THE ESTIMATION OF MEAN GRAVITY ANOMALIES AT SEA FROM OTHER GEOPHYSICAL PHENOMENA

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The estimation of mean gravity anomalies at sea from other geophysical phenomena

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A review of the literature shows there exists a correlation between free-air anomalies and depth of water in ocean areas, on both a global and a local scale. Woolland shows that a more fundamental relationship exists with crustal age, and Daugherty has some success in predicting gravity from crustal age in the North East Pacific region. A prediction using collocation is performed with the same data, and similar results are obtained.
However, the success of this approach is limited by the complexity of the tectonic and geophysical components in the ocean’s sub-strata. It appears that geophysical prediction techniques at sea will be of less importance than previously in areas where reliable satellite altimetry is available. In such cases, the gravity field deduced from altimetry will assist in the interpretation of the tectonic and geophysical nature of the ocean floor and its sub-strata.
Foreword

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1. Introduction

1.1 The Nature of the Problem

Large regions of the ocean (amounting to about 30% of the earth's surface) are inadequately surveyed for gravity data (Figure 1). To improve present models of the global geoid it is necessary to rectify this situation and to extend our knowledge of the gravity field into these regions. A number of different methods have been suggested for this purpose, including:

(i) least-squares prediction using the auto-covariance function of the known global gravity field. This is basically an extrapolation process, with all the inherent shortcomings of extrapolation. Some researchers have preferred to adopt a zero anomaly value with a large variance, rather than use a predicted value (Gaposhkin, 1973) so that the estimated value has little influence on the determination of the potential coefficients;

(ii) computation of $\Delta g$ from the geoid undulation (or its approximation) measured by altimetry from GEOS-3, using either collocation (Rapp, 1977) or by a solution of the inverse Stokes' Equation combined with collocation (Gopalapillai, 1974);

(iii) methods which imply gravity information from known geophysical properties of the ocean floor. The correlation of these properties with gravity anomalies can be derived from areas of good gravity data, and this correlation is applied to known geophysical data in areas lacking in gravity information. Thus an improved estimation of the gravity field in this area is obtained.

It is the purpose of this report to concentrate on this last method. After all, in this approach one is looking for a direct cause and effect relationship, and if it is found that the anomalous gravity field is closely related to some widely known geophysical property, such as the depth of the ocean, then this information can help greatly in the extension of the gravity field into unsurveyed areas.

1.2 Requirements for a Successful Prediction

Woollard and Strange (1966, p. 96) have succinctly outlined the elements needed for a successful prediction. These are repeated here.

"The degree of success achieved in any prediction process depends on the following:
(a) Recognition of factors influencing the quantity to be predicted.

(b) Recognition of the magnitude of the effect of each individual factor on the quantity to be predicted.

(c) Recognition of the interaction between individual factors and knowledge of the degree of interdependence between factors.

(d) Availability of data concerning each factor.

(e) One's philosophy of approach as to whether a given factor varies in a random or a discrete way."

It is proposed to review the literature which has explored the correlations between various geophysical phenomena in order to gain some insight into the various factors which have been used as correlators with the gravity anomaly. Much of this literature is concerned with geophysical interpretation, i.e. the inference of geophysical properties of the ocean floor and its sub-strata from the gravity anomalies. This inference is in the opposite direction to the prediction of gravity from geophysical phenomena being treated in this report. However, the two approaches are very closely connected in that techniques of gravity measurement and of the modelling of the geophysical properties are common to both, and what is learned from the geophysical research can certainly be used in the gravimetric applications.

Before reviewing the literature, it is necessary to define the terms and explain the models which are used in the discussions of ocean floor geophysics that follow.

