PARAMETRIC DECAY OF EXTRAORDINARY ELECTROMAGNETIC WAVES INTO TWO UPPER HYBRID PLASMONS

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The effects of self-generated magnetic field in laser produced plasmas on the parametric decay of an extraordinary electromagnetic wave into two upper hybrid plasmons is examined for arbitrary magnetic field intensity and arbitrary ratio $k/k_0$. Due to the presence of magnetic field, the linear Landau damping is greatly reduced and the spectrum of unstable modes is significantly modified for $kA > 0.2$. 

$k \lambda < \lambda_0$
I. INTRODUCTION

A dc magnetic field of several megaGauss is generated spontaneously near the critical density layer of a laser irradiated plasma.\textsuperscript{1-4} Because of the plasma expansion, the self-generated magnetic field spreads out to the underdense regions of the plasma. It has been measured experimentally\textsuperscript{5-7} and in computer simulations.\textsuperscript{8} In the present paper we study the effects of this magnetic field on the parametric decay of the laser light into two plasma waves near the quarter critical density.

The nonlinear process of an electromagnetic wave decaying into two plasma waves in a homogeneous and unmagnetized plasma was first studied by Goldman\textsuperscript{9} and Jackson.\textsuperscript{10} Goldman analyzed the process by examining the Green function for Poisson's equation when the pump induced energy of the particles is small compared to the thermal energy. He showed the existence of the instability by keeping the pump wavenumber finite since the dominant part of the nonlinear susceptibility is proportional to it, and found the threshold for the instability. Jackson used linearized Vlasov equation allowing the pump intensity to be well above threshold. He showed that the system is stable in the dipole approximation; however, if the pump wavenumber is finite then the most unstable perturbations are those for which the wave vector of the decay wave lies in the plane determined by the propagation and polarization vectors of the pump wave and bisects the right angle between them. He found a threshold condition one order of magnitude higher than that calculated by Goldman. More recently, there has been renewed interest in the parametric decay of laser radiation into two plasmons in an inhomogeneous plasma.\textsuperscript{11-14} Rosenbluth\textsuperscript{11} used WKB approximation to derive the threshold condition.
for the growth of the decay waves in order to overcome the convective
loss out of the three-wave resonance region. Lee and Kaw\textsuperscript{12} showed
the absolute nature of this parametric instability. Liu and Rosenbluth\textsuperscript{13}
used an alternative method to analyze the instability instead of the
usual WKB approximation and found the growth rate, the absolute instability
condition, the threshold condition imposed by plasma inhomogeneity,
and the saturation level of the plasma waves due to pump depletion.
Schuss\textsuperscript{14} considered the problem of an electromagnetic wave obliquely
incident on the density gradient and found that the threshold of the
absolute instability decreases as the angle between the density gradient
and the propagation vector approaches 90°. Experimentally, the decay of
electromagnetic wave into two plasmons has been observed through
measurements of plasma emissions at the three-halves harmonic
of the incident laser frequency from the quarter critical density
layer.\textsuperscript{15-17}

In this work we investigate further the linear instability
properties by including the effects of the self-generated dc magnetic
field on the decay of laser light into two upper hybrid plasmons.
These effects might be relevant since the magnetic field changes
the plasma dispersive properties, for instance, if the wave vectors
of the decay waves are in the plane perpendicular to the magnetic field
the growth rate is finite even for $k\lambda_D^{-1}$, $\lambda_D$ is the Debye length,
contrary to the unmagnetized plasma where the decay waves are heavily
Landau damped for $k\lambda_D^{-1} > 0.2$ and the instability is turned off.

In Sec. II, we derive the system of nonlinear coupled equations
describing the decay of an extraordinary mode into two upper hybrid
waves. The dispersion relation is solved to find the growth rate,
in Sec. III. In Sec. IV, we present the conclusions and the numerical
results on the variation of growth rate with magnetic field intensity, pump and decay wave wavenumbers, and the angle between these wave vectors.

