CAVITATION DYNAMICS:

III. THRESHOLDS AND THE GENERATION OF TRANSIENT CAVITIES

by

H. G. Flynn

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Acoustical Physics Laboratory
Department of Electrical Engineering
University of Rochester
Rochester, N. Y. 14627

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**Title:** Cavitation Dynamics: III. Thresholds and the Generation of Transient Cavities

**Authors:** H. G. Flynn

**Performing Organization:** Acoustical Physics Laboratory, Dept. of Elec. Eng., University of Rochester, Rochester, N. Y. 14627

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**ABSTRACT:**

Dynamics of small argon bubbles under the influence of an acoustic pressure field have been studied using a mathematical formulation derived previously (J. Acoust. Soc. 57, 1379-1396 (1975)). Calculation of the maximum pressure in a collapsing bubble and work done by expanding bubbles show there are two cavitation pressure thresholds -- one for the onset of cavitation activity and the other for a decrease in cavitation activity.
Phenomena associated with acoustic cavitation in liquids are observed to change abruptly in magnitude with small changes in the amplitude of the acoustic pressure or intensity. The amplitudes at which changes occur are called cavitation thresholds. Investigators have noted that, once such thresholds have been exceeded, the phenomenon being observed rises to a maximum and then decreases. L. D. Rozenberg\(^1\) was perhaps the first to point out the existence of these maxima.

Many experimenters have determined both the existence and magnitude of cavitation thresholds, using erosion of solids, cavitation noise, chemical reactions and biological effects as the cavitation phenomena under observation.\(^2\) The work of Kaufman, Miller, Griffiths, Ciaravino and Carstensen\(^3\) and of Clarke and Hill\(^4\) may be cited as examples of the use of biological effects in threshold measurements, while that of Neppiras and Coakley\(^5\) may be cited as an example of use of cavitation noise. For brevity, the term "cavitation activity" will be used to denote the wide range of phenomena associated with cavitation fields or zones and used by various experimenters to demonstrate the existence of thresholds as functions of pressure or intensity.

Attention will be limited here to the remarkable papers of Monakhov, Peshkovskii, Popovich, Pomichev, Chinyakov and Yakovlev\(^6\) and Brandt, Yakovlev and Peshkovskii\(^7\). These workers observed two cavitation thresholds -- one for the onset of cavitation activity and a second for a marked decrease in cavitation activity (except
for the radiation of noise). Monakhov et al distinguished three regimes of cavitation. Below the first threshold, there was no cavitation activity. Above the second threshold, cavitation activity (as evidenced by erosion, for example) virtually ceased, large bubbles were observed and the liquid resembled boiling water. Monakhov et al carried out their experiments at a single acoustic frequency (17.8 kHz) in water that presumably contained gas "seeds" or "nuclei" having a wide distribution of initial radii. In the terminology suggested by this author\(^2\), the first regime of Monakhov et al would be a zone containing only stable cavities while the regime bounded by the two thresholds would be a zone dominated by transient cavities. In the third regime, the zone would again be dominated by bubbles that behave much as stable cavities.

An objective of this paper is to seek a dynamical basis for the existence of such thresholds and of the observe maxima in cavitation activity. This paper is the third in a series on cavitation dynamics. In the first\(^8\), hereinafter referred to as CD:I, a mathematical formulation for predicting the motion of, and other quantities associated with, a small cavity set into motion by an acoustic pressure field in a liquid was derived. This formulation consists of a set of differential, integral and algebraic non-linear equations that take into account compressibility and viscosity of the liquid, heat conduction across the interface and surface tension. The equations have been programmed for solution on a digital computer. In the second paper\(^9\), hereinafter referred to as CD:II, this formulation was used to study the free pulsations of small argon-filled cavities in water. The results presented in CD:II (and in an expanded report\(^10\)) are fundamental to the interpretation of calculations reported here.

The limitations on the usefulness of such a mathematical model include the assumptions that the speed of sound in the liquid is constant (that is, the variational pressure is a linear function of the variational density), that the cavity retains its spherical shape throughout its motion, that the amount of gas in the cavity remains constant and that the gas in the cavity behaves as an ideal gas.

In the calculations reported here, a pre-existing seed of argon
in water grows into a cavity under the influence of a uniform, sinusoidal acoustic field characterized by a pressure amplitude, \( P_A \), and a frequency, \( f_A \).

