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Technical Rep. No. 53
Civil Engng. Res. Rep. ~~No.~~ 78-SM-8

LINE CRACK SUBJECT TO ANTIPLANE SHEAR

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Research Sponsored by the
Office of Naval Research
under
Contract N00014-76-C-0240
Modification No. P00002

Technical rept.

Jul 78

July 1978

Approved for public release; Distribution Unlimited

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LINE CRACK SUBJECT TO ANTIPLANE SHEAR[1]

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ABSTRACT

Field equations of nonlocal elasticity are solved to determine the state of stress in a plate with a line crack subject to a constant antiplane shear. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip. By equating the maximum shear stress that occurs at the crack tip to the shear stress that is necessary to break the atomic bonds, the critical value of the applied shear is obtained for the initiation of fracture. If the concept of the surface tension is used, one is able to calculate the cohesive stress for brittle materials.

1. INTRODUCTION

In several previous papers [1] - [4] we discussed the state of stress near the tip of a sharp line crack in an elastic plate subject to uniform tension and in-plane shear. The field equations employed in the solution of these problems are those of the theory of nonlocal elasticity. The solutions obtained did not contain any stress singularity, thus resolving a fundamental problem that persisted over half a century. This enabled us to employ the maximum-stress hypothesis to deal with fracture problems in a natural way. Moreover it has been possible to predict the atomic cohesive stresses by introducing the experimental values of the surface energy.

[1]The present work was supported by the Office of Naval Research

The present paper deals with the problem of a line crack in an elastic plate where the crack surface is subject to a uniform anti-plane shear load. This problem, classically, is known as the Mode III displacement. We employ the field equations of nonlocal elasticity theory to formulate and solve this problem. The solution, as expected, does not contain the stress singularity at the crack tip and therefore a fracture criterion based on the maximum shear stress hypothesis can be used to obtain the critical value of the applied shear for which the line crack begins to become unstable. If the concept of the surface energy is introduced, it is possible to calculate the cohesive stress holding the atomic bonds together. Estimates of cohesive stress are also given for perfect crystalline solids.

In section 2 we present a resumé of basic equations of linear nonlocal elastic solids. In section 3 the boundary-value problem is formulated, and the general solution is obtained. In section 4 we give the solution of the dual integral equations, completing the solution. Calculations for the shear stress are carried out in a computer, and the results are discussed in section 5.

2. BASIC EQUATIONS OF NONLOCAL ELASTICITY

Basic equations of linear, homogeneous, isotropic, nonlocal elastic solids, with vanishing body and inertia forces, are (cf. [5]):

$$t_{k\ell,k} = 0 \tag{2.1}$$

$$t_{k\ell} = \int_V [\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)e_{rr}(\underset{\sim}{x}')\delta_{k\ell} + 2\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)e_{k\ell}(\underset{\sim}{x}')]dv(\underset{\sim}{x}') \tag{2.2}$$

$$e_{k\ell} = \tfrac{1}{2}(u_{k,\ell}+u_{\ell,k}) \tag{2.3}$$

where the only difference from classical elasticity is in the stress constitutive equations (2.2) in which the stress $t_{k\ell}(\underset{\sim}{x})$ at a point $\underset{\sim}{x}$ depends on the strains $e_{k\ell}(\underset{\sim}{x}')$, at all points of the body. For homogeneous and isotropic solids there exist only two material moduli, $\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and $\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ which are functions of the distance $|\underset{\sim}{x}'-\underset{\sim}{x}|$. The integral in (2.2) is over the volume V of the body enclosed within a surface ∂V.

Throughout this paper we employ cartesian coordinates x_k with the usual convention that a free index takes the values (1, 2, 3), and repeated indices are summed over the range (1, 2, 3). Indices following a comma represent partial differentiation, e.g.

$$u_{k,\ell} = \partial u_k / \partial x_\ell$$

In our previous work [6, 7] we obtained the forms of $\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and $\mu'(|\underset{\sim}{x}-\underset{\sim}{x}|)$ for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found very useful

$$(2.4)\quad \begin{aligned} (\lambda',\mu') &= (\lambda,\mu)\alpha(|\underset{\sim}{x}-\underset{\sim}{x}|) \quad , \\ \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) &= \alpha_0 \exp[-(\beta/a)^2(\underset{\sim}{x}'-\underset{\sim}{x})\cdot(\underset{\sim}{x}'-\underset{\sim}{x})] \ , \end{aligned}$$

where β is a constant, a is the lattice parameter, and α_0 is determined by the normalization

$$(2.5)\quad \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)dv(\underset{\sim}{x}') = 1 \ .$$