1.3 Definitions

a) The Earth's Crust and Upper Mantle

A simple model of the earth's crust and upper mantle is illustrated in Figure 2. It will be noticed that three principle zones can be identified: a sedimentary layer, the crust consisting of both granitic and basaltic material and the upper mantle of ultrabasic rock. The sedimentary layer in mid-ocean areas varies in depth (0.1 km to 1 km) and density and in some areas is non-existent. The mean density of the upper crust is usually taken to be 2.67 gm cm\(^{-3}\) (cf. 2.7 to 2.9 gm cm\(^{-3}\) in Figure 2). Its thickness decreases with increase in distance from the continental margin, and the ocean basin floor is found to be between 5 to 10 km. The upper mantle is usually assumed to have a density of 3.2 to 3.3 gm cm\(^{-3}\) (Pick, Picha and Vyskocil, 1973, p. 212-214; Vogt, Schneider and Johnson, 1969, p. 557).
SCHEMATIC VIEWS OF CONTINENTAL AND OCEANIC CRUSTS

Figure 2: Schematic View of Continental and Oceanic Type of Earth Crust (from Pick, Picha and Vyskocil, 1973, p. 213)

Figure 3: Idealised Illustration of Sea Floor Spreading and of Isostatic Compensation
It is necessary to explore the theories of the tectonic processes of the ocean floor in order to appreciate the inter-relationships between the various geophysical parameters which will be referred to below (Section 2.1). Since the early 1960's (Hess, 1960; Vine and Mathews, 1963), it has been thought that new material forming the crust may be generated along the sub-oceanic ridge lines (see Figure 3). This theory is based partly on the evidence provided by the changing magnetic orientation of crustal material. "The ocean crust has been created by dike injections...at the axis of the mid-oceanic ridges. As the mantle derivatives are injected into the axial crust and cool, they must acquire a magnetization depending upon the ambient strength and direction of the geomagnetic field...The order of magnitude of sea-floor motion had to be several cm/year if continental movements were to be explained by sea floor spreading." (See Vogt, Schneider and Johnson, 1969, p. 557.)

An associated phenomenon is that, as the crust spreads from the axial zone of crustal genesis and cools, it subsides (see Figure 3). Hence, as will be seen later (Section 2.2 (iv)) a relationship is found to exist between crustal age and ocean depth thus establishing one of the interactions mentioned in (c) of Section 1.1.

(b) Accuracy of Gravity at Sea

Early gravity measurements at sea had an expected accuracy of ±5 to ±8 mgal (Anderson, 1962, p. 54), i.e. about one order of magnitude lower than for land measurements. This was mainly due to errors in position, the motion of the ship bearing the gravimeter, the Eötvös effect and the difficulty in estimating the drift of the gravimeter accurately (Nettleton, 1976, p. 116—124). More recently, the use of the gyro-stabilized platform and satellite navigation has improved this accuracy to about ±3 mgal (Lucas, 1971, p. 8-9; Khan et al., 1971, p. 1).

Observed gravity is corrected for the usual latitude and drift factors, and also a (potentially) large Eötvös correction must be applied. The result is the free-air anomaly as gravity is measured at sea level (effectively). Individual measurements can then be combined in some standard technique to obtain mean free-air anomalies for, say, the $5^\circ \times 5^\circ$ or $1^\circ \times 1^\circ$ blocks which are required for global gravity data banks. The accuracy of these mean values is dependent on the nature of the ocean floor, the amount of data and the method used in obtaining the mean, but for a well represented $1^\circ \times 1^\circ$ block the accuracy can be less than ±10 mgals. For areas with little or no known data, an accuracy of about three times this is possible; Rapp and Rummel (1975) assumed a standard deviation of ±30 mgals for $1^\circ \times 1^\circ$ means in the U. S. Calibration Area, this being the r.m.s. of the free-air gravity anomalies in the area.
(c) Bouguer and Isostatic Anomalies

(i) Bouguer Anomalies

The normal Bouguer correction implies the removal of an infinite slab of material of assumed density whose thickness is the height of the observation point above mean sea level. This correction enables one to compare gravity anomalies at sea level after the attraction of material above this level has been removed. The aim of a 'Bouguer' correction at sea is similar. The intent is to remove the effect of the density contrast between seawater and bedrock so that gravity effects from sources below the sea floor are more apparent.

The correction is computed in the usual way except that the density contrast between sea-water and rock (usually taken to be 1.64 gm cm$^{-3}$) is used (Nettleton, 1976, p. 280). Thus:

\[
\Delta g_{\text{Boug}} = \Delta g_{\text{fa}} + 0.0687 \cdot h'
\]

where $h'$ is the depth of the ocean (see Figure 3). To compute the $1^\circ \times 1^\circ$ mean value the mean free-air anomaly and mean depth are used instead of the discrete value. Terrain corrections could also be applied to correct for the fact that adjoining $1^\circ \times 1^\circ$ depths are different to the computation value and thus the model of an infinite slab used in computing the correction is inadequate.

