U.S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

BOTTOM STATISTICS
FEASIBILITY REPORT NO. 1

By

H. T. Loeser

USL Technical Memorandum No. 2329-59-69

17 April 1969

This document is subject to special export controls
and each transmittal to foreign governments or
foreign nationals may be made only with prior approval
of the U.S. Navy Underwater Sound Laboratory.

DISTRIBUTION STATEMENT A
Approved for public release; Distribution Unlimited
# USL Tech Memo
No. 2320-50-69

**BOTTOM STATISTICS**

**FEASIBILITY REPORT**

**TABLE OF CONTENTS**

1.0 Introduction
2.0 General
3.0 Problem Areas
   3.1 Pressure Sensor
      3.1.1 General
      3.1.2 Model 8160 Vibrotron
      3.1.3 Hytech Pressure Sensor
   3.2 Acoustic Transducer
      3.2.1 General
      3.2.2 Pressure Effects
      3.2.3 System Design
   3.3 Vehicle Dynamics
      3.3.1 General
      3.3.2 Effect of Pitch
      3.3.3 Effect of Roll
      3.3.4 Effect of Trailing
      3.3.5 Dynamic Kiting
      3.3.6 Cable Angles
      3.3.7 Vehicle Tow Staff Angle
      3.3.8 Hydrodynamic Tests in Pool
      3.3.9 Hydrodynamic Tests at NSRDC
      3.3.10 Cable Oscillations
         3.3.10.1 General
         3.3.10.2 Vertical Oscillations
         3.3.10.3 Lateral Oscillations
      3.3.11 Snap Action
   3.4 Doppler Sensors
   3.5 Telemetry
   3.6 Computer Analysis of Data
   3.7 Shipboard Installation
4.0 Schedule
5.0 General Discussion
6.0 References

**APPENDIX I** - Effect of Trail Distance on Track Error due to Helmsman's Error
SUMMARY

The feasibility of obtaining detailed bottom statistics is investigated. It is concluded that the most cost effective method is to use a vehicle towed within 300 feet of the bottom which obtains a profile by means of high frequency sonar and determines its depth by means of a precision depth gage. The track of the vehicle is determined by doppler acoustics.

Such a vehicle at present appears feasible although detailed test data on the precision of the depth sensor has not yet been obtained due to difficulties in establishing reference pressures to the precision desired.

The system as presently configured would be used on a typical oceanographic ship such as the AGOR USNS SANDS.
BOTTOM STATISTICS
FEASIBILITY REPORT NO. 1

1.0 INTRODUCTION

In certain projects at this Laboratory, it is necessary to obtain statistical information on the bottom roughness in deep ocean water and rough terrain.

The vertical resolution required for this use is 0.2 feet and the horizontal resolution is 2 feet. This information is needed to supplement information obtainable from surface vessels and, therefore, instrumentation stability over distances of 5 nautical miles is needed.

All possible sources of this information were investigated, and a survey of possible methods of obtaining this information were reviewed. This is discussed in reference 1.

The method offering the most hope of accomplishment was to use a high frequency acoustic sensor to sense the bottom and a precision depth meter to determine coincidently the depth of the sensor. The vehicle for transporting these instruments would be towed from a surface vessel.

A feasibility study has been established to determine if it is feasible to obtain this statistical information. Studies to date have provided sufficient optimism to warrant several investigations for confirming critical areas. This report describes the investigations to date, their results and planned future work.

2.0 GENERAL

Reference 1 describes the method being investigated and the major problem areas. However, a brief description of the system will avoid unnecessary referral to other documents.

The system consists of a 1Mhz narrow beam projector mounted in a towed body which contains a high resolution depth sensor and a doppler speed sensor. In addition, a forward looking, collision avoidance sonar, a magnetic compass and two angle indicators are provided.
The principal problems arise from the resolutions required of the instrumentation which in turn impose difficult stability requirements on the vehicle dynamics and hydrodynamics.

3.0 PROBLEM AREAS

3.1 PRESSURE SENSOR

3.1.1 GENERAL - The pressure sensor resolution requirements approach the present limit of the state of the art.

Two pressure sensors have been ordered. One has been delivered. In neither case does the manufacturers specification meet the requirements of the project. However, these gages have been used at resolutions approaching the desired resolution on a project at APL.

The sensor that has been delivered is a model 8160 Vibrotron manufactured by United Control of Redmond, Washington.

To meet the requirements of the project, it is necessary that it have a resolution of 0.2 feet and a good stability over a period of several hours.

3.1.2 MODEL 8160 VIBROTRON

A number of tests have been made on the 8160 Vibrotron. They are summarized below:

3.1.2.1 INITIAL CALIBRATION

The calibration certificate for this gage is shown in Figure 1. This shows that the average sensitivity of the gage is

\[
\frac{1.302 \text{ Hz}}{10,000 \text{ psi}} = 0.13 \text{ Hz/psi}
\]

Therefore \(0.1 \text{ psi} \rightarrow 0.013 \text{ Hz} \rightarrow 2.7 \text{ in.}\)

3.1.2.2 GAGE DRIFT

A rough recheck of the gage at USL gave an average difference from this calibration of 3.2 Hz on 23 January, 20 days later. This corresponds to a drift of 25 psi \(\rightarrow 55 \text{ ft/20 days.}\)
During a series of tests made during 13 February over an eight hour period, a pressure of 1037.0 psi was checked eight times as accurately as possible. These checks were:

```
Pressure = 1037.0 psi
```

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Frequency</th>
<th>Δ Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11101.647</td>
<td>-.018</td>
</tr>
<tr>
<td>2</td>
<td>11101.675</td>
<td>+.010</td>
</tr>
<tr>
<td>3</td>
<td>11101.648</td>
<td>-.017</td>
</tr>
<tr>
<td>4</td>
<td>11101.698</td>
<td>+.033</td>
</tr>
<tr>
<td>5</td>
<td>11101.679</td>
<td>+.014</td>
</tr>
<tr>
<td>6</td>
<td>11101.648</td>
<td>-.017</td>
</tr>
<tr>
<td>7</td>
<td>11101.675</td>
<td>+.010</td>
</tr>
<tr>
<td>8</td>
<td>11101.648</td>
<td>-.017</td>
</tr>
</tbody>
</table>

The average drift is .017 Hz which corresponds to about 3\(\frac{1}{2}\) in. Maximum drift is .033 or about 7 in.

3.1.2.3 Resolution

Over a period of three days, various sensitivity measurements were attempted. The results were:

```
Test Press Frequency Δ Mean
No. psi psi Hz/psi From Mean
1 1037 .176 .013 .074 0
2 1037 .176 .015 .085 +.011
3 1037 .086 .004 .047 -.027
4 1037 .086 .005 .058 -.016
5 1037 .086 .006 .070 -.004
6 1037 .086 .004 .047 -.027
7 1037 .176 .015 .085 +.011
8 1037 .176 .014 .079 +.005
9 4038 .086 .006 .070 -.004
10 2000 .086 .007 .082 +.008
11 2000 .086 .007 .082 +.008
12 8037 .176 .020 .112 +.038
```

The average sensitivity was .074 Hz/psi or .0028 Hz/in. The greatest difference from this average was .038 Hz/psi or .0014 Hz/in and the average difference was .013 Hz/psi or .005 Hz/in. It should be noted...
that the apparent sensitivity is only about one-half that of the sensitivity over the full range. This is probably caused by limitations in the test equipment. The probable limitation on resolution of this equipment is about 0.02% according to its manufacturers. This means that the equipment has a borderline response for the variations attempted. The expected pressure differential based on the applied weights probably did not materialize in fact. Therefore, the apparent low sensitivity. In addition, the signals contained considerable noise. It was diagnosed that this noise was mechanical in nature. It was very evident that personnel movement in the area strongly affected the signal. From this it was postulated that wind on the building and other causes could account for the signal noise.

It is quite possible to measure the output frequency to .001 Hz. It is, therefore, probably possible to obtain a resolution to 0.2 in. The tests give evidence of a low drift rate and good repeatability. However, it will require more stable test conditions to demonstrate this ability. Arrangements have been made to have the gage calibrated at the National Bureau of Standards-Gaithersburg, Maryland.

