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INVESTIGATION OF POSSIBILITY OF MEASURING PARAMETERS OF DISPERSED PARTICLES IN COMBUSTION PRODUCTS OF COAL BY LOW-ANGLE SCATTER OF LIGHT

By

E. P. Zimin, A. M. Krugenskiy, Z. G. Mikhnevich

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*ye initially, after vowels, and after ū, ū, ē elsewhere.
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RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

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INVESTIGATION OF POSSIBILITY OF MEASURING PARAMETERS OF DISPERSED PARTICLES IN COMBUSTION PRODUCTS OF COAL BY LOW-ANGLE SCATTER OF LIGHT

E. P. Zimin, A. M. Krugenskiy, Z. G. Mikhailovich

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ABSTRACT Discussing the possibility of measuring the parameters of ash particles in the combustion products of coal by the method of light scatter. Studied are the different aspects of the well-known method of low angles, which makes it possible to measure the distribution function of the particles by dimensions. Obtained are the correlations of some of the characteristics of a scattered
light flux (sum fluxes of scattered radiation, discrete values of light-scattering flux, and extreme values of functions obtained by transformation of the scatter indicatrix) with the integral parameters of the particles (sum surface of particles per unit volume and volume concentration). This approach makes it possible to reduce the volume of measuring and calculating operations in comparison to determination of the integral parameters by the method of low angles. Presented here are the results of the experiments for a gas stream with ash particles. END ABSTRACT

Symbols

A - ash content, %

a - diameter of dispersed particles, m

b - amount carried away

c - instrument constant

F - light-scattering flux, W

G - specific volume of gaseous combustion products, m³/kg
g - scattering volume, $m^3$

i - intensity of incident light flux, $W/m^2$

$L$ - distance to plane of registration, $m$

$l$ - length of scattering volume, $m$

$N$ - registered concentration of particles, $m^{-3}$

$n$ - spectral density of registered concentration, $m^{-4}$

$P$ - particle distribution function

$r$ - stream radius, $m$

$N$ - volume concentration of particles

$v$ - spectral density of volume concentration, $m^{-1}$

$\theta$ - scatter angle
A characteristic feature during combustion of solid fuel is the two-phase nature of the working medium, associated with the presence of a condensed dispersion phase in the combustion products. This condensed phase in the combustion products of coal is represented by dispersed ash particles, whose mean diameter lies within the limits of 10-20 microns. According to the standard classification, two-phase media with such particles include coarsely dispersed ash. The presence of dispersed particles in the working medium is a determining factor for a number of physicomechanical processes and, in the final analysis, affects the output characteristics of any combustion device. In connection with this the creation of diagnostic methods for the parameters of the condensed dispersion phase of coal...
(carbon) combustion products is of direct practical interest in the study of solid fuel combustion processes.

At the present time sampling methods which employ subsequent laboratory analysis are widely used to determine the parameters of coarsely dispersed particles. However, such methods are not very operational and, moreover, introduce perturbation into the two-phase flow, disturbing its structure. Recently methods which use the effects of the interaction of electromagnetic radiation with dispersed particles have received more recognition. Such methods make it possible to obtain information on the dispersed phase without disturbing the structure of the flow (flux).

In describing the physical properties of the dispersed phase of two-phase media it is convenient to use the spectral density of the registered concentration of particles $n(a)$, which is equal to the number of particles whose dimensions lie within a single interval of dimension $a$. However, for applied problems it is more convenient to determine the dispersed composition of the particles by their volume concentration in the two-phase medium $V$ and by the distribution function $R(a)$, which determines the volume percent of particles with a dimension greater than $a$ as a function of quantity $a$. If the particles are spheres, then the volume parameters $V$ and $R(a)$ are expressed in terms of $n(a)$ as follows:
\begin{equation}
V = \frac{1}{6} \int_0^\infty a^n n(a) da
\end{equation}

and

\begin{equation}
R(a) = \frac{50\Pi}{3V} \int_a^\infty a^n n(a) da
\end{equation}

