MATHEMATICAL MODEL FOR AMBIENT NOISE IN THE DEEP OCEAN

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I. INTRODUCTION

Measurements of the ambient ocean noise for sea states greater than zero show that the noise is angular dependent. This implies that the spatial gain of a beam forming sonar is dependent on the orientation of the sonar array. There is, therefore, an optimum gain available for a given sonar system operating in a particular environment which is orientation dependent. This optimum orientation can be predicted from theory providing the ambient noise field properties are known. However such is not the case. To date there is ample data in the frequency band from 0.5 kHz to 1.5 kHz but very little data above this frequency. In addition, environmental conditions such as bottom and surface conditions, velocity profiles, inhomogeneities, etc. are not well chronicled for the measured data. However the latter situation is not critical if one were interested in finding average values for the ambient noise by measurements over a long period of time since these differences average out. This approach is very costly, however, since a large quantity and variety of data is required.

Another approach to the problem can be taken which is cheaper and can give estimates of the ambient noise in agreement with large data set averages. This approach recognizes that the far field noise anisotropy is basically due to the surface disturbance and attempts to calculate the surface source from data
as a function of frequency, sea state (or wind scale) and angle. However the observed noise data and its environmental conditions which are used to calculate the surface sources must be well known to insure that the source function represents the real world. Thus one can predict, quite accurately, the noise anisotropy for a particular environment if one knows the source corresponding to the surface disturbance. This source can be calculated from a completely different set of boundary conditions since the source is uniquely related to the sea state conditions only. TRACOR has taken the latter approach to predicting the ambient anisotropic noise. This document is an offshoot of some of this work done in conjunction with systems performance studies. There is now a requirement to establish a library of source data versus sea state as a function of frequency and angle so that systems modeling problems for a large variety of conditions can be solved.
II. THE DIRECTIONAL AMBIENT NOISE MODEL

The directional ambient noise field is assumed to be the superposition of two noise fields,

\[ N(\theta_R, \phi) = N_1(f) + N_2(\theta_S, f, \phi) \]  \hspace{1cm} (1)

where \( \theta_R \) = elevation angle of the received noise in the vertical plane, \( 0^\circ \) along the horizontal axis, and positive angles toward the sea bottom.

\( \phi \) = azimuthal angle of the received noise, positive angles being measured counterclockwise.

\( N_1 \) is a noise field that is isotropic and which corresponds to that which would be measured in a sea state zero, perfectly calm, deep ocean. Its magnitude is solely a function of frequency. 

\( N_2(\theta_R, \phi) \) is assumed to originate from a uniform distribution of noise generators which lie at the sea surface.

Its magnitude and shape will be a function of frequency, \( f \), depth, \( h \), and ocean environment. Measurements show that it is, to within about 1 dB, independent of azimuthal angle but this dependence can be handled by the theory if desired. The total directional noise field for a particular sea state neglecting \( \phi \) dependence, is

\[ N(\theta_R, f, h) = N_1 + N_2(\theta_R, f, h) \]
Suppose that a receiver is placed at a depth \( h \) in the deep ocean (Fig. 1). Let \( E_0 \) equal the noise power per unit solid angle being emitted from a point source on the ocean surface. Let \( dA_0 \) be the area intersected by a cone of rays which subtend the solid angle at unit reference distance. Then the energy crossing the area \( dA_0 \) per unit time and contained within the cone is

\[
E_0 = I_0 \, dA_0
\]

(2)

where \( I_0 \) = the intensity at unit reference distance of the surface element in the direction \( \theta_0, \varphi_0 \).

The assumption of ray acoustics is that the small cone of energy leaving the point source element will diverge or converge as the rays trace out their respective paths but the total power transmitted is constant for a constant number of rays. In this case the intensity, \( I \), at the point \( R \) where the receiver is located, must satisfy the equation

\[
I_0 \, dA_0 = I_R \, dA
\]

or

\[
I_R = I_0 \left( \frac{dA_0}{dA_R} \right)
\]

(3)

where \( dA_R \) = incremental area of the cone of rays at the receiver, which corresponds to the total number of rays in the solid angle described by \( dA_0 \).
The spreading loss from a point S on the surface, to a point R, at the receiver, is defined as (see Figure 1)

$$N_{(S \rightarrow R)} = \frac{I_o}{I_R} = \frac{dA_R}{dA_o}$$  \hspace{1cm} (4)

so that

$$I_R = \frac{I_o}{N_{(S \rightarrow R)}}$$  \hspace{1cm} (5)