(ii) Isostatic Anomalies

The theory of isostacy assumes that there is a state of equilibrium at a certain level ($T = 30$ km) below the earth's surface. In the Airy version of the theory this is achieved by variations of thickness in the crust, which is assumed to have a uniform density throughout. Thus in areas of low relief (A in Figure 3) the crust is thinner than normal, and vice versa in areas of high relief (B in Figure 3). The net effect is that hydrostatic pressure at depth $T$ is equal under both A and B (see Heiskanen and Vening Meinesz, 1958, p. 135-142).

The thickness of the anti-root ($t'$ in Figure 3) is:

\[
t' = (\rho_c - \rho_w) / \Delta \rho h'
\]

where

\[
\rho_c = \text{density of the crust} (\approx 2.67 \text{ g cm}^{-3})
\]

\[
\rho_w = \text{density of sea water} (\approx 1.03 \text{ g cm}^{-3})
\]
\[ \Delta \rho = \text{density contrast between crust and mantle (} \approx 0.6 \text{ g cm}^{-3} \) \\
\]
\[ h' = \text{depth of water} \]

The total thickness of the crust under the ocean \((T_0)\) is therefore:

\[
(1.3) \quad T_0 = T - t' - h' 
\]

Corrections can now be applied to the observed free-air gravity anomaly to account for the fact that the column beneath the observation point differs from a standard column of 30 km depth and \(\rho = 2.67 \text{ g cm}^{-3}\).

Other theories using the theory of isostasy will not be treated here. The interested reader is referred to (Ibid., p. 131-142) and Kivioja (1963, p. 17-22).

2. Geophysical Factors Related to Gravity Anomalies in Ocean Areas

2.1 Introduction

The oceans are composed of three morphological units: continental margins, mid-oceanic ridges and ocean basins. Mid-oceanic ridges constitute about 30% of the oceanic area (Vogt, Schneider and Johnson, 1969, p. 562) and, because of their role in the tectonic process (Section 1.3(a)), present a complex picture in the interpretation of anomaly-causing structures. The ocean basins may possess features such as abyssal plains, mid-ocean canyons, ocean rises, micro-continents, seamounts and fracture zones (Ibid., p. 562-573). All but the last feature are either tectonically quiet or, as in the case of seamounts, are isolated and may have little effect on mean values of height and gravity for 1°x 1° blocks. The fracture zones can have a depth of 1 km and may cover extensive areas. They involve crust and upper mantle, and depending on magnitude and extent, are likely to complicate the relationship which may otherwise exist between geophysical features of the ocean floor and the gravity field.

It is necessary to place limits on the extent of the present investigation into the correlation between gravity anomalies and geophysical properties. For example, there seems little value in reviewing findings at continental margins as most of these are already well surveyed for gravity (as witnessed by the fact that the literature deals heavily with geophysical interpretation from gravity in these regions). Investigation into the fine structure in localized areas (such as the Hawaiian Emperor Seamount chain, see Lucas, 1971; Watts, 1976) will probably
have limited application and will generally not be included in the survey. Papers which explore the gravity relationships to such tectonic properties as lithospheric flexure, plate thickness, mantle convection and heat flow are also omitted as data on these matters is limited to areas well surveyed for gravity, and the conclusions concerning their relationships are still a matter of conjecture.

The obvious place to concentrate research, then, is in ocean basins which are uncomplicated by fracture zones or large sea mounts. In the section which follows, a number of papers are reviewed. These are not limited to ocean basins but are broader in scope so that general relationships (and their complications) can be seen in perspective. Following this review some numerical tests will be made in an ocean basin area to help evaluate some of the relationships which are found to exist in the literature.

2.2 Review of Relationships Between Gravity Anomalies and Geophysical Phenomena

(a) Global Relationships

A number of researchers have analyzed the correlation between geophysical or geomorphological phenomena and the global gravity field in ocean areas of the earth. This is a broader approach than is required in this present investigation but may be useful in providing a 'bias' (or datum shift) to predicted anomalies in ocean areas devoid of gravity data.

