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MATCHED FILTERING OF CONTINUOUS SIGNALS BY THE PRODUCT OF TRANSFORMS TECHNIQUE USING SAW CHIRP FILTERS

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Matched filtering can be implemented in a variety of ways using an assortment of devices. One straightforward technique employs the Si-on-LiNbO₃ structure as a programmable correlator. A more versatile configuration, however, can be realized by the utilization of chirp devices. With these devices, signal Fourier (chirp) transforms are readily obtainable and correlation follows with an inverse transformation of the product of transforms. This paper demonstrates the selective removal of jamming noise (by transform gating) for a continuous (Cont.)
PSK Barker encoded binary input signal along with some probability of error data for this system. The output of this system, in the absence of noise, consists of a continuous stream of plus and minus correlation peaks. The technique for transforming continuous signals, based upon the knowledge of the region of transform validity, will be discussed.
MATCHED FILTERING OF CONTINUOUS SIGNALS BY THE PRODUCT
OF TRANSFORMS TECHNIQUE USING SAM CHIRP FILTERS* 

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Abstract 

Matched filtering can be implemented in a variety of ways using an assortment of devices. One straightforward technique employs the SinonLiIMc structure as a programmable correlator. A

more versatile configuration, however, can be realized by the addition of chirp devices. With these devices, signal Fourier (chirp) transforms are readily obtainable and correlation follows with an inverse transformation of the product of transforms. This paper demonstrates the selective removal of jamming noise (by transform gating) for a continuous BSK Barker encoded binary input signal along with some probability of error data for this system. The output of this system, in the absence of noise, consists of a continuous stream of plus and minus correlation peaks. The technique for transforming continuous signals, based upon the knowledge of the region of transform validity, will be discussed.

Introduction

Chirp filters have proven to be extremely versatile devices specifically when utilized in a real-time Fourier (chirp) transformation system. 1-9 When properly implemented the output of such a system is an accurate time domain representation, over a finite period of time, of the Fourier transform (i.e. complex frequency spectrum) of the time-limited input signal. For signals much longer than the impulse response of the chirp filter, a time limiting function is impressed upon the signal and the resultant transform envelope is appropriately weighted. The output region of transform validity is a function of the width of the time limited input signal and the interaction time of the chirp filter. Furthermore, the magnitude of the resultant Fourier transform is the actual magnitude of this complex function and the phase is an intimate part of the phase of the modulating chirp. Although a complex function, the real and imaginary parts of the transform are in phase quadrature so that the output is always real, as it must be. These real and imaginary parts can be isolated from one another by use of properly phased chirps in a dechirping operation.

Since the Fourier transform obtainable with these devices is a satisfactory representation of a signal's frequency spectrum, in real time, it is extremely desirable to want to adjust input signal characteristics by direct modification of the signal's frequency spectrum, after which the modified signal can be recovered by an inverse transformation. 10 One simple form of spectrum modification is time gating, which can be used to realize adaptable filters by applying the Fourier transform and a timed pulse or pulses to a mixer or fast rf switch. Such a technique has been demonstrated for the selective removal of a narrowband interference from finite noncontiguous encoded data bits. 11 When the signal is isolated and of duration less than the interaction time of the chirp filters employed in the chirp transformer, the appropriation of its Fourier transform is straightforward with no interference from neighboring signals. However, contiguous data streams must be handled in a systemized orderly fashion, otherwise the output of the system would be erroneous. Since the entire signal transform is impossible to obtain, one must be satisfied with systematically taking the Fourier transform of small slices of the signal and performing operations on each of the individual transforms of the resultant continuous transform stream. Specifically the signal must be split up into two (or more) alternating streams so that transformation can be performed on each slice of the signal without interference from neighboring slices. This process will be elaborated upon in the next section and a representative system will be described showing actual results obtained. In particular this system will demonstrate the implementation of a contiguous data correlation receiver realized by multiplying the transforms of the coded data bits with the reference code transforms and then performing inverse transformations to result in the desired correlations. System performance will then be evaluated in terms of actual probability of error data obtained for the correlation receiver.

