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The roots of this work are from a paper by Richard Simon [1], "Stationary Properties of a Two-Echelon Inventory Model for Low Demand Items." A subsequent paper by Kruse and Kaplan [2] claimed to correct Simon's results, but Kruse [3] points out that Simon's results are, nevertheless, correct in spite of the apparent logical error in their derivation. He also notes that both Simon's and Kruse and Kaplan's results reduce to a simple and intuitive form.

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ABSTRACT (CONT)

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AN EXACT N ECHELON INVENTORY MODEL: THE SIMPLE SIMON METHOD

by

U.S. ARMY
INVENTORY
RESEARCH
OFFICE

MARCH 1979

ROOM 800
U.S. CUSTOM HOUSE
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TECHNICAL REPORT

BY

W. KARL KRUSE

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1. Background

The roots of this work are from a paper by Richard Simon [1], "Stationary Properties of a Two-Echelon Inventory Model for Low Demand Items." A subsequent paper by Kruse and Kaplan [2] claimed to correct Simon's results, but Kruse [3] points out that Simon's results are, nevertheless, correct in spite of the apparent logical error in their derivation. He also notes that both Simon's and Kruse and Kaplan's results reduce to a simple and intuitive form.

We take a new look at the model in this paper with the intent of providing a more revealing analysis than appears in either [1] or [2]. We then show how these 2-echelon results can be extended to model an N-echelon inventory system in a manner similar to the way the METRIC model uses Palm's Theorem to accommodate an indefinite number of echelons.

With apologies to Dr. Simon, we call this the Simple Simon model, not only because it is an apt descriptor, but also because that designation, as we have learned, is irrepressible.

2. System Description

Simon's original work was done for RAND and, consequently, uses Air Force terminology. We will continue that. There is a lowest echelon, composed of several independent bases, which is the source of failures for a particular item. When an item fails, it is inspected at the base and a decision is made to repair the item at the base, repair the item at the depot, or forego repair and dispose of the carcass. Spares may be held at base or depot. If the failed item is to be repaired at the base, a replacement item is supplied from the base spare stock, and the failed item enters the spare stock after being repaired itself. Spare stock at the base is issued on a FIFO basis and all demands are backordered until filled. If the item is not to be repaired at the base, a spare is still supplied from base stock and a replenishment spare is requested from the depot. Demands on the depot are also filled with FIFO priority and backordered until filled. Items which the depot repairs go into its spare stock. Other replenishment comes from an exogenous source with an infinite supply.

3. Assumptions and Definitions

For the reader's convenience, we repeat the assumptions and definitions from [1] and [2] below:

r_j = the probability that an item failed at base j will be repaired at base j

ρ = the probability that a failed carcass that is not base reparable will be depot reparable

$D_j(t),\ t \geq 0$ = the demand process at base j (j = 1,2,...J). This is assumed to be a simple Poisson process with rate λ_j.

$D_j^B(t),\ t \geq 0$ = the stochastic process of units repaired at base j.

$D_j^D(t),\ t \geq 0$ = the stochastic process of units sent to the depot from base j for repair.

$D_j^C(t),\ t \geq 0$ = the stochastic process of units condemned at base j.

$$D_o^D(t) = \sum_{j=1}^{J} D_j^D(t)$$

$$D_o^C(t) = \sum_{j=1}^{J} D_j^C(t)$$

$$D_o(t) = D_o^C(t) + D_o^D(t)$$

$B_j(t)$ = the number of backorders at location j at time t; j = o denotes the depot.

R_j = the deterministic repair time at facility.

t_j = the deterministic delivery time from depot to base j.

t_o = the deterministic procurement lead time from the exogenous source to the depot. We assume $R_o < t_o$.

S_j-1, S_j = replenishment policy used at base j.

s_o, S_o = replenishment policy used at depot.

$N(t_1,t_2) = N(t_2) - N(t_1)$ for any stochastic process.

4. Stationary Distributions of Base Backorders

As a consequence of the S_j-1, S_j inventory policy at base j we have that

$$B_j(t) = X_j(t) + Y_j(t) - S_j$$

where

$X_j(t)$ = amount due in to base j at time t from the depot.

and $Y_j(t)$ = amount in repair at base j at time t.

$B_j(t) < 0$ indicates stock on hand.

