FOREIGN TECHNOLOGY DIVISION

SUPersonic flow in the area of antisymmetric thin cruciform wings with supersonic leading edges in a horizontal plane, with consideration of flow separation on the edges

By

Stefan Staicu

Approved for public release; distribution unlimited.
EDITED TRANSLATION

SUPERSONIC FLOW IN THE AREA OF ANTISYMMETRIC THIN CRUCIFORM WINGS WITH SUPERSONIC LEADING EDGES IN A HORIZONTAL PLANE, WITH CONSIDERATION OF FLOW SEPARATION ON THE EDGES

By: Stefan Staicu

English pages: 16

Source: Buletinul Institutului Politehnic, Vol. 33, Nr. 1, 1971, pp. 103-118

Country of Origin: Romania

Translated by: SCITRAN

Requester: FTD/TQTA

Approved for public release; distribution unlimited.

THIS TRANSLATION IS A RENDERING OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.
SUPersonic flow in the area of antisymmetric thin cruciform wings with supersonic leading edges in a horizontal plane, with consideration of flow separation on the edges

Author: Stefan Staicu

1. General Considerations

A study is made of the flow in the supersonic regime in the area of a thin cruciform wing with an antisymmetric distribution of incident angles. The horizontal plane has supersonic leading edges and flow separation is considered along the subsonic leading edge line of the vertical plane.

We shall therefore consider a cruciform wing composed of two simple delta wings perpendicular to one another, and refer it to a system of Cartesian axes $Ox_1x_2x_3$ with an origin in the wing apex and with the axis $Ox_1$ in the direction of the unperturbed stream $U_\infty$ (figure 1).

Figure 1
Let $\alpha$ be the equal incidence angles of opposite signs along the two halves of the horizontal wing, $+\beta$ the incident angles for the upper part of the sheet and $-\beta$ for the lower part.

Analogous to what happens for a thin cruciform wing with complete subsonic edges, in the present case the flow separates at the subsonic leading edges of the sheets, producing a vortex system located on the right and left sheets as a function of the incident angles $\alpha$ and $\beta$, which produces anti-symmetric motion.

Thus the flow is modified by the existence of two vortex apexes situated antisymmetrically with regard to the axis of symmetry and with the same intensity and sign (figure 1). Below we shall denote by ($-r'_0, t'_0$) the coordinates for the physical plane of the vortex core under which the apex formed at the edge of the sheet is concentrated.

Since the study of flow is becoming more and more complicated, we shall try below to find the effects of these vortices on the wing and on the sheet. Therefore we shall assume that the effect of flow separation at the leading edges of the sheet and the formation of vortex cores is to create a complex field of vertical and horizontal velocities which will modify the flow in such a way that the velocities are zero at the edge.

Under these considerations the flow in the area of the cruciform system remains conical longer and can be treated using the methods of conical flow theory for wings [1].

Proceeding as if the cruciform wing had complete subsonic edges, we shall consider that an actual thin cruciform wing, which in a way has certain finite velocities at the edges because of the effect of stream separation, is equivalent, from an aerodynamic point of view, to a fictitious thin wing with a convenient variety of incident angles (or velocities of lateral perturbation). In order to study the motion with ease, with the method of conical motion, we shall divide the fictitious thin wing into three component wings.
1. The thin cruciform wing has antisymmetric incidence on the sheets and is thus variable, so that there is some pressure modification and of normal perturbation velocities on the sheet in the vicinity of the leading edge. A fictitious thin wing is obtained with a finite velocity even at the subsonic edges of the plates, but equal and of opposite sign on the two sides, which does not agree with experimental findings.

2. A cruciform wing of "symmetric thickness" with equal sheet slopes and with the same signs as the first incidents of the component wing. This wing, combined with the first, will form a tall cruciform wing which will have different pressures on the two halves of the sheet, somewhat approaching the real situation.

3. A third wing will have a "symmetric thickness" with variable slope, so that in combination with the second wing a mean slope of zero will be found, corresponding to an actual thin wing. This wing will have the role of total compensation for the aerodynamic effects of the second wing in the field of normal perturbation velocities.

If we superimpose these three component fictitious wings, we shall actually be totalling their aerodynamic effects. We obtain a cruciform wing equivalent to a real one, but with consideration of separation on the edge.