Continuous-Time Processing

There are a variety of receiver structures utilizing Fourier transformation and matched filtering. 4 All of these structures permit the adaptable notch filtering of a narrowband jamming signal and are otherwise optimum structures for reception in white Gaussian noise. The signal may be transformed, gated and then inverse transformed before
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can be studied in order to develop an understanding of the steps required to insure accurate correlation at the output to the receiver. Both \( f(t) \) and \( g(t) \) (a receiver-generated replica of the input signal where \( f(t) \) represents the incoming signal plus noise), existing over the range \( 0 < t < \Delta \), are first premultiplied by the chirp

\[
m(t) = \cos(\omega_s t - 2\pi^2 t^2)
\]

After these chirp modulated signals are passed through chirp filters having impulse responses of \( h(t) \), the outputs are given by

\[
A(t) = \int (\omega_s - 2\pi^2 t) f(2\omega_s t^2 - \omega_s t - 2\pi^2 t^2) e^{-j2\pi\omega_s t} dt + c.c. \quad (3)
\]

\[
B(t) = \int (\omega_s - 2\pi^2 t) g(2\omega_s t^2 - \omega_s t - 2\pi^2 t^2) e^{-j2\pi\omega_s t} dt + c.c. \quad (4)
\]

where

\[
F(\omega) = \int f(t)e^{-j\omega t} dt \quad (5)
\]

and

\[
G(\omega) = \int g(t)e^{-j\omega t} dt \quad (6)
\]

The Fourier transforms (and the complex conjugates) of these signals have been individually obtained with a time to frequency correspondence given by \( \omega_s t^2 + \omega_s t - 2\pi^2 t^2 \), valid over the region \( \Delta < t < \Delta + T \). These transforms are modulated by chirps that are identical to the impulse response of the chirp filter. The transform region of validity is determined as that time during which the entire time-limited signal is totally overlapped by the impulse response of the chirp filter in the convolution integral defining the output, so that the limits of this integral can be set to plus and minus infinity without error. Although the output begins \( \Delta + \Delta \) seconds after the signal ends, must be \( T + \Delta \) seconds wide, the valid region is only \( \Delta \)-seconds duration preceeded and proceeded by \( \Delta \)-wide invalid regions resulting from the sliding transform of the signal. In a sliding transform of this form every point not only represents a frequency component of a different delayed portion of the signal but the signal is also disappearing totally from the integral. In invalid region sliding transforms can also be considered as diagonal slices of a 3-dimensional running transform (magnitude versus frequency versus delay) of the signal. Some conditions on the magnitude of \( \Delta \) can now be considered. Since the invalid regions of the transformation can only be detrimental to the correlation process, if allowed to remain, they must be gated out prior to Fourier inversion. Since, however, it is known that the next signal bit begins at time \( t = \Delta \) with its total output beginning at time \( t = 2\Delta \), it can be seen that the overlap of the two outputs is \( \Delta \)-wide. It is therefore necessary that the condition

\[
T < \Delta < \Delta \quad (7)
\]

be met, otherwise the invalid transformation regions of neighboring bits will overlap into the valid regions and cannot be gated out. Furthermore, if \( R_f = 1/2\Delta \) is \( \omega_{zoom} \) and \( B \) is the minimum signal bandwidth required for satisfactory correlation characteristics, then

\[
T > B \frac{RT}{R_C} \quad (8)
\]

This simply states that the time duration of the valid region of the signal transform be at least wide enough to accommodate the bulk of the signal spectrum. It is not enough, however, that this region be of the proper width, but the signal transform must also be centered within this time window. The portion of the signal transform 'viewed' through this window is a function of the starting frequency of the modulating chirp so that, to center the transform, the following condition must be met, namely

\[
\omega_s t_0 + \frac{T + \Delta}{2} \quad (9)
\]

