Plasma Diffusion in a Space-Simulation Beam-Plasma Discharge

E. P. Szuszczywicz, D. N. Walker, and J. C. Holmes

E. O. Hulburt Center for Space Research
and
H. Leinbach

NOAA/ERL/Space Environment Laboratory
Boulder, Colorado 80302

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Electron density and temperature profiles have been measured in a large-facility beam-plasma-discharge (V_B = 1.3 KeV, I_B = 63 ma) at low pressure (P = 2-4(10^-6)Torr) and low magnetic fields (B = 1.2 G). The results follow an exponential behavior and are found to be in good agreement with a simple diffusion model. The favorable comparison between theory and experiment yields the first estimate of cross-field electron diffusion (D_e(non) = 2.2(10^6)cm^2/sec) under conditions intended to simulate SpaceLab-borne beam-plasma investigations. This value for D_e (Continues)
orders of magnitude larger than would be expected for a cross-field collisional diffusion process, a result which is identified with the turbulent state of collective beam-plasma interactions.
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I. INTRODUCTION

Beam-plasma interactions have received increased attention in recent years largely as a result of considerations bearing on vehicle neutralization during space-borne accelerator experiments, enhanced beam-plasma ionization processes, and in general, collective phenomena initiated by beam injection into neutral gas and charged-particle environments. With plans for Shuttle-borne experiments directed at controlled beam-plasma interactions and the use of beams to probe ionospheric/magnetospheric electric and magnetic fields, it became important to conduct laboratory simulations which supported the Shuttle plans and helped establish definitive Shuttle-borne experiments.

As part of just such a plan involving a continuing investigation of large-facility beam-plasma interactions (Bernstein, et al, 1975,1977,1978) the Naval Research Laboratory (NRL) participated in a recent series of experiments at the NASA Johnson Space Center (JSC). The NRL

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contribution involved the direct measurement of electron
density and temperature under varying conditions of beam-
plasma and neutral gas parameters. Reported here are
the first large facility profiles of electron density and
temperature under conditions of enhanced beam-plasma
ionization...conditions which have come to be known as the BPD,
the "beam-plasma-discharge" (Getty and Smullin, 1963;
Bernstein et al., 1978). The observed density profile is
found to be in good agreement with a two-dimensional diffu-
sion model and this result makes possible the first estimate
for the cross-field electron diffusion coefficient in a
large facility BPD. In the following Sections the experi-
ment is described, the theoretical model is defined, and
the diffusion coefficient is derived.

II. EXPERIMENTAL RESULTS

The Experiment. The technique of the pulsed-plasma-
probe (P3) was employed to measure the beam-plasma profile.
The P3 provides simultaneous measurements of $N_e$, $T_e$, $V_o$,
and $\delta N_e(+P_n(k))$ and is particularly useful under dynamic
plasma conditions and in environments that can contaminate
electrode surfaces (Holmes and Szuszczewicz, 1975;
Szuszczewicz and Holmes, 1975, 1976, 1977). Both these
conditions prevailed to various degrees in the JSC experi-
ments.

The experimental configuration (similar to that
described by Bernstein et al., 1978) is illustrated in

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Figure 1. The P3 was mounted on a traversal mechanism positioned at approximately 8 m above the injection point of the beam. The tungsten cathode gun was operated at 63 mA and 1.3 KV DC with the chamber pressure in the $2-4 \times 10^{-6}$ torr range. A combination of coil current and the Earth's field established vertical and horizontal component B-fields at $B_z = 1.21$ gauss and $B_{xy} = 0.27$ gauss, respectively. Under these conditions a BPD was established and the probe traversal mechanism was exercised to determine the plasma's radial profile out to 4.6 m. The resulting measurements of $T_e$ and $N_e$ are presented in Figure 2.

Following the diffusion-model density-profile predictions (discussed below) the experimental curves were fit with exponential distributions found to be

$$T_e \left[10^3 \text{ K}\right] = 6.93 \exp \left(-r/1.99\right) \tag{1}$$

$$N_e \left[10^6 \text{ cm}^{-3}\right] = 7.01 \exp \left(-r/2.49\right) \tag{2}$$