Consequently, any method of diagnosing the dispersion composition of particles \textit{presumes measurement} of \( n(a) \) or direct measurement of the quantity \( \frac{\Pi}{6} \int a^n n(a) da \). The latter is obtained, for example, by the sieve method (for a given density of the substance of the particles). Extremely important here is the integral parameter \( V \), even in the absence of information on the dispersed composition of corresponding particles. Thus, the diagnostic methods which provide direct measurement of this parameter are of immediate practical interest. Also important are such integral parameters as the registered concentration of particles \( N \) and the sum surface of the particles per unit volume \( S \), determined by the following expressions:

\begin{equation}
N = \int_0^\infty n(a) da
\end{equation}

\begin{equation}
S = \Pi \int_0^\infty a^n n(a) da
\end{equation}
Knowledge of the parameter \( V \) makes it possible to determine the coefficient of entrainment \( b \), which describes the fraction of the unburned fuel component which is carried away from the combustion device with the gaseous combustion products, through relationship \( b = 100 \frac{pG}{V} \).

1. Method of Low Angles

The method of low angles (MLA), proposed by K. S. Shifrin [1] to determine the function \( n(a) \), is based on the effect of the low-angle scatter of light on coarsely dispersed spherical particles, whose diameter is significantly greater than the wavelength of the incident radiation. The angle dependence of the light flux scattered on the polydisperse system of such particles (scatter indicatrix) is described by the following expression

\[
F(\theta) = \frac{C}{\Theta^2} \int_0^\infty J_1\left(\frac{\Pi a}{\lambda} \theta\right) a^3 n(a) da
\]

which represents the integral Fredholm equations of the first order. The solution to this equation, obtained in [1] by using the properties of the generalized Fourier transforms of [2], takes the form of
Constant $C$ in equation (5) can be expressed as follows

$$C = \log(4)$$

The function in $n(u)$ thus calculated makes it possible to determine the values of $V, R(a), N,$ and $S$ by equations (1)-(4).

The method of low angles has been rather widely used in the practice of scientific research on two-phase systems. A survey of early studies in which MLA were used is given in [3]. A typical system for obtaining measurements by this method includes a monochromatic radiation source, an optical system for collimating and diffracting the beam, and a light detector. A laser is currently used as the source, thus eliminating the need for a monochromator.

The original equation (5) was obtained under the assumption that for all scattered particles the following conditions are observed: $i = \text{idem}, \omega = \text{idem}, \theta = \text{idem}$. Strict observation of the last two conditions can be assured by the use of a long-focal lens between the diagnosed object and the light detector. However, as shown in [4], with the ratio $L/L < 0.2$, these conditions are observed sufficiently well and measurements can be conducted without using the lens.
The first of the conditions may not be observed in the case of substantial light attenuation along the scattered volume. However, the effect of light attenuation can be considered as follows. In the presence of attenuation, equation (5) must be written in the form of

\[ F(\theta) = \frac{e^{-c}}{\theta^2 I} \int_0^I \left[ \int_0^\infty J_i \left( \frac{\Pi a}{\lambda} \theta \right) a^2 n(a) da \right] \exp \left( \int_0^I -\xi dy \right) dy \]

Assuming that the two-phase medium is homogeneous and assuming that

\[ i(y) = i(0) \exp \left( -\int_0^y -\xi dy \right) \]

we get

\[ F(\theta) = \frac{ci(0)\exp(-\xi)}{\theta^2 I} \int_0^I J_i \left( \frac{\Pi a}{\lambda} \theta \right) a^2 n(a) da \]

Consequently, to consider light attenuation when the method of small angles is used in the inversion equation (6), we must use the intensity of the incident radiation which has passed through the two-phase medium in the capacity of \( i \).

The above ideas are correct for homogeneous two-phase media. If the distribution of the parameters of the condensed phase across the flux is heterogeneous, then formula (6) gives the averaged values of the registered concentration and the distribution function of the particles with respect to dimensions in the direction of the sounding.
beam. If the flux of dispersed particles has axial symmetry, then by measuring the indicatrix of scatter on a sufficient number of chords, we can get the distribution of the indicated parameters with respect to the radius of the flow. The method of determining the radial change in any physical quantity with respect to integral chord measurements in an axisymmetrical cylindrical stream (Abel method) is described in [5] and [6].