Kivioja and Lewis (1966) used worldwide 5° x 5° mean elevations and depths in an Airy-Heiskanen isostatic model to compute free-air anomalies for each 5° x 5° block. Although no comparison with known gravity data is given, the geoid maps computed by Stokes formula from both the generated and known data were compared. The authors concluded that a large part of geoid undulation is due to bathymetric, topographic and isostatic masses, and felt that the differences revealed information about the hidden density distributions inside the earth. It is dangerous to draw too broad a conclusion from this study, however, as both solutions contained common data, the generated data being used in areas where there was no surveyed data.

Woollard (1969, p. 286) noted that geoidal anomalies defined by the then current harmonic representation of satellite data bore no relationship to surface mass distribution, but notes a correlation between surface gravity and tectonic activity.

Lambeck (1971) also pointed out the presence of positive free-air anomalies over 19 ridges selected from all major oceans of the world, and developed lithospheric models to explain the variation in gravity over these ridges (Ibid.,
Anderson et al. (1973) confirmed this correlation, but found no correlation between spreading rates and observed crustal depths. Roufosse and Gaposhkin (1976) also noted the correlation of Δg with mid-oceanic ridges and also in volcanic areas but, when comparing short wavelength features of the global Δg field with topography concludes that the correlation was poor, particularly in ocean regions (see also Roufosse, 1977).

It is obvious from this survey that there is no simple general relationship which could be used on a global scale to predict Δg from geophysical data in ocean areas. As recognized by Khan et al. (1971), it is necessary to restrict the study to tectonically quiet and uniform areas in order to limit the variables which influence the gravity values in ocean areas.

(b) Relationship with Topography or Bathymetry

Many investigators have noted a correlation between depth of water (or sea floor topography) and gravity in various ocean regions. These relationships are complex when over tectonically active ridges. But in ocean basins one would expect positive gravity anomalies in shallower than normal areas (and vice versa) providing the density of the sub-strata remains more or less uniform throughout the region.

Taiwani and Le Pichon (1969) analyzed 5° x 5° mean anomalies and depths in both the North and South Atlantic and found a very strong correlation between topography and gravity in the North Atlantic while in the South Atlantic, the correspondence was less apparent. (The gravity field in the South Atlantic at that time was not well defined.) Mathews et al. (1969) in a more localized analysis in the Peak and Freen Deeps in the North Atlantic felt that "large anomalies...are mainly due to the topography and that there is an almost uniform mass distribution below the sea floor" (Ibid., p. 533). Dehlinger (1970) corroborates this relationship in the North Pacific ocean basins, noting that most of the topographic highs (usually seamounts) are characterized by positive free-air anomalies, although he felt that some of the Δg's could not be correlated with topographic features, and that the correlation was more on a local than a regional basis. In fact, he concludes that topography is a poor to unsatisfactory guide for estimating regional anomalies, even where the extent of isostatic equilibrium has been determined (Ibid., p. 363).

It has been suggested that this relationship between ocean floor topography and gravity anomalies holds even in tectonically active regions. Early studies over the mid-oceanic rise of the North Atlantic showed that isostatic equilibrium existed in this ridge area and that topography accounted for most of the anomalies here (van Andel and Bowen, 1968). And in 1973, Anderson and others showed a positive correlation between free-air anomalies and differences in crustal depths of the mid-oceanic ridge systems. However, they found no correlation between spreading rate and gravity, and felt that no uniform relationship held for all oceans between spreading rate and observed crustal depths (Anderson et al., 1973).
The foregoing suggests there is correlation between sea-floor topography and gravity anomalies, particularly in the Atlantic Ocean and for local areas generally. However, it appears that no simple relationship holds for large areas in the Pacific Ocean.

