The number of test samples used to characterize the crack propagation parameter N of glass and ceramic materials determines the confidence in failure predictions for these materials. A Monte Carlo simulation technique was used to study the effect of sample size on the statistical reproducibility of the fatigue constant N from dynamic fatigue data. It is shown that the statistical variability in N can be large especially for sample sizes of less than 100. It is recommended that before meaningful conclusion can (continued)
be drawn regarding the effect of a test variable on \( N \) as measured from the dynamic fatigue experiment, the statistical reproducibility of \( N \) should be evaluated.

The fatigue behavior of optical glass fibers was found to be strongly dependent on humidity with the fatigue resistance decreasing with an increase in humidity. The preform used in the production of glass fibers was found to affect the fatigue behavior. Fibers made from a natural quartz preform had significantly poorer fatigue resistance than fibers made from synthetic quartz preforms. The fatigue behavior determined by the dynamic and static fatigue experiments were in agreement, indicating that the accelerated dynamic fatigue test can be used to estimate fatigue lifetimes under static loading. Proof tests results showed that when slow unloading rates were used in the proof test, the after-proof strength distribution was significantly weaker than the initial distribution and could not be predicted from fracture mechanics theory. These results emphasize the importance of having good proof test controls.
LIFETIME PREDICTIONS FOR OPTICAL GLASS FIBERS

by
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by
Mechanical Engineering Department
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Amherst, MA 01003
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FOREWORD

This report describes the results of an experimental program oriented toward a better understanding of lifetime predictions for optical glass fibers. Some of the progress made toward this goal is summarized in the following two technical papers comprising this report.

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</table>
The dynamic fatigue experiment, where strength is measured as a function of stressing rate, is becoming an increasingly popular test for measuring the crack propagation parameter $N$ for ceramic materials.\textsuperscript{1-11} Data from the dynamic fatigue test are fitted to the equation:

$$S_m = C \sigma^{1/(N+1)}$$

where $S_m$ is the median strength, $\sigma$ is the stressing rate, and $N$ and $C$ are constants describing fatigue susceptibility. $N$ is determined from Eq. (1) by the least squares fit of the data. Figure 1 shows dynamic fatigue data for soda-lime glass in water where the median strength was measured at 7 stressing rates ranging from 0.01 to 53 MPa/s with 50 samples per stressing rate. From this data $N$ was found to be 18.4.

Somewhat surprisingly, dynamic fatigue data for a given material tested under similar conditions result in a relatively large range of values for $N$. For example, $N$ from 13 to 19 have been reported for soda-lime glass in a moist environment,\textsuperscript{1,4,6,7,12} 15 to 25 for optical glass fibers in a moist environment,\textsuperscript{8} and 35 to 68 for alumina in an aqueous environment.\textsuperscript{9-11} The present note reports some results on the statistical reproducibility of $N$ as determined from dynamic fatigue experiments.
Dynamic fatigue tests on an "ideal" material were simulated on a computer using Monte Carlo technique. It was assumed that samples of this ideal material had a Weibull inert strength distribution whose slope and scale parameters, $m$ and $S_0$ are known. The fatigue parameters, $N$ and $B$ were also assumed to be known. The object of the computer experiments was to assess how accurately one can determine the value of $N$ using only a limited number of samples. In the computer simulated study, a given number of samples at a specific stressing rate, $\sigma$, were chosen randomly by selecting their failure probability, $F$, from a uniform distribution between 0 and 1. Then the fatigue fracture strength, $S$, was calculated according to the fracture mechanics relationship:

$$
\ln S = \frac{1}{N+1} \left[ \ln B + \ln (N+1) + (N-2) \left( \frac{1}{m} \ln \ln \frac{1}{1-F} + \ln S_0 \right) + \ln \sigma \right]
$$

Once a set of strength values at several different stressing rates were obtained, the median values were determined and Eq. (1) was then used to calculate $N$. By iterating this procedure 100 times, a distribution for $N$ was generated which represents the statistical reproducibility of $N$ as determined from the dynamic fatigue tests. For a given material/environment system, i.e. for a given set of $m$, $S_0$, $N$ and $B$, the important variables studied were the stressing rate range, the number of samples per stressing rate, and the number of stressing rates that were used in a given test. The number of stressing rates chosen in a given test varied from 2 to 7.