2. Axial Perturbation Velocities

Below we shall determine the axial perturbation velocities for these three fictitious component cruciform wings necessary to calculate the pressure distribution on these four arms as though we were determining the aerodynamic properties of real wings. We shall now indicate the flow variables and the planes used.

Starting from the physical plane yOz (figure 1) for the coordinates

3
and by transformation
\[ \eta = \frac{y}{1 - B^2 x^2}, \quad z = \frac{z \sqrt{1 - B^3 (y + z^2)}}{1 - B^2 z^2}, \quad (B = \sqrt{\frac{M^0}{M^*} - 1}), \]
we get the auxiliary plane
\[ x = \eta + i z. \]
represented in figure 2.

Figure 2.

Starting with figure (2), the height of the sheet and the ordinate of the core position of the vortex created at the leading edge of the sheet, \( x \) will be given in the auxiliary plane by the following formulae:
\[ b = \frac{h}{\sqrt{1 - B^2 x^2}}, \quad \gamma_0 = \frac{t_0}{\sqrt{1 - B^2 x^2}}. \]

From plane \( x \) we shall plot \( X \) in the complex plane (figure 3) through suitable transformation

Figure 3.

which is situated in a horizontal plane on the cruciform wing, similar to a plane delta wing. However, in order to define the axial perturbation velocities in primary form, we shall apply the method of hydrodynamic analogy in the plane \( \chi \) (figure 4), defined by the corresponding transformation
\[ z^* = \frac{1 + \mathbb{A} X}{1 - \mathbb{A} X}. \]
1. **Antisymmetric Thin Cruciform Wing with Variable Incidence**

As a result of the effects of these two antisymmetric vortices, the normal perturbation velocities at the surface of the cruciform wing are modified to correspond to a real wing, and therefore with the fictitious thin wing defined above. We shall consider that the vertical velocity on the wing with the supersonic edges is not modified, because of the existing vortices, but that the lateral one on the sheet \( v' \) will have the following variations:

\[
\begin{align*}
v_0 &= -\beta_0 U_\infty, (y = 0, -t_s \leq z \leq t_s), \quad (7a) \\
v' &= v'(z) = -\beta'(z) U_\infty, |z| = 0, y \xi(-h, -t_s) \cup (h, t_s), \quad (7b)
\end{align*}
\]

such that the velocity at the sheet edges becomes

\[
v_1 = -\beta_1 U_\infty. \quad (8)
\]

2h represents the opening of the sheet and \( t_s \) is a coordinate which limits the interval of variation of the lateral velocity on the sheet.

The continual variation in velocities \( v' \) or of the respective incidence angles corresponds to continuous distribution of elementary edges situated on the surface within the interval under consideration, while their contributions in the expression of axial perturbation velocity from point \( X \) in plane (6) will be

\[
d\mathbf{u}_i = q'_i(X) \left( \ln \frac{\gamma - \gamma_i}{\gamma + \gamma_i} - \ln \frac{\gamma - 1}{\gamma} - \ln \frac{\gamma - 1}{\gamma} \right) d\chi_i. \quad (9)
\]
in which \( \chi' \) is the abscissa of the edge stream in the wake of the wing:

\[
\chi' = \pm \sqrt{\frac{1 + \delta T}{1 - \delta I}}, \quad T' = \delta \gamma - \gamma'.
\]

Below we shall use the source distribution on the wing \( q'_i(\chi' \gamma) \) as the most simple, corresponding to the conditions imposed by the problem on lateral velocity \( v' \). Thus, looking in plane \( X \), the intensity of the source located in the wake of the sheet will be

\[
q_i(T) = q_i \frac{T}{b}, \quad T \in (-T_0, T_0).
\]

