such as simultaneous initiation of a cylinder from both ends or initiation as in this type of projector, no correction should be made for using pre-formed fragments. It may be that the C/M should be increased a certain amount, but insufficient data precludes that conclusion.

Once again, obtaining the deflection angle of the fragments is a very difficult task. From the experiments presented in Ref 2, it is evident that having the plastic ring (see Fig 22) around the cubes cut down dispersion greatly. The plastic ring had a 12.7 cm diameter and the cubes were 5.72 cm in length. In these instances, the solid angle for 75 percent of the fragments at a distance of 24 feet ranged from 1.3 to 9.1 millisteradians. It appears that, in general, by enclosing the intended fragments in a similar plastic ring whose diameter is twice the length of the fragments arranged in a square, similar dispersions should be obtained. Therefore, in designing an end projector type warhead, the fragments should be enclosed by some sort of ring, if a small dispersion is desired.

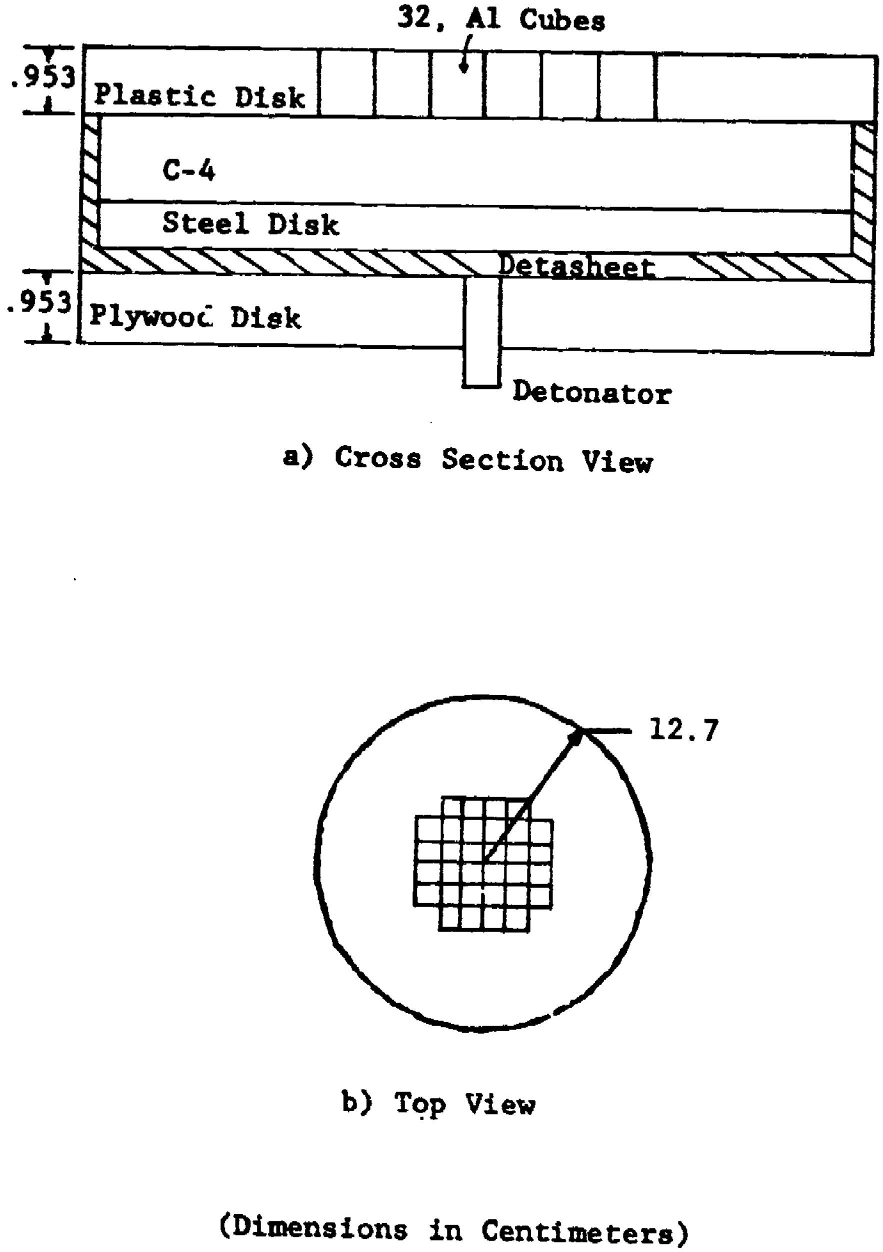


Fig 22. Configuration for Asymmetric Sandwich

V Conceptual Warhead Designs

Analysis of conceptual preliminary designs of warheads can now be made using the information gained in the previous sections. For example purposes, warhead designs which will give initial fragment velocities of 3000 m/s will be considered. Also, it is assumed that the warhead is to be fitted in an air-to-air missile and is detonated at a distance of 15 meters from the target.

Velocity of Fragments at Target

It is important to know what the final velocity of the fragments will be when they reach the target. If they are too low, that is, if the kinetic energy is low, they will not penetrate the target. Since the warhead is not likely to be fitted at the nose of the missile, the fragments will have to go through some of the missile's components when the warhead detonates. Consequently, the fragments will leave the missile at a lower velocity than 3000 m/s, not including the missile's own velocity which is imparted to the fragments. The warhead is made of pre-formed steel cubes (.953 cm) and it is assumed that they would go through a number of components which are comparable to a 2.54 cm aluminum sheet (exact thickness would be known in an actual design). If plugging type failures are assumed, the residual velocity of the fragments can be found.

$$V_R = \frac{M_F}{M_F + M_0}\left[V_P^2 - V_{50}^2\right]^{1/2} \quad \text{(Ref 1)} \qquad (33)$$

where

V_R = residual velocity (m/s)

V_P = initial fragment velocity (m/s)

M_F = mass of steel fragment (=6.7 g)

M_0 = mass of Al plug punched by the fragment (=6.4 g)

V_{50} = ballistic limit, velocity at which 50% complete penetration and 50% partial penetration of the target plate can be expected