(c) Relationship with Depth Anomalies

The depth anomaly is defined as the difference between the measured depth and a depth value generated by an empirical model based on crustal age (Anderson et al., 1973, p. 403). Sclater et al. (1975) examined the correlation between $1^\circ \times 1^\circ$ and $5^\circ \times 5^\circ$ mean depth anomalies and gravity anomalies in the relationship between these two data sets. However, for the $1^\circ \times 1^\circ$ means, the correlation averaged through the whole area was poor and could not be used with confidence in prediction (Ibid., p. 1036), although certain local areas (e.g. South of Iceland and near the Azores) show strong correlation. Marsh and Marsh (1976) noticed visual correlations between the residual depth anomalies and $\Delta g$ in the central and eastern Pacific oceans. They also found a series of linear positive and negative 'residual' anomalies across the Pacific. (A residual anomaly was defined here as that field remaining after a 12 degree and order field is subtracted from a field defined to degree and order 22). This suggested to them the presence of longitudinal convective rolls beneath the Pacific plates which may be significant when developing a prediction model for the Pacific region.

(d) Relationships with Crustal Age

The theory of sea floor spreading provides a model which enables one to understand the interrelationships between some geophysical parameters. As shown in Section 1.3 (a) both the magnetic properties and depth of the ocean floor can be seen as a function of age. Both these properties are relatively easy to measure and a large body of data is available for them. Furthermore, as reported in Woollard and Daugherty (1974, p. 5), Woollard has found "systematic interrelationships between depth of water, thickness of crystalline rock crust, mean velocity of the crust and velocity of the mantle in the Pacific region, and has observed that these interrelationships change with crustal age and past crustal spreading rate".

In one of the few attempts to predict gravity from other geophysical information, Daugherty has used crustal age derived from (i)magnetism and (ii) water depth to predict $1^\circ \times 1^\circ$ and $5^\circ \times 5^\circ$ mean gravity anomalies in the northeast Pacific. The $1^\circ \times 1^\circ$ areas involved are classified according to topographic or tectonic type in (Daugherty, 1975, p. 8), and shows that about 65% of these areas are undisturbed by major topographic or tectonic features. His prediction method is outlined below.
(i) An area in the north Pacific bounded roughly by $6^\circ \leq \phi \leq 4^\circ$, $210^\circ \leq \lambda \leq 255^\circ$ was chosen as a test area as it was considered to have adequate data and had relatively uncomplicated topographic and tectonic characteristics.

(ii) The $1^\circ \times 1^\circ$ mean free-air gravity anomalies ($\Delta g_{\text{fa}}$) were obtained from the Department of Defence (DoD) gravity library. Ocean floor ages and $\Delta g_{\text{fa}}$ were mapped, and the $\Delta g_{\text{fa}}$'s averaged over 5 million year (m.y.) intervals.

(iii) These 5 m.y. averages were now plotted against age up to 80 m.y., beyond which lack of data precluded any meaningful analysis. A series of polynomials were now fitted to this plot and the best fitting polynomial assumed to represent the $\Delta g$ vs. age relationship.

(iv) $1^\circ \times 1^\circ$ mean values of $\Delta g_{\text{fa}}$ were now 'predicted' from the $1^\circ \times 1^\circ$ mean crustal ages. A comparison of the predicted vs. known $\Delta g$'s enabled a statistical assessment of the success of the prediction.

Results and Comments

The root mean square (RMS) of the differences in (iv) ranged from $\pm 10.5$ mgal to $\pm 12$ mgal, the former for when ages were interpolated directly from an age map and the latter computed from the age-depth relationship referred to in (c) above. This is about a 40% improvement over the RMS of the gravity anomalies themselves ($\pm 17$ mgal) and compares favorably with the accuracy of the 'known' mean anomalies, which varies between $\pm 5$ to $\pm 25$ mgal and has a mean of about $\pm 13$ mgal.

Unfortunately, the success of this technique was limited. When applied to a new data set in the South East Pacific (where nearly 50% of the set had uncertain topographic type) it gave disappointing results, and it was necessary to conclude that in this region, there is no apparent correlation between $\Delta g$ and age (Daugherty, 1975, p. 19). The complexity of the ocean floor structure (Woollard et al., 1975, p. 3) and the uncertainty of the gravity data (Daugherty, 1975, p. 19) were cited as possible reasons for this. In fact, the points (a) to (d) in Section 1.2 remained unsatisfied and an unsuccessful prediction resulted.

