NOISE IN CHARGE COUPLED DEVICES

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The four general sources of noise in CCD are reviewed which are: signal input, signal output, dark current and trapping noise. The trapping noise in a Surface Channel CCD (SCCD) is mainly due to interface traps and is the dominant noise in this device. In the Buried Channel CCD (BCCD), the low density of bulk traps gives much less trapping noise. Theoretical RMS noise levels at the output are about 500 electrons/packet in a SCCD and 100 to 300 electrons/packet in a BCCD with comparable dynamic ranges. Noise in SCCD can be reduced by reducing interface state density during device fabrication.
This scientific report is the result of a in-depth review of literature dealing with noise in CCD (Charge Coupled Devices) undertaken by James F. Detry from June 1, 1976 to September 30, 1977. Its content is identical to that submitted by him as the thesis for the Master of Science degree in Electrical Engineering. One of the reason for this review work is the statement made in several published papers that there is no 1/f noise from surface states in SCCD (Surface Channel CCD), giving the impression that interface states are unimportant or cannot contribute to low frequency noise in CCD devices. This impression contradicts expectations based on simple physical considerations of the CCD geometry and operation conditions. Indeed the review concludes that interface state noise is the principal mechanism which makes the SCCS less sensitive to low level signals than BCCD (Bulk Channel CCD). This report also serves the purpose of a short review of the large number of published papers on CCD noise. Such a review was not available to us previously.

C. T. Sah
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I. INTRODUCTION

Charge Coupled Devices (CCDs)\(^1\) are conceptually simple devices, consisting basically of a string of MOS capacitors.\(^2\) By applying the appropriate clocking voltages to these capacitors, charge packets are transferred from one potential well to the next. This enables the CCD to be used for, among other things, digital memory\(^3\), analog delay lines\(^4\), or analog imaging devices.\(^5\)

The cross section of a CCD is shown in Figure 1. Each cell stores one charge packet, and the number of capacitors (or gates) per cell is equal to the number of clock phases (three in this case).

There are two types of CCDs: the Surface channel CCD (SCCD), where the charge is stored at the oxide-semiconductor interface; and the Buried channel or Bulk CCD (BCCD)\(^6\), where the charge is stored away from the interface in the bulk of the semiconductor. The SCCD can accommodate a larger signal than the BCCD and is easier to fabricate, but must operate with a fat zero* to achieve necessary efficiency. The BCCD, on the other hand, is faster, does not need a fat zero, and has less internal noise than the SCCD. It is the internal noise of the CCD which sets a lower limit on the usable signal level in CCD applications.

* The fat zero is a circulating background charge, about 10\% of the maximum signal. The interface traps with long emission time constants are kept filled by the fat zero and so cannot contribute to transfer losses.
Figure 1
Cross Sectional View of a CCD
II. SOURCES OF NOISE

There are four major noise sources in CCDs: input noise from injecting the signal into the CCD, output noise from amplifying and removing the signal, dark current noise due to the random thermal generation of electron-hole pairs in the signal storage area, and trapping noise due to the effect of localized states within the semiconductor band gap located in the semiconductor itself or in the oxide near the oxide-semiconductor interface.

The first three noise sources operate in the same manner for either the SCCD or the BCCD, but the trapping noise differs greatly. It is this difference which causes a higher noise level in the SCCD. Typical trapping noise levels at the output would be about 400 noise electrons per packet in a SCCD and less than 100 in a BCCD.*

* Representative numerical values of all noise sources can be found in Table 8.2 on page 34.
III. INPUT NOISE

3.1 Origins of Input Noise

There will be a noise associated with any attempt to inject a signal into a CCD. This same noise will also occur during the injection of a flat zero in the case of the SCCD. If the signal is introduced optically, as in the case of an imaging device, there will be shot noise due to the quantum nature of photons. In this case the mean square of the number of noise electrons is

$$\langle q^2 \rangle = N_s$$

where $N_s$ is the number of signal electrons. In practice $N_s$ could be as large as $10^7$, depending on the gate size.

It is not expected that a flat zero would be introduced optically because there is another method of signal injection, the electrical method, which has a much lower noise associated with it. The electrical signal injection takes place at the first potential well of the CCD or, in an array-type CCD, at the first potential well of each row. The charge is input through one or two MOSFETs as shown in Figures 2 and 3, respectively.

3.2 Derivation of Electrical Input Noise

The input is a MOSFET with the input diode acting as the source, the input gate as the gate, and the potential well under the first clock phase acting as a virtual drain. When operated in saturation, a MOSFET has mean square thermal noise current per unit bandwidth

$$\langle i^2 \rangle = 4kT (\frac{2}{3}\beta)$$

(3.2)
Figure 2
Single Gate Input

Figure 3
Floating Diffusion Input
where \( g \) is the conductance of the MOSFET channel. From this noise current the charge variance of the packet can be obtained by following the derivation of Carnes and Kosonocky.\(^9\) The variable of interest is the fluctuation of charge on the capacitor shown in Figure 4. The mean square noise voltage per unit bandwidth is

\[
\langle v^2 \rangle = \frac{\langle i^2 \rangle}{|G|^2} \quad (3.3)
\]

where

\[
G = g + j \omega C \quad (3.4)
\]

The lowest frequency at which the input MOSFET can contribute noise is \( f = pf_c \) (where \( f_c \) is the clock frequency and \( p \) is the number of phases). This comes about because the charge packet will be transferred after a time \( 1/pf_c \), so that any fluctuation taking longer than \( 1/pf_c \) cannot affect the charge in the packet.

The total noise voltage is obtained by substituting (3.4) into (3.3) and integrating over frequency

\[
\langle v^2 \rangle = \int_{pf_c}^{\infty} \langle v^2 \rangle df = \int_{pf_c}^{\infty} \frac{4kT(\frac{2}{g})}{2\pi p f_c} \frac{d\omega}{g^2 + \omega^2} \quad (3.5)
\]

This can be broken up into two integrals

\[
\langle v^2 \rangle = \frac{4kT}{3\pi C} \left\{ \int_{0}^{\infty} \frac{g/c}{g^2 + \omega^2} \ d\omega - \int_{0}^{\infty} \frac{g/c}{g^2 + \omega^2} \ d\omega \right\} \quad (3.6)
\]

The first integral is simply \( \pi/2 \). In the second integral let

\[
y = \omega C/g \quad (3.7)
\]
Figure 4

Input Noise Equivalent Circuit
The integral becomes
\[ \frac{2\pi f_c C}{g} \int_{0}^{\pi} \frac{dy}{1+y^2} \] (3.8)

In order for the low noise operation to take place the RC circuit must be allowed to reach equilibrium, meaning that
\[ \frac{1}{\tau} = \frac{g}{C} \gg \omega_c \tau \] (3.9)

or
\[ 2\pi f_c \frac{C}{g} \ll 1 \] (3.10)

So the integrand =1 over limits of 0<y<<1 and therefore the integral is insignificant in comparison with \( \pi/2 \). Putting \( \pi/2 \) in the brackets of (3.6) gives
\[ <V^2> = \frac{2}{3} \frac{kT}{C} \] (3.11)

The mean square number of noise electrons is then
\[ <q^2> = \frac{C^2}{e^2} <V^2> = \frac{1}{e^2} \frac{2}{3} kTC \] (3.12)

where \( e \) is the magnitude of the electron charge.

