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INTERIM REPORT

THE INFLUENCE OF AN ACOUSTICALLY THIN DOME UPON THE PARAMETRIC GENERATION EFFICIENCY OF A PLANAR ARRAY (U)

Project No. 11131601 Task No. 16833

Submitted to

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ABSTRACT

Numerical results are presented for a planar thin dome model showing how the dome influences the parametric generation efficiency at steering angles of 0~~°~~, 20~~°~~, and 45~~°~~ deg. An extension of the model to the thick dome case is also included. During the next quarter, a comparison will be made between the thick and thin dome cases in order to develop a more quantitative criterion for judging whether a dome is acoustically thin for the higher primary frequencies of a parametric sonar.

LIST OF SYMBOLS

$\hat{P}_{2-}$	time independent pressure amplitude of difference frequency signal
P_o	initial pressure level of each primary
k_-	difference frequency wave number
α_1, α_2	attenuation constants at the primary frequencies
θ	steering angle
ϕ	field points angular coordinate
ψ	virtual source point's angular coordinate
$R \doteq$	$r-r_o \cos\left(\phi-\psi\right)$
$R_o \doteq$	$r+r_o \cos\left(\phi-\psi\right)$
r	range to field point from dome origin
r_o	range to virtual array source point from dome origin
β	nonlinearity parameter of sea water

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ω_1, ω_2	angular primary frequencies
d	dome-transducer spacing
ρ_o	density of sea water
c_o	small signal sound speed in sea water

$$D = i\eta \left[e^{-i\eta d} + \frac{\eta}{\omega^2 \rho} \left(-\sigma\omega^2 + HK^4 \right) \sin \eta d \right]$$

$$K = k \sin\theta$$

$$\eta = k \cos\theta$$

$$\sigma = \rho_s h$$

h	dome plate thickness
ρ_s	dome density

$$H = \frac{Eh^3}{12(1-\nu^2)}$$

ν	Poisson's ratio
E	Young's Modulus
D^*	complex conjugate

1.0 INTRODUCTION

During the third quarter of the Nonlinear Dome Effects Study some numerical results have been obtained which give insight into the influence of a thin dome upon parametric generation efficiency.

2.0 THIN DOME RESULTS

The infinite planar dome-transducer model is shown in Fig. 1. The transducer is simultaneously excited at two frequencies.

Our simplified model led to the following integral for the normalized difference frequency pressure distribution[1]

$$\frac{\hat{P}_{2-}}{P_o} = A \int_0^{\pi}\int_0^{r_o^*} \left(\frac{e^{i k_- R}}{\sqrt{R}} - \frac{e^{i k_- R_o}}{\sqrt{R_o}} \right) e^{-(\alpha_1+\alpha_2) z} \cdot e^{-i k_- r_o \sin(\psi+\theta)} \, r_o dr_o \, d\psi . \tag{1}$$

Fig. 2 shows the relationship between R_o, R, r, r_o and the angles ϕ and ψ. In Eq. (1) A is given by