The system of notation adopted in CD:I will be used in this paper. An asterisk (*) is used to denote a quantity in some consistent set or units. In this notation, \( p^* \) is a pressure in bars and \( p = p^*/p_n^* \) is the non-dimensional pressure where \( p_n^* \) is the reference pressure (here taken to be 1 bar). Thus \( P_A^* \) is the non-dimensional pressure amplitude, \( P_A^*/P_n^* \). The quantity \( R = R^*/R_n^* \) is the non-dimensional radius of the cavity, \( R^* \) its radius in centimeters and \( R_n^* \) the initial radius of the cavity in centimeters. The frequency, \( f^* \), is the non-dimensional frequency \( f^*/f_n^* \) where \( f^* \) is the frequency in Hertz and \( f_n^* = R_n^*/a_n^* \) and \( a_n^* \) is the equilibrium speed of sound in the liquid.

Once set into motion, the cavity passes through a series of maxima and minima and its motion may or may not be periodic. The complexity of this non-linear motion results from the tendency of the cavity to pulsate both at the driving frequency, \( f_A^* \), and at some resonance frequency, \( f_r^* \), of free pulsation as determined in CD:II. When the motion is periodic, there is a least period, \( T_L^* \), in which the motion repeats itself. This least period, \( T_L^* \), must be an integral multiple of both the acoustic period, \( T_A^* = 1/f_A^* \), and the period of free pulsation, \( T_r^* = 1/f_r^* \); that is, for a periodic motion, \( T_L^* = m T_A^* = n T_r^* \) where \( n \) and \( m \) are integers. Of particular interest is the case where \( m = 1 \) and \( T_L^* = T_r^* = n T_A^* \). The motion is then said to contain a subharmonic order \( n \) (or, in terms of frequency, \( f_L^* = f_r^* = f_A^*/n \) and there is said to be a subharmonic of order \( 1/n \)).

Most motions of cavities in an acoustic pressure field are quasi-periodic; that is, the radius-time curve shows a slowly varying time interval that approximates a least period, \( T_L^* \), and the maxima and minima in such a period change slowly in amplitude and phase from one such quasi-period to the next.

The non-linear motion of a cavity is thus a combination of a free pulsation and a driven pulsation, both of which contain a fundamental and associated harmonics. The driven pulsation has the period, \( T_A^* \), of the acoustic field, but the period of the free pulsation, \( T_r^* \), depends on the amplitude of motion, as shown in CD:II. In general, in a least period, \( T_L^* \), the radius time curve will be quite complicated. However, when the amplitude of
motion is such that the period, \( T_\nu \), of free pulsation equals the acoustic period, \( T_A \), then the motion consists of a single maximum and a single minimum in a least period, \( T_L = T_A = T_\nu \). The frequency \( f_\nu \) at which this coincidence takes place is the non-linear resonance frequency for a cavity of initial radius, \( R_n^* \), in an acoustic field specified by the pair \((P_A', T_A)\). This non-linear resonance frequency is always less than the linear resonance frequency, \( f_0 \), for a cavity pulsating with very small change in radius.

Curves of the resonance frequency, \( f_\nu' \), as a function of the maximum radius, \( R_0 \), are shown in Fig. 2 and Reference 10 for various values of initial cavity radius, \( R_n^* \). When a cavity of initial radius, \( R_n^* \), is driven at an acoustic frequency, \( f_A' \), these curves tell us the maximum radius, \( R_0' \), at which the specified \( f_A' \) equals some resonance frequency, \( f_\nu' \), of that cavity. In a non-linear pulsation at a frequency close to some \( f_\nu' \), the radius-time curve has an unique maximum, \( R_\nu \), corresponding to the specified pressure amplitude, \( P_A' \). This pressure amplitude at which the acoustic frequency \( f_A' \), equals a resonance frequency, \( f_\nu' \), will be called the resonance pressure, \( P_\nu' \).

We shall find that the resonance pressure, \( P_\nu' \), determines one cavitation threshold. Another cavitation threshold is defined through use of a function called the dynamical threshold radius, \( R_t' \), described in CD:II for free pulsations. When the maximum radius, \( R_0' \), of an expanding cavity of initial radius, \( R_n^* \), exceeds the threshold radius, the cavity is a transient cavity. When such a cavity collapses, inertial forces in the surrounding liquid generate rapidly increasing kinetic energy that is either stored in the compressible liquid or converted into internal energy of the cavity contents. Ultimately, the inward motion is halted by the pressure in the cavity and part of the stored energy radiated as a shock wave. Most of the phenomena summed up as cavitation activity are brought about by transient cavities.

Determination of the threshold radius, \( R_t' \), requires partition of the acceleration of the cavity interface into two functions: the pressure function, \( PF \), and the inertial function, \( IF \). When a cavity starts to contract from a maximum, \( R_0' \), \( PF \) first decreases, passes through a minimum and then increases. \( IF \) is a function of
the maximum radius, $R_o$, at the start of collapse, and the value of $R_o$ for which IF intersects this minimum is the threshold radius, $R_t$.