3.3 Discussion of Electrical Input Noise

For the method which uses just one MOSFET, \( C \) is the capacitance of the first gate of the CCD. The two-MOSFET method uses a floating diffusion between two input gates for intermediate charge storage. In that case the mean square charge fluctuation is doubled because the two transfers are un-
correlated, giving

\[ <q^2> = 2 \frac{1}{e^2} \frac{2}{3} kTC \]  

(3.13)

But now \( C \) is the capacitance of the floating diffusion, which can be made much smaller than that of a CCD gate, resulting in less overall noise. Typical numbers are \( <q^2> = 3000 \) electrons for single gate input and 2000 for the floating diffusion input. Though it has less noise, the floating diffusion method has drawbacks. The smaller capacitance limits the signal to a smaller maximum size, and the input is not as linear as the single gate method. However, these disadvantages do not matter if the electrical input is used for a fat zero only, as in the case of an imaging device. For that particular application the fat zero input will be constant, rendering the nonlinearity irrelevant, and the magnitude will be well within the limit imposed by the floating diffusion capacitance.

It was shown by Sah, Wu, and Hielscher that the equivalent noise conductance used in equation (3.2) \( \frac{2}{3g} \) is the theoretical lower limit. The true value is a function of oxide thickness, gate voltage, and impurity concentration, and can be several times larger than \( \frac{2}{3g} \). This could explain the excess noise found experimentally.

1/f noise is also present in the MOSFET, but since it is less than the thermal noise at about \( 10^4 \) Hz and continues to decrease at higher frequencies, 1/f noise should be negligible in the megahertz range of the clock frequency.

The electrical input noise is predicted to be independent of signal size, but in practice it is not. The main reason is that as the signal size decreases, the area it occupies decreases, so the effective capacitance decreases.
citance decreases. For very small signals \( N_s^2 kTC/e^2 \) the noise becomes roughly shot noise. For signals greater than this limit (about \( 10^7 \) electrons) the mean square noise almost levels off, rising slowly in a SCCD, but growing in proportion to the signal size in a BCCD. The change in capacitance is much more drastic in a BCCD because as the signal size increases, it not only expands in volume, it also moves closer to the surface of the semiconductor.

It might be assumed that this same noise would accompany every charge transfer which takes place in a CCD (as indeed it does in a bucket brigade device) but such is not the case. The reason for the absence of this noise in the charge transfer process is that the CCD itself does not provide the conduction electrons. The thermal noise above was due to the random scattering of the charge carriers, electrons, in the conducting channel by the phonons. In the case of the CCD, the carriers are the electrons which make up the signal packet. These will experience scattering during transfer, but at the end of the transfer period, when the total charge transferred is "evaluated," there are almost no electrons left in the channel to be scattered. What little charge is left behind is trapped charge, not the free charge whose thermal noise comes from phonon scattering. However, fluctuations in the trapped charge are also a source of noise. That noise will be treated later.
IV. OUTPUT NOISE

The usual method of turning the CCD charge packets into an output signal is by an on-chip MOSFET. The output equivalent circuit is shown in Figure 5. The charge packet is shifted from the last storage cell to a reverse biased p-n junction. The diode is in turn connected to the gate of a MOSFET. The current through the output MOSFET is the output signal. Before the next charge packet can be output, the diode must be reset to its original biasing level. Noise is introduced during the reset process. Resetting the diode through another MOSFET produces exactly the same type of noise as the input circuit given by (3.12). Thus, the output noise is

\[ \langle q^2 \rangle_o = \frac{1}{e^2} \frac{2}{3} kT C_o \]  

(4.1)

Here \( C_o \) is the capacitance of the output node, about 0.15pF typically. Since \( C_o \) is much larger than the input capacitance, the output noise is also larger; \( \langle q^2 \rangle \) is on the order of 16,000.

Additional noise is present due to the channel of the output MOSFET. This noise can be referred back to the gate through the expression

\[ \langle q^2 \rangle = \frac{\langle i^2 \rangle C^2}{e^2 g^2 o} \]  

(4.2)

where \( g_o \) is the transconductance of the output MOSFET, and \( \langle i^2 \rangle \) is the mean square noise current of the MOSFET channel given by

\[ \langle i^2 \rangle_n = \langle i^2 \rangle_{\text{thermal}} + \langle i^2 \rangle_{1/f} \]  

(4.3)

The MOSFET noise current has two components, the thermal noise of the channel and the 1/f noise due to trapping at the semiconductor-oxide surface. The equivalent noise (4.2) is quite small with \( \langle q^2 \rangle \) less than 10.
Figure 5

Equivalent Circuit of CCD Output
White, Lampe, Blaha, and Mack devised an output scheme called Correlated Double Sampling (CDS) which can eliminate the reset noise. Once the output capacitance has been reset, the MOSFET through which it was charged is turned off, becoming a very large resistance. This large resistance means no high frequency noise will be present at the output node. CDS makes use of this fact by sampling the output signal once after the resetting takes place, and again after the signal charge has been received. Since no high frequency noise can be present during this time, the reset noise is highly correlated in the two measurements and vanishes when their difference is taken. Though this process eliminates the reset noise, it doubles the output channel noise because that noise source is not correlated during the two measurements.
V. DARK CURRENT NOISE

CCDs do not operate at equilibrium. The semiconductor surface of the semiconductor-oxide interface is kept at a potential large enough to cause inversion, but there is not enough time between clocking pulses to allow carriers to be collected at the interface. However, minority carriers are continually generated thermally and collected by the potential wells of the CCD, giving rise to the leakage or dark current. Each charge packet receives the same average dark current, but there will be noise due to the variance of this additional charge. Since it is thermally generated, this current will have a shot noise spectrum. The total amount of dark current collected by a packet in one clock period will be all the current which reaches one of the p elements of a cell during that period. So the mean square noise is equal to the total number of electrons received,

$$<q^2> = \frac{1}{e^2} \frac{PA}{f_c} J_D$$

(5.1)

where A is the area of one gate and $J_D$ is the dark current density. In the case of imaging, the same relation would apply, with the clock period replaced by the imaging period.

$$<q^2> = \frac{1}{e^2} pAT_i J_D$$

(5.2)

where $T_i$ is the imaging time.