II. NONLINEAR COUPLED EQUATIONS

Consider an electromagnetic wave

\[ \vec{E}_0(\vec{x},t) = \frac{\hat{E}_0}{2} [\exp(i\vec{k}_0\cdot\vec{x}-i\omega_0 t)+c.c.] \] (1)

in a magnetized plasma with the uniform magnetic field \( B_0 \) along the z-direction. Assuming the electromagnetic pump to be an extraordinary mode, we find electron oscillations along and perpendicular to the direction of propagation with velocities to be

\[ v_{0x} = -i \frac{e}{m} \frac{\omega_0 E_0 x - i\Omega_e E_0 y}{\frac{\omega_0^2}{2} - \Omega_e^2} \] (2)

and

\[ v_{0y} = -i \frac{e}{m} \frac{\omega_0 E_0 y + i\Omega_e E_0 x}{\frac{\omega_0^2}{2} - \Omega_e^2} \] (3)

and number density

\[ n_0 = \frac{n_0 v_{0x} \cdot \vec{v}_0}{\omega_0} \] (4)

where \( \Omega_e = eB_0^2/mc \), \( n_0^0 \) is the particle density of the unperturbed electron fluid, \( e \) is the electron charge, and \( c \) is the speed of light. The electrostatic component of the electric field can be written in terms of the electromagnetic component as

\[ E_{0x} = -\frac{\varepsilon_{xy}}{\varepsilon_{xx}} E_{0y} \] (5)

where

\[ \varepsilon_{xx} = \frac{\omega_0^2 - \omega_{eh}^2}{2\omega_0^2 - \Omega_e^2}, \quad \varepsilon_{xy} = i \frac{\Omega_e}{\omega_0} \frac{\omega_p^2}{\omega_0^2 - \Omega_e^2} \] (6)

and

\[ \omega_p^2 = \frac{4\pi n_0^0 e^2}{m}, \quad \omega_{eh} = \omega_p^2 + \Omega_e^2 \] (7)
The pump wave magnetic field is given by

$$\mathbf{B}_0 = \frac{e}{\omega_0} \mathbf{k}_0 \times \mathbf{E}_0 \, .$$  \hspace{1cm} (8)

The perturbed density fluctuations for the decay waves are obtained from the equation of continuity which, after Fourier analysis, gives

$$n_{\pm n}(\mathbf{k}, \omega) = \frac{n_0^{\mp} \mathbf{v}_0 + \frac{1}{2} n_{\mp}^{\mp} \mathbf{v}_0^*}{\omega} + \frac{1}{2} \frac{n_0^{\mp} \mathbf{v}_0^*}{\omega} \, .$$  \hspace{1cm} (9)

and

$$n_{- n}(\mathbf{k}, \omega) = \frac{n_0^{\mp} \mathbf{v}_0 + \frac{1}{2} n_{-}^{\mp} \mathbf{v}_0^*}{\omega} + \frac{1}{2} \frac{n_0^{\mp} \mathbf{v}_0^*}{\omega} \, .$$  \hspace{1cm} (10)

where $\mathbf{k} = \mathbf{k} - \mathbf{k}_0$ and $\omega = \omega - \omega_0$ according to the resonance conditions.

The anti-Stokes component is considered off-resonant for this parametric process. The perturbed velocities $\mathbf{v}_+ \, \text{and} \, \mathbf{v}_-$ are calculated from the equations of motion

$$\frac{\partial \mathbf{v}_+}{\partial t} + \frac{1}{2} \frac{\mathbf{v}_0^* \cdot \mathbf{v}_+}{m_0} + \frac{1}{2} \frac{\mathbf{v}_+ \cdot \mathbf{v}_0^*}{m_0} = - \frac{T}{m_0} \mathbf{v}_+ \, .$$  \hspace{1cm} (11)

$$- \frac{e}{m} \mathbf{E}_+ x_{\mathbf{n}}^+ - \frac{\mathbf{v}_+ \times \mathbf{B}_0}{2mc}$$

and

$$\frac{\partial \mathbf{v}_-}{\partial t} + \frac{1}{2} \frac{\mathbf{v}_0^* \cdot \mathbf{v}_-}{m_0} + \frac{1}{2} \frac{\mathbf{v}_- \cdot \mathbf{v}_0^*}{m_0} = - \frac{T}{m_0} \mathbf{v}_- \, .$$  \hspace{1cm} (12)

where $T$ is the electron thermal energy, and $\mathbf{E}_+$ and $\mathbf{E}_-$ are the perturbed electric fields. Fourier analyzing Eqs. (11) and (12) for $(\mathbf{k}, \omega)$ and $(\mathbf{k}, \omega)$, respectively, we get the equations for the components of $\mathbf{v}_+$ and $\mathbf{v}_-$ which together with Eqs. (2)–(4) are substituted into Eqs. (9) and (10) to get the expressions for the perturbed density oscillations for the two decay waves. Substituting these expressions for $n$ and $n_-$...
into Poisson's equations for the perturbed electrostatic potentials \( \phi \) and \( \phi_- \) we get the system of coupled equations

\[
\varepsilon \phi = -\frac{4\pi e}{k} n_{NL} \quad (13)
\]

and

\[
\varepsilon_- \phi_- = -\frac{4\pi e}{k_-} n_- \quad (14)
\]

where \( n_{NL} \) and \( n_- \) are the nonlinear contributions to the density perturbations,

