NARROWBAND JAMMER SUPPRESSION IN SPREAD SPECTRUM SYSTEM
USING SAW DEVICES

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### AN ANALYSIS OF A BINARY SPREAD SPECTRUM PSK COMMUNICATION SYSTEM OPERATING IN THE PRESENCE OF A SINE WAVE JAMMER

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NARROWBAND JAMMER SUPPRESSION IN SPREAD SPECTRUM SYSTEM USING SAW DEVICES

L.B. Milstein† P.K. Das‡‡ D.R. Arsenault‡‡

Abstract
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1. INTRODUCTION
In this paper, a spread spectrum communication system employing SAW devices as real-time Fourier transformers to allow filtering operations to be performed by multiplications in the "frequency domain" will be modeled and analyzed. In particular, a system operating in the presence of a narrowband interferer and employing a bandpass filter to eliminate the interference will be analyzed and compared to a similar system employing a notch filter. Such systems have been described in the past and various degrees of experimental results have been presented ([1]-[4]). A detailed analysis of this system is provided in [5], and the results of that analysis are presented below.

2. DESCRIPTION OF SYSTEM
A block diagram of the system is shown in Figure 1. The input consists of the sum of the transmitted signal s(t), the additive thermal noise n(t), and the interference i(t). The Fourier transform of the input is taken, the transform is multiplied by the transfer function of some appropriate filter \( H_c(w) \), the inverse transform of the product is taken, and the resulting waveform put through a detection filter matched to s(t). The details of how the Fourier transform is implemented using the chirp filter shown in Figure 1 are presented in many references (see e.g. [1]). It will just briefly be noted here that the chirp filters are assumed implemented with a tapped-delay line of total length \( T_1 \) seconds.

In [5], it was shown that if a time limited signal, say \( f(t) \), assumed nonzero for \( t \in [0,T] \), is inputted into the above system, the output will be given by

\[
 f_0(T) = \int_0^T f(\lambda) \left[ h_R(\lambda) \ast s(\lambda) \right] d\lambda 
\]

(1)

where \( \ast \) denotes convolution and \( h_R(t) \) is the inverse transform of \( H_R(w) \), \( H_R(w) \) being the transfer function of the filter into which it is desired to pass the received waveform. \( H_R(w) \) is assumed to be real, and its relation to \( H_c(w) \) shown in Figure 1 is

\[
 H_c(w) = 4 H_R(w) \cos 2\pi t/T_1 . 
\]

That is, one uses \( H_c(w) \) given above when one wants to filter with \( H_R(w) \).

3. NARROWBAND INTERFERENCE
To use (1) to determine the performance of a wideband system being interfered with by a narrowband signal (eg. a jammer), assume the transmitted signal is a binary waveform where each information-bearing symbol has superimposed upon it the seven-bit PN sequence \(-1 +1 -1 -1 -1 -1 -1 \), with \( T \) the duration of each PN code chip. The noise \( n(t) \) is additive

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white Gaussian noise (AWGN) with two-sided spectral density $\gamma_0^2/2$, and the interference $I(t)$ is the tone $\alpha \cos(\omega_0+\delta\omega)t$, where $\alpha$ is a constant and where $\omega_0$ is the carrier frequency of the transmitted signal. With this model, the output statistic equals

$$
\int_0^T [s(t)+n(t)+I(t)]h_R(t)^*s(t)\,dt \tag{3}
$$

Denoting the signal, noise, and interference terms in (3) by $S(t)$, $N(t)$, and $I(t)$ respectively, the noise term $N(t)$ is clearly a Gaussian random variable with mean and variance given by

$$
E(N(t)) = 0 \tag{4}
$$

and

$$
\sigma_N^2 = \frac{\gamma_0^2}{2} \int_0^T [h_R(t)^*s(t)]^2\,dt \tag{5}
$$

respectively, and the probability of error of the system is given by

$$
p_e = \frac{1}{2} \left[ \phi \left( \frac{S_0+I_0}{\sigma_N} \right) + \phi \left( \frac{S_0-I_0}{\sigma_N} \right) \right] \tag{6}
$$