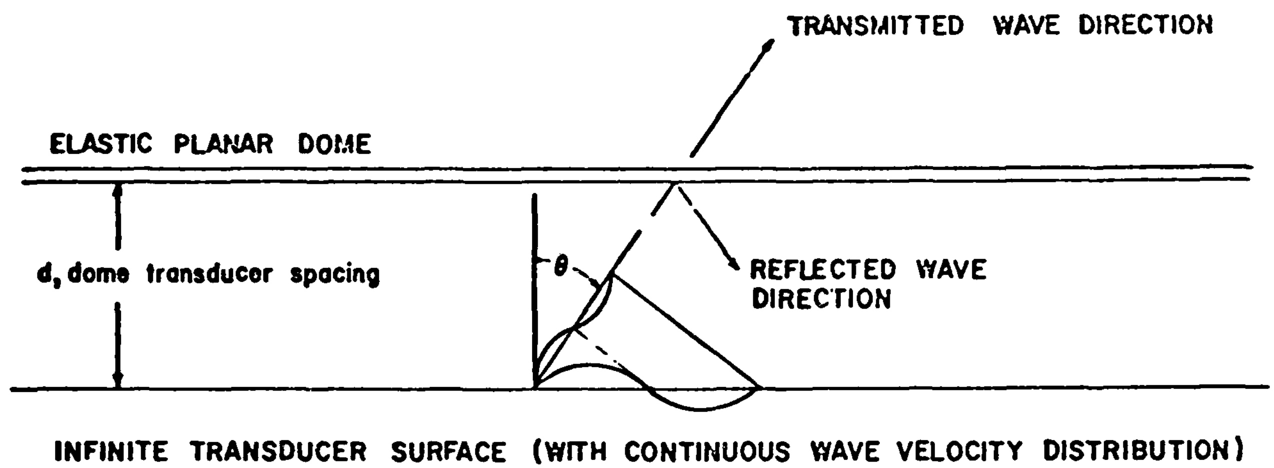

FIG. 1 - GEOMETRY OF INFINITE PLANAR DOME – TRANSDUCER MODEL

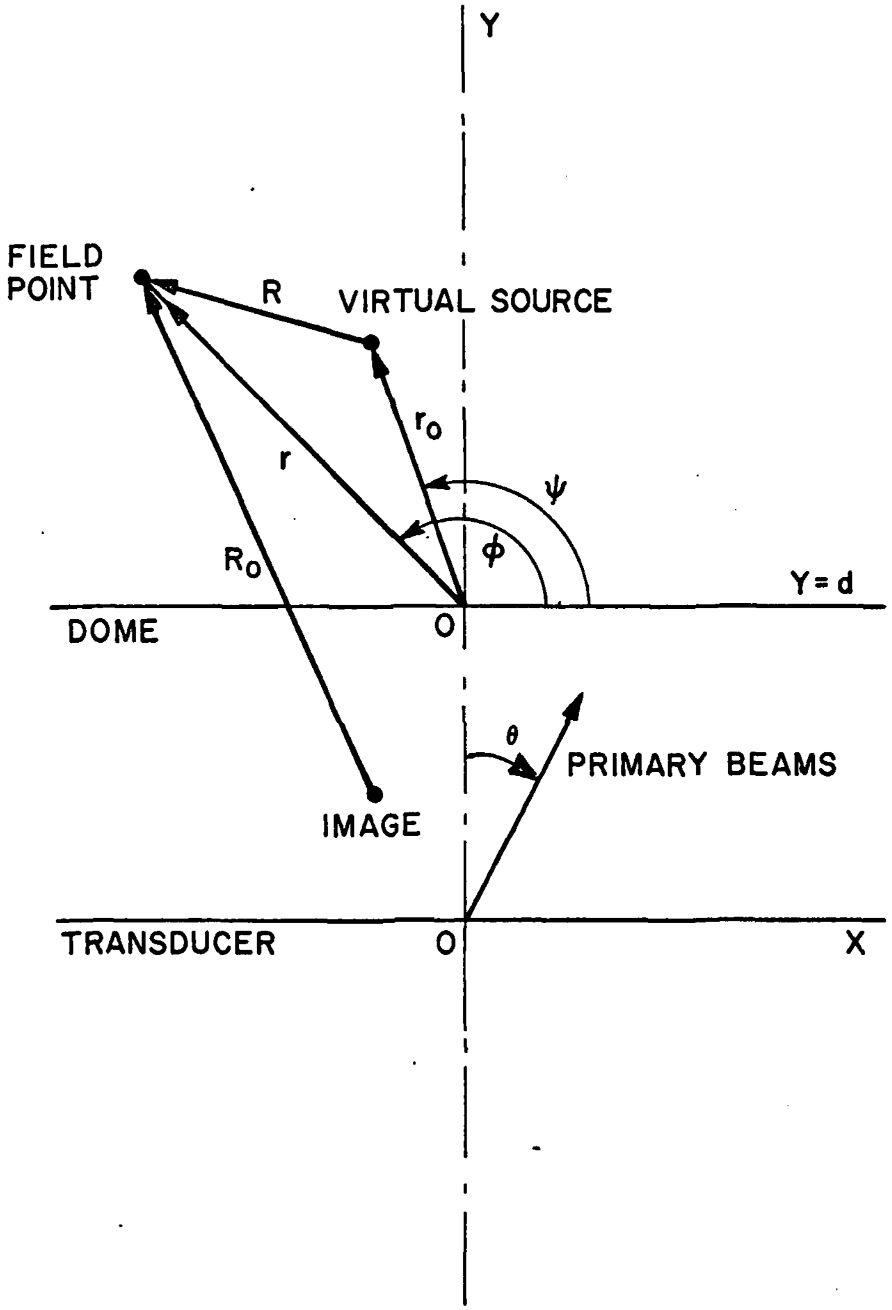

FIG. 2 GEOMETRY PERTINENT TO Γ_2 AND THE CALCULATION OF $P_2(\omega_-)$.

