MODELING OF ROOM RESPONSE TO AIR BLASTS

Final Report

by

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**Title:** Modeling of Room Response to Air Blasts

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**Abstract:**
A relatively simple model has been developed for the prediction of pressure response within a one room structure subjected to air blast. The model is based on an acoustic analysis by Vaidya and a room fill model developed by Rempel. These models have been combined, together with a model for the entering diffracted shock, to produce a model which can predict the pressure-time response at any spatial location within the room. Good agreement is
obtained when the model predictions are compared with measurements obtained from full scale field tests and scaled shock tube tests.
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I. INTRODUCTION

Prediction of the structural damage sustained by buildings and structures subjected to air blast requires a detailed knowledge of the pressure history both inside and outside the structure. Previous models for the pressure response inside a structure have relied on empirical pressure histories outside the structure and a simple one-dimensional flow model for the pressure within the structure\(^{(1)}\). This model has been relatively successful at predicting the general rise and decay of pressure within the structure, but fails to adequately predict the high frequency pressure response. Further, the model is not capable of predicting the pressure at different locations within the room since it assumes that the pressure within the room is uniform. These limitations are particularly noticeable during the early time response of the room and at the back wall for rooms with relatively large openings.

A new model has been developed which removes these limitations. It is based on a combination of an acoustic model developed by Vaidya\(^{(2)}\) for the response of a room subjected to a sonic boom, the room fill model of Rempel\(^{(1)}\) and a model for the entering diffracted shock.

The new model gives considerably improved predictions for the early time response and is capable of predicting the pressure response at any location within the room. The development of the analytical model is given in Section II and the model predictions are compared with both field and shock tube measurements for a one room structure in Section III.

II. ANALYTICAL MODEL

The pressure in the room is calculated using two separate models,
each of which produces reasonable results in different time regimes. The two models were then combined using a physical argument for the time regime in which each model was dominant.

The early time behavior was calculated from an acoustic model developed by Vaidya (2) with the addition of the effect produced by an entering diffracted shock. The response at later times was calculated using a one-dimensional flow model developed by Rempel (1). The predictions for each of these time regimes are then "blended" together using a rise time equal to the period of the natural (Helmholtz resonator) frequency for the room.

A. Acoustic Model

The acoustic model is that developed by Vaidya (2) for the response of rooms subjected to sonic booms. The success of this model lies in its ability to properly treat the multiple reflection of waves within the room; the process which dominates the early time characteristics of the room response. For the moderate overpressures which exist in the far field of a blast, the variation in the speed of sound is small and the acoustic model provides a reasonable approximation for blast behavior.

Vaidya presents an exhaustive analysis of the problem using several methods. We will present only a brief discussion of the Green function formulation used in this study. The reader is cautioned that an unusual number of typographical errors are contained in the original papers.

The theory is developed by matching the velocity and velocity potential in the opening into a closed room. The velocity and the velocity
potential due to an incident wave of constant frequency $\omega$ is determined from the Green function for an infinite half space. This is matched in the opening to the velocity and potential determined from the Green function for the interior of the room. Vaidya assumes that the incident wave of amplitude $A$ is "block reflected" from an infinite front wall which leads to an input amplitude of $2A$. For a front wall of finite size with a relatively small clearing time, it is more appropriate to use $A$ as the input amplitude as found experimentally, both in this study and that of Vaidya.

With these modifications the potential inside the room generated by an incident wave $A e^{-i(\omega t - kz)}$ is given by

$$
\phi = A \sum_{m,n} \sigma_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \frac{\cos [k^*_m (d - z)]}{\cos (k^*_m d) - Q_{mn} \sin (k^*_m d)}
$$

(1)

where the normalizing parameters $Q_{mn}$ satisfy the condition

$$
\sum_{m,n} \sigma_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} = 1 \quad \text{for} \ x, y \ \text{in the opening}
$$

$$
= 0 \quad \text{otherwise}
$$

(2)

The reflection factor for the window, $Q_{mn}$ is given by
\[ Q_{mn} = \frac{k^*_\text{mn} ab}{S^2} (2 - \delta_{0,m}) (2 - \delta_{0,n}) \iint_{S_{x',y'}} \iint_{S_{x,y}} G(x,y,0|x',y',0) \]

\[ x \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, dx' \, dy' \, dx \, dy \]

(3)

where \( G(x,y,z|x',y',z') \) is the Green function for the infinite half-space \( z < 0 \). In the above equations \( k^*_\text{mn} \) is the cut off wave number given by

\[ k^*_\text{mn}^2 = k^2 - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2 \]