An approximate numerical value for the V_{50}, ballistic limit, can be found by using an equation obtained to fit .30 cal armor-piercing projectiles because an exact value could not be found from the sources. Assuming that the fragments go through 2024-T3 Al,

$$V_{50} = A\left[\frac{\text{thickness of plug}}{\text{plug diameter}}\right]^{.7}$$

$$= 950\left[\frac{2.54}{.953}\right]^{.7} = 575.5 \text{ m/s} \qquad \text{(Ref 3)} \quad (34)$$

$$V_R = \frac{6.7}{13.1}\left(3000^2 - 575.5^2\right)^{1/2} = 1506 \text{ m/s} \qquad (35)$$

The velocity at 15 m from the point of detonation can be found by using an aerodynamic drag law

$$V_M = V_0 \exp\left(-\frac{C_D \rho A}{2M} x\right) \qquad \text{(Ref 1)} \qquad (36)$$

where

V_M = fragment velocity (m/s)

x = distance from point of detonation (=15 m)

V_0 = initial velocity (=1506 m/s)

C_D = drag coefficient (=1.66)

ρ = air density (=.00123 g/cc)

A = fragment presented area (=.908 cm^2)

M = fragment mass (=6.7 g)

For rotating cubes, C_D was obtained from Ref 7. For computational purposes, it is assumed that the warhead is detonated at sea level. Therefore, the fragments will have a velocity of

$$V_M = 1224 \text{ m/s} \tag{37}$$

However, due to end effects, the fragments towards the edges will have lesser velocities. From experimental data for the first end projector discussed in the previous section, the velocity of the fragments located at a distance of x/L=.25 from the edge is reduced by approximately 17 percent. In this case, these fragments would attain a velocity of approximately 1030 m/s.

These velocities, given favorable relative velocities between the target and the missile, should be sufficiently high to penetrate thin aircraft skins and components not protected by armor.

If a sufficient number of fragments pierce and destroy a vulnerable area (cockpit, engines, fuel tanks, etc...), the target will be destroyed. Therefore, by using the modified Gurney equation for an end projector type warhead (Eq 32), one finds that the velocity of approximately 50 percent of the fragments will be greater than 1030 m/s, a velocity deemed sufficiently high to destroy a target. However, sources were not found to confirm this. Experimental testing would have to be carried out for fragments at that velocity against targets of interest. For illustrative purposes, it is assumed that a warhead which imparts an initial velocity of 3000 m/s to the fragments near the middle of the warhead surface would be desirable. Conceptual designs of different geometries which produce fragments at 3000 m/s will be investigated.

Example 1

The first design of an end projector is a simple one using Comp B as the explosive charge and 32, .953 cm steel cubes enclosed in a plastic ring in a configuration like the one shown in Fig 10 and Fig 20. Based on similar projector tests seen in the previous section, enclosing fragments in a plastic ring should give a small dispersion at 15 m. Also, the warhead is given a 12.7 cm diameter to fit inside an air-to-air missile.

To find the characteristics of the warhead, Gurney's equation for an open-faced sandwich is used (Eq 22). Solving for C/M, with V_M = 3000 m/s , the warhead will have a C/M = 4.47 . The other root of the equation is negative and is therefore neglected. Since pre-formed fragments are used, the C/M must be increased by 20 percent, or to C/M=5.36 , to compensate for the loss of energy between the separating fragments. Also, it can be observed that, from the first end projector tests, the method still overestimates the velocity. To reproduce the data points at the center of the graph (Fig 20) more exactly, a value of $C/M = 1.85$ has to be used for that experiment rather than $C/M = 2.94$, the actual value. This corresponds to a decrease of approximately 37 percent of the actual C/M. Consequently, to obtain the most exact estimate of the velocity in this first design, the C/M to be used in the equation should be $C/M = 4.47 \times 1.37 = 6.12$.

The total weight of the steel cubes is $M_s = 214.4$ g.

The weight of the plastic ring (same composition as the one used in Ref 2) is $M_p = 156$ g.

Total weight of upper level, $M = M_s + M_p = 370.4$g.

Therefore, the required weight of Comp B is $C = 2266.9$ g.

The density of Comp B is $\rho = 1.75$ g/cc (Ref 4).

The height of the Comp B cylinder is 10.2 cm.

Therefore, a warhead with $L/D = .881$ and $C/M = 6.12$ would have a velocity distribution with the center fragments at approximately 3000 m/s (see Fig 24a). The C/M for this warhead is quite high. It even falls outside the range of applicability of the Gurney theory. This warhead would not be selected because its high C/M would probably cause the fragments to break up.

Example 2

A slightly more complex example is presented. The effects of the addition of a tamper weight, or driver plate, to the first design are investigated. The configuration of the warhead is the same as that of an asymmetric sandwich and Gurney's equation (Eq 20) for that geometry is used to find the dimensions of the components of the warhead, L/D, and C/M.

Although there does not appear to be a physical optimum for C/M and N/M for a particular design, it is possible to use the equation to obtain a "best" design for a given set of specifications, such as fragment velocities to be attained, maximum warhead size, and production cost of the warhead, etc... The results of the equation are tabulated in Table XI for a few values of C/M and N/M. The non-dimensional velocity can be obtained from it, and given an explosive, i,e, $\sqrt{2E}$, the fragment velocity can be obtained for that particular C/M and N/M.

From the table, it can be seen that as C/M and N/M increase, the velocity increases. Although the largest N/M used was 10, larger values give increasing velocities. The table was assembled for $.1 \leq C/M \leq 10$, but since the Gurney equations give best results for $.1 < C/M < 5$, the velocities obtained for C/M = 10 are largely approximate and are tabulated only to show that $V_M/\sqrt{2E}$ does in fact increase as N/M and C/M increase. The data are plotted in Fig 23. The solid lines indicate increasing velocities with increasing C/M and N/M. The dotted lines represent a warhead where the total weight was kept constant at 3 kg. N and M were varied while C was kept constant; this was repeated for three different values of C. It can be observed

Table XI

Values of $V_M/\sqrt{2E}$ for Various C/M and N/M

N/M \ C/M	0.1	0.5	1.0	2.0	5.0	10.0
0.1	0.117	0.355	0.558	0.818	1.175	1.392
0.5	0.185	0.427	0.608	0.837	1.167	1.379
1.0	0.222	0.480	0.655	0.866	1.168	1.369
3.0	0.269	0.536	0.745	0.943	1.196	1.361
5.0	0.284	0.593	0.781	0.982	1.223	1.370
10.0	0.311	0.655	0.866	1.095	1.270	1.400

that for a fixed total weight and a fixed amount of explosive, the velocity increases as the tamper weight, N, increases and the fragment mass, M, decreases. Also, the larger the amount of explosive, the higher the velocity. These curves would be different for a different total weight. They are useful in designing a warhead. For instance, if the specifications called for a warhead with a maximum weight of 3 kg, fragment velocities of at least 2000 m/s, and Comp B as the explosive to be used, the graph would give the different combinations possible which would satisfy those specifications. In this case, those combinations are:

C (kg)	C/M	N/M	M (kg)	N (kg)
0.5	@ 1	@ 4	0.5	2.0
1.0	@ 1.25	@ 1.5	0.8	1.2
1.5	@ 1.5	@ 0.6	1.0	0.5

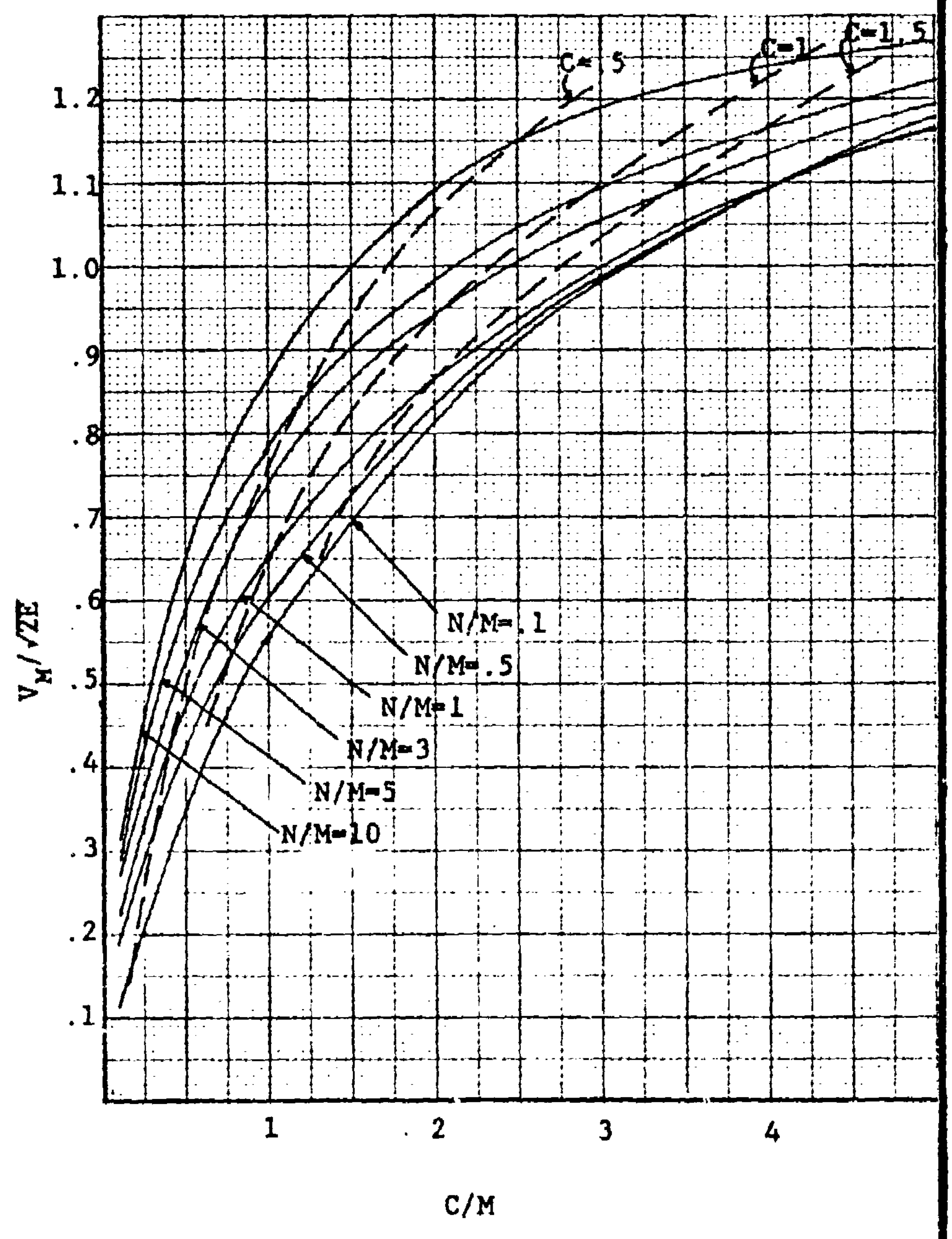


Fig 23. $V_M/\sqrt{2E}$ vs C/M for Asymmetric Sandwich

The selection could then be made on which of these combinations meets best the specifications.

Returning to the second example where a fragment velocity of 3000 m/s is required and Comp B is to be used, the solid lines in Fig 23 can be used to find possible combinations to meet the velocity of $V_M/\sqrt{2E} \geq 1.12$, i.e. $V_M/\sqrt{2E}$ equals $3000/2680 = 1.12$. Some combinations are :

C/M	N/M
2.5	20.0
3.0	7.0
3.5	4.0
4.0	2.5

More possibilities exist but for $C/M<2.5$, large N/M's are required, which would probably make the design more difficult, and for $C/M>4$, the Gurney equation starts to lose its applicability. The values of $C/M = 3.0$ and $C/M = 4.0$ will be used design the warhead; they can be compared to find which design is likely to be the best. Since preformed fragments are to be used, the C/M ratio must be increased by 20 percent. For the first design with $C/M = 3.0$, the actual C/M ratio to be used becomes 3.6.

From example 1, $M = 370.4$ g . Therefore, $C = 1333$ g.
The height of the Comp B disk would thus be 6 cm.
The steel tamper weight would be $N = 7M = 2593$ g .
The height of the steel disk would therefore be 2.54 cm.
The warhead would have the following characteristics:

$L/D = 0.756$

$C/M = 3.6$

$N/M = 7.0$ (see Fig 24b)

If $C/M = 4.0$ is chosen, the warhead characteristics are:

$L/D = 0.779$

$C/M = 4.8$

$N/M = 2.5$

Height of Comp B disk = 8 cm

Height of steel disk = 0.943 cm (see Fig 24c)

Of the two designs, the first one would be preferable because less Comp B is required, given that the cost of Comp B is far greater than steel. Compared to design 1, it is seen that the addition of a tamper reduces the amount of explosives required to produce the same velocities. Also, as the tamper gets heavier, the amount of explosive required gets less.