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega - \Omega_e^2} \frac{k_\perp^2}{k^2} - \frac{\omega_p^2 k_\parallel^2}{\omega k^2} \quad (15)
\]

and

\[
\varepsilon_- = 1 - \frac{\omega_p^2}{\omega_-^2} - \frac{\omega_p^2}{\omega - \Omega_e^2} \frac{k_-^2}{k_-^2} - \frac{\omega_p^2 k_-^2}{\omega_- k_-^2} \quad (16)
\]

\[
v_e^2 = \frac{T}{m}, \quad \omega_p^2 = \frac{4\pi n_0^2 e^2}{M},
\]

and \( M \) is the ion mass. The wavenumbers \( k_\parallel \) and \( k_- \), \( k_\perp \) and \( k_- \) refer to the components of \( \mathbf{k} \) and \( \mathbf{k}_- \) parallel and perpendicular to the magnetic field, respectively. The ions are considered cold and unmagnetized since the frequencies of the pump and decay waves are much larger than the ion cyclotron frequency. Equations (13) and (14) can be rewritten as

\[
\varepsilon \phi = (\alpha_1^+ + \alpha_1^-) \phi_- \quad (18)
\]

and

\[
\varepsilon_- \phi_- = (\alpha_1^+ + \alpha_1^-) \phi \quad (19)
\]
where

\[ \alpha_1^\prime = \frac{\omega_p^2}{2k_2\omega(\omega^2 - \omega_e^2)(\omega^2 - \omega_e^2)} \left( \frac{k_{-1}^2 v_{e}^2}{\omega^2 - \omega_e^2} - \frac{k_{-1}^2 v_{e}^2}{\omega^2} \right) \]

\[ \left\{ \mathbf{k} \cdot \mathbf{\hat{v}}_0 (\omega - \omega_e^2) \mathbf{k}_0 \cdot \mathbf{k}_- + (\omega - \omega_e^2) \mathbf{k}_0 \times \mathbf{k}_- \right\] 

\[ + \omega_0 \mathbf{k}_0 \cdot \mathbf{k}_- \mathbf{v}_0 \mathbf{k}_0 \times \mathbf{k}_- \]

\[ + \mathbf{k}_0 \cdot \mathbf{\hat{v}}_0 \left( \omega - \omega_e^2 \right) \mathbf{k}_0 \cdot \mathbf{k}_- + \frac{\Omega_e (\omega - \omega_e^2)}{\omega} (\omega - \omega_e^2) \mathbf{k}_0 \times \mathbf{k}_- \]

\[ + 1 \mathbf{v}_0 \times \mathbf{k}_0 \mathbf{\hat{v}}_0 (\omega - \omega_e^2) \mathbf{k}_0 \times \mathbf{k}_- \]

\[ + \frac{ie}{m} \frac{\mathbf{k}_0 \times \mathbf{k}_0}{\omega_0} \left[ 1 \mathbf{v}_0 (\omega + \omega_e^2) \mathbf{k}_0 \cdot \mathbf{k}_- \right] \]

\[ + (\omega - \omega_e^2) \mathbf{k}_0 \times \mathbf{k}_- \mathbf{\hat{v}}_0 \]

\[ \alpha_1'' = \frac{\omega_p^2}{2k_2\omega\omega} \left( \frac{k_{-1}^2 v_{e}^2}{\omega^2 - \omega_e^2} - \frac{k_{-1}^2 v_{e}^2}{\omega^2} \right) \]

\[ \left\{ \mathbf{k} \cdot \mathbf{\hat{v}}_0 (\mathbf{k}_0 \cdot \mathbf{k}_- - \frac{\omega}{\omega} \mathbf{k}_0 \cdot \mathbf{k}_-) + \mathbf{k}_0 \cdot \mathbf{\hat{v}}_0 \frac{\omega}{\omega_0} \mathbf{k}_0 \cdot \mathbf{k}_- \right\} \] 

(20)

(21)

and \( z = \frac{z}{z} \). \( \alpha_1'' \) and \( \alpha_1'' \) are obtained from \( \alpha_1' \) and \( \alpha_1'' \), respectively, by making the following interchanges

\[ \alpha_1'' = \alpha_1' \left[ (\mathbf{v}_0, \omega) \leftrightarrow (\mathbf{v}_0, -\omega), (\mathbf{v}_0, \omega_0) \leftrightarrow (\mathbf{v}_0, -\omega_0), (\mathbf{v}_0^*, \mathbf{v}_0^*) \right] \] 