There are three sources of dark current: carriers generated in the depletion region, carriers generated in the bulk within a diffusion length of the depletion region, and carriers generated at the semiconductor-oxide interface. The dark current is usually dominated by the interface
contribution. So it would be expected that SCCDs, in which many of the interface traps are filled by the stored signal charge, would have a smaller dark current than BCCDs, in which the surface traps are free to act as generation centers. This is indeed the case. The BCCD may have two or three times the dark current of a SCCD. Dark current can be reduced by using a lower clock voltage, resulting in a smaller depletion width, but only at the expense of reduced transfer efficiency. During imaging typical devices would have $<q^2> = 60$ for a SCCD or 200 for a BCCD. During a single transfer $<q^2> < 1$ for both devices.
VI. TRAPPING NOISE

6.1 Sources of Trapping Noise

There are three types of traps which cause noise in a CCD: bulk traps, interface traps, and oxide traps. SCCDs carry the charge at the semiconductor-oxide interface and so are subject to noise from the interface and oxide traps. The BCCD trapping noise is due to bulk traps only, because although the interface states are close enough to generate a dark current, the signal is not physically at the interface and so cannot be trapped by those states.

6.2 Derivation of Oxide Trapping Noise

The wide range of oxide trap time constants causes 1/f noise in the steady state current of a MOSFET. In a SCCD the same mechanisms are present, but when a charge packet fluctuates under the gate, the variance in charge is obtained by integrating the 1/f noise over the appropriate spectrum as pointed out by Barbe. For imaging applications the frequency range of interest is \( f > 1/T_i \). The results give a \( \langle q^2 \rangle \) proportional to \( T_i \) for a small \( T_i \), and the natural log of \( T_i \) for large \( T_i \).

As a function of frequency, the mean square charge fluctuation per unit bandwidth is

\[
\frac{4kT N_{TO}^A}{2\pi f_B} \{ \tan^{-1}(2\pi f_B \exp(\beta x)) - \tan^{-1}(2\pi f_B \exp(-\beta x)) \}
\]

(6.1)

where the following definitions apply:

- \( \exp(\beta x) \) is the emission rate of the oxide traps as a function of \( x \) (the distance into the oxide from the interface).
- \( N_{TO} \) is the oxide trap density per eV.
\( \tau_s \) is the shortest oxide time constant, corresponding to \( x=0 \) and is equal to \( N_{TTS} \) (where \( N_{TTS} \) is the interface trap density per eV).

\( l \) is the oxide thickness.

The factor \( \exp(\beta l) \) is so large that for all frequencies of interest,

\[
\tan^{-1}(2\pi f_s \exp(\beta l)) = \pi/2
\]  

Integrating over frequency

\[
\langle q^2 \rangle = \int_{1/T_s}^{\infty} \frac{4kT_{TTS}A}{2\pi B} \left( \frac{\pi}{2} - \tan^{-1}(2\pi f_s) \right) df
\]  

Making the substitution \( y = 2\pi f_s \)

\[
\langle q^2 \rangle = \frac{4kT_{TTS}A}{2\pi B} \int_{2\pi s/T_i}^{\infty} \frac{\pi}{y} \tan^{-1}y dy
\]  

When \( 2\pi s/T_i < 1 \) the integral can be broken into two parts

\[
\langle q^2 \rangle = \frac{4kT_{TTS}A}{2\pi B} \left\{ \int_{2\pi s/T_i}^{\infty} \frac{\pi}{y} \tan^{-1}y dy \right\}
\]

For the first integral \( \tan^{-1}y \) can be expanded as

\[
\tan^{-1}y = y - \frac{1}{3}y^3 + \frac{1}{5}y^5 - \cdots
\]  

Keeping only the leading term, the first integral of (6.5) becomes
Since \( \frac{2\pi s}{T_i} << 1 \) the last term of (6.7) can be ignored. For the remaining integral \( (y > 1) \) \( \tan^{-1} y \) can be expanded as

\[
\tan^{-1} y = \frac{\pi}{2} - \frac{1}{y} + \frac{1}{3y^3} - \frac{1}{5y^5} + \cdots
\]

(6.8)

Keeping only the first two terms of the series, the second integral of (6.5) becomes

\[
\frac{4kTN_{TTO}}{2\pi\beta} \int_{1}^{\infty} \frac{\frac{\pi}{2} - \left( \frac{\pi}{2} - \frac{1}{y} \right)}{y} dy = \frac{4kTN_{TTO}}{2\pi\beta}
\]

(6.9)

The total \( \langle q^2 \rangle \) is the sum of (6.7) and (6.9).

\[
\langle q^2 \rangle = kTA \frac{N_{TTO}}{\beta} \ln \left( \frac{T_i}{2\pi s} \right) \text{ for } \frac{T_i}{2\pi s} \gg 1
\]

(6.10)

When \( \frac{T_i}{2\pi s} << 1 \) there is only one integral to be evaluated. Using the expansion of (6.8)

\[
\langle q^2 \rangle = \int_{\frac{2\pi s}{T_i}}^{\infty} \frac{4kTN_{TTO}}{2\pi\beta} \frac{\frac{\pi}{2} - \left( \frac{\pi}{2} - \frac{1}{y} \right)}{y} dy
\]

(6.11)

\[
\langle q^2 \rangle = \frac{kTN_{TTO}}{\pi^2\beta} A \frac{T_i}{T_s} \text{ for } \frac{T_i}{2\pi s} << 1
\]

(6.12)

To find the noise due to oxide traps during CCD operation, \( T_i \) is replaced by \( 1/pf \) which is the length of time the packet stays under a given gate. The imaging operation would probably take long enough to put \( T_i \) in the first category with the logarithmic term, while clock rates are high.
enough to put normal operation in the linear region given by Equation (6.12). Typical values of trapping noise are $<q^2> = 300$ for an imaging period (from Equation (6.10)) and only about one for a single clocked transfer (from Equation (6.12)). If a clock time falls between the two regions above, a reasonable approximation is

$$<q^2> = kT \frac{N_{TTO}}{2} A \ln\left(1 + \frac{T_1}{2\pi T_s}\right)$$

(6.13)

This expression is exact for large $T_1$ but is too big by a factor of $\frac{\pi}{2}$ for small $T_1$.