Anderson, Gocht, and Sarota,\(^1\) have shown that the spreading loss from a point S to the point R is related to the backward spreading loss from R to S along the same path by

$$N_{(S \rightarrow R)} = \frac{\cos^2 \theta_R}{\cos^2 \theta_S} N_{(R \rightarrow S)} = \frac{C_R^2}{C_S^2} N_{(R \rightarrow S)}$$  \hspace{1cm} (6)

where \(C_R\) = velocity of sound at the receiver.

\[ C_S = \text{velocity of sound at the sea surface.} \]

The intensity of the received noise may now be written in terms of the received angle and the spreading loss from the receiver to the surface element as

$$I_R (\theta_R) = \frac{I_o \cos^2 \theta_S}{\cos^2 \theta_R N_{(R \rightarrow S)}} = \frac{I_o C_S^2}{C_R^2 N_{(R \rightarrow S)}}$$  \hspace{1cm} (7)
From Physics of Sound in the Sea, the spreading loss from R to S can be shown to be

\[
\frac{dA_o}{dA_R} = \frac{x \frac{\delta x}{\delta \theta_R}}{\cos \theta_R} \sin \theta_S
\]

where \( x = \) horizontal surface range
\( \theta_R = \) angle at the receiver
\( \theta_S = \) angle of ray at the sea surface

The noise power propagated from a point source surface element will also suffer absorption in its journey to the receiver, so that the intensity received at angle, \( \theta_R \), along the direct path (no surface or bottom bounces) is

\[
I(\theta_R) = \frac{I_o c_S^2 e^{-\alpha s}}{C_R N} = \frac{I_o c_S^2 \cos \theta_R e^{-\alpha s}}{C_R x \frac{\delta x}{\delta \theta_R} \sin \theta_S}
\]

where \( s = \) direct path length to surface element
\( \alpha = \) attenuation coefficient

Equation (9) gives the intensity received via the direct path, at angle \( \theta_R \), from a group of point source noise radiators at the sea surface. A more realistic assumption, however, is to assume that the noise originating at the sea surface consists of a continuous distribution of noise generators distributed over the ocean surface. In accordance with this
viewpoint, assume that each infinitesimal area $S$ of the ocean surface generates noise power per unit solid angle of (see Figure 2)

$$dE_o = \sigma G(\theta, \phi) \, dS$$  \hspace{1cm} (10)

where

$$\sigma = \lim_{\Delta S \to 0} \frac{\Delta I_o}{\Delta S} = \text{surface noise density}$$

$$G(\theta_s, \phi_s) = \text{normalized intensity of the noise radiated surface element in the direction } (\theta_s, \phi_s).$$

then the noise intensity received is given by

$$dI_R(\theta_R, \phi) = \frac{dE_o \, C_s^2 \cos \theta_R \, e^{-\alpha s}}{C_R \sin \theta_S \, \left| \frac{\partial x}{\partial \theta_R} \right|}$$  \hspace{1cm} (11)

$$= \frac{\sigma \, G(\theta_s, \phi_s) \, C_s^2 \cos \theta_R \, e^{-\alpha s} \, dS}{C_R \sin \theta_S \, \left| \frac{\partial x}{\partial \theta_R} \right|}$$

The surface element $dS$ (see Figure 2) can be expressed in terms of $\theta_R$ and $\phi$ as

$$dS = x \, dx \, d\phi$$

But $x$ is a function of $\theta_R$ and the depth, $h$, so that for constant depth
\[ dx = \frac{\partial x}{\partial \theta_R} \left|_{h = h_0} \right. \ d\theta_R \]

and

\[ dS = x \frac{\partial x}{\partial \theta_R} \ d\theta_R \ d\phi \]

The final expression for \( dI_R(\theta_R, \phi) \) becomes in terms of the solid angle \( d\Omega \), since \( d\phi \) is the same for the source and receiver,

\[
dI_R(\theta_R, \phi) = \frac{C_S^2 \sigma G(\theta_S, \phi_S) \ e^{-as} \cos \theta_R \ x \ \left| \frac{\partial x}{\partial \theta_R} \right| \ d\theta_R \ d\phi}{C_R^2 x \ \left| \frac{\partial x}{\partial \theta_R} \right| \ sin \ \theta_S}
\]

\[
= \frac{C_S^2 \sigma G(\theta_S, \phi_S) \ e^{-as}}{C_R^2 \ sin \ \theta_S} \ d\Omega \quad (12)
\]

