MEMORANDUM REPORT ARLCD-MR-78007

DYNAMIC MODELING OF
POST FAILURE CONDITIONS
OF REINFORCED CONCRETE SUBJECTED TO BLAST

MICHAEL F. LEONDI

APRIL 1979

U.S. ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER WEAPON SYSTEMS LABORATORY
DOVER, NEW JERSEY

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
The views, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

Destroy this report when no longer needed. Do not return to the originator.
A dynamic modeling analysis of post-failure fragments of reinforced concrete subjected to blast overpressures was performed. The analysis related the physical (constitutive) properties as well as the geometric relationships of the model and the prototype. Results indicated that a replica model can provide rational test data if all the mechanical and geometric properties of the reinforced concrete element are reflected in the model.
ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Dr. W. E. Baker of Southwest Research Institute and Messrs. J. Marsicovete, J. Moroney, and E. Krajkowski of ARRA COM for valuable contributions and assistance in the preparation of this report.
# TABLE OF CONTENTS

| Introduction | 1 |
| Problem Definition and Solution | 1 |
| Mechanics of Fragmentation | 1 |
| Dynamic Modeling of Reinforced Concrete Elements | 3 |
| PI Terms from Dimensional Homogeneity | 3 |
| Scale Factors | 5 |
| Conclusions and Recommendations | 6 |
| References | 6 |
| Distribution List | 11 |

## Tables

<table>
<thead>
<tr>
<th></th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>List of parameters from explosive fragmentation of reinforced concrete elements</td>
</tr>
<tr>
<td>2</td>
<td>PI terms - explosive fragmentation</td>
</tr>
</tbody>
</table>

## Figure

<table>
<thead>
<tr>
<th></th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Explosive fragmentation.</td>
</tr>
</tbody>
</table>
INTRODUCTION

In support of an Army-wide modernization program, the Army Armament Research and Development Command, Dover, NJ, is engaged in the development of safety criteria as a project entitled, "Safety Engineering in Support of Ammunition Plants."

Prior to investigating the explosive fragmentation of reinforced concrete structures (one aspect of the safety engineering program), a feasibility study of sub-scale dynamic modeling was conducted to determine:

1. The applicability of sub-scale modeling to random mass fragments of reinforced concrete subjected to excessive blast loads.

2. The physical relationships between the model and the prototype in constitutive properties, panel geometry, and boundary conditions as well as the resulting fragment weight/shape, density, and distribution.

PROBLEM DEFINITION AND SOLUTION

In munitions manufacturing facilities, reinforced concrete plays an essential role as a building material for both dividing walls (between quantities of explosive) and exterior walls and roofs.

Mechanics of Fragmentation (ref. 1)

When a reinforced concrete element is substantially overloaded by a blast wave, the element fails and concrete fragments (post-failure) are displaced at high velocities (fig. 1).

Failure of an element is characterized by the dispersal of concrete fragments formed by the cracking and displacement of the concrete between the donor and receiver layers of the reinforcement. As an element deflects and the concrete begins to crush, the compression stresses normally resisted by the concrete are transferred to the reinforcement. With increased deflections, these compression forces tend to buckle the reinforcement outward, initiating rapid disintegration of the element.
The velocity of individual fragments varies and depends on:
(1) the magnitude of the excess (or blast) impulse minus the flexural
impulse capacity of the element (area under the resistance-time
curve), (2) the mass of the fragment, (3) the location of the frag-
ment prior to collapse, (4) the interaction between fragments during
their flight, and (5) the strength and time history of the compres-
sive stress wave transmitted through the wall as the blast wave is
reflected. Although the velocities of individual fragments differ,
the average translational velocity, \( V_g (avg) \), of the debris after
complete failure can be approximated from the excess impulse, \( i_e \), and
the unit mass, \( m \), of the barrier; i.e., the momentum of the wall
after collapse is numerically equal to the excess impulse, \( i_e = mv_f \)
(avg).