3.1.2.4 TEMPERATURE SENSITIVITY

The temperature coefficient was calibrated at the factory over a temperature range of from 30°F to 105°F. The change in frequencies was as follows:

<table>
<thead>
<tr>
<th>T°F</th>
<th>f_p=0</th>
<th>Δf</th>
<th>Δf/°F</th>
<th>in/°F</th>
<th>f_p=10,000</th>
<th>Δf</th>
<th>Δf/°F</th>
<th>in/°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>11,228</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9,924</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>70</td>
<td>11,235</td>
<td>7</td>
<td>.175</td>
<td>35</td>
<td>9,933</td>
<td>9</td>
<td>.225</td>
<td>45</td>
</tr>
<tr>
<td>105</td>
<td>11,239</td>
<td>11</td>
<td>.148</td>
<td>30</td>
<td>9,939</td>
<td>15</td>
<td>.200</td>
<td>40</td>
</tr>
</tbody>
</table>

The temperature drift may be a factor both in calibration and use. This is overcome relatively simply however, by insulating the sensor to slow the drift rate due to temperature.

3.1.2.5 SUMMARY OF VIBROTRON TESTS

The tests, due to mechanical noise, did not define the resolution, drift and temperature coefficient of the gage. However, they have been given confidence that the gage approaches the capabilities needed for this project.
3.1.3 HYTECH PRESSURE SENSOR

A Model 4016 Pressure Sensor manufactured by Bissett-Berman, San Diego, California, is on order. Its specifications in all respects are better than the Vibrotron. It is, however, larger. In particular the temperature coefficient on sensitivity is given as 5 PPM/°C which for 20,000 ft is

\[
\frac{5 \times 20,000}{10^6} \text{°C} \quad \text{or} \quad 1 \text{ft} = 1.2 \text{in/°C}
\]

which is an order of magnitude improvement over the vibration. This should eliminate the temperature problems.

3.2 ACOUSTIC TRANSDUCER

3.2.1 GENERAL - The principal questions relative to the 1 mHz transducer are:

1. Will it operate at 20,000 ft.?
2. What is its beamwidth?
3. What is its maximum power output?
4. What reflectivity coefficient is to be expected?
5. What is its sensitivity as a receiver?
6. What attenuation is experienced when passing through mylar?

3.2.2 PRESSURE EFFECTS

Three 1 mHz transducers were supplied by Westinghouse Underseas Division for our use in testing.

Two of these were installed in a high pressure chamber at USL. One was used as a projector, the other as a receiver. See reference 4.

They were put in operation at 0 psi and the pressure was gradually increased to 10,000 psi. During the pressure increase an acceptably low change in signal amplitude occurred. The resonant frequency changed slightly, requiring a shift in input frequency to obtain maximum output amplitude.

These transducers are being tested for the other characteristics mentioned.
3.2.3 SYSTEM DESIGN

Conferences with Mr. R. J. Wimberger concluded that the Echo Sounder, including the logic circuits would be G.F.E. and that a synchronizing signal from the data conversion system to the high frequency acoustic sonar system would call for a read out of 15 parallel binary bits with an output voltage of 0-5 v. The electronics will be housed in a 3½ " I.D. x 30" long pressure resistant stainless steel cylinder. The pulse power for the system will be 100 watt - 20 microsec at 28 v.

3.3 VEHICLE DYNAMICS

3.3.1 GENERAL - It is essential that the instruments be installed in a dynamically stable body if accurate bottom information is to be obtained. Vertical movement is compensated by the pressure transducer, however, bad pitching or rolling can introduce spurious information into the data.

3.3.2 EFFECT OF PITCH

The fore and aft velocity of the vehicle will be sensed by the Doppler System seven times per second. Therefore any surging of the vehicle will be detected and automatically taken into account when distance is determined by integration.

The effect of pitching of the vehicle may be compensated in a similar manner provided the frequency-amplitude product is less than some value, dependent on distance to the bottom. Referring to the fig., the effect of pitching a small angle $\alpha$ about an apparent pitch center $d$ feet from the bottom with a frequency $f$ may be discussed as follows, assuming the pitch angle varies harmonically, i.e.

$$\alpha = \alpha_{\text{max}} \sin \omega t$$

WHERE:  $2\pi f = \omega$

THE PITCH ERROR, $e$ IS $d \sin \alpha$

OR $e = d \sin[\alpha_{\text{max}} \sin \omega t]$

$\dot{e} = \omega d \sin[\alpha_{\text{max}} \cos \omega t]$

$\ddot{e} = \omega \dot{d} \sin[\alpha_{\text{max}} \cos \omega t]$

$\dot{\alpha}_{\text{max}} = \pm d \alpha_{\text{max}} f \omega \approx \pm \frac{d \alpha_{\text{max}} f \omega}{2\pi}$

\[ e_{\text{max}} = \pm d \alpha_{\text{max}} f \omega \approx \pm d \alpha_{\text{max}} f \omega \frac{2\pi}{2\pi} \]
Now if the average velocity of the vehicle is \( v \)-ft/sec., and if \( \hat{E} \) equals or exceeds this velocity, the sonar will sample some areas intensely while skipping over other areas. If we assume a data rate, \( \gamma \) of 7 times per second and a \( v = 2.0 \) ft/sec, the sweep velocity \( v_s \) will be:

\[
v_s = v - \hat{x} = v - \frac{d \alpha_m f}{2 \pi}
\]

The maximum sweep velocity is:

\[
v_s = v + d \alpha_m f \cdot 2 \pi
\]

The sample distance \( S_s = \frac{\nu_s}{2} = \frac{\nu}{2} + \frac{d \alpha_m f \cdot 2 \pi}{2} \)

IF MAXIMUM \( S_s = 2.0 \) FT. THEN:

\[
2.0 = 0.286 + 26.9 \alpha_m f
\]

or

\[
\frac{1.714}{26.9} = \alpha_m f = \frac{.0063 \text{ RAD/SEC}}{}\]

\[
T = \frac{\frac{1}{f}}{2.11} = \frac{2 \pi \frac{3.4}{\sqrt{32.2 \times 30}}}{21.4} = \frac{.14}{1.44} = \frac{.0063}{1.44}
\]

\[\alpha_m \cdot 1.44 = .0063\]

\[
\alpha_m = .0044 \text{ RADIANS}
\]

\[
= 0.25^\circ
\]

If the bridle is 30 feet long, to cause a 25° angle change, the bridle connection would have to move 30 \times .254 = 7.62 ft horizontally on either side of its midpoint or nearly 1.25 ft. The disturbing
forces at the surface are not expected to have a horizontal component of this magnitude and the attenuation through a long length of cable will reduce the disturbance to less than 2 inches.

The only disturbance likely to cause pitching moments are the vertical surges, hoists and lowerings. It has been observed in the tests performed in the USL pool that raising the vehicle had the effect of eliminating pitch angle. The most noticeable pitch angles were observed when the vehicle went through a change from up to down. At this time, the tension in the support cable is reduced. If the downward velocity equals the terminal velocity, then the stabilization effect of the bridle and cable is lost and the vehicle may take a large angle. The maximum downward velocity is likely to be about 270 ft/min or 4.5 ft/sec. If the projected area is 14 ft² and the $C_d = 0.5$,

\[ \text{drag} = 0.5 \times (4.5)^2 \times 14 = 1400 \text{#} \]

since the weight of the vehicle is estimated at 2200#, this reduces the effective weight to 800#. Sufficient stability should exist under these conditions. If a problem is encountered, however, it will only reduce the speed at which the vehicle may be lowered and will not effect the speed at which it may be raised to avoid an obstacle.

Since the vehicle will have some pitching motion, it must be sensed and accounted for in the computer calculations. This compensation is relatively simple since the distance along the track, $x^1$, traveled by the vehicle differs from the spot illuminated, by the distance $e$. Therefore, $x$ the track distance of the illuminated spot is:

\[ x = x^1 + e \text{ or } x = x^1 + d \sin \alpha \]

Since $\alpha$ will be given by the pitch angle sensor, to within an accuracy of $\pm 0.1$ degree, the error due to this effect is $d \times 0.0017$ or 1.5 ft/sec. Therefore, the error in $x^1$ due to pitching will be about 0.5 ft maximum. In addition, any "skips" due to pitching will be less than 2 feet, or within the discrimination of the acoustic sensor.