The above development is for those rays which arrive at the receiver by direct paths. In addition to noise arriving over the direct path from a surface element at horizontal range \( x_1 \), noise may arrive at the angle \( \theta_R \) from surface elements located at ranges \( x_2, x_3, \ldots, x_i \) with corresponding path lengths of \( s_2, s_3, \ldots, s_i \). (See Figure 3). If such rays strike the sea
Figure 3
surface or bottom they will experience corresponding energy losses. Accordingly, for rays arriving from the surface which have struck both the bottom and surface, the total noise received from all such elements is

\[
dI(\theta_R, \phi) = \frac{\sigma G(\theta_S, \phi_S) C_s^2 \sin \theta_S}{C_R^2} \left[ e^{-\alpha_s} + \gamma \beta e^{-\alpha_{s2}} + \ldots + \gamma^{k-1} \beta^{k-1} e^{-\alpha_{sk}} + \ldots \right]
\]

\[
= \frac{\sigma G(\theta_S, \phi_S) C_s^2 \sin \theta_S}{C_R^2} \sum_{k=1}^{M} \gamma^{k-1} \beta^{k-1} e^{-\alpha_{sk}}
\]

where

\[
\gamma = \text{bottom loss coefficient}
\]

\[
\beta = \text{surface loss coefficient}
\]

For those rays which suffer surface loss but do not strike the bottom, the noise intensity is given by equation (13) with the bottom loss coefficient set equal to one. For rays which have no direct path to the surface without first striking the bottom, the noise intensity is given by

\[
dI(\theta_R, \phi) = \frac{\sigma G(\theta_S, \phi_S) C_s^2 \sin \theta_S}{C_R^2} \sum_{k=1}^{M} \beta^{k-1} \gamma^{k-1} e^{-\alpha_{sk}}
\]

(14)
The total noise intensity per unit solid angle is obtained by summing the sea state zero noise per unit solid angle and the appropriate expression from equation (13) or (14)

\[ N(\theta_R, \phi) = N_1(\theta_R) + N_2(\theta_R) \]  \hspace{1cm} (15)

The total noise received from all directions is obtained by integrating \( N(\theta_R, \phi) \) over all solid angles at the receiver.

\[ N_T = \mathcal{J} N(\theta_R, \phi) \, \, d\Omega \]  \hspace{1cm} (16)
III. COMPUTER PROGRAM

A computer program for calculating equation (15), the noise per unit solid angle, and equation (16), the total noise intensity, has been written and perfected for the Univac 1108 digital computer. Additional features of the program are the ability to solve for $\sigma G(\theta)$, the product of the surface noise density and the noise source radiator pattern, from experimentally measured data and to give the directional noise field over a frequency band. Major points of the computer model are as follows.

For any specified depth, frequency, sea state, bottom loss, velocity profile, and $\sigma G(\theta_{S})$, the directional noise field in the vertical plane is calculated in increments of $1^\circ$ from $-89^\circ$ (directly overhead) to $+89^\circ$ (toward the sea bottom). The results of this noise field computation are then used to calculate the total noise intensity by performing a numeric integration over all solid angles at the receiver. If experimentally measured data of $N(\theta)$ is read into the program, along with pertinent environmental and velocity profile data, the program will solve for $\sigma G(\theta)$ and give the results in increments of $1^\circ$. If the directional noise over a frequency band is desired, rather than single frequency data, the directional noise field is calculated for frequency increments of specified length, and the resultant noise field is obtained by integrating over the bandwidth.
IV. ANALYSIS OF EXISTING DIRECTIONAL NOISE FIELD DATA

The computer model, based upon the results of Section II, has been used to analyze the rather fragmentary experimental directional ambient noise data which now exists. The results of this analysis to be discussed below, give a better physical understanding of the directional ambient noise and give support to the general validity of the model.