Also,

\[
i^2 = C_u \frac{(pHd^3 f_d s)}{H} + C_f d^2 v_f^2
\]

where

\( i \) = applied unit blast impulse \\
\( pH \) = reinforcement ratio in the horizontal direction \\
\( d_c \) = distance between the centroids of the compression
and tension reinforcement. \\
\( f_d s \) = dynamic design stress for the reinforcement \\
\( H \) = span height \\
\( v_f \) = maximum velocity of the post-failure fragments \\
\( C_u \) = impulse coefficient for ultimate deflection \( \chi_u \) \\
\( C_f \) = post-failure fragment coefficient

The reflected shock wave is also transmitted through the wall
as a compressive wave, and reflects from the rear surface as a ten-
sion wave which partially cancels the incoming compressive wave. If
the shock strength is great enough and its decay rate rapid enough,
a net tensile stress will occur at some distance from the rear sur-
face. The concrete (which is weak in tension) will fail and spall
off. The thickness of the spall and its velocity can be estimated
from knowledge of the reflected wave properties and the constitutive properties of the concrete.

Dynamic Modeling of Reinforced Concrete Elements

The following exercise defines the physical similarity between reinforced concrete elements in the model and the prototype subjected to blast overloading as described in reference 2. The constitutive, geometric, and mechanical parameters dictated by the scope of the problem are given in table 1.

PI TERMS FROM DIMENSIONAL HOMOGENEITY

By inspection (already dimensionless)

\[ \pi^{13} = \psi \]
\[ \pi^{14} = K \]

Technique from reference 2: Matrix Solution

\[ F_{0L0T0d} \rightarrow W^{a1} R^{a2} \rho_c^{a3} \rho_s^{a4} C_c^{a5} T_c^{a6} U_s^{a7} T_s^{a8} L_c^{a9} L_s^{a10} L_A^{a11} E_s^{a12} E_c^{a13} W_F^{a14} V^{a15} \]

Using matrix algebra (identity submatrix)

Form a matrix:

\[
\begin{pmatrix}
W & R & \rho_c & \rho_s & C_c & T_c & U_s & T_s & L_c & L_s & L_A & E_s & E_c & W_F & V \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
F & +1 & 0 & +1 & +1 & +1 & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 \\
L & +1 & +1 & -4 & -4 & -2 & -2 & -2 & +1 & +1 & -2 & -2 & +1 & 3 \\
T & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
Matrix algebra steps:

Step 1 - Divide row 3 by row 2.

Step 2 - Subtract new row 3 from row 1, and write new row 1.

Step 3 - Subtract row 1 from row 2.

Step 4 - Multiply row 3 by row 4 and add to row 2. Write new row 2.

Therefore, identity submatrix in the following matrix:

\[
\begin{array}{cccccccccccccccc}
  & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
F & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 \\
L & 0 & +1 & 0 & 0 & -3 & -3 & -3 & +1 & +1 & +1 & -3 & -3 & 0 & +3 \\
T & 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Then, by inspection

\[
a_1 = -a_5 -a_6 -a_7 -a_8 -a_{12} -a_{13} -a_{14} \\
a_2 = 3a_5 + 3a_6 + 3a_7 + 3a_8 - a_9 -a_{10} -a_{11} + 3a_{12} + 3a_{13} -3a_{15} \\
a_3 = -a_4
\]

Substituting in the original equation

\[
F^0L^0T^0d \quad \begin{array}{c}
w^{-a_5-a_6-a_7-a_8-a_{12}-a_{13}-a_{14}} \\
R^{-3a_5+3a_6+3a_7+3a_8-a_9-a_{10}+a_{11}} \\
+3a_{12}+3a_{13}-3a_{15} \\
x_r^{a_{14}a_{15}}
\end{array}
\]

\[
\begin{array}{ccccccccccccccc}
  & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
F & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 \\
L & 0 & +1 & 0 & 0 & -3 & -3 & -3 & +1 & +1 & +1 & -3 & -3 & 0 & +3 \\
T & 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Then, by inspection

\[
a_1 = -a_5 -a_6 -a_7 -a_8 -a_{12} -a_{13} -a_{14} \\
a_2 = 3a_5 + 3a_6 + 3a_7 + 3a_8 - a_9 -a_{10} -a_{11} + 3a_{12} + 3a_{13} -3a_{15} \\
a_3 = -a_4
\]