Throughout these calculations the oxide trap density, $N_{TTO}$, has been treated as a constant, but it is not. $N_{TTO}$ is a function of $x$, the distance into the oxide. Through the time constant, $\alpha \exp(\beta x)$, $N_{TTO}$ can be treated as an effective trap density which is a function of frequency. Since $N_{TTO}$ varies sufficiently slowly it can be taken outside the integral (when imaging time changes by a factor of 1000, the effective $N_{TTO}$ changes by only a factor of 10).10

6.3 Derivation of Interface Trapping Noise

The interface traps are distributed in energy in the energy gap. When a signal comes in, they are filled to a level20

$$E_0 = kT \frac{a}{\ln\left(\frac{a}{N_{\text{ns}} A}\right)}$$

(6.14)

with a time constant

$$\tau_{\text{fill}} = (k N_{\text{ns}} A)^{-1}$$

(6.15)
where \( E_0 \) is the energy below the conduction band, \( k_{ns} \) is the two-dimensional electron capture rate, \( A \) is the area of the gate, \( N_s \) is the number of electrons in the packet, and "\( a \)" comes from the electron emission rate

\[
e_{ns} = a \exp(-E/kT) \quad (6.16)
\]

When the traps discharge, their fluctuations appear as noise in the signal. The variance of this noise is derived by the method of Tampse.\(^7\) The trapping effects must be integrated over the band gap, and the result is a constant for reasonably large signals.

The probability of a trapped carrier being emitted in time \( t \) is

\[
P = 1 - \exp(-e_{ns}t) \quad (6.17)
\]

At a given energy, the distribution of empty and full interface states will be binomial with variance per unit energy per unit area

\[
P(1-P)n_{TS}(E) \quad (6.18)
\]

where \( n_{TS}(E) \) is the filled trap density at energy \( E \). The total mean square charge fluctuation is found by multiplying by the area and integrating over the band gap.

\[
<q^2> = A \int_{E_0}^{E_g} n_{TS}(E) P(1-P)dE \quad (6.19)
\]

\( n_{TS}(E) \) can be replaced by \( N_{TTS} \) if the limits are changed.

\[
<q^2> = A \int_{E_0}^{E_g} N_{TTS} P(1-P)dE \quad (6.20)
\]

\( P \) is a function of time, and the result is evaluated at the end of the
transfer cycle. The charge has a time $t/pf_c$ to transfer, so

$$P = 1 - \exp\left(-\frac{n_s}{pf_c}\right) \quad (6.21)$$

Substituting (6.16) and (6.21) into (6.20) gives

$$<q^2> = AN_{TTS} \int_{E_0}^{E_g} \left[ 1 - \exp\left(-\frac{a}{pE_c}\exp(-E/kT)\right) \right] \exp\left(-\frac{a}{pE_c}\exp(-E/kT)\right) dE \quad (6.22)$$

Letting

$$u = \frac{a}{pE_c} \exp(-E/kT) \quad (6.23)$$

$$<q^2> = AN_{TTS} \int_{u_g}^{u_0} \left( \exp(-u) - \frac{\exp(-2u)}{u} \right) du \quad (6.24)$$

where

$$u_0 = u\bigg|_{E=E_0} = \frac{k_n}{n_s} \frac{S}{A} \quad (6.25)$$

$$u_g = u\bigg|_{E=E_g} = 0 \quad (6.26)$$

Breaking down (6.24) into two integrals

$$<q^2> = kTN_{TTS} \left\{ \int_{u_g}^{u_0} \frac{\exp(-u)}{u} du - \int_{u_g}^{u_0} \frac{\exp(-2u)}{u} du \right\} \quad (6.27)$$

In the second integral of (6.27) let $v = 2u$. Then

$$\int_{u_g}^{u_0} \frac{\exp(-2u)}{u} du = \int_{2u_g}^{2u_0} \frac{\exp(-v)}{v} dv \quad (6.28)$$
Since \( u \) and \( v \) are dummy variables, this can be combined with the first part of (6.27) giving

\[
<q^2> = A_k T N_{TS} \left\{ \int_{u_0}^{2u_0} \frac{\exp(-u)}{u} du - \int_{2u_0}^{\infty} \frac{\exp(-u)}{u} du \right\} \quad (6.29)
\]

If \( u_0 > 2u_g \) (as it almost certainly will be)

\[
<q^2> = A_k T N_{TS} \left\{ \int_{u_0}^{2u_g} \frac{\exp(-u)}{u} du - \int_{2u_g}^{\infty} \frac{\exp(-u)}{u} du \right\} \quad (6.30)
\]

Since \( u_0 = 0 \) over the range of the first integral, \( \exp(-u) = 1 \). So the first integral of (6.30) becomes

\[
\int_{u_0}^{2u_g} \frac{1}{u} du = \ln(2) \quad (6.31)
\]

For the second integral of (6.30), if \( u_0 \) is large enough the whole integral is negligible. (For \( u_0 = 2 \) the integral is less than .06 and it drops off very rapidly for larger \( u_0 \).) To justify the size of \( u_0 \) from (6.25), consider typical values

\[
k_{ns} = 0.01 \text{(cm}^2\text{s}^{-1}) \quad , \quad p = 2 \quad , \quad f_c = 10^6 \text{ Hz} \quad , \quad A = 10^{-6} \text{ cm}^2 .
\]

For \( u_0 > 2 \), \( N_s \) need only be 400 electrons. Not only would a signal this small be lost in the noise, but the use of a fat zero insures that the charge packet is much larger than that in order to prevent unacceptable transfer losses. Putting (6.31) into (6.30) then gives

\[
<q^2> = A_k T N_{TS} \ln(2) \quad (6.32)
\]
For an extremely small signal or a very high clock frequency, \( u_o \) will be small and is equal to \( t/t_{\text{fill}} \) so that the assumption under which (6.32) was derived — that all traps filled to \( E_o \) — is not valid. The original integral, (6.19), has been numerically evaluated by Carries and Kosonocky\(^9\) for the case of small \( u_o \) with the result that \( \langle q^2 \rangle \) is proportional to \( u_o \).

\[
\langle q^2 \rangle = AkTN_{\text{TTS}} \ln(2)u_o \quad \text{for} \quad u_o << 1
\]  

(6.33)

These results can be combined to give, as an approximate solution,

\[
\langle q^2 \rangle = AkTN_{\text{TTS}} \ln(2) [1 - \exp(-u_o)]
\]  

(6.34)

where \( 1 - \exp(-u_o) \) can be thought of as the fraction of traps filled. This fraction will be approximately one except in the case of a very small signal or extremely high clock frequency.