Taking this distribution of sources into consideration and applying the formulas established in conical motion theory [1], the axial perturbation velocity of the first component wing will be obtained by adding up the contributions of all elementary distributed edges. Thus, proceeding in plane \( X \) through transformation (6), where we determine the contribution of the subsonic edges of the sheets and the supersonic ones of the horizontal planes, we shall obtain the following expression

\[
n'_I = A_{1s} \frac{1}{X} \sqrt{1 - \delta^s X^2} + \frac{2}{\pi} K_{10} \text{arc cos} \left[ \frac{1}{2 \delta (L - X)} \right] - \text{arc cos} \left[ \frac{1}{2 \delta (L + X)} \right] + \frac{2}{\pi} q_i \int_{-T_0}^{T_0} \frac{1}{2 \delta (X + T)} \right] dT,
\]

which, as a result of calculation, becomes:

\[
n'_I = A_{1s} \frac{1}{X} \sqrt{1 - \delta^s X^2} + \frac{2}{\pi} K_{10} \text{arc cos} \frac{1}{\delta (L - X)} + \frac{1}{\pi b} \int \frac{2}{\delta} \left( T_0^2 - X^2 \right) \left[ \text{arc sin} \left( \frac{1}{\delta^s T_0 X} \right) - \text{arc sin} \left( \frac{1}{\delta T_0 X} \right) \right] dT_0.
\]

Taking the equation (2) into consideration, we shall obtain the following expression of the axial perturbation velocity for the points on the wing

\[ (x=\delta y, \quad s=0) \]

\[
u_t = \nu_0 \sqrt{1 - B^2 v^2} + \frac{2}{\pi} K_{10} \text{arc cos} \left( \frac{1}{1 + B^2 v^2} \left( 1 - B^2 v^2 \right) \right) + \frac{1}{\pi b} \int \frac{2}{B} \sqrt{(1 - B^2 y^2) \left( 1 + B^2 v^2 \right)} \text{arc cos} \left( \frac{1}{1 + B^2 v^2} \left( 1 - B^2 v^2 \right) \right) dT_0.
\]
but on the surface of the sheet, where we observe that
\[ \eta = y = 0, \quad x = i \frac{z}{\sqrt{1 - B^2 z^2}}, \]
we get
\[ u_{tp} = a_{18} \sqrt{1 - B^2 h^2} + \frac{2}{\pi} K_{18} \arccos \left\{ \frac{1}{2} \frac{1 - B^2 h^2}{1 - B^2 h^2} \right\} + \frac{q_t \sqrt{1 - B^2 h^2}}{\pi h (1 - B^2 z^2)} \left( \frac{1}{B^4 \left( \sqrt{1 - B^2 h^2} \right)^2} \right) \arg \chi \left( \frac{1}{1 - B^2 h^2} \right), \]
(16)

2. Cruciform Wing of Symmetric Thickness with Slopes Equal to the Incidence of the First Component Wing

We shall introduce the double cruciform wing with a symmetrical thickness plate in order to remove the accentuated pressure apices on its intrados.

Proceeding in the same way as in the case of a thin wing, we shall obtain the following expression for the axial perturbation velocity in plane \( X \):
\[ u_x = \frac{2}{\pi} Q_{16} \arg \chi \left\{ \frac{1}{\beta X} + \frac{q_t}{\pi h} \left( \frac{1}{2} \left( T_0 - X^2 \right) \right) \left( \arg \chi \left( \frac{1}{2} \frac{1 - \beta^2 t_0^2}{1 - \beta^2 t_0^2} \right) \right) \right\} + \arg \chi \left( \frac{1}{2} \frac{1 - \beta^2 t_0^2}{1 - \beta^2 t_0^2} \right) \right\} \]
3. Cruciform Wing of Symmetric Thickness Compensating for Slope

We shall compensate for the effect of the wing thickness resulting from the superposition of the first and second wings by introducing on the wing surface a new distribution of sources of a form which will return the wing to a mean zero thickness. Normal velocity variations on sheet \( v'' \), created by a new distribution of origins, will correspond to a "wing compensating for a slope" of symmetric thickness.