Figure 2 illustrates how the statistical reproducibility of $N$ for soda-lime glass in water depends on total sample size used in the dynamic
fatigue experiments. In this figure the standard deviations, $\varepsilon$, were calculated using the customary formula:

$$\varepsilon^2 = \frac{\sum_{i=1}^{J} (N_i - \bar{N})^2}{J - 1}$$

where $J$ is the number of times the simulated experiment was repeated, $N_i$ are the individual $N$ values, and $\bar{N}$ is the mean value. It is seen that the statistical variability of the crack propagation parameter $N$ depends strongly on sample size and can be large for sample sizes less than 100. For example, for a sample size of 100, the $\pm 2 \varepsilon$ limits ranged from about 14 to 23. The results also showed that for the same range of stressing rates (maximum to minimum) and the same total number of samples, the best reproducibility of $N$ occurs for the case where $N$ is determined from strength measurements at 2 stressing rates corresponding to the maximum and minimum. For the case of multi-stressing rates, the reproducibility depended mainly on the total number of samples and not on the number of samples chosen per stressing rate. However, testing at only two stressing rates would not necessarily be recommended since there would be no way of knowing if one of the data sets contains a systematic error.

The statistical uncertainty in $N$ for a given sample size was found to depend on the stressing rate range and also on the Weibull slope parameter, $m$. As the stressing rate range and $m$ decreased, the uncertainty in $N$ increased. Figure 3 compares the $\pm 2 \varepsilon$ limits for $N$ as a function of $m$, keeping the other fatigue parameters constant, and illustrates that low $m$ values, corresponding to greater variability in strength, result in larger uncertainties in $N$. 
The variability in N also depends on the fatigue resistance of the material (large N values generally represent materials with a greater fatigue resistance). For example, Fig. 4 compares the results on the reproducibility of N of a vitrified grinding wheel material \( N = 43.2 \) with that for soda-lime glass \( N = 18.4 \) where the stressing rate range and m was taken to be the same for the two materials. It is evident that the uncertainty in N is much greater for the more fatigue resistant vitrified grinding wheel material.

In summary, it was found that the statistical reproducibility of N as determined from the dynamic fatigue experiment depends on the total number of samples, stressing rate range, Weibull slope parameter m, and the fatigue resistance of the material. This statistical uncertainty in N can be large, especially for sample sizes less than 100. Thus, it is recommended that before meaningful conclusions can be drawn regarding the effect of a test variable on N as measured by the dynamic fatigue experiment, the statistical reproducibility of N should be evaluated. For example, the ranges of N values quoted at the beginning of this note for soda-lime glass, optical glass fibers, and alumina can be shown to be well within \( \pm 2 \sigma \) range expected from the dynamic fatigue test and it would be imprudent to attribute a physical meaning to the difference between the high and low values of N in these ranges. This investigation of the statistical reproducibility due to sampling variability in fatigue strength testing is continuing and the complete results and its implications to failure predictions will be reported in a forthcoming paper.

Acknowledgement

This research was supported by Office of Naval Research.
References

Figures

Figure 1. Dynamic fatigue data for soda-lime glass in water.

Figure 2. Statistical reproducibility of $N$ as a function of the number of samples randomly chosen from the fatigue population of soda-lime glass where the appropriate constants are $N = 18.4$, $B = 0.18 \text{ MPa}^2 \cdot \text{s}$, $m = 8.2$, $S_0 = 138.0 \text{ MPa}$ and the stressing rate range is .01 to 53 MPa/s. The number of stressing rates used in determining $N$ is as indicated.