With this value of \( \omega_s \), the valid portion of the transform represents the signal spectrum over the frequency interval \(-5(\Delta - \Delta) < \omega < 5(\Delta - \Delta) \) where the chirp modulation extends over the interval \( 2\Delta < \omega < 2\Delta \) independent of \( \omega_s \). Both the positive and negative portions of the signal transforms should be retained otherwise, with the negative portion eliminated, the output would represent the autocorrelation of the complex function \( f(t) + j \hat{f}(t) \), where \( \hat{f}(t) \) is the Hilbert transform of \( f(t) \). Before continuing with the discussion of the correlating system of Fig. 3, a few additional comments should be made. First of all, it is straightforward to show that the chirp convolution process will result in terms representing the Fourier transform and its complex conjugate due specifically to the sum terms of the complex multiplication. However, the output also consist of integrals resulting from the differences between these integrals represent the Fresnel transform and complex conjugate of the signal given by

\[
F_B(2\omega_s t^2 - \omega_s t - 2\pi^2 t^2) = \int [f(t)e^{j\omega t}] e^{-j(2\omega_s t^2 - \omega_s t - 2\pi^2 t^2)} dt \quad (10)
\]

With \( \omega_s = \omega_{zoom} + \beta(T, \Delta) \), the valid portion of this transform represents the frequency interval \( 2\omega_{zoom} + \beta > \omega > 2\omega_{zoom} - \beta \). Since the Fresnel transform is the Fourier transform of \( f(t) e^{j2\beta t} \) or the convolution of \( F(\omega) \) and \( e^{j2\beta t} \), which will reside, in bulk, at the origin, the above integral results in the negligible high frequency portion of the Fresnel transform during the valid interval and can be ignored. This argument is supported by Fig. 4, where every successive trace represents a smaller value for \( \omega_{zoom} \) As \( \omega_{zoom} \) decreases the Fourier window represents a higher frequency portion (DC moves to the left) while the Fresnel window represents a lower frequency portion (DC moves to the right). As expected the Fresnel Transform makes its appearance in the final trace. Note that when a signal transform is intentionally taken by using identically sloped chirps the Fourier high frequency components will also be there. This will not occur, however, if the Fresnel is obtained by an rf modulation and chirp device combination. The second comment concern the fact that from a general rule of spectrum analysis a finite signal must have an infinite frequency spectrum. Although this is true, the dominant portion of almost any practical signal will reside in a well defined region of the frequency spectrum. The windowing of the Fourier transform will result in a convolution of the signal with a sinc function upon inversion, but in this case the distortion would be small. Since the convolution of a sinc function with two separate signals that are later convolved is the same as convolving this sinc function with the convolution of the two signals, i.e. 

\[
\sin(f_1 + f_2) = \sin(f_1)\sin(f_2) \quad (12)
\]

then the resultant correlation from our receiver will also be convolved with this sinc function which should be negligible for spectrums concentrated on one portion of the frequency axis.

Returning to the process depicted in Fig. 3, it is seen that the reference transform term is modified prior to multiplication with the signal transform term. This modification results in the selection of the reference
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transform form given by

$$C(t) = e^{i\left((2\omega_t - \omega_0 - 2\Delta) + 2\Delta t + 2t^2\right)} g(t) + c.c.$$  \(12\)

$$i + \Delta < t < i + T$$

Although multiplication with \(A(t)\) and further processing results in many terms, only those of importance, that lead to terms which fall within the passband of the transform inversion chirp filter, will be discussed. 'Multiplication' of \(A(t)\) and \(C(t)\) using a mixer results in the term with \(f_I\) modulation given by

$$D(t) = e^{i\left((2\omega_t - \omega_0 - 2\Delta) + 2\Delta t + 2t^2\right)} g(t) + c.c.$$  \(13\)

$$i + \Delta < t < i + T$$

Multiplying this by the chirp modulation \(e^{i(2\omega_t - \omega_0 - 2\Delta) + 2\Delta t + 2t^2)}\) results in the terms

$$J(a,n,2\Delta) g(t) = e^{i\left((2\omega_t - \omega_0 - 4\Delta) + 2\Delta t + 2t^2\right)} g(t) + c.c.$$  \(14\)