$$A = \frac{\beta P_o k_-^{3/2} \omega_1 \omega_2 \sqrt{2\pi} \; e^{i\left(\frac{\pi}{4} + dk_- \cos\theta\right)}}{\rho_o c_o^4 D_1 D_2^*} \qquad (2)$$

and z represents the attenuation path length of the primary beams to the source point as shown in Fig. 2.

$$z = \frac{r_o \sin\psi}{\cos\theta} \qquad (3)$$

In order to evaluate the integral in Eq. (1) it was first reexpressed in rectangular coordinates. Integration with respect to x was done analytically and integration with respect to y was done numerically using Gaussian quadrature. A representative field point was taken at 2500 yards along the y axis from the dome. The virtual array was arbitrarily truncated at the range

$$r^* = \frac{1}{\alpha_1 + \alpha_2} \qquad (4)$$

where α_1 and α_2 are the attenuation coefficients at the primary frequencies. This is justified since we are only interested in relative amplitudes of pressure as the steering angle and dome parameters are varied.

Table I gives values for $\frac{\hat{P}_{2-}}{P_o}$ for three choices of the steering angle θ for a ¼" steel plate placed 10" in front of the transducer. The no dome case is obtained by letting the dome thickness h be zero which simplifies D_1 and D_2^* . Originally, it had been thought that a comparison could be made with the infinite plane wave solution to Burgers' equation which was quoted in the Semi Annual Report[1]. This is not feasible because we would have to evaluate Eq. (1) in the limit as $r^* \longrightarrow \infty$.

Table I gives $\frac{\hat{P}_{2-}}{P_o}$ computed by Eq. (1) at a point 2500 yards from the dome along the y axis as a function of steering angle and primary frequency. Even though we have assumed that the dome is acoustically thin, it is evident that the dome significantly reduces the parametric generation efficiency for steering angles of 0^o and 20^o. For a steering angle of 45^o with primary frequencies of 50 kHz and 60 kHz there is actually a small enhancement. We have no reason to doubt this particular calculation. Both the difference frequency and the steering angle appear under the integral sign in Eq. (1) and so it is not intuitively obvious how

Steel Plate Dome $\frac{1}{4}''$ Thick 10'' From Transducer			
Primary Frequencies kHz	Steering Angle θ		
	0^o	20^o	45^o
50,60	1.9 10^{-2}	3.5 10^{-5}	8.5 10^{-5}
40,60	2.75 10^{-3}	6.5 10^{-6}	5.9 10^{-6}
40,70	1.21 10^{-2}	3.6 10^{-6}	1.32 10^{-5}
No Dome (h = 0)			
	0^o	20^o	45^o
50,60	3.7 10^{-1}	1.5 10^{-4}	6.7 10^{-5}
40,60	2.5 10^{-1}	2.1 10^{-4}	7.7 10^{-5}
40,70	4.1 10^{-1}	2.0 10^{-4}	7.2 10^{-5}

TABLE I. $\frac{\hat{P}_2}{P_o}$ on the y axis (see Fig. 1) at r = 2500 yards. SPL at transducer is 111.6 dB ref 1μ bar.

$\frac{\hat{P}_{2-}}{P_o}$ will vary with these two parameters.

In conclusion, we can say that in most cases the amplitude of $\frac{\hat{P}_{2-}}{P_o}$ is quite sensitive to variations in ω_-, ω_1, ω_2 and θ.

3.0 EXTENSION OF THE MODEL TO THE THICK DOME CASE

For conventional sonars the assumption that the dome is acoustically thin has been a good one. Parametric sonars require two high frequency primaries. Thus the thin dome assumption may not always be justified. In the past, a dome has been assumed to be acoustically thin if its thickness is $\frac{1}{10}$ th of the radiating wave length. It would be worthwhile to check the validity of such a criterion. Consequently, the model has been reformulated for a thick dome. The integral in Eq. (1) is unchanged. Only the coefficient A is modified and this is included in Appendix A.

4.0 ACKNOWLEDGEMENT

The authors wish to thank Mr. E. MC Donald for skillfully executing the numerical integration of Eq. (1).