Figure 3. Statistical reproducibility of $N$ as a function of the Weibull slope parameter $m$, keeping the other constants the same ($N = 18.4$, $B = 0.18 \text{ MPa}^2 \cdot \text{s}$, $S_0 = 138.0 \text{ MPa}$).

Figure 4. Comparison of the statistical reproducibility of $N$ for a vitrified grinding wheel material ($N = 43.2$, $B = 1.1 \times 10^{-5} \text{ MPa}^2 \cdot \text{s}$, $m = 15$, $S_0 = 53 \text{ MPa}$) to that of soda-lime glass ($N = 18.4$, $B = 0.18 \text{ MPa} \cdot \text{s}$, $m = 15$, $S_0 = 138.0 \text{ MPa}$).
\textbf{% EXPECTED VALUE (N)}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Figure 3}
\end{figure}

- \textbf{m=4.2}
- \textbf{m=8.2}
- \textbf{\pm 2 STD. DEV.}
Fatigue of Optical Glass Fibers

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1.0 INTRODUCTION

One of the key problems in the design of a high strength, optical-fiber communication cable is the long-term mechanical reliability of the glass fibers. Unfortunately, glass fibers exhibit delayed failure (commonly known as static fatigue) and a wide variability in fracture strength that can cause a significant numbers of fibers to fail at moderate stress levels.\(^1,2\) Thus, it is important that the possibility of failures either be statistically predictable or be eliminated by proof testing.

The objectives of the research reported herein were: to demonstrate the applicability of fracture-mechanics theory in predicting fatigue failure of optical glass fibers before and after proof testing; to determine the effects of relative humidity on the fatigue strength of optical glass fibers; to study the effectiveness of proof testing in improving the after-proof strength distribution of optical glass fibers.

2.0 EXPERIMENTAL PROCEDURE

The fatigue behavior of three optical glass fibers was determined by the dynamic fatigue and static fatigue test techniques in which strength and time-to-failure are measured as a function of stressing rate and applied stress, respectively. The dynamic fatigue test environments were
ambient air at 23°C and 55% RH and moist air at 23°C and 95% RH. The
environment for the static fatigue tests was ambient air at 23°C and
about 75% RH. The three optical fibers studied were all obtained from
ITT Corporation, Roanoke, Va. and were:

Fiber I = Type PS-105 Doped silica core (index of refraction = 1.48)
optical fiber with protective polymer coatings of silicon
and Hytrel. This fiber was delivered June 1977. It is
not known if this fiber was proof tested.

Fiber II = Type PS-105 Doped Silica core (index of refraction = 1.48)
optical fiber with protective polymer coatings of silicon
and Hytrel. This fiber was proof tested to 2700 MPa and
delivered January 1978.

Fiber III = Type GG-02 Glass Graded index multimode optical fiber with
a Doped Silica core (index of refraction = 1.48), a boro-
silicate cladding (index of refraction = 1.45), and a fused
silica outer coating. This fiber had protective polymer
coatings of silicon and Hytrel. The fiber was factory
proof tested to 2070 MPa and delivered January 1978.

In addition, the fracture strength after proof testing for Fibers I
and II was determined as a function of a fast and slow unloading rate
from the proof stress. Fiber I samples were loaded to a proof load of
6.2 kg at a crosshead speed of 83.3 m/s (.5cm/min), corresponding to a
stressing rate of 18.2 MPa/s, then rapidly unloaded (83.3 m/s). This
proof load gave an average proof stress of 4260 MPa and was chosen to give 33% failure in the proof test. The proof test survivors were divided into two groups. With one group the inert strength was measured in dry N\textsubscript{2} at 23°C and with the other group the strength in ambient air (23°C and 55% RH) at a stressing rate of 19.2 M\textsubscript{Pa}/s was determined. One hundred samples of Fiber III were also proof tested in ambient air. These samples were loaded to a proof stress of 5177 MPa corresponding to a 33% failure probability, at a stressing rate of 192 MPa/s then the survivors were alternately unloaded at a slow unloading rate of 8.33 m/s (5 cm/min) and a fast unloading rate of 10.7 mm/s (i.e. 1 sec to unload), respectively. The proof test survivors, having been divided into the two unloading groups, were reloaded to fracture at a stressing rate of 192 MPa/s in an ambient air (23°C, 55% RH).