A typical value of the noise due to the surface traps is \( \langle q^2 \rangle = 90 \) for a single transfer.

6.4 Derivation of Bulk Trapping Noise

The noise due to bulk traps is similar to that due to surface traps, except the bulk traps are at discrete energies. Therefore Equation (6.22) becomes a summation.\(^{17}\)

\[
\langle q^2 \rangle = V_s \sum_i N_{\text{TTB}}^i \exp(-e_i^i/nB_c/p_f) (1 - \exp(-e_i^i/nB_c/p_f))
\]  

(6.35)

where \( V_s \) is the volume occupied by the signal, \( N_{\text{TTB}}^i \) is the density of the \( i \)-th type of trap, and \( e_i^i \) is the emission rate of the \( i \)-th type of trap.

For a single type of trap the noise peaks at \(.25V_sN_{\text{TTB}}^i \) when \( \exp(-e_i^i/nB_c/p_f) \)
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= 1/2, or \( f_c = e^{i \frac{n(2)}{p}} \). It can be seen that, for the discrete emission rates of the different bulk traps, the noise will peak at certain clock frequencies.

\( V_s \) is not a constant. For signals larger than about 10% of a full well \( V_s = N_s/N_D \), where \( N_D \) is the impurity density of the buried channel diffusion. \( V_s \) is, of course, limited to a maximum as the potential well fills.

It is important to note that, while the interface trapping noise is fairly constant, the noise due to bulk thermal generation and trapping is a function of both clock frequency and signal size. Varying the clock frequency will give maxima and minima, while varying the signal size will change the volume almost linearly, making the noise proportional to the signal.
VII. THE TRANSFER PROCESS

7.1 Transfer Losses

Whenever a charge transfer takes place in a CCD there are losses. These transfer losses are mostly due to charge which is trapped and not released in the time allotted for transfer. Transfer losses will distort the signal, but they are deterministic and not random noise. It is the fluctuations in these losses which produce noise.

In any good device, the fractional loss per transfer \((L)\) will be on the order of \(10^{-4}\) or less. For a reasonably large signal the accumulated losses over many transfers are the limiting factor on the maximum number of transfers. These losses attenuate both the signal and any noise present. However, the noise picked up by the charge packet near the output will not be attenuated as much as the noise picked up near the input, since the attenuation is roughly proportional to the number of transfers. Most devices are designed such that the product of the number of transfers, \(n\), and \(L\) is less than 0.1. Since noise suppression due to the losses becomes important only for \(nL < 1\), it is negligible.

7.2 Spectral Density of Transfer Process Noise

The CCD has a unique form of noise in that, during the transfer process, a fluctuation of \(\delta q\) in one packet causes a corresponding fluctuation of \(-\delta q\) in the following one, since charge must be conserved. No matter what the spectral density of the noise in an individual packet, that packet is only sampled once, so the spectral density of the output depends on the correlation (or lack of such) between different packets. The spectral density of the transfer noise was derived by Thornber and Tompsett. The noise spectrum is obtained by assuming a DC input signal and taking the Fourier transform of the output signal. Derived on the following pages, the spectral density of the transfer process noise is proportional to \(1-\cos(2\pi f/f_0)\).
In a CCD with \( p \) phases, there are \( p \) gates per cell, and one charge packet per cell. \( q^m_s,\mu \) is the notation for the transfer noise charge introduced during the \( \mu \)-th phase of the \( s \)-th transfer cycle as charge flows from the \( \mu-1 \) to \( \mu \)-th phase of the \( m \)-th cell. If there are \( N \) cells in the CCD, the total number of transfers is \( Np = n \). The point when the accumulated noise is of interest is in the last phase of the \( N \)-th cell, for it is there that the output sampling occurs. The noise output during the \( r \)-th transfer cycle is

\[
\Delta Q^N_r = \sum_{m=1}^{N} \sum_{\mu=1}^{p} \left( q^m_s,\mu - q^m_{s-(N-m)-1} \right)
\]

where the two terms represent the noise introduced in a given location and the negative of the noise left from the previous packet when it was in that location, respectively. The output current from the last cell is

\[
i(t) = \sum_r \Delta Q^N_r g(t-r t_0)
\]

where \( t_0 \) is the clock period and \( g \) is defined as:

\[
g(t-r t_0) = \begin{cases} 0 & \text{when } t < r t_0 \\ 1/b t_0 & \text{when } r t_0 < t < (r+b)t_0 \\ 0 & \text{when } t > (r+b)t_0 \end{cases}
\]

The charge is clocked out at the \( r \)-th clock cycle, beginning at time \( r t_0 \) and ending at time \( (r+b)t_0 \). Since the vast majority of the charge transfers quickly, \( b \) will be very small. (See Figure 6.)

The Fourier component of \( i(t) \) at frequency \( f_h = h/t \) is
Figure 6

Output Current Due to $g(t-r t_o)$
The noise current spectral density is

$$S_f(f_h) = \lim_{T \to \infty} \frac{2T}{a_h a_h^*}$$  \hspace{1cm} (7.5)

Substituting (7.2) into (7.4)

$$a_h = \frac{1}{T} \left[ \sum_{r=1}^{T} \Delta Q_r \frac{(r+b)t_0}{r t_0} \exp(j2\pi t h/T) dt \right] \hspace{1cm} (7.6)$$

It can be seen now that the upper limit of \( r \) must be \( T/t_0 \). Taking the integral inside the summation and substituting Equation (7.3)

$$a_h = \frac{1}{T} \left[ \sum_{r=1}^{T/t_0} \Delta Q_r \frac{(r+b)t_0}{r t_0} \exp(j2\pi t h/T) dt \right] \hspace{1cm} (7.7)$$

where it is assumed \( \Delta Q_r \) is independent of time. Following through with the integration

$$a_h = \frac{1}{T} \left[ \sum_{r=1}^{T/t_0} \Delta Q_r \frac{T}{j2\pi h t_0} \exp(j2\pi rt_0 h/T)(\exp(j2\pi h t_0 h/T)-1) \right] \hspace{1cm} (7.8)$$

