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Interim Report Number 5
7 December 1964 through 13 January 1965.
Appendix I.
Listening Tests Part II.

Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.

Attention: Code 1632D, Dr. W. A. Youngblood

Dear Sir:

This report is the fifth interim report under this contract and
covers the work performed in the period 7 December 1964, through
13 January 1965.

During this period the analysis of the listener detection be-
behavior has been completed. The results are reported in Appendix
I included with this report.

The comparison between the efficiencies of the spatial correlator
and the replica correlator has continued. The number of echo
cycles included in the study has been extended.

The determination of coincidence of false alarms in the spatial
correlator processor and the replica correlator processor has
been run on the computer. Results are being assembled and were
to have been presented at the Conference on Signal Processing
and Recognition Differential scheduled at Raytheon, Portsmouth,
for 28 January 1965.

A number of complete echo cycle correlograms have been passed
through a symmetrical AGC. The resulting output has been passed
through the false alarm rate program and this output will be
used to construct detection curves for comparison with the
spatial correlator detection curves.

DISTRIBUTION STATEMENT A
Approved for public release.
The reports giving the results of these studies will be prepared in the next reporting period.

Sincerely,

B. M. Brown
Project Director

Enclosure: Appendix I

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APPENDIX I

LISTENING TESTS PART II

INTRODUCTION: The partial report on the listener experiment which was included as an appendix to the third interim report dated 9 November 1964, presented listener detection probability as a function of false alarm probability for pulse lengths of 70 ms, 150 ms, and 250 ms. Both FM pulses with 100 c/s bandwidth and CW pulses were employed. The summary given in that appendix stated that the listeners detected as well as a filter having a 50 c/s bandwidth with the ability to track the FM slides. Since that time the listener behavior for pulses of 20 ms and 5 ms have become available. In addition the predicted detection behavior for filters with 40 c/s and 100 c/s bandwidth has been compared with the listener behavior. The complete summary of listener behavior is presented in this appendix.

THE LISTENER MODEL: The listener detection model described in the earlier report was based upon standard signal detection theory. The detection probability was computed for a given threshold by computing the probability that signal power plus noise exceeded the threshold. The signals were assumed to be steady for a time \( \tau \); the noise power was assumed to obey the Rayleigh distribution. The false alarm probability was found by assuming that the noise alone exceeded the same threshold.

The noise bandwidth \( B_n \), determines the length of time \( 1/B_n \) between independent noise samples; the receiver bandwidth \( B \) (the equivalent listener bandwidth) determines the length of time \( 1/B \) between independent noise samples available to the listener. In a section of noise having duration \( T \), there will be

\[
n = T/(1/B) - k = TB - k
\]  

(1)
opportunities for the listener to respond incorrectly - that is, n opportunities to produce a false alarm. n is less than TB by k where k is chosen to represent the number of opportunities the listener has to detect. Its value is

\[ k = \tau/(1/B) = \tau B \],

(2)
because if \( \tau \) is the duration of the signal and each independent noise sample are received each \( 1/B \) seconds, there will be \( \tau B \) opportunities while the signal exists for the noise to be low enough that the listener can detect the signal.

In the previous report on the listener behavior \( \tau B \) was greater than unity and B was assumed to be less than \( B_n \). For this situation the listener bandwidth B will pass all of the signal power and the fraction \( B/B_n \) of the noise power. The output signal-to-noise ratio with this model is

\[ \left( \frac{S}{N} \right)_{\text{out}} = 10 \log \frac{B}{B_n} + \left( \frac{S}{N} \right)_{\text{in}} \, \text{dB} \].

(3)
The detection is assumed to take place with thresholding techniques using the output signal-to-noise ratio, computed by (3) at the input to the detector. The first term in the right member is called the processing gain.

When \( \tau B < 1 \), that is, when the pulse length is less than the reciprocal of the listener bandwidth, all of the signal power can no longer pass through the filter. If the listener passband is assumed ideal (flat response within the band B and zero outside), the energy which passes through the filter is proportional to the fraction

\[ \varphi = \frac{2}{\pi} \int_{0}^{TB} \left( \frac{\sin \xi}{\xi} \right)^2 d\xi \].