3.3.3 EFFECT OF ROLL

During the model tests, the models showed no inclination to roll. This is due to there being no off-center forces to cause rolling. No appreciable rolling is anticipated. Rolling, if it did occur, would move the sensed spot in an oscillatory fashion on either side of the vehicle track. Due to the stochastic nature of the bottom, the track observed would probably be statistically equivalent to the vehicle track.
3.3.4 **EFFECTS OF TRAILING**

The helmsman will probably not be able to steer a perfectly straight course.

To estimate the expected excursions of the bottom vehicle when the ship on the surface deviates from a straight course, it is convenient to make some simplifying conservative assumptions.

Due to the length of the towline and the resistance of the water, the vehicle will act somewhat like a damped pendulum swinging from the towpoint. Since the length of this pendulum is 10,000 to 20,000 ft, its period is on the order of:

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}} = \frac{2\pi}{\sqrt{\frac{20000\text{ft}}{32.2\text{ft/s}^2}}}$$

$$= 50\pi \approx 157\text{ sec}$$

Since the speed of the ship is on the order of 1 to 2 feet per second, the distance traveled is only from 150 to 300 ft. per cycle. In addition, the water provides a good deal of damping. Therefore, it is very unlikely that pendulum action will cause appreciable variations in the track.

The most likely variations will be due to helmsman error.

A relatively simple analysis of this error, using a conservative model of the situation is given in the Appendix.

This model assumes that the vehicle is towed toward the ship at all times and that the ship is following a sinusoidal path. Since the actual path cannot be predicted, this approach can only give one an understanding of the relative magnitudes of the errors involved.

Imposed on this simple solution, of course are: a) the pendulous swing of the vehicle, b) the drag of the cable through the water which will damp out the movement of the vehicle.

The analysis in the Appendix indicates that if the ship has a sinusoidal motion with an excursion of 1000 ft either side of its mean course, has a trail distance of two miles and a period of roughly one nautical mile, the excursion of the vehicle would be about 83 ft. This is approximately 8% of the ship's motion.
The graph, Figure 1, shows how this maximum excursion varies with changes in the trail distance and periodic distance.

The amplitude of the vehicle excursion is, under the simplifying assumptions, proportional to the ships eccentricity by the factor "f" which is called the response amplitude operator.

To decrease the response, the trail distance should be increased, the periodic distance should be decreased and/or the ships eccentricity should be decreased.

The periodic distance and ships eccentricity are a function of the navigation of the ship. The trail distance is a function of the system design and water depth.

For the representative dimensions given in the Appendix, the maximum excursion is acceptable.

3.3.5 DYNAMIC KITING

The sailing of the vehicle from the ships course due to hydrodynamic forces is one of the more difficult phenomena to overcome in the vehicle design.

Even at the slow speeds at which the vehicle operates, the hydrodynamic forces dominate in the horizontal plane. If the shape of the vehicle generates an unbalanced side force as it moves through the water, there is nothing to prevent its moving sideways until it is at an angle where the horizontal component of the towline force balances it. What might happen at this point is that the vehicle would turn slightly and the process would reverse to the other side.

The kiting phenomena were observed during model testing and successfully avoided by installing a drag strip at the end of the tail to cause a high drag at the stern breaking up any tendency to create air foil action.

This kiting was particularly noticeable with the spherical model even when a small tail fin was added. The elliptical model was far better in this respect and trailed with no kiting provided a tail with drag strip was used.
3.3.6 CABLE ANGLES

The angle at which the cable would pay out if it had no vehicle at its lower end is called the critical angle. This angle is a function of the weight per foot of the cable and its drag per foot. For the cable selected, this angle is plotted on Figure 1. At a speed of 1 knot, the cable angle from the vertical is about 23°. The secant of 23° is about 1.086 therefore at 1 knot approximately 21,720 ft of cable will be required to reach a depth of 20,000 ft. A more complete discussion including the effect of a weight on the end of the cable is given in reference 3. At the greater depths the cable dominates. Therefore, the effect of the towed body on the depth and trail distance will be small over all.

3.3.7 VEHICLE TOWSTAFF ANALYSIS

The towstaff angle is the angle which the vehicle weight to drag ratio generates in the cable at the point where it connects to the vehicle. In place of a towstaff, this vehicle uses a bridle for convenience in handling aboard ship. This bridle can be adjusted to effectively change the towstaff angle if the towing speed is to be changed. The expected weight in water of the towed body is about 220#. The drag at 1 knot will be about 37#. Therefore the weight to drag ratio is 6. See Figure 2. The towstaff angle will therefore be about 10°. The length of cable affected by this body weight is only about 50 to 100 ft. The remainder of the cable will lie at the critical cable angle.

3.3.8 HYDRODYNAMIC TESTS IN POOL

During December 1968 a series of tests of 1/4 scale models was run in the pool at USN/USL. These tests were qualitative in nature and aimed at selecting a shape for further quantitative tests to be run at NSRDC.

Four basic models were tested:

a. Sphere 12" dia SKD-49988
b. Elliptical 12" dia x 7" wide SKD-49990
c. Elliptical 12" dia x 4" wide SKD-49991
d. Elliptical with trunnion 12" dia x 7" wide SKD-49989
These models were tested with an assortment of vanes, staffs, tails and bridle as follows:

<table>
<thead>
<tr>
<th>Sketch No.</th>
<th>Vanes</th>
<th>Staffs</th>
<th>Fins</th>
<th>Drag Strip</th>
<th>Bridle</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKD-50014</td>
<td>24&quot;</td>
<td>3&quot;</td>
<td>3&quot;</td>
<td>1/4&quot;</td>
<td>5/4&quot;</td>
</tr>
<tr>
<td>SKD-50101</td>
<td>30&quot;</td>
<td>6&quot;</td>
<td>6&quot;</td>
<td>1&quot;</td>
<td>1/2&quot;</td>
</tr>
</tbody>
</table>

The model weights without attachments are given below:

<table>
<thead>
<tr>
<th>Model</th>
<th>Wt. in Air lbs.</th>
<th>Wt. in Water lbs.</th>
<th>W/B Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>43-1/4</td>
<td>9</td>
<td>1.26</td>
</tr>
<tr>
<td>7&quot; Elliptical</td>
<td>32</td>
<td>9.9</td>
<td>1.6</td>
</tr>
<tr>
<td>7&quot; Ell. w/yoke</td>
<td>36</td>
<td>15-1/2</td>
<td>1.75</td>
</tr>
<tr>
<td>4&quot; Elliptical</td>
<td>28</td>
<td>16</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The models were towed across the pool while being raised and lowered.

Results of USL Pool Tests:

It was noted in these tests that the sphere suffered from extensive eddy forces which caused it to sway from side to side. Later, with a tail fin, this was reduced. It was noted that with a short staff, the models would pitch about the connection to the towline. The longer the
staff, the less the pitching effect. With the 30 inch staff, this pitching was almost eliminated.

The 7 inch model with the trunnion and fins performed no better than the 7 inch model without fins therefore it was eliminated.

The 4 inch elliptical model tended to "kite" i.e. develop lift on one side which drove it off track. The 7 inch elliptical model performed better than the others and in a highly satisfactory manner when it was fitted out with the proper attachments. It was found that when a tail fin fitted with drag strips was used and a long staff it tracked well and exhibited no pitching during perturbations from the towline.

However, since space aboard the surface vessel would be restricted, the scaled up towstaff would be extremely awkward. The overall height of the arrangement would be about 15 ft. Therefore, an alternate was devised consisting of a three part bridle. Since the bridle would travel over a sheave, it could be made suitably long without difficulty. This was tried in the pool and performed well.

Many of the pool tests were photographed at twice normal speed to scale up the motion to simulate full scale timing when the motion picture was projected. These motion pictures corroborate the good performance of the 7" x 12" ellipsoid with tail fin.

3.3.9 HYDRODYNAMIC TESTS AT NSRDC

The models, together with various appurtenances, were taken to NSRDC for more exact tests in the circulating water tunnel. The 4 inch elliptical model was not tested because preliminary arrangements of the vehicle showed that there was insufficient space within the envelope for the required equipment.

The circulating water tunnel was excellent for these tests because it permitted careful and complete viewing of the motions of the models as well as quantitative information on drag. The motion pictures were again taken at twice normal speed to simulate full scale when projected. See reference 5.