III. THEORETICAL CONSIDERATIONS

For purposes of comparison, a simple two-dimensional diffusion model is developed which follows the approach used in the study of small laboratory arc plasmas by Bohm et al., (1949a). The approach suggests the consideration of a differential plasma element of thickness $dr$ and of length $h$ parallel and equal to the length of the beam as depicted in Figure 3. The differential element is taken outside the domain of the primary beam and is assumed to be homogeneous.
along its length. The equilibrium density then arises from the balance of charge accretion by differential drift across the element, i.e., \( j_e(r) = j_e(r+\text{dr}) \), and end losses (Z=0 and h) due to axial diffusion. Requiring \( dN_e/dt = 0 \) and assuming an axial diffusion loss directly proportional to density (i.e., differential end losses = \( 2 a_z^e N_e \text{dr} \)), the balance between production and loss requires

\[
\frac{dj_e}{dr} = -2a_z^e N_e \delta r
\]

which is rewritten as

\[
\frac{dj_e}{dr} = -2a_z^e N_e / h
\]

Similar arguments for the plasma's ion component yield

\[
\frac{dj_i}{dr} = -2a_z^i N_i / h
\]

Independent of Eqs. (3) and (4) the diffusion currents \( j_e \) and \( j_i \) can be described by their respective mobility equations

\[
j_e = -D_e^\perp \left\{ \frac{dN_e}{dr} + \frac{eN_e}{kT_e} \frac{dV}{dr} \right\}
\]

and

\[
j_i = -D_i^\perp \left\{ \frac{dN_i}{dr} + \frac{eN_i}{kT_i} \frac{dV}{dr} \right\}
\]

where \( V \) is an electric potential and \( D_e^\perp \) and \( D_i^\perp \) are the perpendicular electron and ion diffusion coefficients, respectively. Taking the one-dimensional space-derivative \( d/dr \) of Eqs. (5) and substituting (4) yields
\[ T_e \left( \frac{a}{hD_e} N_e - \frac{d^2 N_e}{dr^2} \right) = -e \frac{d}{k} \left( N_e \frac{dV}{dr} \right) \]  

(6a)

\[ T_i \left( \frac{\alpha}{hD_i} N_e \frac{d^2 N_e}{dr^2} \right) = e \frac{d}{k} \left( N_e \frac{dV}{dr} \right) \]  

(6b)

when \( N_e = N_i \) and the terms have been rearranged. Addition of Eqs. (6) eliminates the potential-dependent term and results in

\[ \frac{d^2 N_e}{dr^2} = 2 \ h^{-1} \ \left\{ \frac{e^{D_e^+} T_e + e^{D_i^+} T_i}{D_e^{D_e^+} (T_i + T_e)} \right\} N_e = \beta^2 N_e \]  

(7)

with a solution

\[ N_e = \left\{ N_e^0 - \frac{(N_e^0 \exp (\beta R) - N_e^R)}{(\exp (\beta R) - \exp (-\beta R))} \right\} \exp (\beta r) \]  

\[ + \left\{ \frac{N_e^0 \exp (\beta R) - N_e^R}{(\exp (\beta R) - \exp (-\beta R))} \right\} \exp (-\beta r) \]  

(8)

In the limit \( R \to \infty \), the solution becomes

\[ N_e = N_e^0 \exp (-\beta r) \]  

(9)

where

\[ \beta = \left\{ \frac{2(e^{D_e^+} T_e + e^{D_i^+} T_i)}{D_e^{D_e^+} (T_i + T_e) h} \right\}^{1/2} \]  

(10)
IV. COMPARISON BETWEEN THEORY AND EXPERIMENT

The agreement between theoretical prediction (Eq.(9)) and experimental results is found to be good (see bottom panel, Fig. 2) and provides the opportunity for determining the effective electron diffusion perpendicular to the magnetic field, i.e., the radial diffusion in the model of Fig. 3. To accomplish this, consider

\[
\lim_{T_1/T_e \to 0} \beta^{-1} = \left( \frac{D^+ n / 2 \varepsilon_z^e}{\tau_e} \right)^{\frac{1}{2}} = 2.49
\]

(11)

where the value on the right side is taken from the exponential in Eq. (2). In addition, consider \( \alpha_z^e \) as the effective velocity of axially diffusing electrons which have sufficient energy to overcome chamber sheath potentials and be completely lost to the system. In this case,

\[
\alpha_z^e = \sqrt{\frac{kT_e}{2 \pi m}} \exp \left( -\frac{eV_c}{kT_e} \right)
\]

(12)

where \( V_c \) is the chamber-sheath potential. With \( V_c = (4.8^{+0.5}) \frac{kT_e}{e} \) (Chen, 1965; Szuszczechewicz, 1972) and \( T_e = 3 \times 10^3 \) °K Eq. (12) yields

\[
\alpha_z^{e(\text{nom.})} = 7.0 \times 10^4 \text{ cm/sec} \quad (13a)
\]

\[
\alpha_z^{e(\text{max})} = 2.8 \times 10^5 \text{ cm/sec} \quad (13b)
\]
Substituting the $a_z^e$ values into Eq. (11) provides

\[
\begin{align*}
D_e^\perp (\text{nom}) &= 2.2 \times 10^6 \text{ cm}^2/\text{sec} \\
D_e^\perp (\text{max}) &= 8.8 \times 10^6 \text{ cm}^2/\text{sec} \\
D_e^\perp (\text{min}) &= 1.3 \times 10^6 \text{ cm}^2/\text{sec}
\end{align*}
\]

as the first experimental estimate of the effective electron diffusion coefficient perpendicular to the magnetic field in a BPD.