Convolving this with \(h(t)\) one obtains

$$r \ast g(t - \frac{a_n - a_p}{2}) 2\Delta < t < 2\Delta + T + \Delta$$

or the delayed version of the correlation of \(r(t)\) with \(g(t)\) valid in the time interval \(2\Delta + T < t < 2\Delta + 2T + \Delta\) with a chirp modulation that can be removed by a successive coherent detection with the same chirp. Note that since the region of validity of this correlation is only as wide as the original signal, only half of the correlation can be viewed. Low level high frequency portions of the transform need not be used so that this much more of the inverse can be seen. But, since the correlation peak is the only portion of the output required in making a bit decision, the sidelobes can be ignored. The overlap of the invalid regions into the valid ones, in this case, lead to the same criterion utilized to previously concerning the size of \(\Delta\). The output correlations, as with the transforms, may be centered within their valid regions by the condition

$$t = \frac{a_n - a_p}{2} 2\Delta < t < \Delta$$

or

$$a_p = a_n - 3(2\Delta - \Delta)$$

With this value of \(a_p\), the valid portion of the output represents the correlation over the time interval \(\Delta < t < \Delta\) so that the center of this window represents the peak of the signal autocorrelation. The correlation chirp modulation for \(a_n = a_p - \frac{1}{2}(\Delta + \Delta)\) extends in frequency over the valid region \(\omega_0 + (\Delta - \Delta) < \omega < \omega_0 + (\Delta + \Delta)\). If one now selects the conventional value of \(\Delta = \frac{1}{2}\), then the valid regions for the transform and correlation are both \(\frac{1}{2}\) seconds wide. Furthermore, if \(i = 0\) then the transform of one bit will exactly coincide with the time of occurrence of the next bit and the correlation will coincide with the bit following this one. Also, the signal modulating chirp \(m(t)\) for the transform modulating chirp \(m(t)\) and the correlation demodulating chirp \(m(t)\) have frequency variations over the \(\frac{1}{2}\) second wide period given by

$$\omega_0 + \frac{1}{2}(\Delta - \Delta) < \omega < \omega_0 + \frac{1}{2}(\Delta + \Delta)$$

(17)

In this case the four alternating chirp streams \(m(t)\) and \(m_2(t)\) can be derived from the two alternating chirp streams \(m_2(t)\) by multiplication with \(\cos(\omega_0 - \omega)\) where filtering of the high frequency chirp occurs at the chip device. It must be mentioned, however, that \(i\), in most (or all) cases, is not negligible and can be an appreciable percentage of the overall chirp filter impulse response. It may therefore be desirable to design devices with initial delays exactly equal to \(\frac{1}{2}\) so that the transforms evolve in the second time period following the bit period and likewise for the correlation. Under these conditions all the chirp streams can be obtained by the technique just mentioned. It is important to note that the mixing operation is a rather crude approximation to the required multiplication of transforms. This can be seen in Fig. 5 where the inputs to a mixer were a triangular \(r\) and a constant \(f_I\). When the constant \(r\) saturates the mixer the output is a fairly accurate representation of the triangular \(r\) (not shown but scribbled onto the picture). For small constant \(r\), however, the output deviates markedly from the proper triangular \(r\). This problem is alleviated by the use of wide band four-quadrant multipliers. This problem does not arise in any of the other mixing processes in the system since one of the signals can be made large enough to always saturate the mixer.

Lastly, there is a item of concern as to the method used in the generation of the chirp streams. Unless stable VCO's are available with equally stable ramp generators to feed these, the best method of chirp generation is the 'impulsing' of chirp devices identical to those in the system. In this way (if the impulses are stable) every chirp in a stream is coherent with one another and their slope and phase variations closely match with every device in the system. Describing of the correlations in this case results in the proper pulse polerity since all chirps were generated and synchronized at the receiver. Large VCO chirp variations cause erroneous polarity fluctuations from a coherent detection or describing process. Furthermore, a slight mismatch, \(\delta\), in chirp slope results in the taking of a Fresnel transform instead of a Fourier. This would appear as follows

$$F_\delta(t) = e^{-i\int (2\omega_t - \omega_0 - 2\Delta) + 2\Delta t + 2t^2) g(t) + c.c.$$  \(18\)

$$i + \Delta < t < i + T$$

so that one obtains a Fourier transform that is convolved with the chirp \(e^{-i\int (2\omega_t - \omega_0 + 2\Delta) + 2\Delta t + 2t^2) g(t) + c.c.$$  \(19\)

A distortion that is appreciable for large \(\delta\).