Taking equation (11) into consideration, we shall choose for the points of this wing

\[ q_t(t) = \frac{k_t}{h} t, \quad (-h \leq t \leq h). \]  

(21)

which in sheet \( X \) become

\[ q_t(T) = \frac{k_t}{\beta b} \sqrt{1 - \beta^2} - \frac{b^2}{\beta T} \left( \frac{b^2 - T^2}{1 - \beta^2 - T^2} \right), \quad (-b \leq T \leq b). \]  

(22)

and we get the following expression of axial perturbation velocity:

\[ u_0 = -\frac{2}{\pi} Q_{10} \arg \text{ch} \left( \frac{1}{\beta b} \right) + \frac{2k_t}{\beta b} \sqrt{1 - \beta^2} - \frac{b^2}{\beta T} \left( \frac{b^2 - T^2}{1 - \beta^2 - T^2} \right) \times \]

\[ \left\{ \arg \text{ch} \left[ \frac{(1 + \beta X)(1 - \beta T)}{2\beta(X - T)} \right] + \arg \text{ch} \left( \frac{1 + \beta X}{1 - \beta^2} \right) \right\} d \left( \frac{b^2 - T^2}{1 - \beta^2 - T^2} \right) = \]

\[ = -\frac{2}{\pi} Q_{10} \arg \text{ch} \left( \frac{1}{\beta X} \right) \frac{b^2}{\beta b} \sqrt{1 - \beta^2} \left[ (1 - \beta^2 b^2) X^2 \arg \text{ch} \left( \frac{1}{\beta X} \right) \right] \]

\[ -(X^2 - b^2) \arg \text{ch} \left[ \frac{1 - \beta^2 b^2}{B^2(X^2 - b^2)} \right] + \]

\[ + \frac{1}{\beta^2} \sqrt{1 - \beta^2 b^2} (1 - \sqrt{1 - \beta^2 b^2}) \sqrt{1 - \beta^2}. \]

Calculating the axial perturbation velocities on wing and sheet we get:

\[ \frac{2}{\pi} \frac{u_{ex}}{Q_{10}} \arg \text{ch} \left( \frac{1 + B^2 b^2}{B^4(b^4 + y^4)} \right) \],

\[ \frac{k_t}{\pi h} \sqrt{1 - B^2 y^2} \left( \sqrt{1 + B^2 b^2} - 1 \right) \sqrt{1 - B^2 y^2} \]

or

\[ \frac{2}{\pi} \frac{u_{ex}}{Q_{10}} \arg \text{ch} \left( \frac{1 - B^2 z^2}{B^4(h^4 - y^4)} \right) - \]

\[ - \frac{k_t}{\pi h} \left[ \left( \sqrt{1 - B^2 y^2} \right) \arg \text{ch} \left( \frac{1 - B^2 z^2}{B^4(h^4 - z^4)} \right) + z^2 \arg \text{ch} \left( \frac{1 - B^2 z^2}{B^4 z^4} \right) + \]

\[ + \frac{1}{B^4} \left( 1 - \sqrt{1 + B^2 h^2} \right) \right] \right\} \].

(25)
Observations

a) By superposing these three component wings we get a real wing for
which the axial perturbation velocity is the expression:

\[ u = u_l + u_r + u_c, \]

which will be antisymmetric to the axis of symmetry Ox_1, continuous and
different from zero at origin O.

b) If we made h\rightharpoonup h_o, (h\rightharpoonup h_o), in the results obtained, we get the case of a
delta wing with forced antisymmetry and with supersonic leading edges.

3. Determination of Constants

We calculate the constants Q_{10}, K_{10}, q_t, k_t in the same way as in a
plane delta wing [3, 9].

Thus, by using some conditions limiting normal perturbation velocities, we
find the constants a_{10}, K_{10} which appear in the expression of the axial
perturbation velocity (13). These equations are found beginning with the
compatibility equations

\[ du = -xd\Phi = \frac{ix}{\sqrt{1 - B^2 x^2}} d\Phi, \]

and considering the variations in velocity at one point on a wing or sheet,
up to a point of zero velocity at the Mach cone. Likewise in the plane delta
wing [3] we shall consider some concentrated sources at the point of their
distribution with intensity Q_t and position Y=T_o given by the equations

\[ Q_t = \frac{1}{2} \frac{a_t}{h} (y^2 - t_0^2), \]

\[ T_o = \frac{2}{3} T_o, \]

which, written in the physical plane, become

\[ Q_t = \frac{1}{2} \frac{a_t}{h} \frac{t^2 - t_0^2}{(1 - B^2 h^2)(1 - B^2 t_0^2)}, \]

