CONSIDERATIONS FOR EFFICIENT WIRE/SURFACE MODELING
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<td>The most significant aspects of a moment method surface patch/wire formulation are speed, accuracy, convergence, and versatility. Techniques for improving these parameters are discussed and applied to a solution based on the piecewise sinusoidal reaction formulation.</td>
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I. INTRODUCTION

In a previous paper [1] the authors presented a moment method solution for wires, plates, and wire/plate attachments. The purpose of this report is to present some details of the formulation and of the computer code, and especially to show how the choice of integral equation and expansion and testing functions impact on the versatility, accuracy, computational efficiency, and ease of use of this and other similar formulations.

Briefly, the solution [1] is based on the reaction integral equation and employs speed-of-light piecewise-sinusoidal (PWS) wire, surface, and attachment dipole expansion modes which are placed on the composite wire/surface geometry in an overlapping array such that continuity of current is enforced. The weighting or test modes are chosen identical to the expansion modes, and thus the solution is a Galerkin method, yielding a symmetric impedance matrix. The electric field integral equation is used, so open as well as closed surfaces can be treated.

This paper will demonstrate the advantages of the PWS modes used from two standpoints. First, it will be shown that they are a good interpolating function requiring relatively few unknowns to describe the current. Second, it will be shown that these modes facilitate the rapid and accurate evaluation of the elements in the impedance matrix.

It must be emphasized that the solution presented in [1] and the techniques presented below are not necessarily the best choices. Further, some of the techniques presented could equally well be applied to solutions employing other types of modes.
II. MODE LAYOUT

This section will describe the layout of expansion modes on a wire/surface geometry. The details of the modes have been given previously [1]. The mode layout will be described with the aid of an example, a monopole on a bent plate as shown in Figure 1. Since this problem involves wire, plate, and attachment modes, the impedance matrix, \([Z]\), can be symbolically shown as in Figure 2a.

![Diagram of monopole on bent plate](image)

**Figure 1.** Geometry for the monopole on a bent plate.

**Wire Modes**

The wire modes are specified using the same method as Richmond [2]. Figure 3a shows the wire broken into three segments, or two PWS dipole modes. Note that while the wire current generally does not vanish at the attachment point, the wire mode current does. Thus the need for an attachment mode, described below.
Figure 2. Impedance matrix structure for the geometry of Figure 1.

(a) Block representation of the entire matrix
(b) Block representation of the P/P block
(c) Detail of P₁/P₁ block showing computed and not computed elements

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- W = WIRE
- P = PLATE
- A = ATTACHMENT

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1 2 3 4 5 6 7 8 9 10 11 12
1 X 0 0 0 0 0 0 0 0 0 0 0
2 X 0 0 0 0 0 0 0 0 0 0 0
3 X 0 0 0 0 0 0 0 0 0 0 0
4 X 0 0 0 0 0 0 0 0 0 0 0
5 X 0 0 0 0 0 0 0 0 0 0 0
6 X 0 0 0 0 0 0 0 0 0 0 0
7 X 0 0 0 0 0 0 0 0 0 0 0 X
8 X 0 0 0 0 0 0 0 0 0 0 0 X
9 X 0 0 0 0 0 0 0 0 0 0 0 X
10 X 0 0 0 0 0 0 0 0 0 0 0 X
11 X 0 0 0 0 0 0 0 0 0 0 0 X
12 X 0 0 0 0 0 0 0 0 0 0 0 X

X = ELEMENT COMPUTED
O = ELEMENT NOT COMPUTED
Figure 3. Layout of the (a) wire, (b) surface dipole, (c) attachment, and (d) overlap modes for a monopole on a bent plate.

Surface Modes

In the computer code a rectangular plate is defined by specifying the coordinates of three consecutive corners, and the plate segmentation in the two orthogonal directions. At present the plate must be rectangular since the surface dipoles are rectangular. Figure 3b shows plate 1 of the bent plate geometry divided into three segments in the 1-to-2 direction and three segments in the 2-to-3 direction. Six overlapping PWS surface dipoles are shown as arrows in the 1-to-2 direction, and six in the 2-to-3 direction. Not shown are the modes on plate 2.
Attachment Modes

The purpose of the attachment mode is to establish continuity of current at the wire/plate junction and also to insure the proper $1/\rho$ singularity of the plate surface current density in the vicinity of the attachment point. Figure 3c shows the PWS attachment dipole consisting of a circular disk monopole with $1/\rho$ surface current density, and a wire monopole with non-zero current at the attachment point. The attachment point must be at least 0.1\(\lambda\) from the edge of a plate, but it need not be at the center, at a corner, or have any special location with respect to the surface dipole modes. A more detailed treatment is required to treat wire attachments at or near an edge. The disk radius does not substantially affect the current or impedance provided that it is chosen between about 0.1 and 0.25\(\lambda\).