System Description

The implementation of the continuous correlation receiver used in this study is depicted in Fig. 6 with actual outputs comparable to Fig. 2 shown in Fig. 7. Fig. 8 shows the correlation quality obtainable when a single small time duration 7-bit Barker Code is applied to a finely tuned non-contiguous version of this system. In this case chirp filters were used to generate the chirp streams and coherent detection followed smoothly. Due to lack of a sufficient number of devices, however, the continuous system was built around a wideband VCO, and although the chirp could be removed, the correlation peaks varied wildly. The six chirp filters in this system were borrowed from Andersen Laboratory. Although these INDOES are not SAW, but bulk wave devices, their performance is equivalent if not identical to their SAW counterparts. All the devices had approximately 120ns wide chirp durations, 6 MHz dispersion over this time interval, 15 MHz center frequency and an initial delay of about 30ns. Although the delay was appreciable, the transforms and convolutions were tuned in such a way as to keep the dominant portions within the bit time intervals. Even though some of each transform and correlation extended into the next bit interval, multi-
System Evaluation

Since the most meaningful performance measure of a digital communication system is the probability of error it would seem adequate to evaluate the correlation system on this premise. Although, as previously noted, error counting was not possible with our contiguous system, probability data was obtained for the non-contiguous version of the system. Our argument is that the contiguous system, as structured, can be recognized as the combination of two non-contiguous systems, and the same probability of data results apply. One may argue that in a contiguous correlating system, intersymbol interference must be accounted for. However, as previously noted, the correlations, as with the transforms, are never allowed to overlap. Therefore, we say that intersymbol interference will not exist and that the non-contiguous data is sufficient to describe the performance of the contiguous system.

For a binary phase-shift modulated signal detected in additive white Gaussian noise, it is well known that the optimum receiver is a matched filter. For such a receiver, the probability of error is given by

$$ P_e = \frac{\pi}{2} \int_{-\infty}^{\infty} \frac{1}{\pi} \left| \frac{e^{-j\pi f^2/2}}{f} \right|^2 df $$

where $E$ is energy per bit of the signal, $\phi(x)$ is one-sided spectral density of the noise,

$$ \phi(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy $$

Employing a matched filter in the receiver optimizes the probability of error performance and a notch filter partially removes large error sources such as narrow-band jammers. Fig. 10 shows the probability of error for a correlating receiver with a signal-to-noise ratio of about 18 dB, the optimal performance. Curve B, within 1/2 dB of curve A, is the actual system performance. Curve C shows the damaging effects of a jammer on the probability of error with a jammer signal ratio of about 18 dB. Curve D shows the drastic improvement in performance by the time gating (band pass filtering) of the major portion of the jammer frequency components from the input signal transform.

Conclusions

A detailed discussion of a Fourier transform correlation receiver was given with emphasis on contiguous operation. Due to the lack of a sufficient number of devices the system could not be built in its desired form, however, its performance was as expected and therefore satisfactory. The probability of error data specifically shows the potential of the SAW system as a jam resistant spread spectrum receiver. The ability to obtain the Fourier transform of a signal at high frequencies using SAW chirp filters, facilitates the performance of spectral processing at the receiver in real time.

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References

Figure 5. Multiple Exposure Demonstrating The Distortion Imparted At A Mixer Output When One Of The Signals Is A Ramp While The Other Is A Low Level, non-saturating RF. The Different Exposures Represent Different RF Levels. A Saturating RF Would Produce The Scribed-In Ramp.

Figure 6. Actual Implementation Of The Contiguous Correlating Receiver Utilizing Six Chirp Devices And A Gated Chirp Generator.

Figure 7. Actual Outputs Of The Contiguous Correlating Receiver.

Figure 8. Barker Code Correlation Obtained From A Well Tuned Non-contiguous Receiver.

Figure 9. Transform-Correlation Pairs Of A Carrier Modulated Encoded Data Stream Showing No Jammer (First Two Traces), A Jammer (Second Two Traces) And The Jammer Removed By Time Gating (Last Two Traces).

Figure 10. Probability of Error Curves For The Non-contiguous Correlating Receiver.