3.0 RESULTS AND DISCUSSION

3.1 Fatigue Behavior

The median, homologous stress, and iterative, bivariant analyses were utilized in determining the fatigue constants B and N from the dynamic and static fatigue data. The reader is referred to references 3 and 4 for a detailed description of these techniques. Table 1 summarizes these results. It is important to note that the data for Fiber I was presented in an earlier paper.\textsuperscript{3}

The overall agreement in the fatigue constants for a given fiber and environment as determined by the various data analysis techniques gives validity to the fracture mechanics approach for analyzing fatigue data of
optical-glass fibers. Since the static fatigue data was obtained in a "shotgun fashion", i.e., time-to-failure was measured at a large number of applied stresses, only the iterative, bivariant analysis was used with this data since it does not require that the time-to-failure distribution be determined at each applied stress.\(^4\)

The effect on fatigue of fiber type, moisture, and test technique can be best seen in terms of a minimum lifetime prediction diagram based on proof testing. The minimum lifetime (\(t_{\text{min}}\)) in service after proof testing is given by:\(^5\)

\[
\ln (t_{\text{min}}^{\alpha_a^2}) = (N-2) \ln (\sigma_P/\sigma_a) + \ln B \tag{1}
\]

where \(\sigma_a\) is the applied stress in service and \(\sigma_P\) is the maximum proof test stress. Thus, a plot of \(\ln (t_{\text{min}}^{\alpha_a^2})\) vs \(\ln (\sigma_P/\sigma_a)\) is a straight line with a slope of \(N-2\) and an intercept of \(\ln B\).

Figure 1 is a minimum lifetime prediction diagram comparing Fibers I, II, and III for ambient air at 23°C and 55% RH. These minimum lifetime predictions are based on the fatigue parameters \(N\) and \(B\) as determined from the iterative, bivariant analysis. This figure clearly shows the similarity in the fatigue behavior of Fibers II and III. Presumably the fatigue behavior of Fiber III is controlled by the fused silica coating and that the small amount of dopants in the fused silica core of Fiber II does not alter its fatigue behavior. The fatigue results of Fibers II and III are in marked contrast to those of Fiber I. The poorer fatigue resistance of Fiber I is thought to be related to the preform used in the glass fiber production.\(^6\) Fiber I is thought to be made from a natural
quartz preform and Fibers II and III from a synthetic quartz preform. Fibers drawn from a natural quartz preform may still contain microreminants of the original crystalline structure that would give rise to micro-defects throughout the volume of the glass fiber. These microreminant defects could enhance subcritical crack growth by effectively increasing crack size when a surface flaw encounters the microreminant defect through subcritical crack growth. Interestingly enough, although the fatigue behavior of Fiber I is distinctly different from Fibers II and III, the median inert strength of all three fibers is similar: 5669 MPa for Fiber I, 5527 MPa for Fiber II, and 5622 MPa for Fiber III.

Figures 2 and 3 show the detrimental effect that humidity has on fatigue. For example, to assure a minimum lifetime of 10 years in service at 23°C and 55% RH under an applied stress of 1000 MPa, Fiber II would have to be proof tested at 1950 MPa; however, for a humid environment the required proof test would be 2450 MPa. Similarly, Fiber III would require a proof test stress of 2000 MPa for 55% RH service environment but 2900 MPa for a high humidity service environment to assure a minimum lifetime of 10 years under an applied stress of 1000 MPa.