Overlap Modes

When two plates intersect additional surface-patch dipole modes are required to allow a nonzero continuous current at the plate-to-plate junction. These modes are termed overlap modes. They are identical to the surface dipoles on plates 1 or 2, except that the dipole angle may differ from 180°. Figure 3d shows two overlap modes at the junction of the bent plate. The edges of the overlap surface dipoles need not coincide with the edges of the surface dipole modes on plate 1 or 2, thus allowing the intersection of plates of different size. The code automatically checks to see which plates intersect and inserts the overlap modes.

Toeplitz-Like Properties

This section will discuss some toeplitz-like properties in computing the mutual impedances between surface dipole modes on the same plate. Figure 2b shows the plate-to-plate block of the impedance matrix of
Figure 2a. The various partitions show plate 1-to-plate 1 surface dipole mutual impedances, plate 1-to-plate 2 mutuals, overlap-to-plate 1 mutuals, etc. Due to the regular nature of the placement of surface dipole modes on a plate, considerable savings in computation time is available in computing the plate 1-to-plate 1 block and the plate 2-to-plate 2 block. For example, consider the plate 1-to-plate 1 block, for which the modal layout is shown in Figure 3b. Note that $Z_{1,2} = Z_{3,4}$, $Z_{1,4} = Z_{3,6}$, $Z_{1,11} = -Z_{4,8}$, etc. On an arbitrary plate it is only necessary to compute the mutual impedances between the first mode in the 1-to-2 direction and all the modes on the plate, and the mutuals between the first mode in the 2-to-3 direction and all 2-to-3 modes on the plate. Figure 2c shows the plate 1-to-plate 1 block in detail. The x represents elements which must be computed, and the 0 represents elements which can be obtained from the toeplitz-like properties. If on a given plate there are $N_{12}$ modes in the 1-to-2 direction and $N_{23}$ modes in the 2-3 direction, then only $N_{12} + 2N_{23}$ of the $(N_{12} + N_{23})^2$ modes need to be computed.

III. MODE CHOICE

The choice of expansion and test modes is based upon three criteria. First, they should be applicable to a wide class of geometries. Second, they should have good convergence properties, requiring as few modes as possible per unit area of surface. Finally, they should facilitate the rapid and accurate evaluation of the impedance matrix. The advantages of the mode choice in [1] with respect to these criteria is discussed below.

Convergence

A rapidly convergent set of modes will minimize the size and thus the time required to compute the impedance matrix and the required storage. In practice, convergence will be the limiting factor in determining
the maximum electrical size of the geometries which can be treated. Figure 4 shows a comparison of the input admittance of a monopole on a small plate computed using 15 PWS modes to that using 136 pulse modes [3]. Noting that 15 PWS show better agreement with measurements than 136 pulse modes, it is clear that the PWS basis converges relatively fast.

IV. COMPUTATION OF IMPEDANCE ELEMENTS

This section will discuss the accurate and efficient evaluation of various elements in the impedance matrix. Two algebraically equivalent, but numerically different, techniques for evaluating the elements will be presented, and their relative advantages in computing various elements described. Since there are three types of expansion and test monopoles (i.e., wire, surface patch, and disk) there are 9 different types of monopole-to-monopole impedances. Five of these involve wire monopoles, require comparatively little computation time, and thus will not be considered. Disk-to-disk impedances occur relatively infrequently and thus also will not be considered. Since we compute only the lower triangular portion of the symmetric impedance matrix surface dipole-to-disk impedances are not computed, and are not considered here. Thus, the discussion below will concentrate on surface-to-surface dipole and disk-to-surface dipole impedances.

A general impedance matrix element \( Z_{mn} \) is defined as [1]

\[
Z_{mn} = -\int \int E_m(e_1,e_2).J_n(e_1,e_2)de_1de_2,
\]

where \( e_1, e_2 \) are independent coordinates on the surface of the \( n \)-th expansion mode. \( J_n \) is the current density of the \( n \)-th expansion mode, and \( E_m \) is the free-space electric field of the \( m \)-th test mode, which can be written as
Figure 4. A comparison of measured input impedance of a monopole on a small plate with that computed using PWS modes and pulse modes.
\[ E_m(e_1, e_2) = \int \int \delta_0(e_1, e_2; t_1, t_2) \cdot J_m(t_1, t_2) \, dt_1 \, dt_2 , \]  

where \( J_m \) is the current density of the \( m \)-th test mode, \( t_1 \) and \( t_2 \) are independent coordinates on the surface of the \( m \)-th test mode, and \( \delta_0 \) is the free-space Green's function. Equations (1) and (2) show that an impedance element in general requires a 4-fold integration, i.e., two to find \( E_m \) and two to integrate over the surface of the \( n \)-th expansion mode. A useful property of the PWS modes is that \( E_m \) is known in closed form for a wire \([4]\) or a surface \([5]\) monopole, and requires only one simple numerical integration for a disk monopole. Each dipole-to-dipole \( Z_{mn} \) is the sum of four monopole-to-monopole impedances.