The use of the MLA in combination with the Abel method for diagnosing the parameters of particles in axisymmetrical two-phase fluxes thus provides the possibility of obtaining the radial distributions of the parameters of the particles.

The accuracy of recovering $n(a)$ by equation (6) by the method of low angles is determined by errors of two types: instrument and methodological. Instrument errors include errors in measuring the light flux and the scatter angle. A study of errors was conducted in [7]-[10]. It was shown in [7] that instrument errors associated with measuring of the light flux hinder reliable calculation of $n(a)$ when $(\Pi a/2) < 30$. Error resulting from replacement of the integral by the sum, according to the data of [8], constitutes $5-10\%$ depending on the value of the step with respect to $\theta$. In [9] and [10] it was shown that if the measurements cover a certain basic interval of angles $(\theta_{\min}, \theta_{\max})$, then the error of determining $n(a)$ within the limits of
0.4 \(a_m < a < 5a_m\) (where \(a_m\) is the modal dimension) does not exceed 5% due to limitation of the range of the angles.

However, such studies were only conducted for completely determinant types of \(n(a)\) functions, since analytical study of the recovery of the function \(n(a)\) was not possible in an arbitrary form. The problem of studying accuracy must be solved by calculations on the computer for a large set of types of \(n(a)\) which are of practical interest. Here it is expedient to divide the work into three stages. In the first stage we studied a small number of characteristic types of function \(n(a)\). This makes it possible to establish "sensitive" values, at which substantial distortion in the information is observed. Such calculations with variations in these "sensitive" parameters are the object of the second stage of studies.

The authors studied the accuracy of recovering function \(n(a)\), assigned in the form of a normal distribution with variable values of the modal dimension \(a_m\) and the square root from dispersion \(\sigma\). This form of the function is close to that obtained in the experiments, which will be described below. By way of illustration Fig. 1 shows the assigned and recovered functions \(n(a)\) for the values \(a_m = 1.2 \times 10^{-5}\) m and \(\sigma = 3 \times 10^{-6}, 2 \times 10^{-6}\) and \(5 \times 10^{-6}\) m. The wavelength \(\lambda\), characteristic of a helium-neon laser and equal to \(6.3 \times 10^{-7}\) m, was used in these calculations.
Experiments were conducted on an airstream 3.3•10^{-2} m in diameter, into which ash particles were introduced. The flow rate of these particles was changed by means of a dosing apparatus. The light source was a helium-neon laser, while the light detector was a photoelectric multiplier, whose signal was transmitted to an amplifier, then to a loop oscillograph. The coordinate of the moving light detector (and consequently, the scatter angle θ) was also recorded on the oscillograph. In Fig. 2 we see the radial change in functions \( r(a) = (\Pi /6)a^n(a) \) and \( R(a) \), obtained during processing of chord measurements produced by the Abel method. The reduced data indicate the possibility of using the MLA for diagnosing ash particles in the combustion products of coal.

2. Method of Two Angles

The use of the MLA for determining the integral parameters of dispersed particles involves a rather significant volume of measurement and calculation operation. However, if the goal of the experiment is to determine the integral parameters, then a method can be proposed which is to a great extent free of this disadvantage.
This method is based solely on the use of the solution to the direct diffraction problem for a polydisperse system of spherical particles. By introducing the parameter

\[ B_j = \int_0^\infty a' n(a) \, da \]

and using the mean-value theorem for the integral, equation (5) can be transformed into

\[ F(\Theta) = \frac{c \omega_{T_x}^{-1}(\Theta) B_j}{\Theta^2} \int_0^\infty J_2 \left( \frac{\pi a}{\Theta} \right) \, da \]

The use of the indicated theorem is possible, since the function \( a^2 J_2 ((\pi a/\lambda) \Theta) \) is continuous, while function \( a' n(a) \) does not change sign in the interval \([0, \Theta]\). Parameter \( B_j \), depending on the value of \( j \), corresponds to the different moments of function \( n(a) \).

The integral parameters of the dispersed phase are easily expressed in terms of the corresponding moments of function \( n(a) \). For example, when \( j = 0 \), we get the registered concentration \( N \), when \( j = 3 \) - the volume concentration \( V = (1/6) B_j \).