(22)
and
\[ a''_{\parallel} = a'_{\parallel} \left[ (k_0, \omega) \leftrightarrow (k_-, \omega_-), (k'_0, \omega_0) \leftrightarrow (-k'_0, -\omega_0), v_0 \leftrightarrow v'_0 \right]. \tag{23} \]

Equations (18) and (19) comprise the system of coupled equations describing the parametric decay of an extraordinary electromagnetic wave into two electrostatic waves. The general dispersion relation is obtained from them, straightforwardly,
\[ \varepsilon \varepsilon_\perp = (a'_\perp + a'_{\perp}) (a''_\perp + a''_{\perp}). \tag{24} \]
Equation (24) allows us to study the decay of an extraordinary electromagnetic wave into

(i) two upper hybrid waves,

(ii) an upper hybrid and a lower hybrid wave, or

(iii) two lower hybrid waves.
The decay (i) occurs at the quarter critical density while channel (ii) at the critical density layer. In the present paper we restrict ourselves to channel (i).

III. GROWTH RATE

For the decay of an electromagnetic wave into two upper hybrid plasmons, the linear dispersion relations (15) and (16), in the limit \( k'_\parallel \ll k \), become
\[ \omega^2 = \omega_p^2 + \omega_e^2 + k^2 v_e^2 \tag{25} \]
and
\[ \omega_-^2 = \omega_p^2 + \omega_e^2 + k_- v_e^2 \tag{26} \]
and Eqs. (18) and (19) reduce to
\[
[w^2-(\omega_p^2+\omega_e^2+k^2v_e^2)]\phi = \left\{ \frac{1}{2} \frac{k_0v_0w_p^2}{w} \right\}
\times \left( 1 + \frac{\omega}{\omega_0} \right) - \frac{1}{2} \frac{k_0v_0w_p^2}{\omega_0^2k^2} \]
\times \left[ \frac{\omega_0(\omega+\omega_0)}{w^2} k_0k_\perp + \frac{\omega_0^2}{\omega_0^2k^2} k_0k_\perp \right] \phi 
\tag{27}
\]
and
\[
[w^2-(\omega_p^2+\omega_e^2+k^2v_e^2)]\phi = \left\{ \frac{1}{2} \frac{k_0v_0w_p^2}{w} \right\}
\times \left( 1 + \frac{\omega}{\omega_0} \right) + \frac{1}{2} \frac{k_0v_0w_p^2}{\omega_0^2k^2} \]
\times \left[ \frac{\omega_0(\omega-\omega_0)}{w^2} k_0k_\perp + \frac{\omega_0^2}{\omega_0^2k^2} k_0k_\perp \right] \phi 
\tag{28}
\]
If we set \( \Omega_e = 0 \) in Eqs. (27) and (28) we get the same equations as Liu and Rosenbluth in their limit \( L \rightarrow \infty \) where \( L \) is the density scale length.

For laser fusion parameters, \( \Omega_e/\omega_p < 1 \), Eqs. (27) and (28) give the dispersion relation
\[
[w^2-(\omega_p^2+\omega_e^2+k^2v_e^2)][w^2-(\omega_p^2+\omega_e^2+k^2v_e^2)] = \]
\[
= \frac{1}{4} \frac{k^2}{w_0^2} \left[ \frac{\omega_0^2}{\omega_0^2k^2} + (\omega_0^2k_\perp^2)^2 \right]. 
\tag{29}
\]

The growth rate is found, from Eq. (29), to be
\[
\gamma = \frac{k_0v_0|2k_\perp-k_0^2|}{4k_\perp \left[ \frac{2\Omega^2}{\omega_p^2} + (k_\perp^2)^2 \right]^{1/2}}. 
\tag{30}
\]
IV. NUMERICAL RESULTS AND CONCLUSIONS

The growth rate decreases slowly with the increasing magnetic field intensity, as seen from Fig. 1. The maximum growth rate is \( \gamma_{\text{max}} = \frac{1}{4} k_0 v_0 \) which holds for \( \Omega_{\text{e}} = 0 \), result that agrees with Liu and Rosenbluth. The linear Landau damping for the upper hybrid waves with finite \( k_\parallel \) is given by

\[
\gamma_L = \frac{\pi}{2} \frac{1}{k \lambda_D^2} \left( 1 - \frac{1}{2} k_\parallel^2 \right) \exp \left( - \frac{\omega_k^2}{k_\parallel^2 v_e^2} \right)
\]

\[
+ \frac{1}{2} k_\parallel^2 \left( (\omega_k - \Omega_e)^2 \right) \exp \left( \frac{(\omega_k - \Omega_e)^2}{k_\parallel^2 v_e^2} \right)
\]

where

\[
\omega_k = \sqrt{\omega_p^2 + \Omega_e^2 + k v_e^2}
\]