These tests simulated the vertical motion of the ship by raising and lowering the model from a crane during the run. The results of these tests showed the superiority of the 7" elliptical model with a 6" or 9" tail fin. The sphere exhibited considerable yaw and sway. Even with the 9 inch fin. This fin actually increased the sway so that the
model appeared to kite, two drogues on bridles were used in an attempt to avoid this kiting action without effect.

The 7 inch elliptical model with the 6" and 9" fins and 1" angles, however, were very steady. The notes made during the testing are given in Table I.

Because of the operational advantages of the smaller, 6" fin and the bridle, the 7" ellipsoid of revolution with the 6 inch tail fin and 5½" bridle was selected as the most acceptable configuration. This configuration was modified to make it more streamlined and give it a larger internal volume. The proposed configuration is shown in SKL 50270.
## TABLE I

NOTES ON TOWED MODEL TESTS AT NSRDC
CIRCULATING WATER TANK

VERTICAL SPEED OF CRANE 1/3 FT/SEC

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Speed in Knots</th>
<th>Model</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 3&quot; fin - 1/2&quot; angles - Runs steady at about 15° - 20° to current about 2 - 3 inch side sway</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>Sphere</td>
<td>Frequency of side sway is faster Aft bridles noticeably bowed</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>Sphere</td>
<td>Frequency of side sway is faster Aft bridles noticeably bowed</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 3&quot; fin - 1&quot; angles - 30° yaw</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 3&quot; fin - 1&quot; angles - yaw straightened out</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 3&quot; fin - 1&quot; angles - yaw straightened out</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1/2&quot; angles - 30° to 40° yaw with kiting</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1/2&quot; angles - 10° to 15° yaw with kiting</td>
</tr>
<tr>
<td>9</td>
<td>1.5</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1/2&quot; angles - small yaw angle</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1&quot; angles - 20° to 30° yaw</td>
</tr>
<tr>
<td>11</td>
<td>1.0</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1&quot; angles - 20° to 30° yaw with kiting</td>
</tr>
<tr>
<td>12</td>
<td>1.5</td>
<td>Sphere</td>
<td>5/4&quot; bridle - 6&quot; fin - 1&quot; angles - 20° to 30° yaw with kiting</td>
</tr>
<tr>
<td>Run No.</td>
<td>Speed in Knots</td>
<td>Model</td>
<td>Comments</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>-------</td>
<td>----------</td>
</tr>
<tr>
<td>13</td>
<td>0.6</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - 10° to 15° yaw with kiting</td>
</tr>
<tr>
<td>14</td>
<td>1.0</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - 10° to 15° yaw with kiting</td>
</tr>
<tr>
<td>15</td>
<td>1.5</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - 10° to 15° yaw with kiting</td>
</tr>
<tr>
<td>16</td>
<td>0.6</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - 45&quot; drogue Bridle - still yaws and kites</td>
</tr>
<tr>
<td>17</td>
<td>1.0</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - drogue 20 in²</td>
</tr>
<tr>
<td>18</td>
<td>1.5</td>
<td>Sphere</td>
<td>54&quot; bridle - 9&quot; fin - ( \frac{1}{2} )&quot; angles - drogue 20 in²</td>
</tr>
<tr>
<td>19</td>
<td>0.6</td>
<td>Sphere</td>
<td>54&quot; bridle - no fin - drogue only Very noticeable sway</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>Sphere</td>
<td>54&quot; bridle - no fin - drogue only About 12&quot; side sway</td>
</tr>
<tr>
<td>21</td>
<td>1.5</td>
<td>Sphere</td>
<td>54&quot; bridle - no fin - drogue only About 12&quot; side sway</td>
</tr>
<tr>
<td>22</td>
<td>0.6</td>
<td>7 EL.</td>
<td>54&quot; bridle - 6&quot; fin - 1&quot; angles - very steady</td>
</tr>
<tr>
<td>23</td>
<td>1.0</td>
<td>7 EL.</td>
<td>54&quot; bridle - 6&quot; fin - 1&quot; angles - slow sway - 12&quot;</td>
</tr>
<tr>
<td>24</td>
<td>1.5</td>
<td>7 EL.</td>
<td>54&quot; bridle - 6&quot; fin - 1&quot; angles - very steady - little sway</td>
</tr>
<tr>
<td>25</td>
<td>0.6</td>
<td>7 EL.</td>
<td>54&quot; bridle - 3&quot; fin - 1&quot; angles - 15° yaw &amp; sway</td>
</tr>
<tr>
<td>26</td>
<td>1.0</td>
<td>7 EL.</td>
<td>54&quot; bridle - 3&quot; fin - 1&quot; angles - 15° yaw &amp; sway</td>
</tr>
<tr>
<td>27</td>
<td>1.5</td>
<td>7 EL.</td>
<td>54&quot; bridle - 3&quot; fin - 1&quot; angles - oscillatory sway - bad</td>
</tr>
<tr>
<td>28</td>
<td>0.6</td>
<td>7 EL.</td>
<td>54&quot; bridle - 9&quot; fin - 1&quot; angles - very steady</td>
</tr>
</tbody>
</table>

USL Tech Memo
No. 2320-50-69
<table>
<thead>
<tr>
<th>Run No.</th>
<th>Speed in Knots</th>
<th>Model</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>1.0</td>
<td>7 EL.</td>
<td>54&quot; bridle - 9&quot; fin - 1&quot; angles - very steady</td>
</tr>
<tr>
<td>30</td>
<td>1.5</td>
<td>7 EL.</td>
<td>54&quot; bridle - 9&quot; fin - 1&quot; angles - very steady</td>
</tr>
<tr>
<td>31</td>
<td>0.6</td>
<td>7 EL.</td>
<td>30&quot; vane - 12&quot; staff - very steady</td>
</tr>
<tr>
<td>32</td>
<td>1.0</td>
<td>7 EL.</td>
<td>30&quot; vane - 12&quot; staff - very steady</td>
</tr>
<tr>
<td>33</td>
<td>1.5</td>
<td>7 EL.</td>
<td>30&quot; vane - 12&quot; staff - with some side sway</td>
</tr>
<tr>
<td>34</td>
<td>0.6</td>
<td>7 EL.</td>
<td>30&quot; vane - 6&quot; staff - steady</td>
</tr>
<tr>
<td>35</td>
<td>1.0</td>
<td>7 EL.</td>
<td>30&quot; vane - 6&quot; staff - steady</td>
</tr>
<tr>
<td>36</td>
<td>1.5</td>
<td>7 EL.</td>
<td>30&quot; vane - 6&quot; staff - small sway</td>
</tr>
<tr>
<td>37</td>
<td>0.6</td>
<td>7 EL.</td>
<td>30&quot; vane - 3&quot; staff - slight sway &amp; yaw</td>
</tr>
<tr>
<td>38</td>
<td>1.0</td>
<td>7 EL.</td>
<td>30&quot; vane - 3&quot; staff - 12° sway 12° yaw</td>
</tr>
<tr>
<td>39</td>
<td>1.5</td>
<td>7 EL.</td>
<td>30&quot; vane - 3&quot; staff - more stable</td>
</tr>
<tr>
<td>40</td>
<td>0.6</td>
<td>Sphere</td>
<td>30&quot; vane - 3&quot; staff - 15° yaw 12&quot; sway</td>
</tr>
<tr>
<td>41</td>
<td>1.0</td>
<td>Sphere</td>
<td>30&quot; vane - 3&quot; staff - 12° yaw 10&quot; sway</td>
</tr>
<tr>
<td>42</td>
<td>1.5</td>
<td>Sphere</td>
<td>30&quot; vane - 3&quot; staff - small rapid yaws &amp; sways</td>
</tr>
<tr>
<td>43</td>
<td>0.6</td>
<td>Sphere</td>
<td>30&quot; vane - 6&quot; staff - 20° yaw 12&quot; sway</td>
</tr>
<tr>
<td>44</td>
<td>1.0</td>
<td>Sphere</td>
<td>30&quot; vane - 6&quot; staff - 12° yaw 10° sway</td>
</tr>
<tr>
<td>45</td>
<td>1.5</td>
<td>Sphere</td>
<td>30&quot; vane - 6&quot; staff - smaller motion</td>
</tr>
<tr>
<td>46</td>
<td>0.6</td>
<td>Sphere</td>
<td>30&quot; vane - 12&quot; staff - 15° yaw 10&quot; sway</td>
</tr>
<tr>
<td>47</td>
<td>1.0</td>
<td>Sphere</td>
<td>30&quot; vane - 12&quot; staff - 20° yaw 20° sway</td>
</tr>
<tr>
<td>48</td>
<td>1.5</td>
<td>Sphere</td>
<td>30&quot; vane - 12&quot; staff - better</td>
</tr>
</tbody>
</table>
3.3.10 CABLE OSCILLATIONS

3.3.10.1 GENERAL - The cable and towed body system will undergo two principal types of vibration, oscillation as a vertical spring and transverse oscillations.

