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SOME THEORETICAL PREDICTIONS AND TEST DATA

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ABSTRACT
Predictions of a new cumulative damage theory established by Hashin and Rotem (HR) are compared with an extensive series of test data for two-level shear strain cycling and a double linear exponential damage rule, all given by Miller and Zachariah (MZ), demonstrating good agreement. While MZ requires determination of parameters by fit to the two level test data the only testing parameter needed for HR is the fatigue lifetime $N_e$ beyond which the fatigue limit occurs. This parameter has here been estimated on the basis of recent advances in understanding cyclic deformation.

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INTRODUCTION

The problem of fatigue lifetime prediction under complicated cyclic loading programs has become known in the fatigue literature as the cumulative damage problem. A new approach to this problem based on the concept of damage curve families and an equivalent loading postulate has been given in [1] with a sequel [2] investigating the significance of the Palmgren-Miner assumption within the frame of the new theory.

The method of analysis established in [1] is of purely phenomenological nature in that all of the information necessary to determine fatigue lifetime is given by fatigue test data of specimens. The problem of fatigue failure under two level loading has recently been considered in detail by Miller and Zachariah [3]. Since a typical specimen spends its fatigue life partially in the crack initiation stage and partially in the crack propagation stage they modeled damage (crack length) accumulation laws in each stage by different exponential laws. The unknown coefficients and exponents in these exponential laws as well as the interface between initiation and propagation stages were determined on the basis of an extensive program of testing.

It is recalled that a related approach has been given by Manson et al [4] which led to the so-called "double linear damage rule".

It is the purpose of the present note to compare the quality of prediction of fatigue life under two level loadings according to the treatments [1] and [3].

THEORY AND TEST DATA

It has been shown in [1] that if:
(1) The material has a fatigue limit defined by stress \( s_e \) and lifetime \( N_e \), fig. 1

(2) The damage curves are straight in log-log or semi-log S-N space and all converge into the fatigue limit, fig. 1

then lifetime in two level cyclic loading \( s_1 \) for \( n_1 \) cycles and \( s_2 \) for \( n_2 \) cycles is given by

\[
\frac{\log(N_2/N_e)}{\log(N_1/N_e)} = \frac{n_1}{n_2} = 1
\]

(1)

here \( N_1 \) and \( N_2 \) are lifetimes for constant \( s_1 \) and \( s_2 \) amplitude cycling respectively as given by the S-N curve. Note that eq. (1) is very versatile in that it applies to all S-N curves and damage curve families which become straight lines in s-log N or in log s-log N space. Also it should be noted that everything applies as well to strain cycling; thus \( s \) should be interpreted as either nondimensional stress or strain.

The lifetime parameter \( N_e \) is easily determined from the S-N curve if it exhibits a clearly discernible break at \( N_e \). This is the case for stress or strain [5] S-N curves of steel and although it has not yet been demonstrated for aluminum alloys and other classes of commercial alloys there is strong evidence that it could exist [5]. However, to uncover the break in the S-N curves for plastic strain cycling it is necessary to perform cycling at very small plastic strain amplitudes. In tests conducted in [3] the plastic strain amplitude was not carried to sufficiently small values to uncover
the break and thus $N_e$. Figure 2 reproduced from [3] which shows the S-N curve for plastic strain in log-log coordinates demonstrates this. It is seen that the S-N curve can be well approximated by a straight line and thus eq. (1) should be applicable; however $N_e$ cannot be read from the curve.