Since no experimental data are available for this type of warhead, it is impossible to determine how accurately the Gurney equation estimates the velocity distribution. It is assumed that the method provides satisfactory estimates for preliminary design.

Example 3

The third example considered is similar to the one in Ref 2. The end projector is treated as two asymmetric sandwiches, as is done in Fig 26. It is assumed that the upper part will be given a velocity of approximately 100 m/s by the Detasheet layer. It is necessary to find the characteristics of the upper part which will give a velocity of 2900 m/s, then those for the bottom part can be found. The upper part is made up of a layer of 32 steel cubes surrounded by a plastic ring, a layer of Comp B detonated simultaneously around the perimeter by Detasheet, 1/16 inch thick, and a steel disk beneath the Comp B. Since this is similar to example 2, $C/M = 3.0$ is chosen. This implies that $N/M = 7.0$. Although two different explosives are involved, Comp B and Detasheet, the amount of Detasheet is small compared to the amount of Comp B and the Gurney energy for Comp B is used in the computations.

It was found from experimental data that for this configuration the velocity is increased due to the amplification of the detonation wave (Ref 19) and that little or no correction is required, even though pre-formed fragments are used. Therefore, in this case, no correction is used.

For the upper sandwich:

Since $C/M = 3.0$, $C = 1111.2$ g .

Also, $N = 2593$ g .

The steel tamper has a 12.38 cm diameter and is 2.78 cm thick.

The height of the Detasheet and Comp B cylinder is 5.28 cm.

For the lower sandwich (Detasheet between steel tamper and upper part):

The weight of the upper sandwich is 4074.6 g.

The weight of the layer of Detasheet between the upper sandwich and the lower tamper is approximately 33 g. In this case, C/M = 0.0081 . To obtain a velocity of 100 m/s for the upper part, a value of N/M = .3 is chosen because it will give a velocity higher than 100 m/s and interpolation in the tables in Ref 6 is easier. Also, because the value of C/M is so small, the answers obtained are highly approximate, since the value is outside the bounds of the Gurney theory. However, since the velocity imparted by the lower part will be very small compared to that from the upper part, the values obtained for C/M, N/M, and the velocity are used to get an estimate of the size of the bottom tamper plate. By using a C/M which falls within the range of applicability of the Gurney equations, more accurate answers would be obtained, but the configuration would be different from the present one if a fragment velocity of 3000 m/s is required. This is not done for this example. Therefore, the warhead, as designed, would have a diameter of 12.7 cm and a L/D of 0.75 (see Fig 24d).

Example 4

Another example which is of interest is one where there are two layers of fragments, with an explosive sandwiched between the fragment layers. This warhead, producing two fragment clouds, would have a higher probability of kill. Assuming a head-on, or beam attack, the first layer would penetrate, or weaken, the target while the second layer would destroy it. The configuration is the same as example 3, but an extra layer of steel fragments enclosed in a plastic ring, backed with a 1/16 inch layer of Detasheet, are added to the warhead. If the same amounts and dimensions as those used in the third design are used, the fragment velocities can be calculated using Gurney's equation for an asymmetric sandwich (Eq 20). The warhead can be split into three different sandwiches to calculate the velocities.

The first part would be made up of the steel tamper on the bottom, a Detasheet layer, and the upper part comprising a steel disk, Comp B, and the layers of fragments. From example 3, $C = 33$ g and $N = 1222$ g .

$$M = M_s + M_{Comp\ B} + M_{f1} + M_{f2} + M_{Detasheet}$$

where

M_s = steel disk = 2593 g

$M_{Comp\ B}$ = 1111.2 g

M_{f1} = fragment layer = 359.8 g

M_{f2} = uppermost fragment layer = 370.4 g

$M_{Detasheet}$ = 33 g

Therefore, $M = 4476.4$ g and consequently, $C/M = 0.0074$, and $N/M = 0.274$. From the tables in Ref 6, the velocity of the upper part is found to be 82 m/s. Again, the C/M falls outside the range of the Gurney equations, and the velocity obtained is approximate. Since no better estimate can be obtained, it is used for the purpose of this example.

The second part is made up of the steel disk, Comp B and two layers of fragments. From example 3, $C = 1111.2$ g , $N = 2593$ g, and $M = 763.2$ g . Therefore, $C/M = 1.46$, and $N/M = 3.4$. Again, from the tables in Ref 6, the velocity of the fragment layers is 2309 m/s. When the initial velocity imparted by the bottom layer of Detasheet is added to the fragments, the velocity is actually 2391 m/s.

The last part is made up of the two fragment layers and the Detasheet layer between them. For this section, C = 33 g, M = 370.4 g, and N = 359.8 g . Therefore, C/M = 0.089 and N/M = 0.971 . The velocity of the uppermost fragments is 497 m/s. For the bottom layer, it is 514 m/s, but in the opposite direction from that of the upper fragments. When the velocities that have already been imparted to the fragments are added, the upper layer has a velocity of 2888 m/s and the other layer has a velocity of 1877 m/s. These are the velocities of the fastest fragments; those at the edge of the projector would be affected by the end effects and would have lower velocities.

The initial velocity of 3000 m/s has not been reached; the best way to attain that velocity would be by increasing the amount of Comp B in the middle layer to 1435 g. This would give the top fragment layer a velocity of 2500 m/s and the required initial velocity would be reached when the other velocities are added. The design would have a length to diameter ratio of 1.03 (see Fig 24e).

These four conceptual designs were used to show how the modified Gurney method can be used in preliminary design. Experimental testing or computer simulation would be necessary before a final design would be accepted.