If we select the two angles \( \Theta_1 \) and \( \Theta_2 \) close enough so that \( a_0 \) can be considered constant in the interval \([\Theta_1, \Theta_2]\), then we get a system of two transcendental equations, the solutions for which are \( a_0 \) and \( B_j \).
Thus, the proposed method of determining the integral parameters of the dispersed phase by the low-angle scatter of light reduces to measuring the flux of scattered radiation on two angles alone.

It should be mentioned that parameter $a_0$ and processing data on light scatter in a polydispersed system of particles by means of the two-angle method is simply an auxiliary quantity, and is generally not expressed in terms of modal dimension. This parameter is equal to the diameter of the particles of a single-dispersed system which is equivalent in light scatter to the studied polydispersed system. It is natural that the lower the dispersion $n(a)$ the better justified will be the assumption of constant $a_0$.

We cannot obtain an analytical expression which describes the dependence of the error obtained in determining the integral parameters by the method of two angles on the values of angles $\theta_1$ and $\theta_2$ under the assumption of a constant $a_0$ in the interval $[\theta_1, \theta_2]$. Therefore, selection of the angles can be optimized by calculating the indicated error for several variations in assigning the pair of angles.

The most natural is selection of angles $\theta_1$ and $\theta_2$ at the very origin of the scatter indicatrix. Calculations conducted on the computer showed that when $\theta_1 = 10^{-2}$ rad and $\theta_2 \ll 4 \cdot 10^{-2}$ rad the error
produced in determining the volume concentration does not exceed 40% for function n(a) corresponding to the normal distribution with the modal diameter 1.2 • 10^{-5} m and \( \sigma = 3 \times 10^{-6} \) m. In this case quantity S is determined with an error of 100%. Other variations of studying the angle pairs was studied in [11]. As one might expect, they proved to be less suitable.

The method of two angles is particularly effective for measuring the radial distribution of the volume concentration and other integral parameters by the radius of axisymmetrical two-phase flows, since within the framework of the Abel method it is sufficient to take chord measurements of the scattered radiation on two angles alone. In this case the substantial reduction in the volume of measurements is obvious.

To remove the hypothesis of constant \( a_0 \) a modification of the method can be proposed in which \( a_0 \) is assumed to be a linear function of the angle of scatter

\[
a_0(\theta) = a_0(\theta_1) + (\theta - \theta_1)q.
\]

In connection with the appearance of the new auxiliary parameter \( q \), measurements of \( F \) are taken on three angles. The third angle can easily be selected as follows: \( \theta_3 = (\theta_1 + \theta_2)/2 \). The system of equations for determining \( a_0(\theta_1) \) and \( q \) takes the form of
Calculations conducted on the computer for \( n(a) \) corresponding to a normal distribution with \( a_0 = 1.5 \times 10^{-5} \) m and \( v = 2 \times 10^{-6} \) m showed that for angles \( \theta_1 = 10^{-2} \) rad and \( \theta_2 < 4 \times 10^{-2} \) rad error in determining \( V \) by the method of two angles does not exceed 40\(^\circ\), while under the hypothesis of the linear dependence of parameter \( a_0 \) this error is no greater than 20\(^\circ\).

In Fig. 3 we see the radial distribution of the value of the volume concentration, obtained in the experiments described above using the method of low angles and the method of two angles. These data indicate the possibility of using the method of two angles for diagnosing integral parameters of the dispersed particles in the combustion products of coal.