2.3 Conclusions

It appears that there is a good correlation between depth and free-air gravity anomalies in ocean basins. A more fundamental relationship between anomalies and crustal age has been suggested and has been used successfully in one ocean area. However, there appears to be no direct relationship which holds generally in all ocean areas, and the inadequacy and inaccuracy of the observed gravity itself precludes further investigation in this field. The unpromising nature of these comments should be viewed in the light of Section 4 where reference is made to alternative methods of predicting anomalies in ocean areas.
3. Collocation of Gravity Anomalies from Crustal Age

3.1 Introduction

Prediction techniques using collocation in either the univariate or multivariate mode (see Moritz, 1972) are now widely used in predicting potential-related parameters. However, in practice, their use has been limited to parameters such as anomalies, geoidal undulations or deflections of the vertical, which are directly related to anomalous earth potential. The question now is whether or not it is possible and feasible to predict gravity by collocation from some geophysical parameter which is not directly related to the anomalous potential.

The theoretical justification for this can be found in Tscherning (1974), where relations assigning a density distribution to the harmonic part of the earth are derived. Tscherning proves it is possible to combine the values of density anomalies, gravity anomalies or other gravimetric quantities in least squares collocation using the covariance models derived therefrom (ibid., p. 13-14).

This derivation is based on a global model, but it is not unreasonable to apply the technique to local fields also. The results of such an application are described below.

3.2 Prediction of Free-Air Gravity Anomalies by Collocation

(a) Covariance Analysis

For purposes of comparison the data set used by Daugherty (1974, Appendices I and II) were used for analysis. The auto-covariance functions (acf) of the free-air anomaly ($\Delta g_{fa}$) and of crustal age (CA) and the cross-covariance function (ccf) of $\Delta g_{fa}$ with CA were computed from the 482 $1^\circ \times 1^\circ$ areas means and the resulting functions illustrated in Figures 4 to 8. The units of CA are million years (m.y.). It will be noted that the acf for $\Delta g_{fa}$ behaves typically, from $\psi = 0^\circ$ to $\psi = 1.5^\circ$, but thereafter is very flat, reflecting the smoothness of the field. $C(\psi) = 0$ at $\psi = 15^\circ 5$ which again shows that there exists positive correlation between anomalies over much larger separations than normally exist over land areas. The acf for CA falls almost linearly from a $C(0)$ value of 400 m.y. and attains zero at $\psi = 16.5^\circ$. The ccf of $\Delta g_{fa}$ with CA is also flat, crossing $C(\psi)$ at $\psi = 15^\circ 5$.

The acf for ocean depth is shown in Figure 7 and is similar to that for CA, at least to $\psi = 10^\circ$, after which it increases slightly and then continues to decompose slowly. The ccf of $\Delta g_{fa}$ with depth is atypical, reaching its maximum at $\psi = 15^\circ 0$ after a local minimum at $\psi = 2^\circ$. Such behaviour is not uncommon in ccf's and was noted by this author in the ccf of $\Delta g_{fa}$ with geoidal undulations in the U. S. Calibration Area (Kearsley, 1977). It is surprising to find the lag at which this maximum occurs is so large. It is hard to accept the implication from this that values $8^\circ$ from a computation point have effectively the same influence on the prediction as values adjacent to it. Use of such functions should be tempered with critical caution.
Figure 5. Auto-Covariance Function for Crustal Age

Figure 6. Cross-Covariance Function: Crustal Age with $\Delta_{\psi}$

$C(\psi) - \text{mgal-my}$
Figure 7. Auto-Covariance Function for Depth

Figure 8. Cross-Covariance Function: $\Delta g_{xy}$ with Depth
(b) Prediction Results

The prediction technique is given in Moritz (1972) and will not be described here. In the prediction the 5 closest 1° x 1° mean crustal ages (as interpolated from the age map) were used to find the mean gravity and its estimated error of the subject 1° x 1° area. The difference between the predicted and 'known' ΔgRA value were used to find accuracy estimates. It should be remembered that the '5 closest' CA's would include the CA for the subject area itself, but the remaining four may not be adjacent to this area as the data set was incomplete.