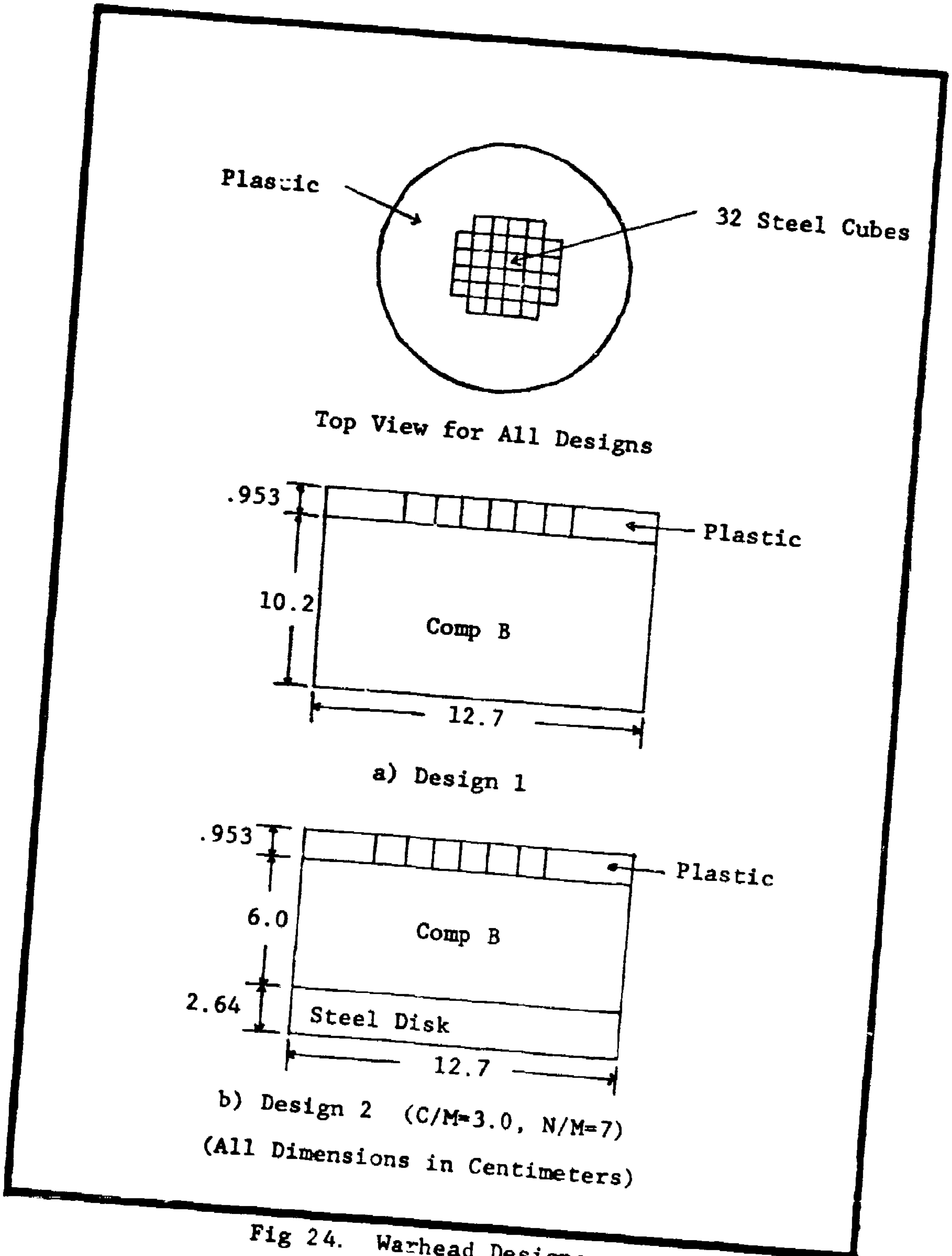


Fig 24. Warhead Designs

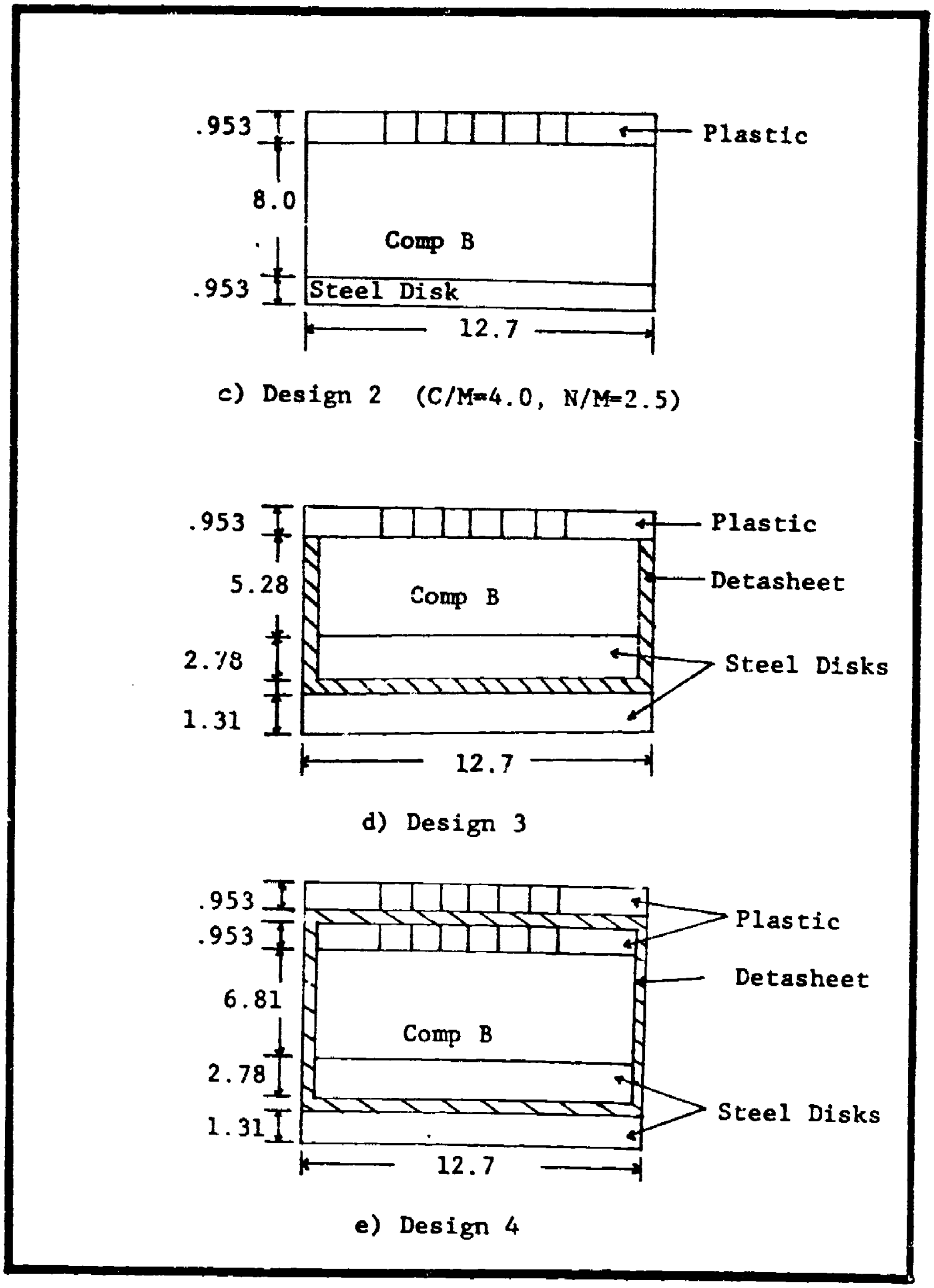


Fig 24 (Cont'd). Warhead Designs

VI Conclusions and Recommendations

It is possible to obtain more accurate estimates of fragment velocity distributions by using modified Gurney equations. Basing these modifications on available experimental data, it is concluded that:

1- the influence of end effects on cylindrical charges with L/D<2 can be modelled adequately using local C/M's which are assumed to be reduced from the actual C/M. It is assumed that a cone of explosive of diameter, D (obtained from the value of L/D), and of height, 2R, is removed from the initiating end and that a cone of diameter, D, and of height, R, is removed from the other end,

2- the influence of end effects on end projector type charges can be adequately modelled by assuming that the amount of explosive contained is a cone of diameter, D, and of height, R (obtained from the value of L/D),

3- for charges initiated at one end, the presence of pre-formed fragments can be modelled by computing the velocity distribution using a C/M which is 80 percent of the actual C/M,

4- for end projector type charges where the explosive is initiated simultaneously around the perimeter of the explosive, no reduction in the C/M is required to correct for the presence of pre-formed fragments,

5- and that use of these modified equations will yield reasonably accurate results for estimates of the velocity distribution. The modified Taylor equation also gives better estimates for fragment projection angles than the usual Taylor equation.

From the information gained in this project, it is recommended that:

1- more experimental data be obtained for a greater variety of L/D and C/M for cylindrical and end projector warheads. This data will serve to improve the accuracy of the modifications or cause them to be changed,

2- the modified Gurney equations be used for preliminary warhead design,

3- and that final design be based on experimental testing or computer simulation.

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Appendix A

Least Square Fit for Gurney Energy, $\sqrt{2E}$

Each explosive has a particular value of Gurney energy. Also, each explosive has its own detonation rate, dependent on density of packing and other factors. Since the energy appears to vary linearly with the detonation velocity, a least square fit (Ref 6) was made. The equation for the straight line is:

$$\sqrt{2E} = 0.52 + 0.28\ D$$

where D is the detonation velocity of the explosive.

The equation, along with experimental data from Ref 4 and Ref 10, are plotted in Fig 25. The graph abscissa starts at 6.6 mm/μs because explosives with lower detonation rates are of little military use. From this graph, an approximate value of the Gurney energy can be found for any explosive, given its detonation velocity.

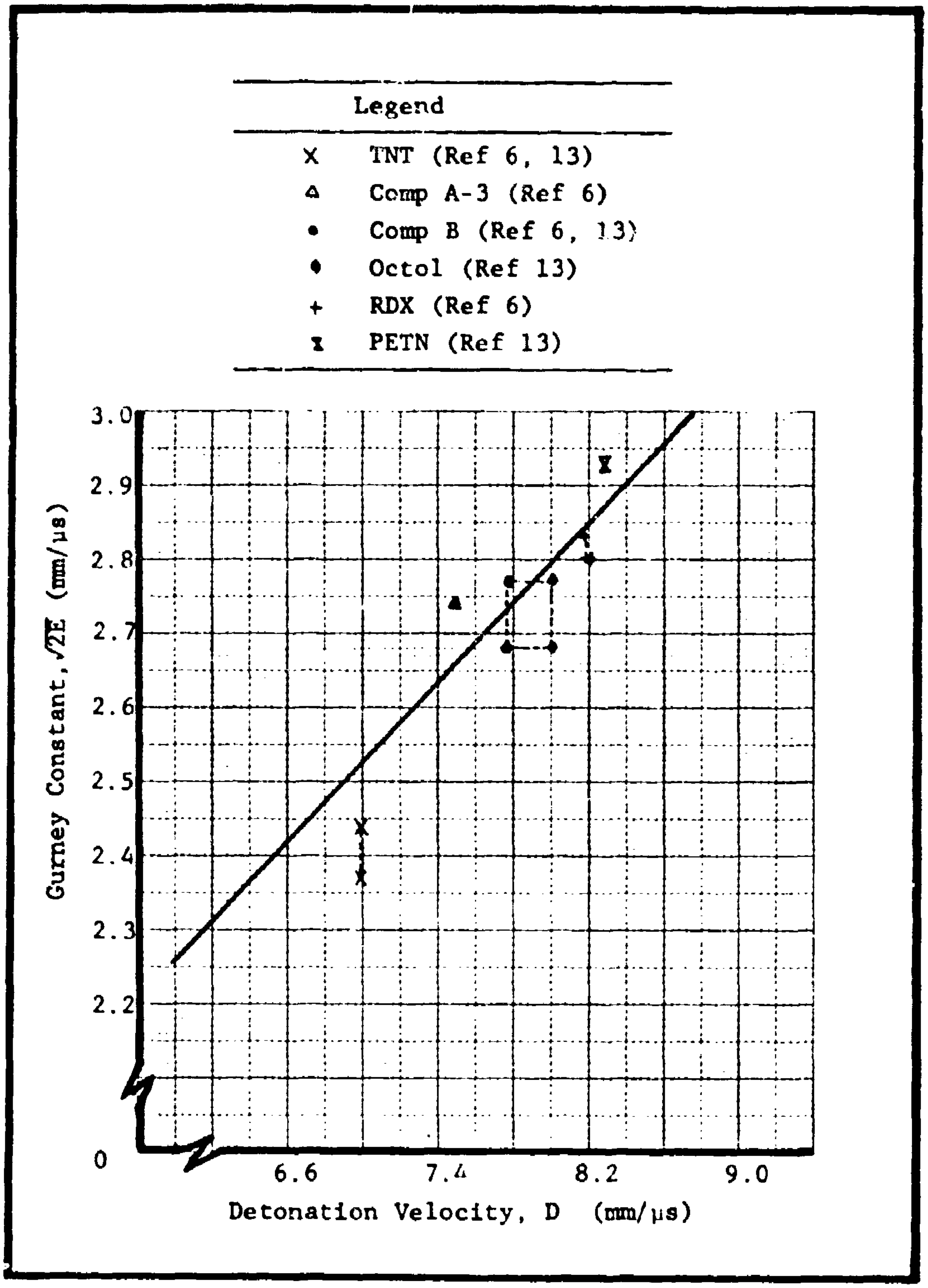

Fig 25. Detonation Rate vs Gurney Constant

Appendix B

Sample Calculation of Fragment Velocity for End Projector

For Shot 43 (Ref 2)