where

$$
\phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} \, dy.
$$

To compute either the noise variance or either of the deterministic terms in (3), one needs to specify a filter $h_R(t)$ and compute

$$
s(t)*h_R(t) = F^{-1} \{S(w)H_R(w)\} \tag{7}
$$

For the narrowband interference problem, there are two obvious filter choices, one an ideal bandpass filter with upper cutoff $w_0+\delta\omega$ and transfer function

$$
H_1(w) = P_{\omega_0} \left( w-w_0 \right) + P_{\omega_0} \left( w+w_0 \right) \tag{8}
$$

where $P_w(x)$ is 1 for $|x| \leq a$ and zero elsewhere, and the other a notch bandpass filter with upper cutoff $w_0+\delta\omega$ and notches at $w_0+2\delta\omega$ and $w_0-2\delta\omega$ with a notch of width $2\delta\omega$ at $w_0$, that is, a filter with transfer function

$$
H_2(w) = P_{\omega_2} \left( w-2\delta\omega \right) + P_{\omega_2} \left( w+2\delta\omega \right) \tag{9}
$$

For the seven-bit PN sequence input signal described above, it is easily shown that its Fourier transform $S_1(w)$ is given by

$$
S_1(w) = \frac{1}{w} \left[ 2 \sin 3\omega_0 t + 2 \sin 5\omega_0 t + 2 \sin 7\omega_0 t \right] + \frac{j}{w} \left[ 2 \cos 3\omega_0 t + 2 \cos 5\omega_0 t + 2 \cos 7\omega_0 t \right] \tag{10}
$$

Using (10) and (8) in (7) yields for the bandpass filter

$$
S(t)*h_R(t) = \frac{1}{\pi} \left[ \sin \omega_0 t \right] \tag{11}
$$

For the notch filter, it can be shown that

$$
s(t)*h_R(t) = \frac{1}{\pi} \left[ \sin \omega_0 t \right] \tag{12}
$$

where $-\frac{1}{2}\sigma_N^2 \sin \omega_0 t \tag{13}

In (11), (13) and (14),

$$
\sin \omega_0 t = \frac{1}{\pi} \int_0^\infty \frac{\sin y}{y} \, dy \quad \text{and} \quad \cos \omega_0 t = \int_0^\infty \frac{\cos y}{y} \, dy.
$$

Expressing $s(t)$ as $s(t) = s_1(t) \cos \omega_0 t$, where $s_1(t)$ corresponds to the baseband information bearing waveform (including the PN code), the output of the final matched filter can be written (neglecting double frequency terms)

$$
\int_0^T \left[ (S_1(t)+iS_2(t))^2 \cos \omega_0 t + n(t) \cos \omega_0 t \right] R_1(t;\omega_2) \, dt \tag{15a}
$$

and

$$
\int_0^T \left[ (S_1(t)+iS_2(t))^2 \cos \omega_0 t + n(t) \cos \omega_0 t \right] R_2(t;\omega_2) \, dt \tag{15b}
$$

for the bandpass filter and notch filter systems, respectively. As a perspective on these results, if the received waveform described above is detected with just a filter matched to the transmitted signal $s(t)$ (i.e., if no attempt is made to remove the interference), the probability of error of the system will be given by

$$
p_{e,MF} = \frac{1}{2} \left[ \phi \left( \frac{\omega_0 t}{\sigma_N} \right) \right] \tag{16}
$$

where $E$ is the energy per bit of the transmitted data.
signal and
\[ I(T) = \frac{a}{2} \int_0^T s_t(t) \cos \omega_0 t \, dt \]
\[ = \sqrt{\pi} \frac{a}{2} \frac{\omega_0}{s} \left[ -2 \sin 3\omega_0 T + 2 \sin 5\omega_0 T + 2 \sin 7\omega_0 T \right] \]
\[ - 2 \sin 6\omega_0 T \cos 7\omega_0 T \right] \]