\[ t_0 = \sqrt{\frac{9 h^2 (1 - B^2 t_0^2) - 4 (h^2 - t_0^2)}{4(1 - B^2 h^2) + 5 (1 - B^2 t_0^2)}} \]

as a function of t_o which limits the source distribution on the first thin
sheet. We shall write the following equations:
where \( U'_t \) is the axial perturbation velocity of the first wing component in the case of sources concentrated in \( z=t' \):

\[ a'_t = \pm \frac{A_{12}}{\lambda} \begin{pmatrix} 1 - \frac{x^2}{\lambda^2} \\ \frac{2}{\pi} \frac{\lambda}{k} \end{pmatrix} \int_{0}^{2\pi} \frac{\lambda}{k} \frac{d\lambda}{\lambda - \lambda^2} \]

(30a)

\[ \begin{align*}
Re B^{\text{cenal Mach}}_{\text{axial 2}} \int_{0}^{2\pi} \frac{1 - \frac{x^2}{\lambda^2}}{\frac{\lambda}{k} - \lambda^2} d\lambda &= -v_1,
\end{align*}
\]

(30b)

(Key: 1- Mach Number; 2- Wing; 3- Sheet.)

Integrating (30a) on the real axis between the limits of \((0, \infty)\) in the complex plane \( \lambda \), we obtain the constant

\[ K_{10} = \frac{a_{10} U'_t}{\sqrt{\beta^2 + 1}} \]

and also the equation

\[ \frac{2}{\pi} \frac{Q_t}{v_1} \int_{0}^{2\pi} \frac{(b^2 + y^2)(1 + B b^2)}{1 + B^2 b^2} \left[ K(k) - \frac{b^2}{\beta^2 - 1} \right] = \frac{2}{\pi} \frac{Q_t}{v_1} \int_{0}^{2\pi} \frac{d\theta}{1 + B^2 b^2} \]

(33)

where the module \( k \) and the parameters \( \rho_1, \rho_2 \) of the complete elliptical integrals which appear are

\[ k = \sqrt{1 - \beta^2}, \quad \rho_1 = -\frac{t^2}{t^2 + b^2}, \quad \rho_2 = -\frac{v_1^2}{b^2 - v_1^2} \]

(34)

From the condition of finite velocity at the subsonic edges of the sheets \((x = \pm t)\) we reduce

\[ a_{10} = A_{13} = 0. \]

(35)

The constant \( Q_t \) is calculated by us by determining \( Q_t \) for the first time from equation (31). Starting with equation

\[ Q_t(t) = \frac{t}{\sqrt{1 - B^2 t^2}} \frac{dv'}{dt} \]

(36)
deduced from the theory of conical movements [1], and taking into consideration
the fact that \( q' \) taken from (11) can also be written thus in plane \( x \),

\[
q'(t) = -\frac{q_1}{b} \frac{t}{(1 - B^2 t^2)^3}.
\]  

(37)

we will write the equations:

\[
v_1 - v_0 = -\frac{q_1}{b} \int_{t_0}^{t} \frac{d t}{(1 - B^2 t^2)^3} \]  

(38a)

\[
v_0 t_0 + v' \int_{t_0}^{t} t d v' = v h.
\]  

(38b)

These equations were written by placing limiting conditions at points
\( t = t_u \) and \( t = h \) for the lateral velocity \( v' \), as well as the condition of real
incidence in order to obtain the mean incidence.

As a result of calculations, we deduce from (38a) and (38b) the equations:

\[
q_1 [\sqrt{1 - B^2 t_0^2} - t_0] [\sqrt{1 - B^2 h^2}] = (v_0 - v_1) h [\sqrt{1 - B^2 t_0^2}],
\]  

(39a)

\[
q_1 \left( \sqrt{1 - B^2 h^2} - \sqrt{1 - B^2 t_0^2} \right) = (v_1 - v) B^2 h^2 [\sqrt{1 - B^2 t_0^2}].
\]  

(39b)