The results are illustrated in Figure 9. The RMS of the differences is ±11.5 mgal, which is not quite as good as that obtained by Daugherty (±10.4 mgal) but is certainly of the same order. It compares well with the theoretical error of prediction (~11.2 mgal). The RMS of the anomalies themselves is ±17.2 mgal, so the prediction represents an improvement of about 30% over an assumption of a zero anomaly for the whole data set.

It must also be remembered that the accuracy of the 'known' gravity data is at times poor (up to ±25 mgal), averaging only about ±13 mgal. An RMS of differences of ±10 to ±11 mgal is therefore quite good.

A further comparison was made between 'known' values and values predicted from the ΔgRA values of the 5 nearest 1° x 1° areas, excluding the subject area. The distribution of the differences is shown in Figure 10. The RMS in this case was ±9.3 mgal, i.e. of the same order as the prediction from crustal ages. It does appear that prediction of Δg from crustal age is successful in this area, approaching the limit set by the accuracy of the known gravity data.

3.3 Testing the Use of Bouguer and Isostatic Anomalies

(i) Bouguer Anomalies

It is common practice to use Bouguer anomalies when predicting in local areas over land, as this field is smoother and more amenable to linear interpolation. The computation of 'Bouguer' anomalies at sea has already been outlined in Section 1.3 and are known to attain high absolute values in ocean areas (e.g. Heiskanen and Vening-Meinesz, 1958, p. 144, 197). However, their magnitude is unimportant in this context. What is needed for successful prediction is a smooth field with small residuals about the mean.

A covariance analysis of the Bouguer anomalies showed that these anomalies would certainly not produce such a field. It is obvious that the correction values vary greatly from one 1° x 1° area to the next and in fact disturb an already smooth field. The variance of this field is very large (2420 mgal²) and the auto-covariance function (acf) drops steeply (cf. the free-air anomaly acf) to cross C (ψ) = 0 at t > 10°. The cross-covariance of Bouguer anomaly with age is also large (about ±10 mgal m.y.). It is obvious that use of Bouguer anomalies will produce poorer results than those obtained using the free-air anomalies, and no predictions were attempted with them.
Figure 9. Distribution of Differences:
\( \Delta g \) (Predicted from Crustal
\( \Delta g \)) Minus \( \Delta g \) (Given)

\[ \text{RMS DIFFERENCES: 11.2 mgal} \]

Figure 10. Distribution of Differences:
\( \Delta g \) (Predicted from Adjacent
\( \Delta g \)) Minus \( \Delta g \) (Given)

\[ \text{RMS DIFFERENCES: 9.3 mgal} \]
(ii) Isostatic Anomalies

It appeared that isostatic anomalies ($\Delta g_{iso}$) would produce more promising results than Bouguer, and even free-air anomalies, as the isostatic correction results from a more sophisticated model which could account for some of the 'noisy' features in the data. The isostatic anomalies for the data set were computed as described in 1.3 (c) and the results subject to covariance analysis.

The isostatic anomalies proved to be almost identical with the free-air anomalies, the largest difference being $\approx 1$ mgal. As a result the statistical analysis is also practically identical ($C(0^\\circ) = 158.3$ mgal$^2$ for $\Delta g_{iso}$, cf. 159.0 mgal$^2$ for $\Delta g_{fa}$). It was obvious that there would be no improvement in accuracy if one used isostatic rather than free-air anomalies, and no such prediction was attempted.

Comments

It appears that, in the test area at least, the ocean acts as a 'filter' to local anomalies in the crust, and that the free-air anomalies are already smooth and suitable for use in the prediction.

It also appears that isostatic equilibrium holds in this area as the free-air and isostatic anomalies are so similar.

Preliminary calculations of the terrain correction showed that this was of such small magnitude that it would not have any impact in improving the prediction. This is due mainly to the size of the separation between adjoining blocks (at least 110 km) and the depths of the ocean being dealt with ($\approx 3$ to 5 km).