3.3.10.2 VERTICAL OSCILLATIONS - The undamped natural frequency of vertical oscillations is:

\[ \omega_n = \sqrt{\frac{K}{W}} = \frac{2\pi}{T} \]

Where:
- \( \omega_n \) = Frequency, Radians/Sec
- \( K \) = Spring Constant Lb/Ft
- \( g \) = Accel. of Gravity Ft/Sec^2
- \( W \) = Weight of Load (including 1/3 of spring) Lbs
- \( T \) = Period of Vibration - Sec.

Since the vehicle with entrained water will weigh about 6,000 pounds and since the cable weighs 0.47 pounds/Ft. in water and 0.58 pounds/Ft. in air, the natural period \( T \), is:

\[ T = 2\pi \sqrt{\frac{W + 6,000}{Kg}} = 2\pi \sqrt{\frac{W + 58 }{32.2 \times 1.94 \times 10^6}} \]

\[ K = k \rho \]  
Where: \( k \approx 1.94 \times 10^6 \)  

or \( T = 2\pi \sqrt{\frac{1.94 l^2}{62.5 \times 10^6} + \frac{W l}{62.5 \times 10^6}} \)
\[ \tau = 2\pi l \sqrt{\frac{194}{82.5 \times 10^6} + \frac{1/8}{l \times 62.5 \times 10^5}} \]

\[ = \frac{2\pi l}{7.9 \times 10^5} \sqrt{194 + \frac{1}{l}} \]

\[ = 8 \times 10^{-4} l \sqrt{194 + \frac{1}{l}} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( % )</th>
<th>( \sqrt{194 + \frac{1}{l}} )</th>
<th>( \tau )-sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>10</td>
<td>3.2</td>
<td>1.53</td>
</tr>
<tr>
<td>1,000</td>
<td>6</td>
<td>2.48</td>
<td>1.99</td>
</tr>
<tr>
<td>6,000</td>
<td>1</td>
<td>1.09</td>
<td>5.23</td>
</tr>
<tr>
<td>10,000</td>
<td>.6</td>
<td>.89</td>
<td>7.1</td>
</tr>
<tr>
<td>18,000</td>
<td>.33</td>
<td>.72</td>
<td>10.2</td>
</tr>
<tr>
<td>24,000</td>
<td>.25</td>
<td>.66</td>
<td>12.7</td>
</tr>
</tbody>
</table>

These values are plotted on Figure 3.

Since the estimated natural period varies from two seconds to twelve seconds, at some time the ship and tow system will be in synchronism. If bad oscillations occur, the best approach is to either wait for a change in the weather or operate temporarily in other depths. Persistent problems will call for a redesign. However, other vehicles have operated successfully with this cable.

The velocity of this type of vibration through the cable is:
USL Tech Memo  
No. 2320-50-69

\[ c = \sqrt{\frac{Y}{P}} \]  
where: \( Y = k \) = Elastic Const. \( \frac{\text{ft}}{\text{ft}} \) 
\( P = \text{mass/ft} = \frac{W}{g} = \frac{\text{sec}^2}{\text{ft}^2} \)

For our cable:

\[ Y = k = 1.94 \times 10^6 \frac{\text{lbs}}{\text{ft}^2/\text{ft}} \text{ (ksi)} \]
\[ P = \frac{0.58 \frac{\text{lbs}}{\text{ft}}}{322 \text{ ft/sec}^2} = 0.18 \frac{\text{sec}^2}{\text{ft}^2} \]

\[ \frac{Y}{P} = \frac{1.94 \times 10^6 \frac{\text{lbs}}{\text{ft}^2/\text{ft}}}{0.18 \frac{\text{sec}^2}{\text{ft}^2}} = 1.075 \times 10^8 \text{ ft}^2/\text{sec}^2 \]

\[ c = \sqrt{\frac{Y}{P}} = 1.035 \times 10^{4} \text{ ft/sec} \]
\[ \approx 19,000 \text{ ft/sec} \]

It is apparent from this that longitudinal motions of the surface end of the cable will be felt very rapidly at the towed vehicle.

3.3.10.3 LATERAL OSCILLATIONS - In addition to the expansion compression type of vibration, the cable will experience lateral vibrations. These lateral motions are caused by sway and surge of the towing ship as well as the component of the heave and pitch of the stern which is normal to the tow cable.

The speed with which these disturbances move down the cable is:

\[ c = \sqrt{\frac{Pc}{k}} \]

Where: \( C = \text{Speed} \)
\( Tc = \text{Tension in cable} \)
\( S = \text{Linear mass density} \)
\( = \frac{\text{Wt/ft}}{\text{Accel. gravity}} \)

Since the cable weighs .58 pounds/ft. in air and has an added
mass of \( \pi \rho a^2 \), \( a = \frac{d}{2} = \frac{66}{2} \), the linear mass density is:

\[
\rho = \frac{.58 \text{ ft}^2}{\text{sec}^2} + \frac{\pi \cdot 2 \cdot (66)^2}{144} \text{ ft}^2
\]

\[
= .018 + .0048 = .023 \text{ ft}^2/\text{sec}
\]

\[
T = 2000^2 + .47l
\]

where \( l \) = length of cable

\[.
. c = \sqrt{\frac{(2000 + .47l)^2}{\text{sec}^2} / \frac{\text{sec}}{\text{ft}}}
\]

\[.
= \sqrt{87000 + 20.5l} \text{ ft/sec}
\]

For various values of \( l \), the speeds are:

<table>
<thead>
<tr>
<th>( l )</th>
<th>( 20.5l ) (87,000 + 20.5l)</th>
<th>( c ) (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>87,000</td>
</tr>
<tr>
<td>1,000</td>
<td>20,500</td>
<td>107,500</td>
</tr>
<tr>
<td>2,000</td>
<td>41,000</td>
<td>128,000</td>
</tr>
<tr>
<td>5,000</td>
<td>102,500</td>
<td>189,500</td>
</tr>
<tr>
<td>10,000</td>
<td>205,000</td>
<td>292,000</td>
</tr>
<tr>
<td>15,000</td>
<td>307,500</td>
<td>394,500</td>
</tr>
<tr>
<td>20,000</td>
<td>410,000</td>
<td>497,000</td>
</tr>
</tbody>
</table>

See Figure 4 for plot of velocities.
The speed of the disturbance decreases as the disturbance approaches the towed body. The equation of the transverse motion is:

\[ y = a \sin \omega (t - \frac{x}{c}) \]
\[ \dot{y} = a \omega \cos \omega (t - \frac{x}{c}) \]

*WHERE: \( y = \text{displacement} \)
\( a = \text{amplitude} \)

The energy in one wavelength is one half the product of mass and velocity at the axis or:

\[ E_w = \int_0^l \frac{1}{2} \dot{y}^2 \, dx = 4 \int_0^{\frac{1}{4} l} \frac{1}{2} \dot{y}^2 \, dx \]

\[ E_w = \frac{2\pi}{4} a_0^2 \omega \cos \omega t \left( - \frac{x}{c} \right) \, dx \]

\[ = -2\pi a_0^2 c \omega \int_0^{\frac{1}{4} l} \cos \left( \frac{x}{c} \right) \, dx \int_0^l \sin \left( \frac{x}{c} \right) \, dx \]

\[ = -2\pi a_0^2 c \omega \left[ \frac{1}{2} \frac{\omega^2}{c^2} + \frac{1}{4} \sin \left( \frac{2\omega l}{c} \right) \right]_0^{\frac{1}{4} l} \]

\[ = \pi a_0^2 c \omega \frac{\pi}{2} \quad \text{and} \quad \Delta E_w = \pi a_0^2 \omega \frac{\pi}{2} \, da_m \]
OR \( a_m = \sqrt{\frac{2E_w}{Sc\omega^{11}}} \)  

\[ \Delta a_m = \frac{\Delta E_w}{\sqrt{2Sc\omega^{11}E_w}} = \frac{\Delta E_w}{\sqrt{2Sc\omega^{11}} a_m} \]

\[ = \frac{\Delta E_w}{Sc\omega^{11} a_m} \]