4. RESULTS AND DISCUSSION

The results of numerically evaluating the previous equations can be seen in Figures (2)-(5). Figures (2) and (3) compare Eqs. (6) and (16) when the interference rejection filter has a transfer function given by Eq. (8), and Figures (4) and (5) show a corresponding comparison when the filter transfer function is given by Eq. (9). The difference in the two figures of each set is the location of the jammer relative to the center frequency of the signal (i.e. the value \( \omega_0 T \)). There are six curves labelled a to f on each figure, and Table I below identifies each of those curves. The notation \( P_e(\omega x) \), for example, means Eq. (6) evaluated for \( \omega x \).

<table>
<thead>
<tr>
<th>Curve</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>( P_e(\omega=0) ) (i.e. no jammer)</td>
</tr>
<tr>
<td>b</td>
<td>( P_e(\omega=4) )</td>
</tr>
<tr>
<td>c</td>
<td>( P_e(\omega=6) )</td>
</tr>
<tr>
<td>d</td>
<td>( P_e(\omega=8) )</td>
</tr>
<tr>
<td>e</td>
<td>( P_e(\omega=4) )</td>
</tr>
<tr>
<td>f</td>
<td>( P_e(\omega=8) )</td>
</tr>
</tbody>
</table>

For the bandpass filter of Eq. (8), the upper cut-off frequency was taken as the jammer frequency minus \( (2\pi/10T) \) (i.e. if the jammer was located at \( \omega_0 + \omega_0 \)) the upper cutoff of \( H_b(\omega) \) was \( \omega_0 + \omega_0 - (2\pi/10T) \), while for the notch filter \( H_n(\omega) \), the upper cutoff was always taken to be \( (14\pi/7) \) (corresponding to the first sidelobe of the PN sequence) and the notch itself was always centered at the jammer frequency and had a width of \( \pm (2\pi/10T) \).

Comparing the figures, one sees that the notch filter with its wider bandwidth invariably outperforms the bandpass filter when the jammer is close to the carrier frequency; and both significantly outperform the matched filter for high jamming levels. On the other hand, the matched filter by itself outperforms either the notch filter/matched filter combination or the bandpass filter/matched filter combination when the jammer is moved further away from the carrier. This is because the signal degradation, due to the band-limiting, is more harmful than the degradation due to the jammer. Clearly, the performance of the system using the notch filter could be improved simply by increasing its bandwidth. The key point is that for a strong enough tone jammer, the notch filter in cascade with the matched filter will always improve the system performance.

5. CONCLUSION

An analysis of the performance of a spread spectrum system employing SAM devices as real-time Fourier transformers has been presented for the case of a binary PN encoded PSK signal operating in the presence of a narrowband jammer. It was shown that for high jamming levels, the presence of the interference rejection filter can significantly improve system performance.

References


Acknowledgement

The authors would like to thank Mr. S. Fellows for generating the numerical results.
Figure 1. Block Diagram

\[ s(t) + n(t) + I(t) \]

\[ \cos[w_a t + \Delta t^2] \]

\[ \cos[w_a t - \Delta t^2] \]

\[ H_c(2\Delta t) \]

\[ 2 \cos[w_a t - \Delta t^2] \]

LPF

Matched Filter

output

Figure 2

\[ P_e \]

\[ 10^{-4} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ 10^0 \]

\[ 10^1 \]

\[ 10^2 \]

\[ 10^3 \]

\[ 10^4 \]

\[ 10^5 \]

\[ 10^6 \]

\[ E_s / \nu_e (\text{dB}) \]

Figure 3

\[ P_e \]

\[ 10^{-4} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ 10^0 \]

\[ 10^1 \]

\[ 10^2 \]

\[ 10^3 \]

\[ 10^4 \]

\[ 10^5 \]

\[ 10^6 \]

\[ E_s / \nu_e (\text{dB}) \]