In the present work we employ the nonlocal elastic moduli given by $(2.4)_2$. Carrying $(2.4)_2$ into (2.5) we obtain

$$\alpha_o = \frac{1}{\pi} (\beta/a)^2 \quad . \tag{2.6}$$

Substituting $(2.4)_1$ into (2.2) we write

$$t_{k\ell} = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\ \sigma_{k\ell}(\underset{\sim}{x}')\ dv(\underset{\sim}{x}') \ , \tag{2.7}$$

where

$$\begin{aligned} \sigma_{k\ell}(\underset{\sim}{x}') &= \lambda\ e_{rr}(\underset{\sim}{x}')\ \delta_{k\ell} + 2\mu\ e_{k\ell}(\underset{\sim}{x}') \\ &= \lambda\ u_{r,r}(\underset{\sim}{x}')\ \delta_{k\ell} + \mu\,[u_{k,\ell}(\underset{\sim}{x}') + u_{\ell,k}(\underset{\sim}{x}')] \end{aligned} \tag{2.8}$$

is the classical Hooke's law. Substituting (2.8) into (2.1) and using Green-Gauss theorem we obtain:

$$\int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\ \sigma_{k\ell,k}(\underset{\sim}{x}')\ dv(\underset{\sim}{x}') - \oint_{\partial V} \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\ \sigma_{k\ell}(\underset{\sim}{x}')da_k(\underset{\sim}{x}') = 0 . \tag{2.9}$$

The contribution to the surface integral from the parts of the surface at infinity would be dropped since the displacement field vanishes at infinity.

3. CRACK UNDER ANTI-PLANE SHEAR

We consider an elastic plate in the $(x_1=x,\ x_2=y)$ - plane weakened by a line crack of length 2ℓ along the x-axis. The plate is subjected to a constant anti-plane shear stress $t_{yz}=\tau_o$ along the surfaces of the crack, Fig. 1. For this problem we have

$$(3.1)\qquad u_1=u_2=0 \quad , \quad u_3=w(x,y) \quad ,$$

$$(3.2)\qquad \sigma_{xz}=\mu\frac{\partial w}{\partial x} \quad , \quad \sigma_{yz}=\mu\frac{\partial w}{\partial y} \quad , \text{ all other } \sigma_{k\ell}=0 \quad ,$$

so that the only surviving member of the field equations (2.9) is

$$(3.3)\quad \mu \int\!\!\!\!\!- \alpha(|x'-x|,|y'-y|)\nabla'^2 w(x',y')dx'dy' - \int_{\ell}^{\ell} \alpha(|x'-x|,|y|)\cdot [\underset{\sim}{\sigma}_{yz}(x',0)\underset{\sim}{]}dx'=0 \quad ,$$

where the integral with a slash is over the two-dimensional infinite space excluding the line of the crack $(|x|<\ell,\ y=0)$. A boldface bracket indicates a jump at the crack line.

When an undeformed and unstressed body is sliced to create a free surface, it will in general be deformed and stressed on account of the long-range interatomic forces. Thus if we are to consider that the plate with a crack is undeformed and unstressed in its natural state then we must apply the boundary conditions on the unopened crack surface.

Under the applied anti-plane shear load on the unopened surfaces of the crack, the displacement field possesses the following symmetry regulations

$$(3.4)\qquad w(x,-y) = -w(x,y) \ .$$

Using this in (3.2) we find that

$$[\sigma_{yz}(x,0)] = 0 \quad . \tag{3.5}$$

Hence the line integral in (3.3) vanishes. By taking the Fourier transform of (3.3) with respect to x', we can show that the general solution of (3.3) is identical to that of

$$\frac{d^2\bar{w}(\xi,y)}{dy^2} - \xi^2\bar{w}(\xi,y) = 0 \quad , \tag{3.6}$$

almost everywhere. Here a superposed bar indicates the Fourier transform e.g.