Since \( E_w = \text{energy/ wave length} \), the energy per ft. is \( E_w/L \)

OR: \[ \Delta a_m = \frac{L}{Sc\omega^{11} a_m} \cdot \Delta \left( \frac{E_w}{L} \right) \]

Where \( \Delta \left( \frac{E_w}{L} \right) = \text{loss of energy/ ft. of length} \)

The energy lost per foot per cycle

The rate of energy loss is:

\[ \frac{Dy}{dt} = \frac{E_l}{E_t} = \text{ft}^2/\text{sec ft} \]

Where: \( D = \text{drag of cable per foot normal to its axis} \)

\[ = \frac{\rho}{2} \cdot C_0 \cdot d \cdot u^2 \]

\( \rho = \text{mass density of water} \)

\[ = \frac{\rho}{2} \cdot C_0 \cdot d \cdot y^2 \]

\( C_0 = \text{drag coeff.} \)

\[ = K_0 \cdot y^2 \]

\( d = \text{dir. ft.} \)

\( u = \text{velocity} \)
OR $K_p \frac{d^3}{dt^3} = \frac{E_L}{dt}$, $T_4 = \frac{\pi}{2\omega}$

OR $E_L = \int K_0 \frac{d^3}{dt^3} dt = K_0 \int a_m^2 \omega^3 \cos^3 \omega (t - \xi) d(\omega t) \frac{\pi}{2\omega}$

$= 4 K_0 a_m^2 \omega^2 \left[ \frac{1}{3} \sin \omega (t - \xi) \left( \cos^2 \omega (t - \xi) + 2 \right) \right]_0$

$= \frac{8}{3\pi} K_0 a_m^3 \omega^2$

$= \text{LOSS OF ENERGY/CYCLE/FT.}$

\[ \therefore \text{The decrease in amplitude due to loss of energy is:} \]

$\frac{d a_m}{dt} = -\frac{L}{5c \omega \Pi a_m} \cdot \frac{8}{3} K_0 a_m^3 \omega^2 \text{ PER CYCLE}$

$= -\frac{8}{3} \frac{L K_0 a_m^2 \omega}{c \Pi S} = -\frac{8}{3} \frac{K_0 a_m^2 \omega}{S \Pi l} = -\frac{16}{3} \frac{K_0 a_m^2}{S \Pi l}$

\[ \text{SINCE ONE CYCLE IS } L \text{ FT.} \]

$\frac{d a_m}{P T} = -\frac{16}{3} \frac{K_0 a_m^2}{S L \Pi} = -\frac{16}{3} \frac{K_0 a_m^2}{S T \Pi} = -\frac{16K_0 a_m \sqrt{S}}{3ST \Pi \sqrt{N_T + w l}} \]
Increase of amplitude due to reduction in wave length and velocity, if no energy were lost:

\[ E_w = \frac{S a^2 c \alpha \pi}{2} \]  
\[ \text{SINCE } C = \sqrt{\frac{W_T + wL}{g}} \]

\[ l = \text{DIST. FROM TOWED BODY} \]

\[ l = L - S \quad L = \text{LENGTH OUT} \]

\[ S = \text{DIST. FROM SURFACE} \]

\[ a = \frac{a^2 \pi \sqrt{g}}{\omega^2 (W_T + wL)} \]

\[ \text{AND } da = \frac{W}{4} \sqrt{\frac{a E_w}{\omega^2 (W_T + wL)^{3/2}}} \]

\[ \text{OR } da = \frac{W}{4} \frac{a m (\pm S)}{(W_T + wL)} \]

Now, the net change in amplitude is:

\[ \frac{\partial a_{\text{net}}}{\partial S} = \frac{\partial a_{\text{net}}}{\partial T} = \frac{W}{4} \frac{a m}{(W_T + wL)} - \frac{16 K_b a_m^2}{3 \sqrt{8} T \sqrt{W_T + wL} - \frac{16 K_b a_m^2}{3 \sqrt{8} T} \left( \frac{W}{4 \sqrt{W_T + wL}} \right)} \]

\[ = \frac{a m}{\sqrt{W_T + wL}} \left( \frac{W}{4 \sqrt{W_T + wL}} - \frac{16 K_b a_m^2}{3 \sqrt{8} T} \right) \]

The amplitude will decrease until the terms in parenthesis are equal, i.e.:

\[ a_{\text{FINAL}} = \frac{\sqrt{g} T W}{6 + K_b \sqrt{W_T + wL} - \frac{16 K_b a_m^2}{3 \sqrt{8} T}} \quad l = L - S \]
i.e., the amplitude will approach $a_{\text{FINAL}}$

For a typical case,

$g = \frac{0.023}{\text{s}^2/\text{Ft}^2}$  \quad T = 6 \text{ sec.}$

$w = 0.47 \text{ #/Ft}$

$W_T = 2000 \text{ #}$

$k_0 = \frac{C_0a}{d} = \frac{66}{12} = 0.055 \text{ #/Ft}^2$

$a_{\text{FIN}} = \frac{3 \times 151 \times 60 \times 0.47}{64 \times 0.055 \sqrt{2000 + 0.47}}$

$= \frac{1.28}{3.53 \sqrt{2000 + 0.47}}$ or $\frac{1.28}{3.53 \times 45} = 0.025 \text{ Ft.}$

(very small)

At the end of the line if $l = 0$. However, if no weight is present, $a_{\text{FIN}}$ becomes large.

Without damping:

$$\frac{\partial a}{\partial l} = \frac{wa_m}{4(W_T + w)}$$

$$\int a \partial l = \frac{w}{4(W_T + w)} \int_S s \quad l = l - S$$

$$= \int_0^S \frac{w}{4(W_T + w(l-S))} \partial s = -\frac{l}{4} \int_{-S}^S \frac{-w ds}{W_T + w(l-S)}$$

$$\log \frac{a}{a_m} = -\frac{1}{4} \log \left[ \frac{W_T + w(l-S)}{W_T + w} \right]$$
\[
\frac{a}{a_m} = 4\sqrt{\frac{W_T + WL}{W_T + WL(l-s)}}
\]

\[
a = a_m \left(\frac{W_T + WL}{W_T + WL(l-s)}\right)^{1/4}
\]

IF \(W_T = 0\) AND \(s = l\) \(a = \frac{a_m}{4}\) i.e. VERY LARGE

With damping:

\[
\frac{\partial a}{\partial l} = \frac{w a_m}{4(W_T + WL)} - \frac{16K_0 a_m^2}{3\sqrt{S} T \sqrt{W_T + WL}}
\]

\[
\frac{\partial a}{\partial s} = \frac{16K_0}{3TW\sqrt{S}} \int_s^{\infty} \frac{-wds}{\sqrt{W_T + WL(s)}}
\]

\[
-a' = \frac{16K_0}{3TWl\sqrt{S}} \left[2\sqrt{W_T + WL(s)}\right]_0^s
\]

\[
a_m - a' = \frac{32K_0}{3TWl\sqrt{S}} \left(\sqrt{W_T + WL(s)} - \sqrt{W_T + WL}\right) = \frac{1}{a_m} - \frac{1}{a}
\]

\[
\therefore a = a_m \left[1 + \frac{a_m \cdot 32K_0}{5TWl\sqrt{S}} (-\sqrt{W_T + WL(s)} + \sqrt{W_T + WL})\right]
\]
IF \( W = \beta WL \) AND \( S = \gamma L \), \( 0 < \gamma \leq 1 \)

\[
a = a_m \left[ \frac{1}{1 + a_m \frac{32Kd}{\pi BTW\sqrt{8}} \sqrt{WL} \left( \sqrt{1 + \beta} - \sqrt{1 + \beta - \delta} \right)} \right]
\]

IF \( L = 10,000 \) FT. \( S = 1,000 \) FT. \( W = 2,500 \) W. \( W = 0.5 \) FT.