C-4:	Weight	= 284 g
	Detonation velocity	= 8.04 mm/μs (Ref 17)
	Gurney constant, $\sqrt{2E}$	= 2.79 mm/μs, from Fig 25
Detasheet:	Weight	= 54 g
	Detonation velocity	= 6.8 mm/μs (Ref 17)
	Gurney constant, $\sqrt{2E}$	= 2.44 mm/μs, from Fig 25

The system is considered as two asymmetric sandwiches. The first is comprised of the plywood disk, Detasheet, and all that is above (see Fig 26a). When the Detasheet detonates, it imparts a velocity to the plywood and the upper sandwich. This upper sandwich is made up of the steel disk, C-4, and 32 Aluminum cubes imbeded in a plastic ring. When the C-4 detonates, it imparts a velocity to the steel disk, and the plastic and fragments. The projector has a 12.7 cm diameter.

The amount of Detasheet, C_{D1}, in the first sandwich is found:

$$\begin{aligned}\text{total area of Detasheet} &= \text{area of disk} + \text{area around sides}\\ &= 126.71 + 76\\ &= 202.71\ \text{cm}^2\end{aligned}$$

$C_{D1} = 54 \times \frac{126.71}{202.71} = 33.75$ g = charge wt of Detasheet

The amount of Detasheet on the sides, C_{D2}, is 20.25 g.

The weight of the steel disk is found:

Density of steel, $\rho_s = 7.75$ g/cc (Ref 16)

$$M_s = \rho_s \, Vol_s = \rho_s \frac{\pi D^2 h}{4} = 624 \text{ g}$$

The mass of the Al fragments is $M_M = 32 \times 2.26 = 72.32$ g

Area of plastic $= \frac{\pi D^2}{4}$ - area of cubes $= 95.74 \text{ cm}^2$.

Density of plastic, $\rho_p = 1.74$ g/cc (Ref 2)

$$\therefore M_p = \rho_p \, Vol_p = 158.79 \text{ g}$$

Therefore, C/M for the first sandwich is

$$M = M_{D1,2} + M_s + M_{C-4} + M_p + M_M$$

$$= 1159.4 \text{ g}$$

$$C_D/M = 0.0291$$

Density of plywood, $\rho_{pl} = 513$ g/cc (Ref 16)

Weight of plywood, $M_{pl} = \rho_{pl} \, Vol_{pl} = 61.91$ g

Therefore, $N_{pl}/M = \frac{61.91}{1159.4} = 0.0534$

From the tables in Ref 6, the speed of the upper part is found to be

$$V_M = \sqrt{2E}\,(0.041) = 93 \text{ m/s}$$

For the upper sandwich (see Fig 26b)

$$C = C_{C-4} + C_{D2} = 304.25 \text{ g}$$

$$C/M_s = .488$$

Mass of plastic and fragments $= M_p + M_M$

$$= 231.11 \text{ g}$$

For this sandwich, $N = 231.11$ g and,

$$N/M_s = 0.3704$$

Again from the tables in Ref 6, the velocity of the fragments is

$$V_M = \sqrt{2E} \ (.816) = 2277 \text{ m/s}$$

But since the Detasheet gave it an additional velocity in the same direction, the total fragment velocity is

$$V_M = 2277 + 98 = 2375 \text{ m/s}$$

Similar calculations can be made for Shot No 66. The computations give an answer for V_M of 2870 m/s.