When a cavity of initial radius $R_n^*$ pulsates in an acoustic pressure field of frequency $f_A'$, its maximum radius increases when the pressure amplitude, $P_A'$, increases. The value of $P_A'$ which causes $R_o$ to equal or exceed $R_t$ will be called the threshold pressure, $P_t$.

It is qualitatively obvious that cavitation activity must increase with acoustic pressure, but must eventually decrease. As the pressure amplitude increases the average volume of a cavity becomes much larger than its equilibrium volume and the cavity spends most of a period, $T_L'$, in such an expanded state. The liquid then becomes much more compressible and this increase in compressibility strongly moderates the violence of collapse. The exciting sound beam is both scattered and absorbed by the increased cross-sections of the cavities and the radiated shock waves from collapsing cavities will likewise be scattered and absorbed by surrounding cavities. Because there may be an enormous number of cavitation events per cm.$^3$ in a cavitation zone, any increase in the average size of cavities may have a drastic effect on cavitation activity. Sirotyuk estimates that there may be as many as $10^6$ cavitation events per cm.$^3$.

The mathematical model of CD: I used in carrying out the calculations reported here predict the motion of a single cavity in an infinite, homogeneous liquid. With this restriction in mind, two quantities have been chosen for calculation they might give insight into the thresholds observed in zones containing many bubbles with a wide distribution of initial radii. These quantities are the maximum pressure, $P_m'$, in a collapsing cavity and the work, $W_E$, done by a cavity on the surrounding liquid in expanding from its minimum radius, $R_m'$, to a maximum radius, $R_o'$. This work is $W_E = W_{E*}/W_n^*$ where $W_n^* = 0.88$ kiloJoules mol$^{-1}$.

The maximum pressure, $P_m'$, determines the initial strength of the radiated shock from a collapsed cavity and $W_E$ measures the transfer of energy to the liquid by the compressed gas in the cavity. In any expansion, most of the work, $W_E$, is done in the initial stage when $R$ is close to $R_m'$. Thus, while $P_m'$ determines the strength of the shock front, $W_E$ determines the width and magnitude of the
shock wave behind the front. Because the least period $T_L'$, may be many times the acoustic period, $T_A'$, the work, $W_E$, is here defined as the average work done by an expanding cavity in an acoustic period, $T_A'$, the average being taken over values in the least period, $T_L'$.

Fig. 1 shows the maximum pressure, $p_m^*$, predicted for a cavity of initial radius $R_n^* = 5 \times 10^{-4}$ cm. as a function of the acoustic pressure amplitude, $P_{A'}$, for three frequencies of the acoustic field. These frequencies, 600 kHz, 300 kHz, and 100 kHz, are approximately equal to $f_0$, $f_0/2$ and $f_0/6$ where $f_0$ is the linear resonance frequency of the cavity.

At the calculated threshold pressure, $P_t$, there is a marked change in the maximum pressure at 300 kHz and 100 kHz. At 600 kHz the inertial function IF always lies above the minimum in PF and $P_t$ is undetermined.

We can draw two conclusions from the location of the resonance pressure, $P_r^*$, on these curves. Most points on these curves are accompanied by an integer. This integer indicates the least period in terms of $T_A'$. Thus $n = 1$ means $T_L = T_A'$, while $n = 4$ means that $T_L = 4 T_A$ and a subharmonic of order 1/4 exists. The location of $P_r^*$ divides each curve into two parts. Below $P_r^*$ only one subharmonic could be found, while above $P_r^*$ there exists a profusion of subharmonics of various orders. At 300 kHz and 100 kHz, the curves of $p_m^*$ abruptly decrease in slope in the vicinity of $P_r^*$, but at 600 kHz the change in slope is much less marked.

Fig. 3 shows the maximum pressure, $p_m^*$, as a function of the acoustic pressure amplitude, $P_{A'}$, for three different cavities. Each cavity is driven at a frequency equal approximately to $f_0/2$ corresponding to its initial radius, $R_n^*$. On all three curves the threshold pressure, $P_{r'}$, marks an abrupt change in the slope of the curve and in the vicinity of the resonance pressure, $P_r^*$, there is an even more pronounced decrease in the slope of $p_m^*$ as a function of $P_{A'}^*$. Again, there are subharmonics in abundance above $P_r^*$ but only one below it.

The maximum pressure curves would lead us to identify $P_t$ with the first cavitation pressure threshold of Monakhov et al and the resonance pressure, $P_r^*$, at which $f_A = f_r$ with the second cavitation pressure threshold. At $P_r^*$ the maximum radius, $R_0^*$, is
the resonance radius for the driving frequency, $f_A$. These conclusions would appear to apply only to cavities driven at frequencies well below the linear resonance frequency, $f_0$.