\( K_d = 0.055 \) FEET\(^2\)/SEC\(^2\) FT. \( S = 0.023 \) FEET/FT.

\( T = 6 \) SEC.

\[
\begin{align*}
\sqrt{W + W} & = \sqrt{2,500 + 500} = \sqrt{3,000} = 87 \\
\sqrt{W + W} & = \sqrt{2,500} = \sqrt{2500} = 50 \\
\frac{32Kd}{\pi BTW\sqrt{8}} & = \frac{32 \times 0.055}{\pi \times 6 \times 15 \times 50} = 0.41
\end{align*}
\]

\[
\therefore a = a_m \left[ \frac{1}{1 + a_m \cdot 0.41 (87)} \right]
\]

\[
= \frac{a_m}{1 + 15.1 \ a_m}
\]

IF \( a_m = 5 \) FT.

\[
a_{1000 \text{ FT}} = \frac{3}{1 + 45.3} = 0.065 \text{ FT}
\]

From these calculations it is apparent that with a reasonably heavy vehicle, the transverse wave disturbances will be small.
3.3.11 SNAP ACTION - The possibility of snap action on the cable when near the surface, either when paying out or hauling in, was investigated. For this purpose, the following parameters were used.

(See reference 2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight in air - est.</td>
<td>2200 #</td>
</tr>
<tr>
<td>Weight in water</td>
<td>2000 #</td>
</tr>
<tr>
<td>Weight incl. ent. water</td>
<td>6000 #</td>
</tr>
<tr>
<td>Period of vertical motion</td>
<td>4 seconds</td>
</tr>
<tr>
<td>of tow point</td>
<td></td>
</tr>
<tr>
<td>Amplitude of motion</td>
<td>8 feet</td>
</tr>
<tr>
<td>Drag coefficient - est.</td>
<td>0.5</td>
</tr>
<tr>
<td>Proj. area, est.</td>
<td>14 Ft.²</td>
</tr>
</tbody>
</table>

Est. terminal velocity occurs when drag = weight in water.

\[
\frac{p}{2} S v^2 C_d = 14.0 v^2 \times 0.5
\]

\[
2000 \# = \frac{p}{2} S v^2 C_d = 14.0 v^2 \times 0.5
\]

\[
286 = v_t^2
\]

\[
17 \text{ FT/SEC} = v_t
\]

Est. velocity of tow point = \[2\pi v_t/\tau\]

\[
= 2 \times 4\pi/4 = 6.3 \text{ FT/SEC}
\]

The body will maintain a strain on the line when moving down.

The motion of the tow point will also cause an effective increase and decrease in the weight of the body as it reaches the top and bottom of its cycle. This acceleration may be estimated as:

\[
\ddot{x} = 4a \pi^2 \tau^2
\]

\[
= 16\pi^2/6 = 9.9 \text{ FT/SEC}^2
\]

This is about \[9.9/32.2 = 0.3089\]
Since the estimated weight in water is 2000 pounds, but the effective mass is 6000 pounds when the weight of entrained water is included, the change in tension will be plus or minus 1900 pounds. Since this is exceeded by the weight in water, slack cable will not result, however, the margin is too small for comfort. During final assembly, additional lead will be added, if necessary, to bring the weight up to about 2500 pounds.

3.4 DOPPLER SENSORS - The vehicle will be fitted out with a doppler navigation system. This system will provide the horizontal coordinates of the vehicle with reference to any time zero. Systems have been designed to operate at these depths and provide this type of information so that this system is presently state-of-the-art. The doppler information will also be processed to provide the operator with vertical velocity information. This information may also be used as a qualitative check on the operation of the depth sensor.

3.5 TELEMETRY - The telemetry system will be a contractor supplied item. However, this system has been reviewed to provide assurance that it may be accomplished. It is also necessary to establish clear cut interfaces with Government furnished equipment.

The data will be digitized, and sent up the coaxial cable. The amount of data to be telemetered does not tax the capability of state-of-the-art telemetry.

Details of this system may be found in the purchase description.

3.6 COMPUTER ANALYSIS OF DATA - The objective of the initial data processing is to establish the track and define the profile of the bottom along the track. The primary information to be obtained is:

a. Nominal track (selected by operator)
b. Average track
c. Bottom elevation, track distance and deviation for all points.

The coordinate system is:

X - distance along track from zero
Y - distance to right of track
Z - elevation from zero
N - distance North from zero
E - distance East from zero
The data input to the computer is given in paragraph 4.3.2.9.2 of the purchase description.

Let time zero be selected at or near beginning of data.

Place all data on quick access memory. Select last data point and obtain trial track slope, i.e. \( m = N/E \)

Obtain the center of gravity of the points:

\[
N_g = \frac{\sum_{n=0}^{n} N_n}{R} \quad N_g = \text{North dist. of C.G.}
\]

\[
E_g = \frac{\sum_{n=0}^{n} E_n}{R} \quad E_g = \text{East dist. of C.G.}
\]

Using this point as the origin and the trial track as the slope, determine the sum of the moments of the deviations, as follows:

Distance of any point, \( n \), from the track = \( d_n \), where:

\[
d_n = \frac{m (E_n - E_g) - (N_n - N_g)}{\sqrt{m^2 + 1}}
\]

The distance of any point, \( n \), from the origin = \( D_n \) where:

\[
D_n = \frac{(E_n - E_g) + m (N_n - N_g)}{\sqrt{m^2 + 1}}
\]

Moment of the deviations is \( M \) where: \( M = \frac{d_n}{D_n} \)

If \( M \neq 0 \), rotate the track by changing "m" and repeat until \( M \) is less than a small number.

This last track is the average track \( (N_T - N_g) = m_T (E_T - E_g) \)

where: subscript "T" refers to average track.

Let \( Z_n \) be the depth of the bottom at data point "n".

Let \( Z_n' \) be the depth of the vehicle at data point "n".

Let \( Z_n'' \) be the height of the vehicle from the bottom at data
point "n", i.e., the bottom sonar.

Then \( Z_n = Z'_n + Z''_n \) (This assumes small \( \alpha_n \))

Let \( X_n \) = distance of illuminated spot along the average track
at data point "n"

\( Y_n \) = deviation of vehicle and illuminated spot from the
average track at data point "n"

\( \alpha'_n \) = pitch angle of vehicle at data point "n"

\( X'_n \) = distance of vehicle along the average track at data
point "n"

The distance of the illuminated spot fore or aft of the vehicle
is:

\[ \alpha'_n \cdot Z'_n \]

Therefore:

\[ X_n = X'_n + \alpha'_n \cdot Z'_n \quad \text{WHERE:} \]

\[ X'_n = \sqrt{m^2 N_g + \epsilon^2} \]

\[ Y_n = d_n \]

If \( \alpha'_n \approx \) small,

\[ Z'_n = Z'_n + Z''_n \cos \alpha'_n \]

The value for \( Z'_n \) may be taken either from word 4, i.e., depth
meter, or word 33, the vertical distance traveled from time zero.

The values for \( X'_n, Y'_n, \) and \( Z'_n \) for each point and the value of
\( N_g, E_g, \) and \( m_\perp \) which define the average track will be stored.

The distance from \( X_0 \) to \( X_R \) will be divided into equal parts,\( \Delta X \) being 0.5 ft. The values at these increments will
be called by subscript, \( \beta \).

i.e., \( X_i = X_0 + i \Delta X \)

and interpolating for \( Y_i \) and \( Z_i \)

\[ Y_i = \frac{(X_i - X_h)(Y_j - Y_h)}{(X_j - X_h)} + Y_h \quad \text{for} \quad h < i \]

\[ Z_i = \frac{(X_i - X_h)(Z_j - Z_h)}{(X_j - X_h)} + Z_h \quad \text{for} \quad j > i \]
The three values at equal distances are now stored, i.e., $X_i$, $Y_i$, and $Z_i$.

3.7 SHIPBOARD INSTALLATION - Although no ship is specifically selected for this system, the system has been reviewed continuously as to its ability to operate from an AGOR such as the U.S.N.S. SANDS.

Several points are pertinent to the installation aboard a ship.