Figure 4 compares the fatigue results of Fiber III as determined in the various test environments. The results not only show the expected shift in the minimum lifetime prediction curves as a function of relative humidity but also show most importantly that the static fatigue results are consistent with the dynamic fatigue results. These results give evidence that the accelerated test (dynamic fatigue) can be used to determine the fatigue parameters needed for long-term failure predictions.
3.2 Proof Testing

The purpose of proof testing is to eliminate weak samples from the strength population so that the after-proof strength distribution will be stronger than the initial distribution. Figure 5 compares the after-proof strength distributions of Fiber I to the initial distributions for the inert and ambient air (23°C and 55% RH) environments. It is evident that proof testing did not significantly improve the strength distributions; however, this result is in agreement with predictions based on fracture mechanics theory. The reason that the after-proof distribution is approximately that of the initial distribution is that although weak samples are being eliminated by the proof test, strong samples are becoming weaker due to crack growth.

Figure 6 compares the after-proof strength distributions of Fiber III for a fast and slow unloading rate to the initial strength distributions. As with Fiber I, a fast unloading in the proof test gave after-proof strength distributions that were in agreement with theory. However, a slow unloading rate resulted in an after-proof strength distribution significantly weaker than the initial. In this case fracture mechanics theory cannot explain the observed after-proof strength distribution. Ritter, et al. found similar results using soda-lime glass microscope slides. That is to say when crack growth was not minimized on unloading, the observed after-proof, strength distributions were much weaker than that predicted by theory. It was thought that in these cases Regions II of the subcritical crack growth curve becomes important. These results emphasize the need for having good proof test controls, namely a fast unloading rate and a relatively dry proof test environment.
4.0 SUMMARY

The fatigue behavior of optical glass fibers was found to be strongly dependent on humidity with the fatigue resistance decreasing with an increase in humidity. The preform used in the production of glass fibers was found to effect the fatigue behavior. Fibers made from a natural quartz preform had significantly poorer fatigue resistance than fibers made from synthetic quartz preforms. The fatigue behavior determined by the dynamic and static fatigue experiments were in agreement, indicating that the accelerated dynamic fatigue test can be used to estimate fatigue lifetimes under static loading. Proof test results showed that when slow unloading rates were used in the proof test, the after-proof strength distribution was significantly weaker than the initial distribution and could not be predicted from fracture mechanics theory. These results emphasize the importance of having good proof test controls.

ACKNOWLEDGEMENTS

This research was supported by Office of Naval Research.
REFERENCES


Table I. Summary of the Fatigue Parameters $N$ and $B$ for Optical Glass Fibers

<table>
<thead>
<tr>
<th>Fiber Environment</th>
<th>Data $^+$</th>
<th>Analysis Technique</th>
<th>$N$ (MPa$^2$.s)</th>
<th>$B$ (MPa$^2$.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Air (23°C and 55% RH)</td>
<td>D.F. Median Homologous Stress</td>
<td>22.2</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>I Air (23°C and 55% RH)</td>
<td>D.F. Iterative, Bivariant</td>
<td>21.7</td>
<td>15.6</td>
<td></td>
</tr>
<tr>
<td>I Air (23°C and 55% RH)</td>
<td>Iterative, Bivariant</td>
<td>18.5</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>II Air (23°C and 55% RH)</td>
<td>D.F. Median Homologous Stress</td>
<td>26.4</td>
<td>16.1</td>
<td></td>
</tr>
<tr>
<td>II Air (23°C and 55% RH)</td>
<td>D.F. Iterative, Bivariant</td>
<td>27.4</td>
<td>16.1</td>
<td></td>
</tr>
<tr>
<td>II Air (23°C and 55% RH)</td>
<td>Iterative, Bivariant</td>
<td>27.1</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>III Air (23°C and 55% RH)</td>
<td>D.F. Median Homologous Stress</td>
<td>36.8</td>
<td>9.1</td>
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<td>35.8</td>
<td>9.5</td>
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<td>III Air (23°C and 55% RH)</td>
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<td>25.2</td>
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<td>10.8</td>
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<td>IV Air (23°C and 75% RH)</td>
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<td></td>
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<tr>
<td>V Air (23°C and 75% RH)</td>
<td>S.F. Iterative, Bivariant</td>
<td>17.5</td>
<td>17.6</td>
<td></td>
</tr>
</tbody>
</table>