Letting \( b \) get very small, \( \exp(j2\pi t h/T) \) expands as \( 1 + j2\pi t h/T \).

$$a_h = \frac{1}{T} \frac{T/t_0}{t_0} \sum_{r=1}^{T/t_0} \Delta Q_r \exp(j2\pi rt_0 h/T) \hspace{1cm} (7.9)$$

Substituting (7.9) into (7.5),

$$S_f(f_h) = 2T \left[ \frac{1}{T} \frac{T/t_0}{t_0} \sum_{r=1}^{T/t_0} \Delta Q_r \exp(j2\pi rt_0 h/T) \right] \frac{1}{T} \frac{T/t_0}{t_0} \sum_{r=1}^{T/t_0} \Delta Q_r \exp(-j2\pi t_0 h/T) \hspace{1cm} (7.10)$$
Substituting (7.1) into (7.10),

\[
S_i(f_h) = \lim_{T \to \infty} \frac{2}{T} \sum_{r=1}^{N-1} \sum_{m=1}^{P} \sum_{\mu=1}^{L} \left( q_{r-(N-m)}^{m,\mu} - q_{r-(N-m)-1}^{m,\mu} \right)
\]

\[
T/t_o \sum_{s=1}^{N} \sum_{k=1}^{P} \sum_{u=1}^{L} \left( q_{s-(N-k)}^{k,u} - q_{s-(N-k)-1}^{k,u} \right) \exp(j2\pi(r-s)t_o h/T)
\]

(7.11)

All noises are uncorrelated except those having the same phase in the same cell during the same transfer period.

\[
\langle q_{a,b,c} \rangle \langle q_{d,e,f} \rangle = \langle q^2 \rangle \delta_{a,d} \delta_{b,e} \delta_{c,f}
\]

(7.12)

This implies \( m=k \) and \( u=u \). Equation (7.11) becomes

\[
S_i(f_h) = \lim_{T \to \infty} \frac{2}{T} \sum_{r=1}^{N-1} \sum_{m=1}^{P} \sum_{\mu=1}^{L} \left( q_{r-(N-m)}^{m,\mu} - q_{r-(N-m)-1}^{m,\mu} \right)
\]

\[
T/t_o \sum_{s=1}^{N} \sum_{k=1}^{P} \sum_{u=1}^{L} \left( q_{s-(N-m)}^{m,\mu} - q_{s-(N-m)-1}^{m,\mu} \right) \exp(j2\pi(r-s)t_o h/T)
\]

(7.13)

The subscripts will be equal when

\[
\begin{align*}
& r-(N-m) = s-(N-m) \quad \text{or} \quad r=s \\
& r-(N-m) = s-(N-m)-1 \quad \text{or} \quad r=s-1 \\
& r-(N-m)-1 = s-(N-m) \quad \text{or} \quad r=s+1 \\
& r-(N-m)-1 = s-(N-m)-1 \quad \text{or} \quad r=s
\end{align*}
\]

(7.14)

Removing the summations over \( r \) and \( s \) by use of (7.14) gives

\[
S_i(f_h) = \lim_{T \to \infty} \frac{2}{T} \sum_{m=1}^{P} \sum_{\mu=1}^{L} \left( q^2 \right) \left( 2 - 2\cos(2\pi t_o h/T) \right)
\]

(7.15)

Going through the summation with every term equal,
\[
S_i(f_h) = \lim_{T \to \infty} \frac{2}{T} N_p \frac{T}{c_0} <q^2> 2(1 - \cos(2\pi \frac{h}{T}))
\]
(7.16)

\[N_p = n, \text{ the total number of transfers.}\]

\[1/c = f_c, \text{ the clock frequency.}\]

\[h/T = f_h, \text{ but when } T \to \infty f_h \to f, \text{ the spectrum becomes continuous, so}\]

\[S_i(f) = 4nf_c <q^2>(1 - \cos(2\pi f/f_c)) .
\]
(7.17)

This is the current spectral density. The charge spectral density is

\[S_q(f) = f^{-2} S_i(f) = \frac{4n <q^2>}{f_c} [1 - \cos(2\pi f/f_c)]
\]
(7.18)

The total mean square charge fluctuation can be obtained by integrating up to \(f_c/2\), which is the upper limit due to the sampling theorem.

\[<q^2>_{\text{total}} = \int_0^{f_c/2} \frac{4n <q^2>}{f_c} [1 - \cos(2\pi f/f_c)] df = 2n <q^2>
\]
(7.19)

This shows that the total mean square noise cannot be obtained by multiplying the noise per transfer \(<q^2>\) by the number of transfers. Each time transfer noise occurs it affects two charge packets, giving rise to twice the total noise expected.

The cause of transfer process noise is the trapping noise that takes place. The dark current, on the other hand, affects each packet separately and its spectral density can be calculated in a similar manner. For such a noise

\[\Delta Q^N = \sum_{m=1}^{N} \sum_{\mu=1}^{P} q^m_{\mu} \delta_{r-(N-m)\mu} \]
(7.20)
If this expression is substituted into Equation (7.10) instead of $\Delta Q^N$ for transfer noise, the result is

$$S(f_h) = \lim_{T \to \infty} \frac{2}{T} \sum_{m=1}^{N} \sum_{\mu=1}^{p} \sum_{r=1}^{T/T_0} \sum_{s=1}^{T/T_0} q_{r-s}(N-m) q_{s}(N-m) \exp(j \frac{2\pi}{T}(r-s)t_h/T)$$

(7.21)

This is nonzero only when

$$r-(N-m) = s-(N-m) \quad \text{or} \quad r=s \quad (7.22)$$

$$S(f_h) = \lim_{T \to \infty} \frac{2}{T} \sum_{m=1}^{N} \sum_{\mu=1}^{p} \sum_{r=1}^{T/T_0} q^2 = 2nf_c q^2 \quad (7.23)$$

$$S_q(f) = \frac{2n}{f_c} q^2 \quad (7.24)$$

This is white noise, as expected from uncorrelated events. The total noise is the integral of the spectral density, $S_q(f)$, over the bandwidth, $f_c/2$, giving

$$<q^2>_{\text{total}} = \frac{2n}{f_c} \frac{f}{2c} = n<q^2> \quad (7.25)$$

Again as expected, the total noise is just the noise during each transfer times the number of transfers.