4. Conclusions

A review of the literature showed that a general relationship existed between gravity anomalies and depth of water in ocean areas. Woollard has suggested that crustal age is a more fundamental parameter to use in prediction, based on the theory of sea floor spreading. Early investigations using a polynomial relationship between crustal age and gravity anomalies showed promise, giving an accuracy of $\pm 10.5$ mgal for a $1^\circ \times 1^\circ$ mean value (Woollard and Daugherty, 1974). However, extension of this approach into a tectonically uncertain area failed, highlighting the fact that research in ocean areas is severely hampered by poor coverage and accuracy of the gravity and lack of knowledge of the nature of the ocean floor and its sub-strata.
Collocation was used to predict free-air gravity anomalies from Daugherty's crustal age data. This produced results comparable to the test mentioned above (±11.2 mgal) and represented an improvement of about 30% over an assumption of zero anomaly for this data set. It also compares favorably with the average accuracy of the known data in this region (±13 mgal). (The uncertainty of this data reaches ±25 mgal in some cases, and one must question its suitability for comparison purposes in such cases.)

The collocation approach has several advantages. It is a simple matter to extend the technique to include a number of parameters (e.g. gravity anomalies and crustal age) in the prediction. Error estimates of the data can be incorporated in the solution, which also provides an estimate of the accuracy of the prediction. Some preliminary analysis of the data is, of course, required to find the relevant covariance functions.

Recent research has shown that the need for the type of prediction discussed in this report has been lessened in areas where reliable satellite altimetry data will be available. Developments in the prediction of gravity anomalies from GEOS-3 altimetry have shown that this technique is capable of a superior accuracy of ±8 mgal for 1° x 1° area means, and ±2.5 mgal for 5° x 5° means (Rapp, 1977; see also Appendix I). Many ocean areas of geodetic interest have been covered by satellite altimetry and, as more data becomes available, will be processed to provide a fairly complete ocean coverage (except at high latitudes) to an accuracy approaching that obtainable from ship borne gravimetry. The anomaly is deduced from direct measurement of the geophysical approach described herein. In fact, it is probable that in the near future geoidal undulations and gravity anomalies obtained from satellite altimetry will aid in the interpretation of large scale (∼5°) or small scale (∼0.3°) geophysical features of the ocean floor and its sub-strata (e.g. Bowin, 1975).
References


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Moritz, H., Advanced Least Squares Methods, Department of Geodetic Science Report No. 175, The Ohio State University, Columbus, 1972.


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Appendix I - Anomaly Comparisons

The table below gives a comparison of a small sample of free-air gravity obtained from various sources for 1° x 1° areas stated.

\( \Delta g^\prime \) are from the DoD library (DMAAC, 1972, 1973) used by Daugherty and this author for their analysis and prediction.

\( \Delta g^\prime \prime \) are from an updated version of the DoD gravity library (DMAAC, 1976).

\( \Delta g^\prime \prime \prime \) are values predicted by Daugherty (1974, Appendix II) from crustal age.

\( \Delta g^\prime \prime \prime \) predicted by collocation from crustal age in this report (section 3).

\( \Delta g^\prime \prime \prime \) predicted by R. H. Rapp (private communication, December 1977) from GEOS-3 altimetry.

<table>
<thead>
<tr>
<th>( \varphi ) (Degrees)</th>
<th>( \lambda )</th>
<th>'Known' Data</th>
<th>Predicted Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \Delta g^\prime )</td>
<td>( \Delta g^\prime \prime )</td>
</tr>
<tr>
<td>39</td>
<td>216</td>
<td>-10</td>
<td>-13</td>
</tr>
<tr>
<td>39</td>
<td>217</td>
<td>-17</td>
<td>-16</td>
</tr>
<tr>
<td>39</td>
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<td>-16</td>
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</tr>
<tr>
<td>40</td>
<td>233</td>
<td>-11</td>
<td>-17</td>
</tr>
</tbody>
</table>

(all gravity values are in mgal).

It is worth noting the large differences between the 'known' data and the generally large uncertainty of the \( \Delta g^\prime \) values. For \( \varphi = 40^\circ \) the \( \Delta g^\prime \prime \prime \) compares well with \( \Delta g^\prime \) (except at \( \lambda = 217 \)), while for the remainder it compares more favorably with \( \Delta g^\prime \). A comparison for such a small sample is meaningless, but it does help to point up the uncertainty of the known data and the apparent improvement in accuracy in the altimetry-derived values.