(4)
This integral can be reduced to

$$\varphi = \frac{2}{n} \left[ \int_0^{TBt} \frac{\sin \frac{\xi}{T}}{\xi} d\xi - \frac{1}{nBt} \right], \quad (4a)$$

and the modification in expected processing gain is

$$10 \log \varphi \quad (4b)$$

THE EXPERIMENT: Steady signals having duration $\tau$ were embedded in noise at specified signal-to-noise ratios, $(S/N)_i$. The observers listened to these signals and responded when they thought they detected the signals. Among the $M$ noise sections in which a signal was embedded, a number $R$ of correct responses was obtained. The ratio

$$D = \frac{R}{M}, \quad (5)$$

was interpreted as the detection probability. From the $N$ noise sections in which no signal had been embedded the probability $F$ per section of at least one false alarm was computed from the ratio

$$F = \frac{W}{N},$$

where $W$ is the number of the empty noise sections in which at least one response was made. Plots of $D$ against $F$ were made for each pulse form and input signal-to-noise ratio.

These experimental plots were overlaid on the graphs obtained from the detection theory model described in the earlier report. The plots for an assumed listener bandwidth of 50 c/s were included in the earlier report. In this report the plots (Figure 1(a) through 1(g)) using an assumed listener bandwidth of 40 c/s are shown. Again the listener detection behavior is
REEL 52
$B_{rec} = 40\%$ $T = 15$ SEC
$B_{noise} = 210\%$ $z = 5$ MS
$n = 600$ $k = 1$

$\left( \frac{S}{N} \right)_{in} = 6$ DB

PREDICTED GAIN = 0 DB
OBSERVED GAIN = -3 DB

△ OBSERVER 2
○ OBSERVER 3
× OBSERVER 4

Fig. 1(a) OBSERVER ROC
REEL 45

Brecoeff = 40 %, T = 15 SEC

Pnoise = 210%, T = 150 MS

n = 594

K = 6.0

Predicted Gain = 7.2 DB

Observed Gain = 7.3 DB

Observer 2

Observer 3

Observer 4

Fig (f) OBSERVER ROC

Detection Probability

False Alarm Probability / 15 SEC

UNCLASSIFIED
UNCLASSIFIED

REEL 3+  
BRDC = 40 S^-1, T = 15 SEC  
BRNOISE = 210 S^-1, T = 250 MS  
N = 590  
K = 10.0  

\[ \left( \frac{\xi^2}{\eta^2} \right)_{0} = 3.2 \]

\[ \left( \frac{\xi^2}{\eta^2} \right)_{\text{IN}} = -4 \text{DB} \]

PREDICTED GAIN = 7.2 DB  
OBSERVED GAIN = 6.8 DB

Fig (9) OBSERVER ROC

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shown by the points overlaid on the predicted curves. The signal-to-noise ratio of the curve along which the points cluster best was chosen as the listener output signal-to-noise ratio.

The processing gains achieved for the assumed 50 c/s bandwidth and the assumed 40 c/s bandwidth are plotted against pulse length in Figures 2(a) and 2(b). The predicted processing gains were computed using (3), (4a), and (4b). The observed processing gains fall somewhat above the predicted gains with the assumed bandwidth of 50 c/s. Agreement is somewhat better between the predicted and observed behavior using an assumed listener bandwidth of 40 c/s.

Figure (3) shows a comparison of the processing gain of an optimum processor \( \text{Gain} = 10 \log B_n T \) and the equivalent processing gain of the listeners for the assumed 40 c/s bandwidth.

The theoretical and experimental ROC curves for a listener assumed bandwidth of 100 c/s were prepared also. These are not included in this report, but there is a discrepancy of 3 dB between the observed and predicted output signal-to-noise ratios for the 70 ms pulses and the 150 ms pulses.

CONCLUSION: Three statements can be made concerning the behavior of listeners in the detection of steady signals in Rayleigh noise:

1. CW pulses and FM slides with bandwidths up to 100 c/s are detected with equal facility.

2. Signal processing by the listeners is equivalent to that obtainable with a narrow band filter with constant bandwidth B followed by a threshold detector.

3. When operating at a center frequency of 800 c/s, the bandwidth B is approximately 40 c/s.
Fig 2  COMPARISON OF OBSERVED AND PREDICTED LISTENER PROCESSING GAINS
Fig. 3 COMPARISON OF PROCESSING GAINS OF AN OPTIMUM PROCESSOR AND LISTENERS
The fact that CW pulses and FM slides are equally detectable with the 40-50 c/s bandpass filter indicates that the listeners in effect can either track the slide or employ adjacent filters with OR gating.

The results presented here confirm the statements made in the earlier report on this subject: that the listeners do not provide optimum processing gain. The ROC curves presented in the earlier report comparing the listeners as signal detectors with the optimum processor also indicate the superiority of the optimum processor.