SPACE-CHARGE FLOW STARTING CONDITIONS IN RF-KEYED CROSSED-FIELD AMPLIFIERS.
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**Title:** Space-Charge Flow Starting Conditions In RF-Keyed Crossed-Field Amplifiers

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ABSTRACT

The performance of cold-cathode distributed emission crossed-field amplifiers (CFA) used in high power radar systems is degraded because of problems with starting delay, power dips, circuit dissipation and low efficiency. A theoretical analysis has been performed at ECOM which explains the reasons for these difficulties. The key lies in the gross features of the CFA space-charge flow which is completely specified by two time-independent parameters, the average secondary emission coefficient of the cathode, and the average electron back-bombardment energy. The parameters characterize the emission capability of the cathode and are a function of CFA interaction space geometry, operating conditions and cathode material. The theoretical analysis has been programmed for a digital computer to provide a design and evaluation tool to improve CFA performance.

SPACE-CHARGE FLOW THEORY

Computation of Average Secondary Emission Coefficient and Back-bombardment Energy

During the initial stages of beam buildup in an rf-keyed CFA, electrons follow predominately cycloidal paths with velocities given by the well-known parametric equations:

\[ \begin{align*}
  \dot{x} &= \frac{V}{dB} (\sin \omega t) \\
  \dot{y} &= \frac{V}{dB} (1-\cos \omega t)
\end{align*} \]

where

- \( \dot{x} \) = velocity component in direction normal to cathode (\( \dot{x} = x = 0 \) at cathode)
- \( \dot{y} \) = velocity component perpendicular to \( \dot{x} \) and in the direction of net beam drift (meters per second)
- \( V \) = spacing between anode and cathode (meters)
- \( dB \) = anode-cathode potential difference (volts)
- \( B \) = constant magnetic field perpendicular to \( x \) and \( y \)
- \( \omega \) = cyclotron frequency

In addition to the DC field between cathode and anode (rf circuit), electrons traverse a time and space-dependent rf field whose components are given by:

\[ \begin{align*}
  E' &= x \text{ component of rf field} \\
  &= \frac{E}{\cosh \beta x} \sin \omega t (2\pi ft \pm 8y) \\
  E'' &= y \text{ component of rf field} \\
  &= -\frac{E}{\sinh \beta x} \cos \omega t (2\pi ft \pm 8y)
\end{align*} \]
An electron which leaves the cathode, traverses a complete cycloid, and returns to bombard the cathode with an impact energy acquired from the rf field which is given by:

\[ J = \text{electron impact energy} \]
\[ J = \int_{E_C}^{E_A} \left( E^2 + E^1 \right) \frac{dE}{E} \text{ (electron volts)} \]  

(6)

where the integral is evaluated over one complete cyclotron period.

\[ J \] for a particular electron is a function of the position of the rf wave with respect to the point of emission at the instant the electron leaves the cathode. To obtain all the possibilities of \( J \), one must integrate (6) for each \( t' \) (the time at which the electron leaves the cathode) in the rf cycle. For each \( J \), there corresponds a value of secondary emission coefficient, \( \delta \), at the point of impact for the electron.

\[ \delta = \text{secondary emission coefficient} \]
\[ \delta = F(J) \]  

(7)

where \( F(J) \) is any good polynomial approximation to the experimental curve obtained for the cathode material in the CFA under consideration.

Assuming that each possibility of \( J \), and corresponding \( \delta \), is equally probable, one can characterize the emission capabilities of the cathode for a CFA with constant cathode-anode spacing for specific operating conditions by averaging \( \delta \) over one rf cycle. The resulting average secondary emission coefficient, \( \bar{\delta} \), is a constant, independent of time for fixed CFA and geometry, frequency, rf input power, anode-cathode voltage, and magnetic field.

\[ \bar{\delta} = \frac{1}{T} \int_{E_C}^{E_A} \delta \frac{dE}{E} \text{ (electron volts)} \]  

(8)

T = rf period

The integral is evaluated over \( T \).

If anode-cathode spacing is not constant, \( \delta \) must also be averaged with respect to the spacing; for most practical cases interaction space variation is one-dimensional and is a function of distance along the cathode. The average bombardment energy for an electron per rf cycle, \( E_b \), is also a constant, independent of time and is given by

\[ E_b = \text{average bombardment energy} \]
\[ E_b = \frac{1}{T} \int_{E_C}^{E_A} J \frac{dE}{E} \]  

(9)

where the integral is evaluated over \( T \).