(4)

\( S \) is the area of the opening, \( a \), \( b \), and \( d \) are the width, height, and depth of the room in the \( x \), \( y \), and \( z \) directions respectively, \( \delta_{ij} \) is the Dirac delta function and the primed quantities are those within the opening. The integral of the Green function in Equation 3 can be related to the mobility \( M \) of the opening to give

\[ Q_{mn} = \frac{k^*_\text{mn} (ab)^3}{S^2} \left( \frac{ik}{2\pi} \right) \left( \frac{\sigma_{\text{mn}}^2}{(2 - \delta_{0,m}) (2 - \delta_{0,n})} \right) \]

(5)

It has been shown by Rayleigh\(^{(3)}\) that the mobility of a circular orifice is equal to its diameter. For this study, we will assume the mobility is equal to the diameter of a circle with the same area as the actual opening.
The time domain response of the room excited by a transient signal is determined from the steady-state solution using Laplace transform theory. The pressure field generated inside the room by an incoming wave \( p_i = Ae^{-i\omega t} \) is given by

\[
P = Ae^{-i\omega t} \sum_{m,n} F_{m,n}(\omega, z) \cos \frac{mnx}{a} \cos \frac{mny}{b}
\]

where

\[
F_{m,n}(\omega, z) = \sigma_{mn} \frac{\cos \left[ k_{mn} (d - z) \right]}{\cos k_{mn} d - Q_{mn} \sin k_{mn} d}
\]

The pressure field generated inside the room due to an incoming impulse, \( p_i = \delta(t) \) is

\[
P_\delta = \sum_{m,n} \cos \frac{mnx}{a} \cos \frac{mny}{b} \mathcal{L}^{-1} \left[ \overline{F}_{mn}(s,z) \right]
\]

where \( \mathcal{L}^{-1} \) is the inverse Laplace transform and the function \( \overline{F}_{mn}(s,z) \) is obtained by the substitution \( s = i\omega \) into \( F_{mn} \). For an incoming wave of arbitrary shape \( p_i = I(t) \) the pressure inside the room is given by

\[
P(x,y,z,t) = \sum_{m,n} \cos \frac{mnx}{a} \cos \frac{mny}{b} \int_{-\infty}^{t} S_{mn}(\tau,z) I(t-\tau) \, d\tau
\]
where

\[ S_{mn} = L^{-1} \left[ \hat{F}_{mn}(s,z) \right], \quad (10) \]

Vaidya has performed the transform for the axial \((m = n = 0)\) mode and finds it can be written as the sum of two terms

\[ S_{00}(t,z) = \sum_n S_{00}^{n,1} + \sum_n S_{00}^{n,2} \quad (11) \]

where \(n\) is an index which represents the number of reflections the wave has undergone. The superscript 1 represents the waves propagating from the window into the room and 2 represents the waves reflected from the back wall. The appropriate timing of these waves is determined by Heaviside step functions as follows

\[ S_{00}^{n,1} = \sigma_{00} \psi_n (t - \frac{2nd + z}{c}) H (t - \frac{2nd + z}{c}) \quad (12a) \]

\[ S_{00}^{n,2} = \sigma_{00} \psi_n (t - \frac{(2n+2)d - z}{c}) H (t - \frac{(2n+2)d - z}{c}) \quad (12b) \]

The progress of the wave is readily traced. At the point \(z\) the first effect occurs at the time \(t = z/c\) \((n = 0)\). The first reflected wave from the back wall occurs when \(t = (2d-z)/c\) and so on. The term \(\psi_n\) is a distorted form of \(\delta(t - z/c)\); distorted due to the passage of the
incident wave through the window.

The higher order modes exhibit a similar behavior. Their form has been derived by Vaidya who gives approximations for their computation.

To calculate the time domain response to an arbitrary input \( I(t) \), the input is first approximated by a sequence of straight line segments. The contribution due to each segment is calculated using Equation 9 with the appropriate time delay and then summed to give the total room response.

B. Shock Diffraction Model

The acoustic model given above determines the pressure response of a room due to an arbitrary input. The response is characterized by a finite risetime and, therefore, is incapable of treating the effect of the sharp shock which diffracts through the opening. The effect of the entering shock is to produce a sharp spike of pressure within the room whose amplitude is approximately that of the incident blast wave and whose duration is characteristic of the time required for a sound wave to propagate across the opening.