Alternatively, the problem of $N_e$ determination can be considered on the basis of the cyclic stress-strain response of the material [5], and the nature of the crack nucleation mechanism in low strain fatigue. It is well known that, in this regime, cracks nucleate in persistent slip bands (PSB) and therefore, if PSB's are absent, there can be no mechanism for fracture. PSB formation is associated with the plateau in the cyclic stress-strain curve of f.c.c. crystals [6]. However, it was not fully accepted or widely understood that there is a strain below which PSBs are absent until Mughrabi conducted his preliminary tests on copper single crystals at very low strains, [5]. Mughrabi, having completed his investigation, concluded that a plastic shear strain amplitude of $6 \times 10^{-5}$ was required to produce a single PSB, [7]. Such findings encouraged Laird [5] to correlate previous reports in the literature for evidence of a fatigue limit in a much wider range of materials than previously considered acceptable. He pointed out that the long-life Coffin-Manson plots by Lukáš and Klesnil [8] clearly supported the existence of a fatigue limit (the "knee" in the S-N curve) in copper and Cu-Zn alloy at a life of $\sim 6 \times 10^6$ cycles, and at a strain amplitude consistent with the PSB threshold. Moreover, a Coffin-Manson plot for a carbon steel with Czech designation 12060 showed a fatigue limit at $3 \times 10^6$ [5]. While the PSB threshold is very precisely established in f.c.c. metals, the phenomena are more complicated in b.c.c. metals because, depending on the strain
rate [9], the cyclic response can be similar to that in f.c.c. metals, or different. In the latter conditions, the mobility difference between edge and screw dislocations is critical [10]. Whilst admitting that more work is needed to explore the subtleties, Mughrabi recently suggested that the strain fatigue limit at ambient temperature in b.c.c. metal single crystals lay in the range $5 \times 10^{-5} < \text{plastic strain amplitude} < 2 \times 10^{-4}$ [11]. Miller and Zachariah [3] provide in fig. 2 a plot of plastic shear strain range versus life. To relate their strains to Mughrabi's monocrystalline limits, it is necessary to multiply the limit by two to account for amplitude, and also by the Taylor factor [10] to account for the polycrystallinity of the steel used by Miller and Zachariah. The life corresponding to the upper limit is $5 \times 10^6$ cycles, that corresponding to the lower is $7 \times 10^7$. Miller and Zachariah attempted to break two specimens in this range but both were 'run-outs'. This suggests that the fatigue limit for their steel is near the upper limit suggested by Mughrabi. Accordingly we choose for $N_e$ a value between 5 and $8 \times 10^6$ cycles.

Miller and Zachariah's [3] test data were obtained by torsional cyclic straining of thin walled steel cylinders. Figures 3-6 summarize their data for failure under two level strain cycling. In each case the indices L and H indicate the lower and higher levels, respectively, in the two level cycling while L-H and H-L indicate the sequence of levels. The symbols $n_L, n_H$ indicate the number of cycles which the specimen spent in low and high plastic strain levels, respectively, while $N_L$ and $N_H$ refer to lifetimes under constant strain cycling from the S-N curve, fig. 2, at the same strain levels.

The dashed curves in figs. 3-6 represent the double cumulative damage rule developed in [3]. The underlying idea is that a fatigue
crack grows at different rates in the initiation and propagation stages and that each of these growth rates can be modeled by an exponential law. As noted above, the values of the necessary parameters entering into the growth laws have been determined in [3] by best fit to the test data thus obtaining the dashed curves.

It should be noted that the blackened triangle and circle test data symbols in the figures represent tests with intermediate annealing.

Also shown in figs. 3-6 are the predictions of eq. (1) with 3 different values of $N_e$ within the range $5 \times 10^6 - 8 \times 10^6$ established above. It is seen that there is quite good agreement between these curves and the curves of [3]. Unlike the approach of Miller and Zachariah, however, the only experimental parameter entering into (1) is the value of $N_e$ for constant amplitude cycling.

CONCLUSION

Miller and Zachariah [3] approach the problem of cumulative damage by attempting to grapple realistically with the failure mechanisms involved. Since these are very complex, evaluating the mechanisms and kinetics of crack initiation and propagation in a variable loading experiment, even one as simple as that of a two-step test, is difficult and tedious. Hashin and Rotem [1] have avoided this difficulty by a purely phenomenological approach, and depend only on the definition of a fatigue limit which can now be estimated for most classes of materials from a knowledge of their behavior in cyclic deformation. In spite of the differences between these approaches, their cumulative damage predictions are concluded to be equally good within the scatter of the limited experimental data available.
REFERENCES


Figure 1 - Linear Damage Curves. Semi-log.
Fig. 2 S-N curve for plastic strain (after [3])
Fig. 3 Fractional lifetimes for two level cycling

$N_H = 720 \quad N_L = 16,000$
Fig. 4 Fractional lifetimes for two level cycling

\[ N_H = 1000 \quad N_L = 200,000 \]
Fig. 5 Fractional lifetimes for two level cycling
\[ N_H = 1000 \quad N_L = 400,000 \]
Fig. 6 Fractional lifetimes for two level cycling

\[ N_H = 900 \quad N_L = 700,000 \]