There are several assumptions made in this section on transfer process noise which are summarized. (i) A constant input signal was assumed, neglecting any effects of a varying input signal on both the total noise and the spectral density. (ii) It was also assumed that each packet went through the same transfers and the same exact physical path. This would be the case in a linear CCD where the signal is electrically input at the first cell. However, for imaging applications, the input is parallel with each cell receiving its own signal. Thus, during imaging operations the part of the signal which starts...
out in a cell near the output would undergo fewer transfers than the packets starting far away from the output. Under those conditions, the first packets clocked out would have much less noise than later packets. There are also array type, for example, corner-turning CCDs, in which different parts of the signal travel separate paths. In that case local variations in dark current could result in variation of the noise between different physical paths.
VIII. TYPICAL NOISE VALUES

In order to compare the relative importance of the various noise sources, it is advantageous to evaluate the results of the preceding derivations by selecting representative values for the parameters and computing \( \langle q^2 \rangle \) for each source. Table 8.1 is a listing of the common values used in the calculations. Table 8.2 shows the noise per transfer and the noise at the output for each source.

All the independent noises add in quadrature. The total number of noise electrons per packet at the output is \( \left( \sum \langle q^2 \rangle \right)^{1/2} \). For an imaging application, typical RMS noise levels are (making use of CDS output and floating diffusion input for the fat zero):

- SCCD: 525 electrons
- BCCD: 325 electrons

For a non-imaging application, the fat zero and signal in a SCCD can be input together, thereby reducing the noise level somewhat. With single gate input and CDS output, typical values of RMS noise are:

- SCCD: 420 electrons
- BCCD: 90 electrons

The SCCD noise is dominated by the interface trap contribution. This noise is almost constant with respect to clock frequency and signal size, so external corrections cannot remove it. In the BCCD the trapping noise is not nearly so dominant, because there are very few bulk traps in comparison with the surface trap density. Furthermore, since there are discrete time constants for the bulk traps near which the noise peaks, the clock frequency can be set between the time constants, minimizing the noise. The BCCD also has the advantage that, unlike the SCCD, its trapping noise...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_s )</td>
<td>( 10^5 ) electrons</td>
</tr>
<tr>
<td>( N_B )</td>
<td>( 10^6 ) electrons</td>
</tr>
<tr>
<td>( N_{TTS} )</td>
<td>( 5 \times 10^9 \text{cm}^2\text{eV} )</td>
</tr>
<tr>
<td>( N_{TTO} / \beta )</td>
<td>( 5 \times 10^3 \text{cm}^2\text{eV} ) for transfer</td>
</tr>
<tr>
<td>( n )</td>
<td>1000 transfers</td>
</tr>
<tr>
<td>( f_c )</td>
<td>( 10^6 ) Hz</td>
</tr>
<tr>
<td>( p )</td>
<td>2 phases</td>
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<tr>
<td>( C_{in} )</td>
<td>( 3 \times 10^{-14} ) F for single gate</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>( 10^{-13} ) F</td>
</tr>
<tr>
<td>( A )</td>
<td>( 10^{-8} ) cm²</td>
</tr>
<tr>
<td>( g_o )</td>
<td>( 10^{-3} ) U</td>
</tr>
<tr>
<td>( i_n )</td>
<td>( 3 \times 10^{-3} ) A</td>
</tr>
<tr>
<td>( \tau_s )</td>
<td>( 10^{-6} ) s</td>
</tr>
<tr>
<td>( T_i )</td>
<td>( 10^{-3} ) s</td>
</tr>
<tr>
<td>( J_D )</td>
<td>( 5 \times 10^{-9} ) A/cm² for SCCD</td>
</tr>
<tr>
<td>( V_s )</td>
<td>( 5 \times 10^{-11} ) cm³</td>
</tr>
<tr>
<td>( T )</td>
<td>300 K</td>
</tr>
</tbody>
</table>

\[
\frac{\sum_{i}^{\infty} N_{TTB} \exp\left(-\frac{e_i}{n_B/p_f}\right)(1-\exp\left(-\frac{-e_i}{n_B/p_f}\right))}{N_{TTB}} = N_{TTB}^4
\]
<table>
<thead>
<tr>
<th>NOISE SOURCE</th>
<th>$\langle q^2 \rangle$ (For a Single Transfer)</th>
<th>$\langle q^2 \rangle_{\text{total}}$ (At the Output)</th>
<th>Spectral Distribution</th>
<th>Numerical $\langle q^2 \rangle_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>SCCE</td>
</tr>
<tr>
<td>fixed zero input</td>
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<tr>
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<td>$\frac{2}{3} e^{-\Delta C_{\text{in}}} \left( N_e - \frac{2}{3} e^{-\Delta C_{\text{in}}} \right)$</td>
<td>$N_e$</td>
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<td>3125</td>
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<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>Floating Diffusion</td>
<td>$\frac{2}{3} e^{-\Delta C_{\text{in}}} \left( N_e - \frac{2}{3} e^{-\Delta C_{\text{in}}} \right)$</td>
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<td>2080</td>
</tr>
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<td></td>
<td>$N_e - \frac{2}{3} e^{-\Delta C_{\text{in}}}$</td>
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<td></td>
</tr>
<tr>
<td>single input</td>
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<tr>
<td>Optical</td>
<td>$N_e$</td>
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<td>White</td>
<td>$10^3$</td>
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<tr>
<td>Single Gate</td>
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<td>White</td>
<td>3125</td>
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<td>$N_e - \frac{2}{3} e^{-\Delta C_{\text{in}}}$</td>
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<tr>
<td>Floating Diffusion</td>
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<td>$2N_e$</td>
<td>White</td>
<td>2080</td>
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<td>$N_e - \frac{2}{3} e^{-\Delta C_{\text{in}}}$</td>
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<td>output</td>
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</tr>
<tr>
<td>Without CDS</td>
<td>$\frac{2}{3} e^{-\Delta C_{\text{in}}} + \frac{1}{2} e^{-\Delta C_{\text{in}}}$</td>
<td>$2 \left( \frac{N_e}{N_0} \right)$</td>
<td>White + $1/f$</td>
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</tr>
<tr>
<td>With CDS</td>
<td>$\frac{1}{2} e^{-\Delta C_{\text{in}}}$</td>
<td>$2 \left( \frac{N_e}{N_0} \right)$</td>
<td>White + $1/f$</td>
<td>7</td>
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<tr>
<td>Transfer</td>
<td>$\frac{pA}{V^2}$</td>
<td></td>
<td>White</td>
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<td>1-cos($2\pi f_{\text{f}}$)</td>
<td>1.73 x $10^3$</td>
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<td>$\frac{N_e^2}{2}$ A T / e V^{1/2}</td>
<td>$2n = qV$</td>
<td>1-cos($2\pi f_{\text{f}}$)</td>
<td>5000</td>
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</tbody>
</table>
will decrease as the signal level drops.