A. THE SURFACE SOURCE FUNCTIONS

If the surface noise source density function, \( \sigma G(\theta) \), is known, then equation (13) and (14) allow the calculation of the directional noise field for any specified velocity profile, depth, bottom loss, surface loss, frequency, and sea state conditions. Becken, Urick, Forster and others (Ref. 1, 2, 4, 6, 9) have postulated normalized \( G(\theta) \) radiators of the form \( \cos^n \theta \), where \( n \) is an integer or fractional power, but thus far no measured data has been used to obtain \( G(\theta) \). If measured directional noise field data, and the associated physical conditions under which it was obtained, are known, then it becomes possible to solve for the source density function, \( \sigma G(\theta) \), from equations (13) and (14). Using measured directional ambient noise field data taken by Axelrod, Schoomer and Von Winkle empirical expressions for \( G(\theta) \) have been obtained from the computer model.
The measured directional noise data of Von Winkle was obtained with the Trident Array, which sits on the ocean bottom at a depth of 14,400 feet about 25 miles south of Bermuda. Data is available for low frequencies from 112 to 1411 Hz and as a function of Beaufort wind scale. The data are averaged over a period of time from Winter thru Summer, and specific velocity profiles for a measured set of data are not available. Average velocity profiles for an area 13 miles from the Trident site have been supplied by Von Winkle* however, and appropriate velocity profiles from this data have been used for obtaining the $G(\theta)$ functions. Bottom loss as a function of grazing angle and frequency were taken from Marsh.$^7$ Surface loss was assumed to be unity for the low frequencies under consideration, and the attenuation as given by Thorp.$^8$ was used. Under these conditions the normalized $G(\theta)$ noise source functions for frequencies of 560, 891, 1122, and 1411 Hz and Beaufort sea state 5 as obtained from the Von Winkle data are given in Figures 4, 5, 6 and 7.

B. PREDICTED DIRECTIONAL AMBIENT NOISE FIELD CHARACTERISTICS

The empirically derived $G(\theta)$ functions as discussed above have been used to study the properties of the directional noise field as a function of depth, velocity profile, bottom loss, and frequency. Computed noise fields, using the empirically

* Private Communication
Figure 6

Computed G(s) Function
Freq. = 1.122 KC
Spring V.W.R. Profile
Marsh Bottom Loss
derived $\sigma G(\theta)$ functions, show that the shape of the noise field is a highly dependent function of velocity profile structure, depth and bottom loss. Typical results are presented below.

1. **Depth Dependence**

Figures 9 through 14 show a typical variation of the noise field with depth, for a frequency of 1411 Hz, the velocity profile shown in Figure 8, and Marsh's bottom loss. The primary change in the shape of the noise field as depth varies occurs around zero degrees elevation angle. As the depth at which the noise field is computed is varied, the noise arriving at the nearly horizontal angles is greatly dependent upon the sound velocity at the receiver relative to the velocity at the ocean surface. When the velocity at the surface is greater than that at the receiver depth, noise from surface will not be able to arrive at the receiver at the nearly horizontal angle, i.e., there exist shadow zones at the surface which do not contribute to the noise field. If the velocity at the receiver is equal to or greater than the surface velocity then there will be no shadow zone areas at the surface and hence no null will appear at the nearly horizontal angles. Greatest variations in the noise field shape as depth changes occur in approximately the first 3000 feet of depth, where the velocity profile is changing most rapidly.
Computed $N(8)$ Noise Field
MS 129 Velocity Profile
Marsh Bottom Loss
Depth = 560 ft.
Freq. = 1.411 KC
Computed Omni Noise Level = 47.532

Figure 10
Computed $N(8)$ Noise Field
MS 129 Velocity Profile
Marsh Bottom Loss
Depth = 975 ft.
Freq. = 1.411 KC
Computed Omni Noise
Level = 47.52

Figure 11
Computed N(θ) Noise Field
MS 129 Velocity Profile
Marsh Bottom Loss
Depth = 3000 ft.
Freq. = 1.411 KC
Computed Omni Noise
= 47.532
Computed $N(\theta)$ Noise Field
NS-129 Velocity Profile
Marsh Bottom Loss
Depth = 6000 ft.
Freq. = 1,411 KC
Computed Omni Noise
Level = 47.532

Figure 13
2. **Velocity Profile Dependence**

A comparison of Figures 9, 16 and 18 along with the accompanying velocity profiles of Figures 8, 15 and 17 gives an indication of the velocity profile dependence of the noise field. The depth of the fields is near the surface in each case. It appears that the characteristic shape of a noise field for a specified profile is largely dependent upon the profile structure. The sharp peaks which arise on both sides of the null in some cases have a physical explanation. These peaks arise because the noise arriving per unit solid angle is coming from a very large surface area, due to the near horizontal aspect angles of the sound rays leaving the sea surface near the shadow zones. Low sea state data taken by G. Fox with Trident array tend to confirm the physical reality of such peaks.