By virtue of the assumptions, $X_j(t)$ and $Y_j(t)$ are independent random variables, and for $t > R_j$

$$\Pr[Y_j(t) = k] = e^{-\lambda_j r_j R_j} (\lambda_j r_j R_j)^k / k!$$

Now $X_j(t)$ for $t > t_j$ is composed of two mutually exclusive and independent elements $X_{j,1}(t)$ and $X_{j,2}(t)$ where

$X_{j,1}(t)$ = amount due in to base j of those demands placed on the depot in $(t-t_j, t)$

and

$X_{j,2}(t)$ = amount due in to base j of those demands placed on the depot prior to $t-t_j$.

Clearly,

$$\Pr[X_{j,1}(t) = k] = e^{-(\lambda_j)(1-r_j)(t_j)} ((\lambda_j)(1-r_j)(t_j))^k / k!$$

The distribution of $X_{j,2}(t)$ remains to be found. Note that $X_{j,2}(t)$ is also the number of backorders at the depot at time $t-t_j$ which are for base j. In [1] and [2], $X_{j,2}(t)$ was called $E_j(t)$ and its distribution was tediously derived by analyzing the composition of the demand sequences in the interval $t-t_o-t_j$ to $t-t_j$ as compared to the depot asset position at time $t-t_o-t_j$. The analyses made a careful distinction between the demand process associated with reparable carcasses and the process associated with condemnation. Motivating this distinction were the realizations that the number of depot backorders at $t-t_j$ were dependent on only $D_o^C(t-t_o-t_j,t-t_j) + D_o^D(t-R_o-t_j,t-t_j)$, and in [2] that $D_o^D(t-t_o-t_j,t-R_o-t_j)$ could affect to whom those backorders at $t-t_j$ were due. However, a seemingly innocuous assumption that the probability that a failed carcass that is not base reparable will be depot reparable, i.e. ρ, is base independent allows a dramatic simplification in the results as well as the analysis. Moreover, as noted in [3], this assumption explains why the results in [1] and [2] are the same upon algebraic reduction.

Suppose there is a demand on the depot and we wish to know the probability that its source was base j. If the demand were associated with a reparable carcass, then because that process is Poisson at each base the probability is

$$\lambda_j(1-r_j)\rho / \sum_k \lambda_k(1-r_k)\rho = \lambda_j(1-r_j) / \sum_k \lambda_k(1-r_k)$$

If the demand were associated with a condemned carcass, the probability is likewise

$$\lambda_j(1-r_j)(1-\rho) / \sum_k \lambda_k(1-r_k)(1-\rho) = \lambda_j(1-r_j) / \sum_k \lambda_k(1-r_k)$$

So, assuming that ρ is base independent means that the type of demand does not affect the probability that the demand comes from a given base.

A natural way to derive the distribution of $X_{j,2}(t)$ is through its conditional distribution on total depot backorders at $t-t_j$.

That is,

$$\Pr[X_{j,2}(t) = k] = \sum_{n=k}^{\infty} \Pr[X_{j,2}(t) = k \mid B_o(t-t_j) = n]\Pr[B_o(t-t_j) = n]$$

Now, given that $B_o(t-t_j) = n$ provides probability information on the number of demands associated with a reparable carcass and the number associated with a condemnation which are backordered at $t-t_j$. But, by the argument above this information is irrelevant for determining the probability a given backorder is for a given base. Consequently, and because the demand processes are Poisson, we have that

$$\Pr[X_{j,2}(t) = k \mid B_o(t-t_j) = n] = \binom{n}{k} P_j^k(1-P_j)^{n-k}$$

where $P_j = \lambda_j(1-r_j) / \sum_k \lambda_k(1-r_k)$

Moreover

$$\lim_{t\to\infty} \Pr[B_o(t) = n] = \sum_{a=s_o}^{S_o} \frac{1}{(S_o-s_o)} \Pr[D_o^C(t-t_o-t_j, t-t_j) + D_o^D(t-R_o-t_j, t-t_j) = a + n]$$

To compute the stationary distribution of $B_j(t)$ then, for $j \geq 1$, involves convoluting the stationary distributions of Y_j, $X_{j,1}$ and $X_{j,2}$. Of course, the distributions of Y_j and $X_{j,1}$ themselves convolute to a Poisson with mean $\lambda_j(t_jR_j + (1-r_j)t_j)$. Unfortunately, the convolution of $X_{j,2}$ with the sum of Y_j and $X_{j,1}$ must be done numerically. To our knowledge, there is no reduction to a simple form.