3. Determining Integral Parameters of Particles by Sum Values of Scattered Light Flux

As already observed, the method of low angles makes it possible, using the measured indicatrix of scatter, to establish function \( n(a) \)
and, consequently, all integral parameters of the dispersed particles. However, there exists the possibility of determining quantities of $V$ and $S$ by the scatter indicatrix, bypassing the awkward stage of recovering $n(a)$. If we use the properties of the Bessel functions [12], then we get the following relationships

$$\int_0^\infty \Theta^k F(\Theta) d\Theta = \frac{c^k \Gamma(2-k) \Gamma\left(\frac{1+k}{2}\right) \left(\frac{\Pi}{\lambda}\right)^{1-k}}{2^{2-k} \Gamma\left(\frac{3-k}{2}\right) \Gamma\left(\frac{5-k}{2}\right)} \int a^{2-k} n(a) da$$

when $k = 0$ and $k = 1$, we get expressions for $V$ and $S$ ($\Gamma(x)$ is the gamma-function) in the form of

$$V = \frac{\Pi \lambda}{8c} \int_0^\infty F(\Theta) d\Theta$$

$$S = \frac{2\Pi}{c} \int_0^\infty \Theta F(\Theta) d\Theta$$

Formally the integral with respect to the angle (7) extends from zero to infinity, but since the scatter is concentrated primarily in the first order of diffraction, integration can be done up to the angle of $\theta_{max}$, which satisfies the condition $(\Pi a_{cp}/\lambda) \theta_{max} = 3.8$, where $a_{cp}$ is the mean diameter of the particles. The very smallest angle $\theta_{min}$ can be selected because of the ability of the measuring system to reliably separate scattered light from incident light. Linear
extrapolation of values $F(\theta)$ from value $F(\theta_{\text{min}})$ to $F(0)$ makes it possible, as numerical experiments have shown us, to greatly improve the agreement of the assigned and obtained values of $V$ and $S$ for equations (8) and (9).

The integral in equation (8) represents the sum light flux, while in equation (9) it represents the total product of the flux and scatter angle.

To illustrate the possibilities of such processing of the scatter indicatrix, Fig. 3 shows the values of the volume concentration of particles $V$ over the radius of the axisymmetrical stream. The values obtained in the above mentioned experiments were calculated by the sum beam of scattered light using the linear extrapolation of function $F(\theta)$ to $F(0)$. The agreement between these data and those calculated by the method of low angles can be considered quite satisfactory.

Apparently the Abel method must be used to determine the integral parameters in axisymmetrical fluxes with use of the correlations expressed by equations (8) and (9).

4. Determining Integral Parameters of Particles by Experimental...
Values of Functions Obtained by Transformation of Scatter Indicatrix

The effect of low-angle light scatter represents one more possibility of determining the most important integral parameter \( V \) with respect to the scatter indicatrix. Actually, by introducing the function \( Z(\theta) \) by means of relationship \( Z(\theta) = 6F(\theta) \), from (5) we get for \( Z(0) \) the following equation

\[
\frac{\lambda}{6c} Z(\theta) = \int J_0 \left( \frac{\Pi a}{\lambda} \theta \right) \left( \frac{\Pi a}{\lambda} \theta \right)^{-1} \Pi \alpha^2 n(\alpha) d\alpha
\]

which, when the average-value theorem is considered for the integral from the product of the two functions, can be written as follows

\[
\frac{\lambda}{6c} Z(\theta) = V J_0 \left( \frac{\Pi a_n(\theta)}{\lambda} \theta \right) \left( \frac{\Pi a_n(\theta)}{\lambda} \theta \right)^{-1}
\]

From equation (11) it is obvious that the maximal value of function \( \frac{\lambda}{6c} Z(\theta) \) should correspond to the maximal value of function \( J_0^2(x) x^{-1} \). Consequently, for the value of the volume concentration \( V \) the following relationship is correct:
For a monodispersed system of particles the argument $x$ can take any values and pass through the value $x$, which is equal to 1.35, where the value of function $J^2(x) x^{-1}$ is maximal and equal to $\approx 0.21$. Then, directly from the maximal value of the function we determine the volume concentration of monodispersed particles $V_m$ from the relationship

$$V_m = \frac{0.795 \lambda}{c} Z(\Theta)_{\text{max}}$$

If ash particles of different dimensions (polydispersed system) are present in the combustion products, then relationship (12) is not fulfilled, since the argument $x$ does not take the value $x_{\text{opt}}$ which corresponds to the maximal value of the function $J^2(x) x^{-1}$. This can be easily demonstrated by analyzing equation (10). Actually, if we assume that $x$ takes the value $x_{\text{opt}}$, then in the integral of (10) for all values of $a$ the value of expression $J^2((\Pi a/\lambda)(\Pi a/\theta)) (\Pi a/\lambda)^{-1}$ should be constant and equal to 0.21. However, this cannot be achieved at a fixed angle $\Theta$. Consequently, in the general case