The values for $D_e^\perp$ are orders of magnitude larger than would be expected for cross-field collisional diffusion. This result in itself is not surprising since a variety of observations indicate that the diffusion of plasma across magnetic lines of force can be substantially enhanced above the collisional diffusion rate when fluctuating fields are present in the plasma (Spitzer, 1962; Kadomtsev, 1965). This is indeed the case in a BPD which has as one of its characteristics the presence of fluctuating electric fields that are associated with large amplitude plasma waves. The enhancement of diffusion in turbulent plasmas goes back to Bohm et al. (1949b) who semiempirically described the rate by what has come to be known as the Bohm diffusion coefficient

\[
D_B = 6.25 \times 10^6 T_e B^{-1} \text{ cm}^2/\text{sec}
\]
where \([T] = eV\) and \([B] = \text{gauss}\). For our nominal temperature of \(3(10^3)\ ^\circ\text{K}\) and a total field \(B_z = 1.2\ \text{gauss}\), Eq. (13) yields \(D_B = 1.3(10^6)\ \text{cm}^2/\text{sec}\). This calculation has not been intended to confirm or deny the validity of Eqs. (14) but has been presented as the only possible comparison with an existing concept of turbulent diffusion. While the Bohm formula appears to apply to a surprising number of different experiments, care must be exercised so that the illusion of universal validity does not automatically develop. It is evident that a first-principles derivation of the coefficient of turbulent diffusion cannot be obtained without a detailed investigation of the plasma, particularly the fluctuation power spectra and associated instability processes (Kadomtsev, 1965; Papadopoulos, private communication 1978). The experimental and theoretical aspects of these details, as they relate to the JSC experiments, are currently underway at NRL (Szuszczewicz and Papadopoulos, private communication 1978).

V. SUMMARY

The first experimental radial profiles of electron density and temperature in a large facility beam-plasma-discharge are found to fit exponential functions defined by

\[
T_e \ [^\circ\text{K}] = 6.93 \ (10^3) \ \exp \left( -r \ [\text{m}] / 1.99 \right)
\]

and
While the present stage of research does not provide a theoretical basis for the temperature distribution, the electron density results do agree with a two dimensional diffusion model which predicts

\[ N_e = N_e^0 \exp \left\{ -\frac{2e^2}{z/D_e} \right\} \]

in the limiting case of large chambers with \( T_i/T_e \to 0 \). With \( e^2 \) taken as the effective velocity of axially diffusing electrons (assigned a nominal value of \( 7 \times 10^4 \) cm/sec) the identity of the experimental profile with the theoretical prediction yields

\[ D_e^\text{(nom)} = 2.2 \times 10^6 \text{ cm}^2/\text{sec} \]

for the radial electron diffusion across a superimposed magnetic field. This is the first such determination in a large-facility BPD and points to a process substantially faster than cross-field collisional diffusion. The reason for this is identified with the presence of large amplitude plasma waves and their associated electric fields.

While enhanced cross-field diffusion appears to be a characteristic of many turbulent plasma environments, the observations reported here are particularly unique in that the applied magnetic field is at best \( 10^3 \) times weaker than in any other turbulent diffusion investigation. This points to the importance of an unequivocal determination...
of the turbulence spectra and the associated instability processes. This work is currently underway at NRL.

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Bernstein, W., H. Leinbach, and B. Baker, "Preliminary results from the electron beam-plasma experiments in the very large vacuum facilities at Plum Brook and Johnson Space Center", NOAA Technical Memorandum ERL-SEL 5 August 1977.


Szuszczewicz, E.P., "Area influences and floating potentials in Langmuir probe measurements", J.Appl.Phys. 43, 874, 1972; $V_c = 4.8kT_e/e$ has been selected as a nominal value since the wall conditions are considered more appropriately identified with the thin sheath conditions in Figs. 3 and 6 of this Ref.


Fig. 1 - Experiment configuration utilized in the large-facility beam-plasma investigations at the Johnson Space Center.
Fig. 2 - Radial profiles of electron density and temperature under conditions of the beam-plasma discharge.
Fig. 3 - Assumed configuration for diffusion-model calculations.