$$\bar{f}(\xi,y) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x,y) \exp\,(i\xi x)\, dx \quad .$$

The boundary conditions are

$$\begin{aligned} &w(x,0) = 0 && \text{for} && |x|>\ell \quad , \\ &t_{yz}(x,0) = \tau_0 && \text{for} && |x|<\ell \quad , \\ &w(x,y) = 0 && \text{as} && (x^2+y^2)^{1/2}\to\infty \quad . \end{aligned} \tag{3.7}$$

The general solution of (3.6) (for $y\geq0$) satisfying $(3.7)_3$ is

$$w(x,y) = (2/\pi)^{\frac{1}{2}} \int_0^{\infty} A(\xi)\, e^{-\xi y} \cos\,(\xi x)\, d\xi \quad , \tag{3.8}$$

where $A(\xi)$ is to be determined from the remaining two boundary conditions. For the non-zero components of the stress tensor we have

$$t_{xz} = -(2/\pi)^{\frac{1}{2}}\,\mu \int_0^\infty A(\xi)\,\xi d\xi \int_0^\infty dy' \int_{-\infty}^\infty [\,\alpha(|x'-x|,|y'-y|)$$

$$-\alpha(|x'-x|,|y'+y|)]\; e^{-\xi y'}\;\sin\,(\xi x')\;dx'\,,$$

(3.9)

$$t_{yz} = -(2/\pi)^{\frac{1}{2}}\,\mu \int_0^\infty A(\xi)\,\xi d\xi \int_0^\infty dy' \int_{-\infty}^\infty [\,\alpha(|x'-x|,|y'-y|)$$

$$+\alpha(|x'-x|,|y'+y|)]\; e^{-\xi y'}\;\cos\,(\xi x')\;dx'.$$

Using $(2.4)_2$ for $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)$, we carry out integrations on x' and y'. To this end we note the following integrals, [8]:

$$I_1 = \int_{-\infty}^\infty \exp(-px'^2)\begin{Bmatrix}\sin\xi(x'+x)\\ \cos\xi(x'+x)\end{Bmatrix}dx' = (\pi/p)^{\frac{1}{2}}\exp(-\xi^2/4p)\begin{Bmatrix}\sin(\xi x)\\ \cos(\xi x)\end{Bmatrix},$$

(3.10) $$I_2 = \int_0^\infty \exp(-py'^2-\gamma y')dy' = \tfrac{1}{2}(\pi/p)^{\frac{1}{2}}\exp(\gamma^2/4p)[1-\Phi(\gamma/2\sqrt{p})\,,$$

$$\Phi(z) \equiv 2\pi^{-\frac{1}{2}}\int_0^z \exp(-t^2)\,dt\;.$$

Hence

$$t_{xz} = -(2\pi)^{-\frac{1}{2}}\,\mu\int_0^\infty \xi A(\xi)\,[e^{-\xi y}\,\mathrm{erfc}\left(\frac{\xi-2py}{2\sqrt{p}}\right) - e^{\xi y}\,\mathrm{erfc}\left(\frac{\xi+2py}{2\sqrt{p}}\right)]\sin(\xi x)d\xi\,,$$

(3.11) $$t_{yz} = -(2\pi)^{-\frac{1}{2}}\,\mu\int_0^\infty \xi A(\xi)\,[e^{-\xi y}\,\mathrm{erfc}\left(\frac{\xi-2py}{2\sqrt{p}}\right) + e^{\xi y}\,\mathrm{erfc}\left(\frac{\xi+2py}{2\sqrt{p}}\right)]\cos(\xi x)d\xi\,,$$

$$p \equiv (\beta/a)^2 \quad,\qquad \mathrm{erfc}(z) = 1-\Phi(z)\;.$$

The boundary conditions $(3.7)_1$ and $(3.7)_2$ now read

$$\int_0^\infty \zeta^{\frac{1}{2}} C(\zeta)\, K(\varepsilon\zeta)\, \cos(z\zeta)\, d\zeta = -(\pi/2)^{\frac{1}{2}}\, T_o \quad , \qquad 0<z<1 \quad ,$$

(3.12)

$$\int_0^\infty \zeta^{-\frac{1}{2}} C(\zeta)\, \cos(z\zeta)\, d\zeta = 0 \quad , \qquad z>1 \quad ,$$

where we set

$$z = x/\ell \;, \qquad \zeta = \xi\ell \;, \qquad \varepsilon = a/2\beta\ell \;,$$

(3.13) $$K(\varepsilon\zeta) = \mathrm{erfc}\,(\varepsilon\zeta) \quad ,$$

$$A(\xi) = \zeta^{-\frac{1}{2}}\, C(\zeta) \;, \quad T_o = \tau_o \ell^2/\mu \quad .$$

To determine the unknown function $A(\xi)$, we must solve the dual integral equations (3.12).