5.0 BIBLIOGRAPHY

1. B. Cary and B. Hamilton, "A Planar Dome Transducer Model" NAVSHIPS Contract NOOO24-73-C-1131 TRACOR Document No. T-73-NJ-4002-U, May, 1973.

2. J. Horton, Fundamentals of Sonar (U.S. Naval Institute, Annapolis, Md., 1957), p. 81.

APPENDIX A

To extend our plane model to the thick dome case we draw upon Brekhovskikh[1] and Van Buren[2]. We retain the same nomenclature as Van Buren[2]. Subscripts L and S refer to longitudinal and shear waves respectfully. Region 1 extends from the dome to the transducer.Region 2 lies inside the dome and region 3 extends beyond the dome. Brekovskikh[1] gives the expressions for the reflection and transmission coefficients. We need to determine the amplitude of the transmitted wave in region 3 in order to compute the virtual array. When media 1 and 3 are identical liquids, as in our case, then the reflection coefficient is given by[1]

$$\frac{R}{I} = \frac{i\left(N^2 - M^2 - 1\right)}{2M + \left(N^2 - M^2 + 1\right)i} \tag{1}$$

and the transmission coefficient by

$$\frac{T}{I} = \frac{2N}{2M + \left(N^2 - M^2 + 1\right)i} \; . \tag{2}$$

Here I, R, and T refer to the amplitudes of the incident, reflected, and transmitted waves. M and N will be defined later. The boundary condition at the transducer face is:

$$\frac{1}{i\omega\rho_1}\left(\frac{\partial\hat{P}}{\partial y}\right)_{y=0} = V_o e^{-ikx\sin\theta_{L1}} \quad . \tag{3}$$

Here $\hat{P}$ is the steady state pressure, k the wave number, ρ_1 the density, ω the angular frequency, and θ_{L1} the angle of incidence in region 1 upon the dome. In region 1, we assume that $\hat{P}$ has the following form:

$$\hat{P} = \left(Ie^{i\eta y} + Re^{-i\eta y}\right)e^{-iKx} \quad . \tag{4}$$

Where $\eta = k\cos\theta_{L1}$ and $K = k\sin\theta_{L1}$ Eqs. (3) and (4) can be used to relate R and I yielding

$$R = I - \left(\frac{\omega\rho_1}{\eta}\right)V_o \quad . \tag{5}$$

By Eqs. (5) and (1), I is found to be

$$I = -\left(\frac{\omega\rho_1 V_o}{\eta}\right)\left[\frac{1}{\frac{i\left(N^2 - M^2 - 1\right)}{2M + i\left(N^2 - M^2 + 1\right)} - 1}\right]. \tag{6}$$

The quantities N and M are defined by

$$N = \frac{Z_{L2} \cos^2 2\theta_{S2}}{Z_{L1} \sin Q_L} + \frac{Z_{S2} \sin^2 2\theta_{S2}}{Z_{L1} \sin Q_S} \tag{7}$$

$$M = \left(\frac{Z_{L2}}{Z_{L1}}\right)\left(\cos^2 2\theta_{S2}\right) \cot Q_L + \left(\frac{Z_{S2}}{Z_{S1}}\right)\left(\sin^2 2\theta_{S2}\right) \cot Q_S \tag{8}$$

and $$Q_L = d\ k_{L2} \cos \theta_{L2} \tag{9}$$

$$Q_S = d\ k_{S2} \cos \theta_{S2} \tag{10}$$

where d is the dome thickness. In the above formulae Z is the acoustic impedance and is defined by

$$Z = \frac{\rho V}{\cos \theta} \quad .$$

The velocities for longitudinal and transverse waves in the dome are given by

$$V_{L2} = \sqrt{\frac{\lambda + 2\mu}{\rho_2}} \tag{11a}$$

and $$V_{S2} = \sqrt{\frac{\mu}{\rho_2}} \tag{11b}$$

λ and μ are the Lamé constants given by

$$\lambda = \frac{E\nu}{\left(1+\nu\right)\left(1-2\nu\right)} \tag{12}$$

and

$$\mu = \frac{E}{2\left(1+\nu\right)} \tag{13}$$

where ν is Poisson's ratio and E is Young's Modulus.