The dynamic range of a device is determined by the maximum and minimum signal levels. It is obvious from the numerical results that the BCCD can handle a smaller signal than the SCCD. This difference is even greater than Table 8.2 would imply. In the case of imaging, for example, when the signal level drops the total noise drops, because the imaging shot noise is reduced. Since this is the most important noise source in a BCCD, the total output noise of the device will drop drastically as the signal level decreases. The SCCD noise is dominated by the interface trapping so the reduced imaging noise will have a much smaller effect on the total.

The SCCD does have the advantage of a larger signal-handling capacity (by a factor of two or three). That is because the BCCD signal charge is stored away from the interface resulting in a smaller effective capacitance. This factor partially offsets the noise level difference, and so both the SCCD and BCCD have approximately the same dynamic range of 90 to 100 db.\(^1\)\(^8\)

The SCCD has the potential for a larger dynamic range than the BCCD if the interface state density can be reduced. Processing improvements could conceivably reduce \(<q^2>\) due to interface trapping noise by an order or magnitude from the results in Table 8.2.

Except for the shot noise in imaging, all noise can be reduced by lowering the temperature. In all cases but one, \(<q^2>\) is proportional to \(T\). The unique case is trapping noise. For that process temperature enters through the emission rates \(e^{i}_{nB} = a_i \exp(-E_i/kT)\) of the traps. If the device is cooled enough so that \(e^{i}_{nB} < \tau_c\) then \(<q^2>\) will be proportional to the maximum rate \(e^{i}_{nB}^{max}\). This can decrease very rapidly with decrease-
ing temperature.

Since the spectral density of the trapping noise, as seen at the output, is proportional to \(1-\cos(2\pi f/f_c)\), much of it can be removed by band limiting the signal. Since the noise is concentrated at the high frequency end, filtering can give noise improvement in excess of the bandwidth reduction. For example, dropping the upper frequency limit from \(f_c/2\) to \(3f_c/8\) (a reduction of 25%) reduces \(\langle q^2 \rangle\) due to trapping by almost 50%.
IX. CONCLUSIONS

The Charge Coupled Device has a low noise level and a wide dynamic range. The SCCD is noisier and has a slightly smaller dynamic range than the BCCD. However, the SCCD can accommodate a larger signal and so has the potential for greater dynamic range than the BCCD. The major source of noise in the SCCD is the interface trapping noise. Processing improvements which reduce the interface trap density could reduce this noise and boost the performance of the SCCD.

What have been treated here are the fundamental macroscopic noise sources in a CCD. There are other sources of noise which have been ignored, including dependence of noise on the shape of the input signal, non-uniform dark current, and non-uniform gate capacitances.

The currents, voltages, and conductances in a CCD vary rapidly with time and hence introduce nonstationary noise. A method of treating nonstationary noise by an extension of the Langevin method was outlined by Thornber. That procedure is quite complicated and has not been carried out by anyone.

For simplicity, the derivations here have assumed a constant signal level. This allowed simple expressions whereby the relative importance of the different noise sources could be easily shown. Because the noise is a function of the signal, its spectral distribution could not be purely white and \(1 - \cos(2\pi f f_c)\), nor will the output noise level be constant. The noise due to a given signal would have to be computed by a method similar to the transfer process noise of chapter 7. This requires extensive calculations for each and many signal spectra.
APPENDIX A

DEFINITION OF TERMS

$q$ Number of noise electrons

$\langle q^2 \rangle$ Mean square number of noise electrons

$N_s$ Number of electrons in a signal packet

$N_B$ Number of electrons in a fat zero

$k$ Boltzmann's constant

$T$ Temperature

$p$ Number of clock phases. Also equal to the number of gates per cell

$f_c$ Clock frequency

$C_{in}$ Input capacitance

$e$ Magnitude of electron charge

$C_o$ Output node capacitance

$g_o$ Transconductance of the output MOSFET

$\langle i_n^2 \rangle$ Mean square noise current of the output MOSFET channel

$A$ Area of one CCD gate

$J_D$ Dark current density

$T_i$ Imaging time

$e_{no}$ Electron emission rate of the oxide traps; equal to $\alpha \exp(\beta x)$ where $x$ is the distance into the oxide from the oxide-semiconductor interface

$\alpha$ A constant in $e_{no}$

$\beta$ A constant in $e_{no}$

$N_{TTO}$ Oxide trap density per eV

$\tau_s$ Shortest oxide trap time constant (corresponding to $x=0$) approximately equal to $N_{TTS}^\alpha$

$N_{TTS}$ Interface trap density per eV
\[ \begin{align*}
\& \text{Oxide thickness} \\
\& \text{a} \quad \text{Constant from } e_{\text{ns}} \\
\& e_{\text{ns}} \quad \text{Interface trap electron emission rate; equal to } a \exp(-E/kT) \\
\& k_{\text{ns}} \quad \text{Interface trap electron capture rate in two dimensions} \\
\& t_{\text{fill}} \quad \text{Filling time constant of the interface traps; equal to } A/c_{\text{ns}} N_s \\
\& E_0 \quad \text{Energy, measured down from the conduction band, to which the interface traps fill} \\
\& V_s \quad \text{Volume occupied by a charge packet in a BCCD} \\
\& \eta_{\text{ITB}}^i \quad \text{Density of the } i\text{-th type of bulk trap} \\
\& e_{\text{nb}}^i \quad \text{Electron emission rate of the } i\text{-th type of bulk trap} \\
\& N_D \quad \text{Impurity density of the buried channel} \\
\& n \quad \text{Total number of transfers} \\
\& L \quad \text{Fractional loss per transfer} \\
\& T_0 \quad \text{Clock period} \\
\& S_i(f) \quad \text{One-sided current spectral density, (number of electrons/second)}^2/\text{Hz} \\
\& S_q(f) \quad \text{One-sided charge spectral density, (number of electrons)}^2/\text{Hz} 
\end{align*} \]
LIST OF REFERENCES