1. Ships speed - operators must have control of ship at speeds as low as one knot.

2. Deck crane - must have A-frame or deck crane operable at sea which can handle the vehicle.

3. Laboratory space - or space for heli hut containing control console and tape recorder must be available.

4. Winch capacity to handle cable.

5. Cable reel with capacity to store sufficient cable with slip ring having low noise level.

The ship has sufficient capability in all of the above respects. However, it is marginal in respect to the cable it can store on its intermediate winch. This winch can stow 30,000 feet of $\frac{1}{2}$" diameter wire rope. However, it is intended to use .66" diameter wire for the towed vehicle. This means that the capacity will be approximately 22,000 feet. This will reach to 20,000 feet depth if the vessel speed is kept below 1.0 knots. See Figure 1. At shallower depths, the speed could be increased if desired.

If it is desired to carry more cable, the deep sea anchoring cable drum may be used.

The ship is fitted with a bow propeller which may be used for this service.

4.0 SCHEDULE

The present planned schedule is to make a procurement decision in June of 1969.
The probable schedule will then be to let contracts for the components by September 1969 and obtain delivery by March 1970. The dates are shown in Figure 5.

5.0 GENERAL DISCUSSION

In general, the study has covered the questions pertaining to the feasibility of the system. No guarantee can be given that systems designed as indicated will perform as specified until actual tests with a prototype have been completed.

Since no definite test data has yet been obtained to define the maximum resolution of either proposed pressure gage, the maximum bottom resolution is open to question. However, tests at USL show that the resolution of the vibrotron is at least about 7 inches or 0.6 foot. Further testing should provide more conclusive data.

The ability to meet the other requirements of the system does not appear to be in question. The cost and time are presently not considered excessive. The installation of the doppler system gives this device an ability no other towed device has, i.e., it will know where it is at all times.

H. T. LOESE, NAVAL ARCHITECT

REFERENCES


5. N.S.R.D.C. Motion Picture Film M-2199 of January 1969, USNUSL Towed Vehicle Program.
APPENDIX

EFFECT OF TRAIL DISTANCE ON TRACK ERROR DUE TO HELMSMAN'S ERROR

Let \( y_2 = a \sin \frac{x_2}{c} \)

\[
\frac{dy_2}{dx_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a \sin \frac{x_2}{c} - y_1}{t}
\]

Since \( y_2 - y_1 \) is small relative to \( t \), let \( x_2 - x_1 = t \)

\[
\frac{dy_2}{dx_1} = \frac{a \sin (x_2 + t) - y_1}{t}
\]

Let \( y = U \xi \cdot \frac{dU}{dx} = U \frac{d\xi}{dx} + \xi \frac{dU}{dx} \)

\[
U \frac{d\xi}{dx} + \xi \frac{dU}{dx} = \frac{a}{t} \sin \left( \frac{x_2 + t}{c} \right) - \frac{U \xi}{c}
\]

Solving:

\[
y = U \xi = \frac{a}{t} \left[ \frac{\sin \left( \frac{x_2 + t}{c} \right) - \cos \left( \frac{x_2 + t}{c} \right)}{\left( \frac{1}{c} \right)^2 + \left( \frac{a}{c} \right)^2} + C_2 e^{-\frac{x}{c}} \right]
\]

or

\[
y = \frac{a}{R^2 + \Theta} \left[ R^2 \sin \frac{x_2 + t}{R} - \Theta \cos \frac{x_2 + t}{R} + \frac{R^2 + \Theta^2}{R^2} C_2 e^{-\frac{x}{R}} \right]
\]

Since \( y = 0 \) when \( x = 0 \),

\[
C_2 = \frac{t R (t \cos \frac{x_2}{R} - R \sin \frac{x_2}{R})}{R^2 + \Theta^2}
\]
\[
  y = \frac{a}{R^2 + t^2} \left[ (R \sin \frac{x+\theta}{2} - t \cos \frac{x+\theta}{2} \right] - \frac{b}{2} \left( R^2 \sin \frac{x}{2} - t \cos \frac{x}{2} \right)
\]

When \( x \) is large this becomes:

\[
  y = \frac{a}{R^2 + t^2} \left[ R \sin \frac{x+\theta}{2} - t \cos \frac{x+\theta}{2} \right]
\]

The maximum excursions of \( y \) occur when

\[
  \sin \frac{x+\theta}{2} \quad \text{or} \quad \cos \frac{x+\theta}{2} = 0
\]

\[
  \therefore y_{\text{max}_1} = -\frac{a \theta}{R^2 + t^2}
\]

\[
  y_{\text{max}_2} = \frac{a R}{R^2 + t^2}
\]

For a typical case:

Trail dist. = 2 mi = \( t = 12,000 \) ft

Ship excursion = \( \pm 1000 \) ft. = \( a \)

If ship period is 6.25 ft then \( R = 1000 \) ft

\[
  y_{\text{max}_1} = 83.7 \text{ ft/mi}
\]

\[
  y_{\text{max}_2} = 7 \text{ ft/mi}
\]

A graph of \( f \) vs \( t \) is given in Fig. 1. Where \( y_{\text{max}} = a + t \)
USL Tech Memo
No. 2320-50-69

FIG 1

Wt. of Cable .47 lb/ft
Dia. = 0.61 in.

Req'd Leth. of Cable

SPEED

1.0

2.0 knots

0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 FT/SEC
PROJECTED AREA = 0.46 ft²
EST. DRAG COEF. 0.47

EST. WT/DRA G RATIO

WT. IN WATER
9.9 ft

EST. WT/DRA G RATIO
FOR VEHICLE

Fig. 2
SPEED OF A DISTURBANCE IN THE TOW LINE — TRANSVERSE DISTURBANCE SHOWN.
LONGITUDINAL DISTURBANCE Vel. 45
ABOUT 10,000 FT/SEC.

FIG. 4

SPEED OF DISTURBANCE — FT/SEC.
MAX EXCURSION OF TOWED VEHICLE UNDER SIMPLIFYING ASSUMPTIONS

MAX EXCURSION = \( Y_{MAX} = 0.6 \times f \)

G.M. MAXIMUM INTEGRAL ENERGY

T = PERIODIC DISTANCE OF OCCUR.

\( T = \text{TRAIL DISTANCE} \)

TRAIL DISTANCE OF TOWED VEHICLE

APPENDIX FIG. 1

10 JUNE 68

N. T. L.
# Bottom Statistics Feasibility Report No. 1 by H. T. Loeser

Dated 17 April 1969

## Technical Memorandum

**UBL Tech Memo No. 2320-50-69**

## Evaluation Report

**Service Test Report**

## Test Report

## Letter

## Other

### Classification

**Unclassified**

### Number of Copies Made

27

<table>
<thead>
<tr>
<th>Copy No.</th>
<th>Description</th>
<th>Copy No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Code 1000/1100</td>
<td>17</td>
<td>Code 2064.2</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>18</td>
<td>2064.2</td>
</tr>
<tr>
<td>3</td>
<td>2010</td>
<td>19</td>
<td>2064.2</td>
</tr>
<tr>
<td>4</td>
<td>2111.5 (A. Ellinthorpe)</td>
<td>20</td>
<td>2064.2</td>
</tr>
<tr>
<td>5</td>
<td>2111.5 (A. Ellinthorpe)</td>
<td>21</td>
<td>2064.2</td>
</tr>
<tr>
<td>6</td>
<td>2111.5 (A. Ellinthorpe)</td>
<td>22</td>
<td>2064.2</td>
</tr>
<tr>
<td>7</td>
<td>2300</td>
<td>23</td>
<td>2064.2</td>
</tr>
<tr>
<td>8</td>
<td>2310</td>
<td>24</td>
<td>2064.2</td>
</tr>
<tr>
<td>9</td>
<td>2320</td>
<td>25</td>
<td>2064.2</td>
</tr>
<tr>
<td>10</td>
<td>2330</td>
<td>26</td>
<td>2321.1 (H. Loeser)</td>
</tr>
<tr>
<td>11</td>
<td>2321</td>
<td>27</td>
<td>2321.1 (H. Loeser)</td>
</tr>
<tr>
<td>12</td>
<td>2321.1 (H. Loeser)</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2322 (R. Reddy)</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>2322 (R. Reddy)</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>23208</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2064.2</td>
<td>32</td>
<td></td>
</tr>
</tbody>
</table>

### Remarks