This sharp pressure pulse also undergoes multiple reflection within the room analogous to the fundamental acoustic modes given in Equation 12. Upon each reflection the pulse will suffer a loss in amplitude due to absorption losses at the wall. The contribution to the total pressure within the room at a particular axial location and time is given by

\[
P_g(z, t) = A \sum (-1)^n \left\{ \alpha^n \exp \left[ -\left( t - \frac{z}{c} \right) \right] H\left( t - \frac{z}{c} \right) + \alpha^{n+1} \exp \left[ -\left( t - \frac{z}{c} \right) \right] H\left( t - \frac{z}{c} \right) \right\}
\]  

(13)
where $A$ is the peak amplitude of the incident blast wave, $\alpha$ is the reflection coefficient, $T$ is the time required for a sound wave to propagate across the opening, $H$ is the Heaviside step function and $z$ is defined by

$$z_1^n = 2n \eta d + z \quad (14a)$$

$$z_2^n = (2n+2) \eta d + z \quad (14b)$$

C. **The Room-Fill Model**

A room fill model which ignores the early-time wave reflection contribution was given by Rempel (1). This model gives good results for times after the initial waves have damped.

This model consists of a step-by-step calculation of the flow into or out of the room based on the principles of isentropic flow in ducts. The calculation is carried out over small successive time intervals. Conditions computed for the end of one time step become the initial conditions for the following step. During the time step it is assumed that conditions both inside and outside the room are constant. The rate of rise of the pressure inside the room is calculated from conservation of energy and momentum using the isentropic equation of state. Rempel gives the rate of rise as

$$\frac{dp_3}{dt} = \frac{\gamma}{\gamma - 1} \left( \frac{p_1}{\rho_1} \right)^{1/2} \left( \frac{p_3}{\rho_3} \right)^{1/\gamma} \left[ 1 - \frac{\rho_3}{\rho_1} \right]^{\gamma - 1} \frac{S}{V_3} \quad (15)$$
where the subscript 1 refers to conditions outside the room and 3 refers to conditions inside the room. Primed quantities refer to values at the beginning of the time step. In Equation 15, $\gamma$ is the ratio of specific heats, $P$ is pressure, $\rho$ is density, $S$ is the area of the opening, $V$ is the volume of the room and $K$ is a discharge coefficient for the opening; taken by Rempel as 0.7.

D. Blending of the Acoustic, Shock and Fill Models

The pressure inside the room as calculated from the acoustic model exhibits large amplitude oscillations of approximately double the peak input pressure. These oscillations occur as a result of the superposition of the waves reflecting back and forth within the room. After the passage of the blast wave, the amplitude decays slowly due to radiation from the opening. The behavior of the diffracted shock is similar to the fundamental mode of the acoustic response. In this model, the walls are assumed rigid and no damping occurs on reflection. In the actual case the damping of the pressure oscillations occurs much more rapidly, not only because of the damping due to reflection at a non-rigid wall, but also because the higher order (non-axial) modes set up waves which propagate back and forth in directions perpendicular to the incident axial wave. These processes tend to "homogenize" the sound energy within the room, damping the orderly oscillations after a period of time. When the sound energy inside the room has become uniform, the room behaves more like a Helmholtz resonator. For blast waves whose pulse length is long compared to the period of the fundamental Helmholtz frequency, the late time behavior of the pressure in the room essentially
follows the outside pressure with a small time delay as predicted by the room-fill model.

To combine the early-time behavior of the acoustic and diffracted shock with the late-time behavior of the room-fill model the pressures predicted by each were blended according to

\[ P = P_{AS} \exp \left[ -(t-z/c)/\tau_H \right] + P_{RF} \left\{ 1 - \exp \left[ -(t-z/c)/\tau_H \right] \right\} \]  

(16)

where \( P_{AS} \) is the pressure predicted by the sum of the acoustic and shock models, \( P_{RF} \) is the pressure predicted by the room-fill model and \( \tau_H \) is the period of the fundamental Helmholtz frequency for the room given by

\[ \tau_H = \frac{2\pi}{c} \sqrt{\frac{V}{M}} \]  

(17)

where \( V \) is the room volume, \( M \) is the mobility of the opening and \( c \) is the speed of sound.

From Equation 16 it is seen that at the point \( z \) the solution is the sum of the acoustic and shock contributions at \( t = z/c \) and reduces to the pure room-fill model as \( t \) approaches infinity. The acoustic and shock model contributions decay exponentially with time constant \( \tau_H \) and the room-fill contribution increases from zero to its full value with a rise time of \( \tau_H \).