By Snell's law, we have

$$\sin\theta_{S2} = \left(\frac{V_{S2}}{V_{L1}}\right)\sin\theta_{L1} \tag{14a}$$

and

$$\sin\theta_{L2} = \left(\frac{V_{L2}}{V_{L1}}\right)\sin\theta_{L1} \quad . \tag{14b}$$

The wave numbers for the transverse and longitudinal waves in the dome are given by

$$k_{L2} = \frac{\omega}{V_{L2}} \tag{15a}$$

and

$$k_{S2} = \frac{\omega}{V_{S2}} \quad . \tag{15b}$$

We can rewrite I from Eq. (6) in the form

$$I = +\left(\frac{\omega \rho_1 V_o}{\eta}\right)\left(a + ib\right) \tag{16}$$

where

$$a = \frac{\left(N^2 - M^2\right)^2 - 2N^2 + 6M^2}{4M^2 + 4} \tag{17a}$$

and

$$b = \frac{2M\left(N^2 - M^2 - 1\right)}{4M^2 + 4} \quad . \tag{17b}$$

Eq. (2) can be rewritten as

$$T = 2NI\left(e - if\right) \tag{18}$$

with

$$e = \frac{2M}{4M^2 + \left(N^2 - M^2 + 1\right)^2} \tag{19a}$$

and

$$f = \frac{-\left(N^2 - M^2 + 1\right)}{4M^2 + \left(N^2 - M^2 + 1\right)^2} \tag{19b}$$

The calculation for $\frac{\hat{P}_{3-}}{\hat{P}_o}$ proceeds just as it did before in the thin dome case. The Green's function is unchanged. The double integral for $\frac{\hat{P}_{3-}}{\hat{P}_o}$ will be the same except that the coefficient A ahead of the integral will now be modified. For the thick dome

$$A = \frac{\beta \hat{P}_o k_-^{3/2} \omega_{31} \omega_{32} \sqrt{2\pi} e^{i\pi/4}}{\rho_1 V_1^4 \eta_{31} \eta_{32}} U . \qquad (20)$$

The first digit of the subscripts denotes the region (3 is beyond the dome) and the second digit indicates the primary frequency. The difference frequency wave number k_- is defined by

$$k_- = k_2 - k_1 . \qquad (21)$$

It is assumed that $\omega_2 > \omega_1$. N and M given by Eqs. (7) and (8) are functions of the frequncy via Q_L and Q_S given by Eqs. (9) and (10). In order to find U we must compute $T_{31} T_{32}^*$ from Eq. (18). From here on, we shall omit the first subscript 3 indicating the region beyond the dome and simply retain the second digit to refer to the appropriate primary frequency. In view of Eqs. (17a,b), we can write

$$T_1 T_2^* = \frac{2 N_1 N_2 \rho_1^2 V_o^2 \omega_1 \omega_2}{\eta_1 \eta_2} \left(H + Ji \right) \tag{22}$$

where

$$H = \left(a_1 a_2 + b_1 b_2\right)\left(e_1 e_2 + f_1 f_2\right) - \left(b_1 a_2 - b_2 a_1\right)\left(f_2 e_1 - f_1 e_2\right) \tag{23a}$$

$$J = \left(f_2 e_1 - e_2 f_1\right)\left(a_1 a_2 + b_1 b_2\right) + \left(e_1 e_2 + f_1 f_2\right)\left(b_1 a_2 - b_2 a_1\right) \tag{23b}$$

The magnitude of $T_1 T_2^*$ is

$$\left| T_1 T_2^* \right| = \frac{2 N_1 N_2 \rho_1^2 V_o^2 \omega_1 \omega_2 \sqrt{H^2 + J^2}}{\eta_1 \eta_2} \tag{24}$$

Now we can define U in Eq. (20) by

$$U = N_1 N_2 \left\{ H + Ji \right\} \tag{25}$$

Since we only want the magnitude of A, then we have by Eq. (20)

$$\left| AA^* \right| = \frac{\beta \hat{P}_o k_-^{3/2} \omega_1 \omega_2 \sqrt{2\pi} \; N_1 N_2 \sqrt{H^2 + J^2}}{\rho_1 V_1^4 \eta_1 \eta_2} \tag{26}$$

APPENDIX A
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1. L. Brekhovskikh, Waves in Layered Media (Academic Press, New York, N.Y., 1960), pp. 68-69.

2. A. Van Buren and M. Breazeale, J. Acoust. Soc. Amer. 44, 1014-1020 (1968).