$$V \geq \frac{0.795 \lambda}{c} Z(\Theta)_{\text{max}}$$

where the equality sign corresponds to a monodispersed system of particles.
The use of equation (12) to determine the volume concentration of the polydispersed system makes it possible to calculate by the measured function \( Z(\theta) \) the value of the volume concentration \( V \), which is always less than the real value of this quantity \( V_r \). The closer the particle system to the monodispersed system, the lower will be the error in determining the volume concentration.

The dependence of the error value in analytical form cannot be found for all types of functions \( n(a) \), and thus it is interesting to estimate this error for certain characteristic types of \( n(a) \). Function \( J_{1/2}(x) x^{-1} \) has a rather flat maximum (Fig. 4) and the error value depends on the ratio of \( a_{\min} \) and \( a_{\max} \) to \( a_c \), where \( a_{\min} \), \( a_{\max} \), and \( a_c \) represent the minimal, maximal, and mean diameters of the particles, respectively. Thus, for example, it can be easily demonstrated that when \( a_{\min}/a_c = 0.8 \) and \( a_{\max}/a_c = 1.2 \), error in determining \( V \) does not exceed 40/0 even for an distribution of particles over the diameters. If, however, \( n(a) \) under these conditions does have a maximum, then error is less.

Values of error in determining \( V \) were estimated for a number of types of functions \( n(a) \) characteristic of ash particles with a normal law of distribution over diameters with \( a_c = 1 \times 10^{-3} \), \( 2 \times 10^{-3} \) and values of \( \sigma \).
For all cases the value of error does not exceed 50%. For \( n(a) \) corresponding to the normal law of distribution of particles over the diameters it is obvious that the condition \( u_m/\sigma = \text{const.} \), the value of error will be constant. In Fig. 5 we see by way of illustration values of \( V \) obtained with transformation of the indicatrix as a function of \( u_m \). The given value \( V_m \) is marked by a dashed line.

For experiments conducted in a gas stream with ash particles, the values of the volume concentrations obtained by the method of low angles and by the extremum of function \( Z(\theta) \) are in good agreement.

By analogy we can propose processing the results of measurements for the extremal points of the functions by the point of function

\[
F(\theta) = (d/d\theta)[F(\theta) \theta^2].
\]

Here the following equality is correct:

\[
\frac{\lambda}{12\varepsilon} E(\theta) = \sqrt{J_1 \left\{ \frac{\Pi a_o(\theta)}{\lambda} \right\} J_0 \left\{ \frac{\Pi a_o(\theta)}{\lambda} \right\} - J_1 \left\{ \frac{\Pi a_o(\theta)}{\lambda} \right\} \left[ \frac{\Pi a_o(\theta)}{\lambda} \right]^{-1} }
\]

The expression for the value of the volume concentration in this case takes the form of

\[
V \geq \frac{0.578\lambda}{c E(\theta)_{max}}
\]
Thus, from the extremal points of certain functions associated with the scatter indicatrix we can determine the volume concentration of particles. Here the step of determining the spectrum of particle diameters is too time consuming.

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Fig. 1. Recovery of spectral density of registered concentration $n(a)$ for different values of $a(a_m = 1.2\times10^{-5}$ m) • $a = 3\times10^{-6}$ m, ▲ $a = 4\times10^{-6}$ m, ○ $a = 5\times10^{-6}$ m — assigned $n(a)$.

Fig. 2. Function of $R(a)$ and $v(a)$ for different values of stream
radius —— R(a), —— v(a).

**Fig. 3. Radial distribution of volume concentration.** • - by MLA, o — by method of two angles, ▲ - by sum light-scattering flux.

**Fig. 4. Curve of function J_{1/2}(x)x^{-1}**.
Fig. 5. Volume concentration as function of quantity \( \sigma \) in determining \( V \) by extremal values of function \( \Theta F(\theta) \) — — — assigned value of \( V \).
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