Next we introduce \( v_1 \) from (39b) and from (28a) and obtain the constant
\( q \) in the following form:

\[
\frac{q_1}{l_o} = \frac{b^2}{\left( \frac{b^2}{l_o} - 1 \right) \sqrt{\left( \frac{b^2}{l_o} - \frac{b^2}{l_o} \right) \left( 1 - \frac{b^2}{l_o} \right)}} \frac{\left[ K(k) - \frac{b^2}{b^2 + \frac{b^2}{l_o}} \Pi \left( \theta_1, k \right) \right]}{a - \beta} + \frac{\pi}{b^2 b^2} \frac{\left( 1 + b^2 b^2 \right)}{\left( 1 + b^2 b^2 \right)} + \frac{\left( 1 + \frac{b^2}{l_o} \right)}{4 b^2 b^2} \frac{\left( 1 + \frac{b^2}{l_o} \right)}{\left( 1 + \frac{b^2}{l_o} \right)}.
\]  

(40)

Equation (39a) was used to determine the velocity \( v_0 \) on the sheet. The
constant \( k_t \) found in expression (23) is determined by beginning with an equation
similar to (38b) and writing the equation for measuring the normal velocities
on the sheets of the three component wings, which will have a mean slope equal
to \(-v\):

\[
-v h = \int_{t_0}^{t} v^* \, d t = \int_{t_0}^{t} v^* \int_{t_0}^{t} t d v^*.
\]  

(41)

Equation (41), in consideration of (21), becomes

\[
3 B^2 h^2 (v - v_1) = k_t \left[ 1 - (1 - B^2 h^2)^{3/2} \right],
\]  

(42)

which, united with (39a), determines the constant \( k_t \):

\[
k_t = \frac{3 q_t}{1 - (1 - B^2 h^2)^{3/2}} \left( 1 - \frac{1}{1 - B^2 h^2} \right).
\]  

(43)
4. Distribution of Pressure and Aerodynamic Properties

Calculation of the coefficient of pressure on the wing and sheet is made by using the formula

\[ C_p = -2 \frac{U}{U_m} = -2 \frac{R_s}{U_m} \phi, \]  

(44)

In which the expressions for axial perturbation velocities \( U_a \) or \( U_p \), obtained from equation (26), are introduced in turn (figure 5).

![Graph showing distribution of pressure and aerodynamic properties](image)

**Figure 5.** Key: 1—linear theory, 2—present theory

The coefficient of lift for both sheets or the wing is found by using formula [2]:

\[ \frac{1}{2} B C_{s_{ma}} = \frac{2}{U_m} \int_0^{l_a} u_{la} dy, \]  

(45)
for the wing region comprised by the interior of the Mach cone,
\[ \frac{1}{2} \left( 1 - \frac{1}{B} \right) C_{\alpha m} = \frac{1}{2} \left( 1 - \frac{1}{B} \right) \frac{4a}{B}, \]  \hfill (46)
for the outside region
\[ C_{\alpha p} = \frac{4}{h U_m} \int_0^h u_{1p} dz, \]  \hfill (47)
for the entire subsonic sheet and
\[ \frac{1}{2} l C_{\alpha a} = \frac{1}{2} \left( 1 - \frac{1}{B} \right) C_{\alpha m} + \frac{1}{2B} C_{\alpha ma}, \]  \hfill (48)
for the entire horizontal wing.

The coefficient of moment of rolling is given by the following formulae:
\[ H C_{m\alpha} = \frac{8}{3l U_m} \int_0^l u_{1a} y dy, \]  \hfill (49)
where the horizontal plane and
\[ H C_{m\beta} = \frac{8}{3h U_m} \int_0^h u_{1p} z dz, \]  \hfill (50)
for the vertical plane, in which \( u_{1a} \) and \( u_{1p} \) are given by (14) and (16).

In order to define the parameter \( \frac{h}{\bar{h}} \) we shall observe for the first time that the position of maximum pressure distribution coincides with that of the center of the vortex core, as is found by experimentation. On the other hand, by basing calculations on the distribution of selected sources we find that the apex of depression on the sheet extrados falls approximately in the center of gravity of the source intensity with position \( t' \), given by (29b).