$^+$ D.F. represents dynamic fatigue and S.F. represents static fatigue.
FIGURES

Figure 1. Minimum lifetime prediction diagram for optical glass fibers I, II, III in air at 23°C and 55% RH.

Figure 2. Minimum lifetime prediction diagram for optical glass fiber II as a function of relative humidity.

Figure 3. Minimum lifetime prediction diagram for optical glass fiber III as a function of relative humidity.

Figure 4. Minimum lifetime prediction diagram for optical glass fiber III comparing the fatigue behavior as determined by the dynamic and static fatigue test techniques.

Figure 5. Strength distributors of Fiber I before and after proof testing compared to that predicted from fracture mechanics theory (from Ref. 3).

Figure 6. Strength distributions of Fibers III before and after proof testing compared to that predicted from mechanics theory.
LIFETIME PREDICTION DIAGRAM
FOR FIBERS I, II AND III
AT A 55% RELATIVE HUMIDITY

\[
\frac{G_p}{G_a}
\]

\[
\ln \left( \frac{G_p}{G_a} \right)
\]

- **Fiber I**
- **Fiber II**
- **Fiber III**

10 years
1 year
1 month

1500 MPa

\( t_{\min} = \frac{G_p}{G_a} \) (s/MPa)
LIFETIME PREDICTION DIAGRAM
FOR FIBER II AS A FUNCTION
OF RELATIVE HUMIDITY

\[ t_{\text{min}} \left( \frac{\sigma}{\sigma_a} \right)^2 \]

\[ \ln \left( \frac{G_p}{G_a} \right) \]

- 55% RH
- 95% RH
LIFETIME PREDICTION DIAGRAM FOR FIBER III AS A FUNCTION OF RELATIVE HUMIDITY
LIFETIME PREDICTION DIAGRAM FOR FIBER III COMPARING FATIGUE PARAMETERS OF DYNAMIC-FATIGUE AND STATIC-FATIGUE
FIBER I BEFORE AND AFTER PROOF TESTING COMPARISON TO PREDICTED BY THEORY

Probabilty (F)

STRENGTH DISTRIBUTION OF FIBER I BEFORE AND AFTER PROOF TESTING COMPARED TO THAT PREDICTED BY THEORY

STRENGTH (MPa)

Initial Distribution

After Proof Test Data

Predicted After Proof Strength Distribution

Dry N2
192 MPa/s

Air
192 MPa/s

Ln STRENGTH (MPa)

Ln (1-F)
STRENGTH DISTRIBUTION OF FIBER III BEFORE AND AFTER PROOF TESTING COMPARED TO THAT PREDICTED BY THEORY

STRENGTH (MPa)

3500 4000 4500 5000 5500 6000

Ln Ln 1/(1-F)

-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0

Ln STRESS (MPa)

-1.4 0.3 0.5 0.7 0.9

SOLID LINE = INITIAL DISTRIBUTION
DASH LINE = PREDICTED AFTER PROOF STRENGTH DISTRIBUTION

X = AFTER PROOF-TEST DATA FOR AIR AT UNLOADING TIMES OF APPROXIMATELY 20 SECONDS
O = AFTER PROOF-TEST DATA FOR A 1 SECOND UNLOAD TIME

FAILURE PROBABILITY

.05 .1 .2 .3 .4 .5 .6 .7 .8 .9