\( \bar{\delta} \) and \( \bar{E}_b \) are easily calculated by digital computer using equations (1) thru (9), and are all that are required to completely specify a gross space-charge flow in a given rf-keyed CFA as will be discussed in the next section. By itself, the coefficient, \( \bar{\delta} \), is useful in determining whether or not a proposed CFA design has inherent starting delay difficulties. Figure 1 illustrates the way in which the coefficient can be used. \( \delta \) is plotted as a function of operating frequency; the operating band of the CFA is defined by upper and lower frequencies \( E^u \) and \( E^l \) respectively. \( \delta \) curves are shown for two different cold-cathode surfaces: barium tungstate and platinum. Whenever \( \delta \) is near unity, the CFA will have starting delay; if \( \delta \) is much below unity it will not start at all. In the figure delay occurs for the platinum cathode near \( E^l \); with a barium tungstate cathode no delay occurs.

Space-Charge Flow Model

As the beam builds up in the rf keyed CFA, electrons tend to cluster in a thick sheath at the cathode and tend to follow laminar paths which are parallel to the cathode and the anode. Under normal operation; electrons are drawn from the outer edge of the sheath to the anode where they are collected as amplification proceeds; it is the sheath which dominates, to first order, the performance of the CFA. Conventional wisdom has regarded the space-charge density, \( \rho_s \), in the sheath as given by an expression of the form

\[ \rho_s = \epsilon_0 \frac{\delta}{m} B^2 \]  

(10)

where \( \frac{\delta}{m} \) = charge to mass ratio of the electron
\[ \frac{\delta}{m} = 1.759(10)^{11} \text{ coulomb/kilogram} \]

and
\[ \epsilon_0 = \text{permittivity of free space} \]
\[ \epsilon_0 = 8.854(10)^{-12} \text{ farad/meter} \]

Expression (10), while useful and approximately correct, provides no explanation for observed experimental differences in operating characteristics when cathode secondary emitting materials are interchanged in a given CFA. The reasonable supposition that the space charge density is enhanced by a factor, \( \bar{\delta} \), in conjunction with a potential depression on the order of \( \bar{J} \) in the electron sheath leads to fruitful conclusions which predict observed power dips, circuit beam interception and efficiency variations as a function of operating frequency. For a flow defined by

\[ \rho = \bar{\delta} \rho' \]  

(11)

the potential difference \( V(x) \) between cathode and anode is given by
The fraction of intercepted power $P_i$ dissipated on the anode (circuit) is given by:

$$ P_i = \frac{K.E.}{eV} = \frac{eB^2}{2mV} \delta (X)(X-x') $$

Expressions (8), (9), (12), (13), (14), (15), (17) and (18) are all that are required to analyze an rf keyed CPA with respect to starting, power output, efficiency and overall circuit interception; the expressions have been programmed in FORTRAN for a Burroughs B-5600 computer to facilitate analysis. The computer program is relatively simple, with the bulk of complexity and running time being taken up by the integrals in (8) and (9). Computation times for a single CPA design case are typically about 45 seconds.

Theoretical predictions based on the above space-charge flow have been compared with experimental data for two high power rf keyed CPA's currently being developed for the Army. Theoretical values were within 12% of the measured values, which indicates the validity of the theoretical approach. All values are extremely sensitive to the accuracy of the secondary emission curve (7), so that agreement to within 12% is rather surprising for an initial check of the theory. In particular, an in-band power dip was predicted with the use of expression (17) i.e., $\eta$ had a minimum at the measured frequency at which the dip actually occurred. Furthermore the theory also predicted that a small change to cathode size could overcome the difficulty, as was verified in practice.

Summary and Conclusions

A space-charge flow has been postulated which predicts observed deficiencies in high power rf-keyed CPA's currently being developed for the Army; theoretical guidance is provided to correct the deficiencies. The flow is critically dependent on the secondary emitting characteristics of the cathode materials used in a given tube. With the aid of a digital computer, the theory provides an analytical and design tool by which a tube designer can evaluate the effect a proposed change in cold cathode material will have on CPA efficiency, power, total circuit interception and starting capability.
FIGURE 1. Typical Average Secondary Emission Curves versus Frequency for an RF Keyed CFA with Fixed Geometry and Input Power.
FIGURE 2. RF KEYED CFA INTERACTION SPACE POTENTIAL DISTRIBUTION WITH SPACE-CHARGE.