To continue we shall use the formula
\[ \frac{h}{\bar{h}} = \frac{1}{1 + 1.7(\beta \pm \Delta \beta)^{1/4}}, \]  \hfill (51)

5. The Simplified Case of Concentrated Sources

Assuming in a simplified way that the normal velocity at the surface of a sheet has a sudden jump into the center of a vortex core, equivalent to a sudden incident jump, we solve the problem from the hydrodynamic point of view by placing several concentrated sources at points \( t'_0 \) and \(-t'_0 \) of intensity \( Q_c \) and \(-Q_c \).
The expressions of the axial perturbation velocities will be as follows: 

\[ u_t = u_0^{\prime} = \frac{A_0}{X} \sqrt{\frac{1}{\delta^4} - \lambda^2} + \frac{2}{\pi} K_{10} \left( \frac{\text{arc cos} \sqrt{\frac{(1 + \delta L)(1 - \delta X)}{2 \delta (L - X)}}}{\lambda} \right) \]

\[ - \text{arc cos} \left\{ \frac{1 + \delta L}{2 \delta (L + X)} \left( \frac{1}{1 + \delta X} \right) \right\} \left[ \frac{2}{\pi} Q_t \left( \text{arg ch} \left\{ \frac{1 + \delta X}{2 \delta (X - T_0)} \right\} \right) \right] \]

\[ - \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X - T_0)} \right\} = \pm Q_t \left( \text{arg ch} \left\{ \frac{1 + \delta X}{2 \delta (X - T_0)} \right\} \right) \]

\[ \pm \frac{2}{\pi} K_{10} \frac{\text{arc cos} \sqrt{\frac{(i^2 + b^2)(1 - b^2 - \lambda^2)}{2 \delta (b^2 + \lambda^2)^2}}}{1 + \delta X} \pm \frac{2}{\pi} Q_t \left( \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X - T_0)} \right\} \right) \]

\[ + \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X + T_0)} \right\} = \pm Q_t \left( \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right) \]

\[ \pm \frac{2}{\pi} Q_t \left( \text{arg ch} \left\{ \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right\} \right) \]

\[ \pm \frac{2}{\pi} Q_t \left( \text{arg ch} \left\{ \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right\} \right) \]

\[ \text{for a lifting cruciform wing,} \]

\[ u_t^{\prime} = \frac{2}{\pi} Q_{10} \text{arg ch} \frac{1 + \delta X}{2 \delta (X - T_0)} \pm \frac{2}{\pi} Q_t \left( \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X - T_0)} \right\} \right) \]

\[ \pm \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X + T_0)} \right\} = \pm Q_{10} \left( \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right) \]

\[ \pm \frac{2}{\pi} \left( \text{arg ch} \left\{ \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right\} \right) \]

\[ \text{for a wing of symmetric thickness, and} \]

\[ u_t^{\prime} = u_t \]

\[ \text{for the third component wing.} \]

The pressure distribution is found by substituting in (45) its expression \( U \) given by (26) in which \( U'_{1}, U'_{t}, \) and \( U'_{c} \) come from (52), (53) and (54). The aerodynamic coefficients are found in the same way as in the case of distributed sources in which constants \( a_{10}, K_{10}, Q_t \) and \( k_t \) appear, deduced from equations

\[ a_{10} = 0 \]

\[ u_0^{\prime} + u_t (h - t_0) = v \]

\[ Q_t = \frac{(v_{0}^{\prime} - u_0^{\prime})}{\sqrt{1 - B^2 t_0^2}} \]

and from (33), and (42):

\[ \frac{Q_t}{U_{\infty}} = -\frac{2}{\pi} \frac{\sqrt{\frac{1}{\delta^4} - \lambda^2}}{\lambda} \left[ K(k) - \frac{b^2}{\lambda} \frac{\Pi (0, k)}{\lambda} \right] a - \beta \]

\[ \frac{2}{\pi} \sqrt{\frac{b^2 - t_0^2}{1 + B^2 t_0^2}} \left[ K(k) - \frac{b^2}{\lambda} \frac{\Pi (0, k)}{\lambda} \right] a - \beta \]

\[ \frac{1}{b} \sqrt{\frac{1 + h^2 b^2}{1 + B^2 t_0^2}} \]

for a lifting cruciform wing, 

\[ u_t^{\prime} = \frac{2}{\pi} Q_{10} \left( \text{arg ch} \left\{ \frac{(1 + \delta X)(1 + \delta T_0)}{2 \delta (X - T_0)} \right\} \right) \]

\[ \pm Q_{10} \left( \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right) \]

\[ \pm \frac{2}{\pi} \left( \text{arg ch} \left\{ \frac{1 + \delta^2}{b^2 (b^2 + \lambda^2)} \right\} \right) \]

\[ \text{for a wing of symmetric thickness, and} \]

\[ u_t^{\prime} = u_t \]

\[ \text{for the third component wing.} \]
in which $k$, $\rho_1$ and $\rho_2$ are taken from (34). The constant $K_{10}$ is the same as in (32).