3. **Bottom Dependence**

The lower half of the directional ambient noise field curves is strongly dependent upon bottom loss as a comparison of Figures 10 and 19 will reveal. Both curves are computed for the same sets of conditions except for bottom loss. In Figure 10 the theoretical bottom loss of Marsh was used. In Figure 19 a bottom loss as published by B. A. Becken was used. The bottom loss affects the shape and magnitude of the lower half of each curve. In addition the magnitude of the upper half of each curve is
Computed N(0) Noise Field
MS 129 Velocity Profile
Marsh Bottom Loss
Depth = 150 ft
Freq. = 1.411 KC
Computed Omni Noise
Level = 47.532

Figure 9
Figure 18

Computed N(6) Noise Field
Summer W Profile
Marsh Bottom Loss
Depth = 328.08
Freq. = 1.411 KC
Computed Omni Noise
Level = 47.25518400
affected for those rays which have arrived after striking the bottom but the effect is much less pronounced for the upper half.

4. A Comparison of Computed Directional Noise Fields with the Measured Becken Data

The only known measured data for directional noise field near the surface are the four curves published by B. A. Becken in his thesis\(^2\). The exact physical conditions, such as velocity profile, bottom loss, etc., under which the Becken data were obtained are not known and therefore exact reproduction of the Becken results is not possible. By making certain assumptions as to the conditions under which the Becken data were taken it is possible to show a general qualitative agreement between the measured data of Becken and the calculated noise using \( \sigma G(\theta) \) calculated from Von Winkle's observed noise.

The Becken data are not single frequency data but are taken over a 750-1500 Hz band. The curves of the restored or "true" field that are published were obtained from the measured data by attempting to remove the array effects of the system used for measurement. The array used by Becken for the noise measurements had a rather "fat" main lobe beamwidth and measurements were taken only in 5° steps. Any attempted restoration of the noise field measured in such a manner will omit much of the fine detail of the noise field structure\(^10\).
Because of the above considerations it was felt that the easiest way to check the measured results of Becken with the computed noise field was to compare the resultant curves of the noise that would be measured at the output of a Becken type array for both Becken's restored data and TRACOR's computed noise field. To do this the Becken array as published in his thesis was simulated on the 1108 Univac computer. Theoretical noise field, for frequencies of 560, 891, 1122, and 1411 Hz, were then computed for conditions of a Pacific Ocean velocity profile assumed to be similar to that under which the Becken data were measured. The results were then integrated over a frequency band from 750 to 1500 Hz to produce the curve shown in Figure 20. Using this curve to represent the noise field in the water and the comparable "corrected" data were integrated over Becken's array beam pattern to obtain the noise out of the array as a function of elevation angle. The results are presented in Figure 21.
Computed Directional Noise over 750-1500 BW
Depth = 550 ft
Sea State = 4

Figure 20
Computed Noise as Would be Measured by a Beeken Type System
Depth = 550 ft.
Sea State = 4

Figure 21
V. AREAS FOR FUTURE INVESTIGATION

There remain many improvements to be made on the developed noise model and further investigation needs to be undertaken to determine the characteristics of the directional noise field. Some of these are as follows:

1. For higher frequencies a surface loss should be incorporated into the model.

2. Improved accuracy could be obtained by incorporating continuous velocity profile rather than straight line approximations for calculating the path length.

3. The contribution of shadow zone areas to the noise field should be investigated. At the present, sharp discontinuities occur in the noise field for the angles at which the shadow zone occurs. Results published in Physics of Sound in the Sea indicate that the sound falls off as a sharp exponential from the shadow zone and that there will be some contribution to the noise intensity from such zones. Presently shadow zone contributions are considered negligible.

4. Solutions for the $\sigma G(\theta_g)$ surface source density noise functions need to be studied further. Because of velocity profile structure it is difficult to obtain accurate solutions for
\( \sigma G(\theta_s) \) for angles near zero degrees. The accuracy for these angles becomes important in predicting the intensity of the noise field for angles arriving nearly horizontal at the receiver.

5. Most important of all, more experimentally measured data, along with the accompanying physical conditions under which it was measured, needs to be obtained. From data measured under the proper conditions, it will be possible to obtain more reliable \( \sigma G(\theta) \) functions as a function of frequency and sea state. Once these have been obtained, it then becomes possible to accurately model the directional ambient noise in a manner analogous to that described above.
REFERENCES