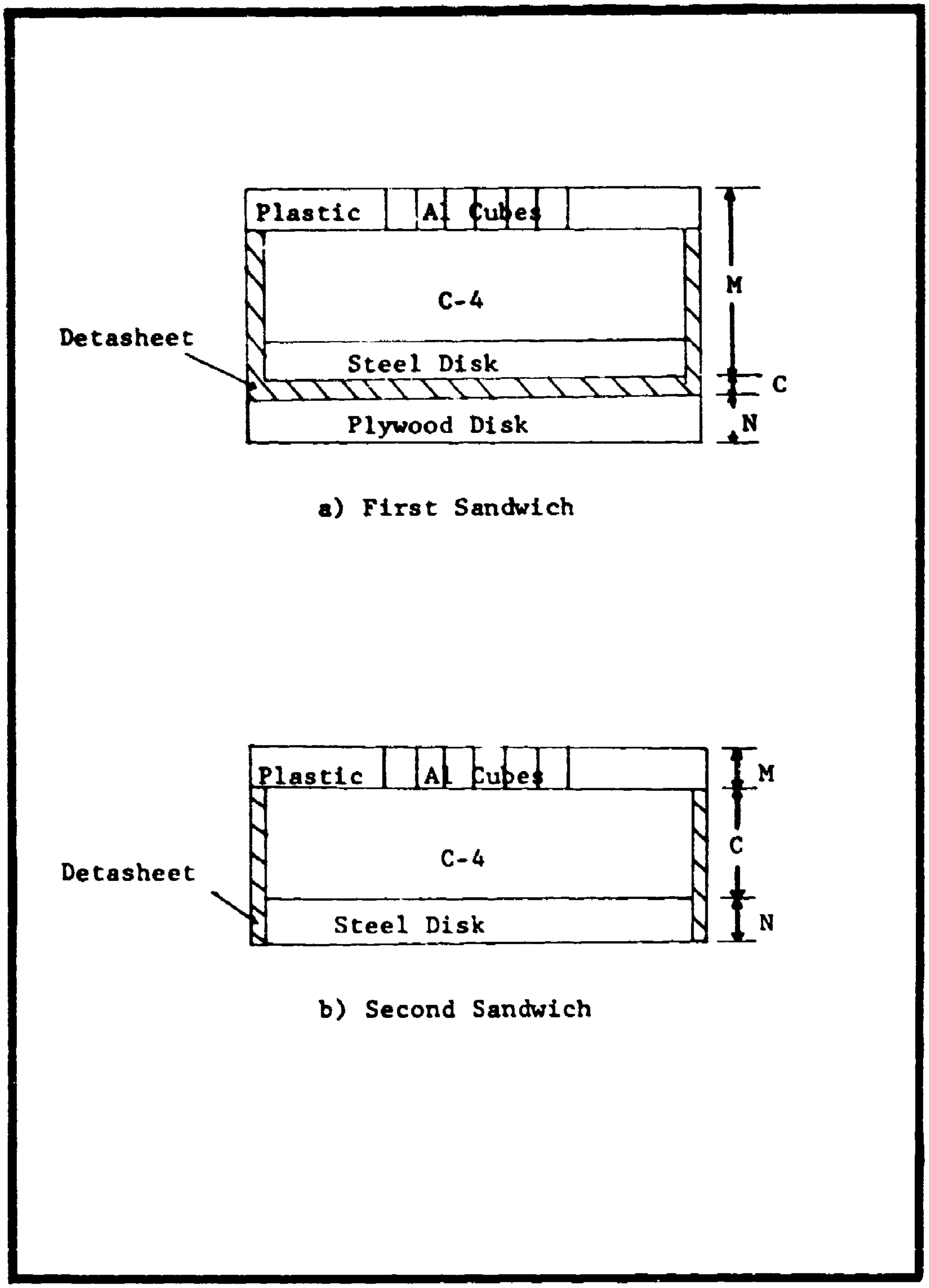

Fig 26. Cross Section of Asymmetric Sandwich

Appendix C

Computer Program for Estimation of Velocity Distribution

```
      [illegible] (INPUT,OUTPUT,PLOT)
C     [illegible] COMPUTES AN ESTIMATE OF THE FRAGMENT  VELOCITY
C     [illegible] BY THE ORIGINAL GURNEY FORMULA FOR A
C     [illegible] THE SECOND METHOD USES AN IMPROVED GURNEY
C     [illegible]

      [illegible] VMOD(20), VL(20),RA(25),TA(25)
      [illegible],F13.3,7X,F10.3,7X,F10.3)
      [illegible] OF MODIFIED AND STANDARD GURNEY AND TAYLO
      [illegible] FILLED WITH OCT(L)
      [illegible] X,17HC/M=,F .E3=.24()
      [illegible] LENGTH,X,[illegible],12HMOD INIT VEL,X,
      [illegible] AND)
      [illegible] VELOCITY=,F .2,1X,5HMM/ S,7X,
      [illegible] ,2,1X,5HMM/ S)
      [illegible] ,1X,5HMM/ S,X,7HDEG,9X,3HDEG)

C     [illegible] CLENGTH TIME CONSTANT
C     [illegible] OF THE EXPLOSIVE.

      [illegible],TAU,RETVEL

      [illegible] / L,CM2
      [illegible]
      XL(I)=[illegible] -1)/2.
      X(I)= XL(I) [illegible]

C     [illegible] THE LOCAL C/M TO REFLECT THE END EFFECTS.
      [illegible]=(1.-[illegible](X(I)/(2.*R),1.-(CL-X(I))/R))**2
```

```
C     V=[illegible] BY GURNEY FORMULA; VMOD=VELOCITY BY IMPROVED
C     GURNEY FORMULA.
C
      V(I)=CM [illegible] ((2.[illegible])/(CM + 2.))**.5
      VMOD(I)=[illegible] ((2. [illegible])/(FN*CM + 2.))**.5
2     CONTINUE
C
C
C     THE ANGLE OF [illegible] OF THE FRAGMENTS IS CALCULATED WITH TAYLOR S
C     [illegible] USING THE VELOCITY OBTAINED FROM THE MODIFIED
C     GURNEY [illegible] BY THE IMPROVED FORMULA BASED ON THE TAYLOR
C     FORMULA, [illegible] EMPIRICALLY (24).
C
      DO 3 I=2,[illegible]
      ALPHA=[illegible](.,Y(I))
      VEL=[illegible]/[illegible](ALPHA))
      ANG=VMOD(I)/(2.*VEL)
      TA(I)=[illegible](ANG)[illegible].3
      VPRIME=(VMOD(I+1)-VMOD(I-1))/(X(I+1)-X(I-1))
      SUM=[illegible] - .[illegible](VPRIME*TAU) - .2*(VPRIME*TAU)**2
      IF (ABS(SUM).GT.1.) SUM=SIGN(1.,SUM)
      [illegible](I)=[illegible](SUM)[illegible].3
3     CONTINUE
      [illegible]
      PRINT [illegible]
C
C     THE [illegible] OUT AND PLOTTED WITH EXPERIMENTAL DATA
C     [illegible] INCLUDED WITH THIS PROGRAM.
C
      DO 10 I=[illegible],2[illegible]
      PRINT [illegible], XL(I), V(I), VMOD(I) ,TA(I), FA(I)
10    CONTINUE
      STOP
      END
```

Appendix D

Conversion Factors

cm	x .3937	=	inch
g	x .00022	=	pound (av.)
g/cc	x .03613	=	lb/cu inch
mm/μs	x 1000	=	m /sec
m/s	x 3.281	=	feet/sec

Vita

Joseph Yves Charron was born on 26 August 1947 in Hull, Province of Quebec, Canada. In September 1966, he was accepted to the Royal Military College of Canada, Kingston, Ontario. He graduated in May 1970 with the Degree of Mechanical Engineering and was granted a commission in the Canadian Armed Forces. He entered the Graduate Aeronautical Study Program at the Air Force Institute of Technology, Wright-Patterson AFB, OH, in January 1978.

Permanent address: DPCO/AERE
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The Gurney energy method, an analytical method with which to compute fragment velocity distributions for continuous casing warheads, is reviewed and a method to model velocity losses due to end effects on cylindrical charges and end projector type warheads is presented. The Taylor formula, which estimates the angle of projection of the fragments, and a modified Taylor formula, which gives

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) cont. better estimates, are also presented. The method is extended for use with casings made of pre-formed fragments. Finally, four conceptual preliminary designs, based on information acquired in the project, are investigated.

The modifications made to the equations provide improved results and the examples confirm that the modified Gurney method is a quick, inexpensive tool for use in preliminary warhead design.