Substituting in the original equation

\[
F^0L^0T^0d \quad \begin{array}{c}
w^{-a_5-a_6-a_7-a_8-a_{12}-a_{13}-a_{14}} \\
R^{-3a_5+3a_6+3a_7+3a_8-a_9-a_{10}+a_{11}} \\
+3a_{12}+3a_{13}-3a_{15} \\
x_r^{a_{14}a_{15}}
\end{array}
\]

\[
\begin{array}{ccccccccccccccc}
  & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
F & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 & 0 & 0 & +1 & +1 & +1 & 0 \\
L & 0 & +1 & 0 & 0 & -3 & -3 & -3 & +1 & +1 & +1 & -3 & -3 & 0 & +3 \\
T & 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Then, by inspection

\[
a_1 = -a_5 -a_6 -a_7 -a_8 -a_{12} -a_{13} -a_{14} \\
a_2 = 3a_5 + 3a_6 + 3a_7 + 3a_8 - a_9 -a_{10} -a_{11} + 3a_{12} + 3a_{13} -3a_{15} \\
a_3 = -a_4
\]

Substituting in the original equation

\[
F^0L^0T^0d \quad \begin{array}{c}
w^{-a_5-a_6-a_7-a_8-a_{12}-a_{13}-a_{14}} \\
R^{-3a_5+3a_6+3a_7+3a_8-a_9-a_{10}+a_{11}} \\
+3a_{12}+3a_{13}-3a_{15} \\
x_r^{a_{14}a_{15}}
\end{array}
\]
Collecting terms of like exponent, we have the model law. This can be expressed as:

\[
\begin{align*}
\frac{(W_f)}{W} & = f_i \frac{\rho_s}{\rho_c}, \\
\frac{V}{R^3} & = \frac{R^3\rho_s}{W} \frac{R^3T_c}{W}, \\
\frac{\psi}{K} & = \lambda L_c L_s L_A R^3E_s R^3E_c
\end{align*}
\]

The law can also be stated in the form of table 2.

**SCALE FACTORS**

Various relations between scale factors can be determined from the dimensionless terms.

\[
\begin{align*}
\pi_1 & \Rightarrow \lambda_{\rho_s} = \lambda_{\rho_c} = 1 \text{ (set equal to 1 for ease in fabricating model)} \\
\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7 & \Rightarrow \frac{\lambda}{\lambda^3R} = \lambda_{C_c} = \lambda_{T_c} = \lambda_{U_s} = \lambda_{T_s} \\
& = \lambda_{E_s} = \lambda_{E_c} \\
\pi_8, \pi_9, \pi_{10} & \Rightarrow \lambda_{L_c} = \lambda_{L_s} = \lambda_{L_A} = \lambda_R \\
\pi_{11} & \Rightarrow \lambda_{W_f} = \lambda W \\
\pi_{12} & \Rightarrow \lambda_{V} = \lambda^3_R
\end{align*}
\]
These interrelations are all satisfied by a replica model\textsuperscript{1} with geometry identical to the prototype. Although wave transmission effects are not shown in scale, they will scale properly because all constitutive properties and densities are unchanged.

CONCLUSIONS AND RECOMMENDATIONS

The preceding similarity exercise has demonstrated that

1. A replica model accurately reflects the performance of a prototype system.

2. Failure modes of a replica model are representative for geometrically similar panels.

3. The model also exhibits similar relationships for fragment mass and velocity, distribution, and debris density.

For economic reasons and ease in testing, it is recommended that subscale replica models be used with a scale factor as small as possible without sacrificing the quality of the model.

REFERENCES


\textsuperscript{1}A replica model is a physical model of a prototype which is geometrically similar in all respects and employs the same materials in the same locations as the prototype (ref. 2).
Table 1. List of parameters for explosive fragmentation of reinforced concrete elements