(31)

and \( \rho_e \) is the electron Larmor radius. From Eq. (31) we infer that if \( \vec{k} \) and \( \vec{k}_\perp \) are in the plane perpendicular to the magnetic field the upper hybrid decay waves cannot resonate with the electrons and, therefore, the linear Landau damping rate vanishes in this case. It means that the growth rate spectrum is significantly modified for \( k \lambda_D > 0.2 \) as compared to the unmagnetized plasma where we would expect the decay waves to be strongly Landau damped. Figure 2 shows the growth rate as a function of \( k \lambda_D \) for various pump powers. The growth rate is finite for all values of \( k \lambda_D \) due to the absence of collisionless damping, except for \( k = k_0 / 2 \cos \phi \) for \( \cos \phi > 0 \) when it vanishes, \( \phi \) is the angle between \( \vec{k}_0 \) and \( \vec{k} \). The growth rate is always positive for \( \cos \phi > 0 \). According to Fig. 1, for laser fusion parameters, \( \Omega_e < \omega_p \), the growth rates for magnetized and unmagnetized plasmas have the same magnitude for \( k \lambda_D < 0.2 \). However, for \( k \lambda_D > 0.2 \) they are substantially different.
due to the inclusion of Landau damping, as can be seen from Fig. 3. For instance, for laser powers up to $10^{12}$ W/cm$^2$ the growth rate vanishes for $k\lambda_D > 0.23$ for an unmagnetized plasma process but it is finite if the magnetic field is present. It is also possible to see from Fig. 2 that as $\cos\phi$ increases the values of $k\lambda_D$ decrease for the maximum growth rate. This is better seen from Fig. 4 where $\phi$ is plotted versus $k\lambda_D$ for a constant growth rate. Figure 5 exhibits the variation of the growth rate with $\cos\phi$. The growth rate vanishes for $\cos\phi = k_0/2k$ if $k_0 < 2k$. For $k_0 > 2k$ the growth rate is finite for all values of $\cos\phi$. Figure 6 shows the proportionality of the growth rate with $k_0\lambda_D$.

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REFERENCES

FIGURE CAPTIONS

Fig. 1 Variation of the growth rate $\gamma \lambda_D/|v_0|$ with the electron cyclotron frequency for $k_0 \lambda_D=0.05$ (0.32 keV), $k \lambda_D=0.002$, and $\cos\phi=0.0$.

Fig. 2 Growth rate $\gamma \lambda_D/|v_0|$ expressed as a function of $k \lambda_D$ for $k_0 \lambda_D=0.05$, $\Pi_e^2/\omega_p^2=0.01$, $\cos\phi=0.0$ (---), 0.6 (---), $-0.6$ (0-0-), 0.9 (----), and $-0.9$ (0-0-).

Fig. 3 Variation of the growth rate $\gamma/\omega_p$ with $k \lambda_D$ for $k_0 \lambda_D=0.1$ (1.275 keV), $\Pi_e^2/\omega_p^2=0.01$, for the following pump powers: (i) $|v_0|/v_e=1.17\ (10^{15}\ W/cm^2)$, $\cos\phi=0.6$ (--------), $-0.6$ (0-0-); (ii) $|v_0|/v_e=3.80\ (10^{16}\ W/cm^2)$, $\cos\phi=0.6$ (-----), $-0.6$ (0-0-). The growth rates follow curves (-----) for unmagnetized plasma.

Fig. 4 Variation of $k \lambda_D$ with $\phi$ (angle between $\vec{\Pi}_0$ and $\vec{k}$) for a constant growth rate $\gamma \lambda_D/|v_0|$ where $k_0 \lambda_D=0.05$ and $\Pi_e^2/\omega_p^2=0.01$.

Fig. 5 The growth rate $\gamma \lambda_D/|v_0|$ expressed as a function of $\cos\phi$ for $k_0 \lambda_D=0.05$, $\Pi_e^2/\omega_p^2=0.01$ and $k \lambda_D=0.03$ (0-0-), 0.15 (----).

Fig. 6 Variation of $\gamma \lambda_D/|v_0|$ with $k_0 \lambda_D$ for $\Pi_e^2/\omega_p^2=0.01$ in the following cases: (i) $k \lambda_D=0.03$, $\cos\phi=0.0$ (---), 0.7 (0-0-), -0.7 (----); (ii) $k \lambda_D=0.15$, $\cos\phi=0.0$ (-----).
Fig. 1
Fig. 2
Fig. 3
Fig. 5
Fig. 6