The combined model has been used to predict the pressure inside...
a full-scale room with a single opening subjected to the blast from an HE explosion and to a 1/24 scale model tested in a shock tube. The results are discussed in the next section.

III. RESULTS

The analysis described in the previous section was used to predict the pressure-time history in a one-room concrete instrumentation bunker with an open door which was tested during Operation Prairie Flat (4). The instrumentation room was subjected to a blast wave with a peak overpressure of approximately 5 psi which was produced by the detonation of 500 tons of TNT.

The test structure was a concrete bunker of 2.44 m height, 2.44 m width and 3.66 m depth interior dimensions. The open door, 0.88 m wide and 2.44 m high was located in the lower left corner of the 2.44 m x 2.44 m wall which faced the blast.

Three transducers were placed in the bunker to record the pressure in the room. The transducer designated position 1 was located in the floor 1.22 m behind the door measured from the inside wall and 0.495 m from the left wall. Position 2 was located in the floor 2.44 m behind the door and 0.495 m from the left wall. The transducer at position 3 was located in the rear wall 1.207 m above the floor and 0.495 m from the left wall.

The measured free-field blast wave was approximated by six straight line segments for input into the computational model. The peak overpressure was 5.12 psi and total duration of the positive phase was approximately 300 msec.
The six segment representation of the measured free-field blast wave is shown in Figure 1. The response of the room at position 1 calculated using only the room fill model of Rempel is shown in Figure 2. For the relatively large opening used in this example, the predicted pressure simply rises to the free-field value in approximately 25 msec, then follows the decay of the free-field pressure. The measured response of the room at position 1 is shown in Figure 3. The peak pressure predicted by the room fill model is 23% lower than the measured value and does not exhibit either the high frequency structure or the longer oscillations resulting in the relatively low pressure observed at approximately 60 msec.

The predicted response calculated from the present model without the contribution due to the diffracted shock is shown in Figure 4. The predicted peak pressure is now 14% greater than the measured value, but the characteristic structure of the response is reproduced, including the decrease in pressure at approximately 60 msec. Note that neither the room-fill nor the acoustic models are capable of predicting the sharp initial spike seen in the measured response. The predicted response due to the complete model, including the diffracted shock is shown in Figure 5. In this case the sharp initial rise is predicted by the model, and several sharp spikes are introduced into the response due to the multiple reflections of the diffracted shock.

It should be noted that the model which was developed requires no empirical input, only the physical dimensions of the room and the free-field pressure is required. To illustrate the range of applicability of the model, the pressure response was calculated for a 1/24 scale model of the same
structure used in the field tests, which was subjected to a blast generated in a shock tube. The results are shown in Figure 6. Note that in this computation, the contribution due to the entering shock was omitted. The agreement between the predicted and measured response is again quite good, particularly in light of the considerable difference in input wave form and scale compared to the field test.

The model which has been developed is capable of providing considerably improved predictions of the pressure response inside a one room structure subjected to air blast, when compared to the predictions of the room fill model. In addition to predicting the high frequency response, it is capable of predicting the pressure at any spatial location within the room. This is of particular importance at the back wall where the wave reflection produces an increase in internal pressure. This can be seen clearly in Figure 7. Also, the pressure on the side walls may rise considerably slower than indicated by Figures 3 or 7 but, unfortunately, no data exists to verify this prediction of the model.
REFERENCES


Figure 1. Six linear segment approximation to the measured free-field blast wave input to the one room structure tested during Operation Prairie Flat (Reference 4).
Predicted pressure response at gage position 1 for the one room structure tested during Operation Prairie Flat (Reference 6), using the model developed herein, but neglecting the effect of the diffracted shock.

Figure 4.
Figure 5. Predicted pressure response at gage position 1 for the one room structure tested during Operation Prairie Flat (Reference 4) using the complete model. Note the initial sharp rise produced by the entering shock.
Experiment: Shock Tube
Probe location: Position 1

Figure 6. Pressure–time history inside a 1/24 scale model of the one room structure tested in a shock tube (Reference 4). The heavy line is the model prediction, the light line with structure is the measured response, and the nearly horizontal light line is the input shock wave. The effect of the entering shock was not included in this prediction.
Figure 7. Pressure response at a position on the back wall of the one room structure tested during Operation Prairie Flat (Reference 4). The heavy line is the predicted response neglecting the entering shock and the light line is the measured response. Note the higher pressures produced upon reflection at the wall.