The work, $W_E$, per $T_A$ done by an expanding cavity is shown in Fig. 4 as a function of the acoustic pressure amplitude, $P_A^*$, for a cavity of initial radius, $R_n^* = 5 \times 10^{-4}$ cm. For this calculated quantity it is even clearer that $P_t$ and $P_r$ are the first and second pressure thresholds, at least for $f_A$ much less than $f_0$.

Fig. 5 shows $W_E$ as a function of $P_A^*$ for three different cavities. Each cavity is driven at a frequency $f_A$ approximately equal to $f_0/2$. For each cavity, $P_r$ is the pressure threshold at which there is a marked decrease in the slope of the curves. For the $5 \times 10^{-5}$ cm. cavity, $P_t$ does not appear to act as a threshold while the point at $P_A^* = 4$ bars does. The significance of this remark lies in the fact that, at this pressure, a quantity called the energy dissipation modulus, $\Delta W/W_m^*$, defined in CD:II, is a maximum. Again, in the curves for $W_E$, subharmonics appear in general only above the second threshold, $P_r$.

The calculations reported here predict the behavior of a single bubble in an infinite, homogeneous liquid, and one must be cautious in seeking quantitative correspondences with experimental results, which in general are statistical averages over many bubbles. The maxima characteristic of cavitation activity do not appear, nor should we expect them to. However, the results clearly give use useful insights into experiments such as those of Monakhov et al. Thus, despite these restrictions, there are several general remarks that can be made about the results reported in this paper:

1. The quantity, $P_t$, is a pressure threshold at which cavitation actively rapidly increases for any driving frequency, $f_A$, well below $f_0$, the linear resonance frequency of a cavity. Tentatively, $P_t$ may be identified as the first cavitation threshold of Monakhov et al.

2. On the other hand, $P_t$ does not appear to be a cavitation threshold when $f_A$ is approximately equal to $f_0$ or $R_n^*$ is less than a micron. When $R_n^*$ is less than a micron, the first cavitation
threshold may occur when the energy dissipation modulus, $\Delta W/W_m$ is a maximum, as suggested in CD:II.

3. A second pressure threshold, $P_{r'}$, occurs when the driving frequency, $f_A'$, equals a non-linear resonance frequency, $f_{r'}$, of the cavity. Tentatively, $P_{r'}$ may be identified with the second threshold of Monakhov et al.

4. At the second threshold, $P_{r'}$, both the maximum pressure and $WE$ change abruptly for driving frequencies less than the linear resonance frequency of the cavity. Curves of both maximum pressure and $WE$ tend to flatten out for pressure amplitudes greater than $P_{r'}$. Changes in the medium due to expanded cavitation bubbles, noted above, may cause these curves to decrease above $P_{r'}$.

5. Subharmonics in general are present only in the region above the second threshold, $P_{r'}$, which may be identified as the region of reduced cavitation activity defined by Monakhov et al.

6. Cavities with initial radii greater than a micron are more effective in producing cavitation activity when driven at frequencies less than their linear resonance frequencies.
REFERENCES

FIGURES

Fig. 1  Maximum pressure in a 5-micron cavity

Fig. 2  Non-linear resonance frequency curve for a 5-micron cavity

Fig. 3  Maximum pressure in three cavities

Fig. 4  Work, $W_E$, done by an expanding 5-micron cavity

Fig. 5  Work, $W_E$, done by three expanding cavities
Fig. 1 Maximum pressure in a 5-micron cavity

$R_n^* = 5 \times 10^{-4}$ cm,

$f_0 = 6.05 \times 10^5$ Hz
Fig. 2 Non-linear resonance frequency curve for a 5-micron cavity
**Fig. 3** Maximum pressure in three cavities
$R_n^* = 5 \times 10^{-4}$ cm.

$f_o^* = 6.05 \times 10^5$ Hz

$W_E$, Work Done by Expanding Cavity per $T_A$

$P_A^*$, Acoustic Pressure Amplitude, $P_A^*$ (bars)

$P_r$
Work, $W_p$, done by three expanding cavities

\[ f_A \approx \frac{1}{2} f_0 \]

<table>
<thead>
<tr>
<th>$R_n^*$ (cm)</th>
<th>$f_A^*$ (Hz)</th>
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<tr>
<td>$5 \times 10^{-3}$</td>
<td>$3.2 \times 10^4$</td>
</tr>
<tr>
<td>$5 \times 10^{-4}$</td>
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</tr>
<tr>
<td>$5 \times 10^{-5}$</td>
<td>$4.7 \times 10^6$</td>
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**Fig. 5** Work, $W_p$, done by three expanding cavities