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$W$</td>
<td>$FL$</td>
<td>Energy in blast source</td>
</tr>
<tr>
<td>2</td>
<td>$R$</td>
<td>$L$</td>
<td>Radial distance of blast</td>
</tr>
<tr>
<td>3</td>
<td>$\rho_c$</td>
<td>$FT^2/L^4$</td>
<td>Density concrete</td>
</tr>
<tr>
<td>4</td>
<td>$\rho_s$</td>
<td>$FT^2/L^4$</td>
<td>Density steel</td>
</tr>
<tr>
<td>5</td>
<td>$C_c$</td>
<td>$F/L^2$</td>
<td>Compressive strength (concrete)</td>
</tr>
<tr>
<td>6</td>
<td>$T_c$</td>
<td>$F/L^2$</td>
<td>Tensile strength (concrete)</td>
</tr>
<tr>
<td>7</td>
<td>$U_s$</td>
<td>$F/L^2$</td>
<td>Ultimate strength (steel)</td>
</tr>
<tr>
<td>8</td>
<td>$T_s$</td>
<td>$F/L^2$</td>
<td>Tensile strength (steel)</td>
</tr>
<tr>
<td>9</td>
<td>$L_c$</td>
<td>$L$</td>
<td>Characteristic geom. of element</td>
</tr>
<tr>
<td>10</td>
<td>$L_s$</td>
<td>$L$</td>
<td>Characteristic geom. of rebar</td>
</tr>
<tr>
<td>11</td>
<td>$L_a$</td>
<td>$L$</td>
<td>Aggregate size</td>
</tr>
<tr>
<td>12</td>
<td>$E_s$</td>
<td>$F/L^2$</td>
<td>Elastic moduli (steel)</td>
</tr>
<tr>
<td>13</td>
<td>$E_c$</td>
<td>$F/L^2$</td>
<td>Elastic moduli (concrete)</td>
</tr>
<tr>
<td>14</td>
<td>$W_f$</td>
<td>$FL$</td>
<td>Energy imparted into fragments</td>
</tr>
<tr>
<td>15</td>
<td>$V$</td>
<td>$L^3$</td>
<td>Vol of fragments</td>
</tr>
<tr>
<td>16</td>
<td>$\psi$</td>
<td>--</td>
<td>Distribution function of fragments</td>
</tr>
<tr>
<td>17</td>
<td>$K$</td>
<td>--</td>
<td>% energy absorbed by element</td>
</tr>
</tbody>
</table>
Table 2. PI terms - explosive fragmentation

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_1 )</td>
<td>( \rho_s / \rho_c )</td>
</tr>
<tr>
<td>( \pi_2 )</td>
<td>( R^3Cc/W )</td>
</tr>
<tr>
<td>( \pi_3 )</td>
<td>( R^3Tc/W )</td>
</tr>
<tr>
<td>( \pi_4 )</td>
<td>( R^3Us/W )</td>
</tr>
<tr>
<td>( \pi_5 )</td>
<td>( R^3Ts/W )</td>
</tr>
<tr>
<td>( \pi_6 )</td>
<td>( R^3Es/W )</td>
</tr>
<tr>
<td>( \pi_7 )</td>
<td>( R^3Ec/W )</td>
</tr>
<tr>
<td>( \pi_8 )</td>
<td>( Lc/R )</td>
</tr>
<tr>
<td>( \pi_9 )</td>
<td>( Ls/R )</td>
</tr>
<tr>
<td>( \pi_{10} )</td>
<td>( LA/R )</td>
</tr>
<tr>
<td>( \pi_{11} )</td>
<td>( W_F/W )</td>
</tr>
<tr>
<td>( \pi_{12} )</td>
<td>( V/R^3 )</td>
</tr>
<tr>
<td>( \pi_{13} )</td>
<td>( \psi )</td>
</tr>
<tr>
<td>( \pi_{14} )</td>
<td>( K )</td>
</tr>
</tbody>
</table>

\( ^1 \) CONSTITUTIVE SIMILARITY

\( ^2 \) GEOMETRIC SIMILARITY

\( ^3 \) SIMILAR ENERGY TRANSFERS

\( ^4 \) SIMILAR MASS TRANSFERS

\( ^5 \) DISTRIBUTION FUNCTION

\( ^6 \) % ENERGY ABSORBED
DONOR (Explosive Charge)  REINFORCED CONCRETE PROTECTIVE ELEMENT

PREDETONATION

LETHAL ZONE

ENERGY AND IMPULSE BALANCES

Blast Energy In = Strain Energy + Sum of Fragment Kinetic Energies
= Sum of Fragment Kinetic Energies
\[ (1 - K) \] (where \( K \) = % Blast Energy Absorbed)

Applied Impulse = Flexural Impulse Capacity + Excess Impulse

ENERGY IN  BLAST LOADING \( P(t) \)  FRAGMENT ENERGY

\[ M_1V_1 \]
\[ M_2V_2 \]
\[ M_3V_3 \]