Observations

a) The positions of the vortices are determined both from $q$ and $\beta$, as is seen in (51).

b) If $\beta = 0$, the antisymmetric flow with vortices is again found.

c) Making $Q_{10} = 0$ in the expression of axial velocity $U$ in the linear theory, we get from the expression for $a_{10}$ calculated in [5], the condition as an antisymmetric cruciform wing in order to have finite velocities at the edge, avoiding the appearance of vortices:

$$
\frac{\alpha}{\beta} = \frac{\pi \sqrt{(l^2 + b^2) (1 + b^2 \rho^2)}}{2 \left[(l^2 + b^2) K(k) - b^2 \Pi(l, k)\right]}. \tag{57}
$$

From the same equation we deduce the supplementary incidence induced by the wing on the sheet when $\beta = 0$:

$$
\Delta \beta = \frac{2 \left[(l^2 + b^2) K(k) - b^2 \Pi(l, k)\right]}{\pi \sqrt{(l^2 + b^2) (1 + b^2 \rho^2)}} \alpha. \tag{58}
$$

introduced in (51).

Bibliography


### DISTRIBUTION LIST

**DISTRIBUTION DIRECT TO RECIPIENT**

<table>
<thead>
<tr>
<th>ORGANIZATION</th>
<th>MICROFICHE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A205 DMATC</td>
<td>1</td>
</tr>
<tr>
<td>A210 DMAAC</td>
<td>2</td>
</tr>
<tr>
<td>B344 DIA/RDS-3C</td>
<td>9</td>
</tr>
<tr>
<td>C043 USAMIA</td>
<td>1</td>
</tr>
<tr>
<td>C509 BALLISTIC RES LABS</td>
<td>1</td>
</tr>
<tr>
<td>C510 AIR MOBILITY R&amp;D LAB/FIO</td>
<td>1</td>
</tr>
<tr>
<td>C513 PICATINNY ARSENAL</td>
<td>1</td>
</tr>
<tr>
<td>C535 AVIATION SYS COMD</td>
<td>1</td>
</tr>
<tr>
<td>C591 FSTC</td>
<td>5</td>
</tr>
<tr>
<td>C619 MIA REDSTONE</td>
<td>1</td>
</tr>
<tr>
<td>D008 NISTC</td>
<td>1</td>
</tr>
<tr>
<td>H300 USAICE (USAREUR)</td>
<td>1</td>
</tr>
<tr>
<td>P005 DOE</td>
<td>1</td>
</tr>
<tr>
<td>P050 CIA/CRT/ADD/SD</td>
<td>1</td>
</tr>
<tr>
<td>NAVORDSTA (50L)</td>
<td>1</td>
</tr>
<tr>
<td>NASA/KSI</td>
<td>1</td>
</tr>
<tr>
<td>APIT/LD</td>
<td>1</td>
</tr>
<tr>
<td>LLL/Code L-389</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ORGANIZATION</th>
<th>MICROFICHE</th>
</tr>
</thead>
<tbody>
<tr>
<td>E053 AF/INAKA</td>
<td>1</td>
</tr>
<tr>
<td>E017 AF/RDXTR-W</td>
<td>1</td>
</tr>
<tr>
<td>E403 AFSC/INA</td>
<td>1</td>
</tr>
<tr>
<td>E404 AEDC</td>
<td>1</td>
</tr>
<tr>
<td>E408 AFWL</td>
<td>1</td>
</tr>
<tr>
<td>E410 ADTC</td>
<td>1</td>
</tr>
</tbody>
</table>

**FTD**

- CCN
- ASD/FTD/NIIS
- NIA/PHS
- NIIS

---

**FTD-ID(RS)T-1859-78**