These computations can be done quickly by computer using an approach which stores the probability arrays of $B_o(t)$, $(Y_j(t) + X_{j,1}(t))$, and $X_{j,2}(t)$. For higher demand items, these arrays will overflow and a more storage conscious approach is needed.

5. <u>Extension to N Echelons</u>

Suppose we consider any imbedded two echelon structure within an N Echelon inventory system of arborescent structure (see figure below) for which the above assumptions hold. With regard to our assumptions, we interpret the term "condemnation" to represent the situation when an item cannot be repaired at the top echelon in the imbedded two echelon structure and must be sent to the next echelon. Further, suppose that the stationary distribution of backorders is known for the upper echelon in the imbedded two echelon structure. Then, we can use the analysis in the previous section to get the stationary distribution of backorders at each lower echelon location. This allows us to go one echelon lower in the structure whereupon the present lower echelon locations each become an upper echelon location in new imbedded two echelon structures.

This method is analagous to the approach used in METRIC to model N echelons except that Simple Simon explicitly uses the distribution of upper echelon backorders while METRIC uses only their expected value thru Palm's Theorem.

THREE-ECHELON ARBORESCENT STRUCTURE

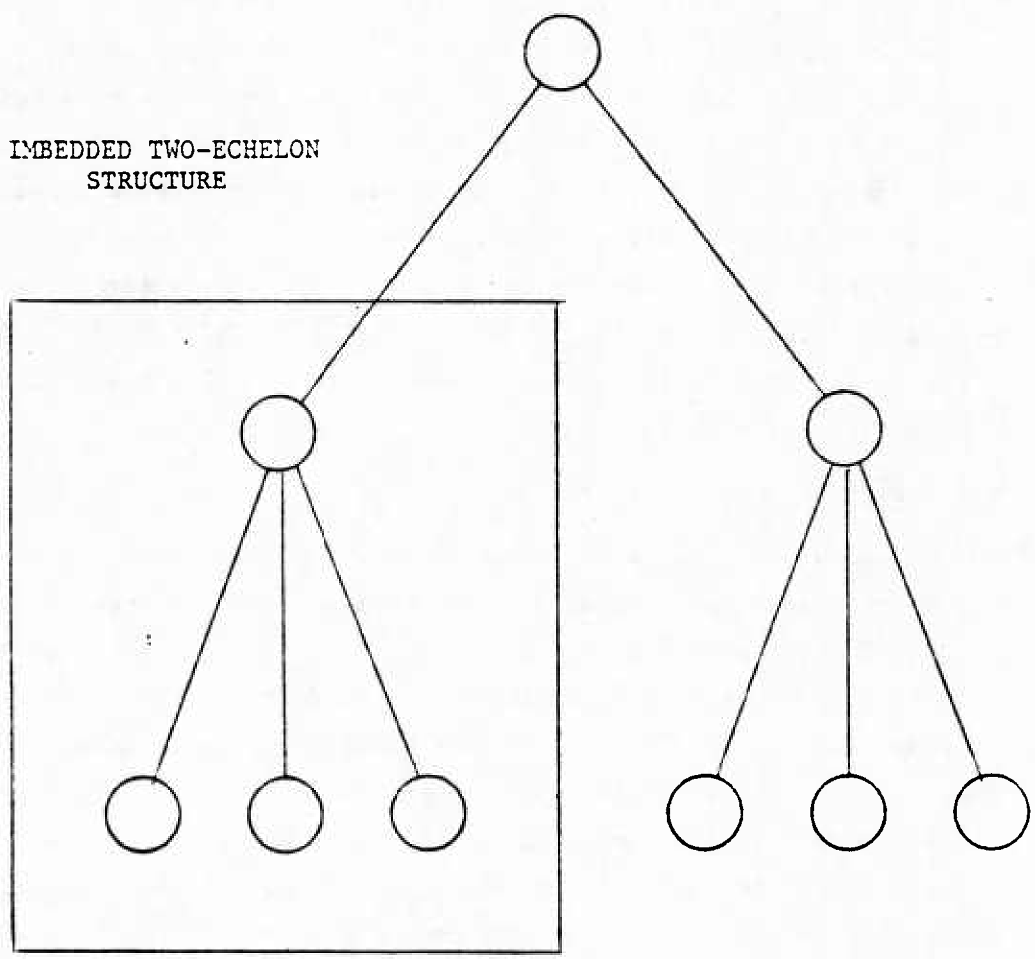

FIGURE 1

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3 W. Karl Kruse, "Letter to the Editor of Operations Research", to appear in early 1979.

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