POST DETONATION  ENERGY ABSORBED

Figure 1. Explosive fragmentation.
DISTRIBUTION LIST

Commander
US Army Armament Research & Development Command
ATTN:  DRDAR-CG
       DRDAR-LC
       DRDAR-LCM
       DRDAR-LCM-S  (15)
       DRDAR-SF
       DRDAR-TSS  (5)
       DRDAR-LCU-P
Dover, NJ 07801

Commander
US Army Materiel Development & Readiness Command
ATTN:  DRCDE
       DRCIS-E
       DRCPA-E
       DRCPP-I
       DRCDI
       DRCISG-S
5001 Eisenhower Avenue
Alexandria, VA 22333

Commander
USDRC Installations and Services Agency
ATTN:  DRCIS-RI-IU
       DRCIS-RI-IC
Rock Island, IL 61299

Commander
US Army Armament Materiel Readiness Command
ATTN:  DRSAR-IR  (2)
       DRSAR-IRC
       DRSAR-ISE  (2)
       DRSAR-IRC-E
       DRSAR-PDM
       DRSAR-LC  (2)
       DRSAR-ASF  (2)
       DRSAR-SF  (3)
       DRSAR-LEP-L
Rock Island, IL 61299
Chairman
Dept of Defense Explosives Safety Board
Hoffman Building 1, Room 856C
2461 Eisenhower Avenue
Alexandria, VA 22331

Project Manager for Munitions Production
Base Modernization and Expansion
US Army Materiel Development & Readiness Command
ATTN: DRCPM-PBM-LA
    DRCPM-PBM-SF
    DRCPM-PBM-EP
Dover, NJ 07801

Director
Ballistic Research Laboratory
ARRADCOM
ATTN: DRDAR-BLE, C. Kingery (2)
Aberdeen Proving Ground, MD 21010

Defense Documentation Center (12)
Cameron Station
Alexandria, VA 22314

Commander
US Army Construction Engineering Research Laboratory
ATTN: CERL-ER
Champaign, IL 61820

Office, Chief of Engineers
ATTN: DAEN-MCZ-E
Washington, DC 20314

US Army Engineer District, Huntsville
ATTN: Construction Division-HAD-ED (2)
P.O. Box 1600 West Station
Huntsville, AL 35807

Commander
Indiana Army Ammunition Plant
ATTN: SARIN-OR
    SARIN-SF
Charlestown, IN 47111
Commander
Kansas Army Ammunition Plant
ATTN: SARKA-CE
Parsons, KS 67537

Commander
Lone Star Army Ammunition Plant
ATTN: SARLS-IE
Texarkana, TX 77701

Commander
Milan Army Ammunition Plant
ATTN: SARMIS
Milan, TN 38358

Commander
Radford Army Ammunition Plant
ATTN: SARRA-IE
Radford, VA 24141

Commander
Badger Army Ammunition Plant
ATTN: SARBA
Baraboo, WI 53913

Commander
Holston Army Ammunition Plant
ATTN: SARHO-E
Kingsport, TN 37662

Commander
Iowa Army Ammunition Plant
ATTN: SARIO-A
Middletown, IA 52638

Commander
Joliet Army Ammunition Plant
ATTN: SARJO-SS-E
Joliet, IL 60436

Commander
Longhorn Army Ammunition Plant
ATTN: SARLO-0
Marshall, TX 75670
Commander
Louisiana Army Ammunition Plant
ATTN: SARLA-S
Shreveport, LA 71102

Commander
Newport Army Ammunition Plant
ATTN: SARNE-S
Milan, TN 38358

Commander
Pine Bluff Arsenal
ATTN: SARPE-ETS
Pine Bluff, AR 71601

Commander
Sunflower Army Ammunition Plant
ATTN: SARSU-O
Lawrence, KS 66044

Commander
Volunteer Army Ammunition Plant
ATTN: SARVO-T
Chattanooga, TN 34701

Weapon System Concept Team/CSL
ATTN: DRDAR-ACW
Aberdeen Proving Ground, MD 21010

Technical Library
ATTN: DRDAR-CLJ-L
Aberdeen Proving Ground, MD 21010

Technical Library
ATTN: DRDAR-TSB-S
Aberdeen Proving Ground, MD 21005

Benet Weapons Laboratory
Technical Library
ATTN: DRDAR-LCB-TL
Watervliet, NY 12189

US Army Materiel Systems Analysis Activity
ATTN: DRXSY-MP
Aberdeen Proving Ground, MD 21005