The second way of computing the impedance elements is to consider the surface-dipole mode current and disk monopole current as being made up of PWS filaments. The order of integration in Equations (1) and (2) can be interchanged to give

\[ Z_{mn} = \int \int z_{mn}(e_2, t_2) \, de_2 \, dt_2 , \]  

where \( z_{mn}(e_2, t_2) \) represents the mutual impedance between two filaments lying in \( e_1 \) and \( t_1 \) directions. That is,

\[ z_{mn}(e_2, t_2) = \int J_n(e_1, e_2) \int \delta_0(e_1, e_2; t_1, t_2) \cdot J_m(t_1, t_2) \, dt_1 \, de_1 \]  

An advantage of the PWS modes is that the \( z_{mn}(e_2, t_2) \) of Equation (4) are known in closed form \([6]\).

Thus, two methods are available for computing \( Z_{mn} \): the use of Equation (1) or Equation (3). Efficient computation makes use of both of these methods, as each has particular advantages in different situations. Some of these advantages will now be described.
Generally, the second method, Equation (3), results in a simpler computer code since only one routine to compute $z_{mn}$ is needed for any combination of test and expansion modes, while the use of the first method, Equation (1), requires three separate routines for $E_m$, the field of each type of test mode.

Another advantage of the second method occurs when computing the impedances between two surface patch monopoles if either the surface current directions are parallel, or the vector transverse to the surface monopoles are parallel. If the expansion and test surface monopoles are each represented by $M$ filaments, then Equation (5) requires $M^2$ evaluations of $z_{mn}(e_2,t_2)$. However, no more than $2M$ of the $z_{mn}$ are different. By storing the $2M$ values prior to computing the mutual impedance, the computation time will be proportional to $2M$ rather than $M^2$. If two parallel patch monopoles are of the same size and in the same plane, then Popovic and Popovic [7] have expressed the mutual impedance in terms of a single numerical integration. The technique described above realizes the time saving advantages of reducing the mutual impedance to a single integral, while applying to a more general class of geometries.

Another time-saving technique deals with the slow convergence of the numerical integration of Equation (3) when computing the mutual impedance of two touching surface monopoles. This slow convergence is a result of the fact that the imaginary part of the mutual impedance of two PWS filaments has a logarithmic singularity as the distance between the two filaments gets small [8]. Thus, the real part of the impedance between two touching surface modes can converge quickly, while the imaginary part converges slowly. For small separation $x$, the reactance between two filaments can be written as [8]

$$X(x) = C_1 + C_2 \ln(x).$$

(5)
The constants $C_1$ and $C_2$ can be easily evaluated and the logarithmic singularity integrated analytically.

This was found to be a very fast and accurate technique, as can be seen from Figure 5, which shows the self-reactance of a surface monopole computed using Equation (1), Equation (3) directly, and Equation (3) with the above two techniques (i.e., only $M$ evaluations of $Z_{mn}$ and extracting logarithmic singularity). Also shown is the computation time for each of the three methods; all times shown in this paper are for the Datacraft 6024 which is about an order of magnitude slower than an IBM370-165.

In using Equations (1) and (3) directly the surface monopoles are separated by $10^{-4}\lambda$ to avoid the singularity. No separation is required if the logarithmic singularity is removed. From Figure 5 it can be seen that not only is the use of Equation (3) with the above discussed two techniques faster than the other methods, but it is also more accurate. Seldom does this good fortune of increased accuracy in less time occur.

So it is seen that the use of Equation (3) can lead to substantial advantages when computing impedance elements. However, one case where Equation (1) has an advantage occurs when computing the mutual impedance between a disk monopole (of an attachment mode) and the set of surface dipoles on a plate which is parallel to the plane of the disk. Here, advantage can be taken of the fact that only the $E$ field of the disk monopole is needed (since $E = 0$ and $E_z \neq 0$), and also $E_\theta$ is independent of $\phi$. Thus, much time can be saved by filling a table of values of $E_\theta$ versus $\rho$. This table can then be interpolated to find $E_m$ on every surface dipole mode on the plate, making the evaluation of Equation (1) very fast.

To illustrate the typical savings in time consider the problem of a monopole in the center of a square plate. Figure 6 shows the time to compute the impedance between the attachment mode and all of the surface dipole modes on the plate by the method of Equation (3) and
Figure 5. Self-reactance and computation time for a .15λ by .2λ surface monopole computed three ways.
Figure 6. Computation time for mutual impedances between an attachment mode on a square plate and all the surface dipoles on the plate, versus the number of surface dipoles.

by the method of Equation (1) augmented with the above table of $E_\rho$ versus $\rho$. Note that Equation (1) can be substantially faster, especially when the number of surface dipoles on the plate gets large. In both cases an 18 by 18 Simpson rule integration was used.
V. CONCLUSION

This paper has demonstrated that in a moment method surface/wire formulation, the piecewise sinusoidal modes have significant advantages from the standpoints of convergence and the fast and accurate computation of the impedance matrix. Also, techniques for reducing the run time of surface/wire codes were presented. To show how the speed of a typical solution can be improved by these methods, the problem of a \( \lambda/4 \) monopole mounted on a \( .54\lambda \) square plate was run without using any of the techniques mentioned in this paper, and again using all of the mentioned techniques. Using the techniques resulted in a factor of twenty decrease in run time, and an increase in accuracy (both runs used the symmetry of the impedance matrix).

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