4. THE SOLUTION OF THE DUAL INTEGRAL EQUATIONS

Recalling the expression

$$\cos\,(z\zeta) = (\pi z\zeta/2)^{\frac{1}{2}}\, J_{-\frac{1}{2}}\,(z\zeta) \;,$$

where $J_\nu(z)$ is the Bessel function of order ν, we write the system (3.12) in the form

$$\int_0^\infty \zeta C(\zeta)[1+k(\varepsilon\zeta)] J_{-\frac{1}{2}}(z\zeta) d\zeta = -T_o z^{-\frac{1}{2}} \;, \quad 0<z<1 \quad ,$$

(4.1)

$$\int_0^\infty C(\zeta)\, J_{-\frac{1}{2}}(z\zeta)\, d\zeta = 0 \quad , \quad z>1 \quad .$$

The kernel function $k(\varepsilon\zeta)$ is given by

$$(4.2) \qquad k(\varepsilon\zeta) = K(\varepsilon\zeta) - 1 = -\Phi(\varepsilon\zeta) .$$

The solution of the dual integral equations (4.1) is not known. However, it is possible to reduce the problem to the solution of a Fredholm equation (cf. [9])

$$(4.3) \qquad h(x) + \int_0^1 h(u)L(x,u)du = -\tfrac{1}{2}(\pi x)^{\frac{1}{2}}T_0 ,$$

for the function $h(x)$, where

$$(4.4) \qquad L(x,u) = (xu)^{\frac{1}{2}}\int_0^\infty tk(\varepsilon t)J_0(xt)J_0(ut)dt .$$

When (4.3) is solved, then $C(\zeta)$ is calculated by

$$(4.5) \qquad C(\zeta) = (2\zeta)^{\frac{1}{2}}\int_0^1 x^{\frac{1}{2}}J_0(\zeta x)\, h(x)\, dx .$$

As discussed in a previous work [4], if we note that ε is extremely small, $k(\varepsilon\zeta)$ may be neglected as compared to unity in (4.1), (see Fig. 2). In this case the zeroth order solution of (4.3) namely $h_0(x) = -T_0(\pi x)^{\frac{1}{2}}/2$ suffices for the calculations when the crack size is larger than 100 atomic distances. In such a case we have

$$(4.6) \qquad C_0(\zeta) = -(\pi/2)^{\frac{1}{2}}T_0\zeta^{-\frac{1}{2}}J_1(\zeta) ,$$

and therefore

$$(4.7)\qquad A_o(\xi) = -(\pi/2)^{\frac{1}{2}}\tau_o J_1(\xi\ell)/\xi\ell$$

The shear stresses are then calculated by (3.11). Interesting among these is the shear stress t_{yz} along the crack line $y = 0$. For this we obtain

$$(4.8)\qquad t_{yz}(z,0)/\tau_o = \int_0^\infty K(\epsilon\zeta)\, J_1(\zeta)\, \cos(z\zeta)\, d\zeta$$

As observed before, this integral converges for all z provided $K(\epsilon\zeta)$ is not approximated by unity for ϵ small. For $\epsilon = 0$ at $z = 1$ we have the classical stress singularity. However, so long as $\epsilon \neq 0$, (4.8) gives a finite stress all along $y = 0$. At $0<z<1$, t_{yz}/τ_o is very close to unity, and for $z>1$, t_{yz}/τ_o possesses finite values diminishing from a maximum value at $z=1$ to zero at $z=\infty$.

For $\epsilon >> 1/100$ the approximate solution given by (4.7) is not very good. However, further improvements can be achieved by the iterative solution of (4.3) with the use of $C_o(\zeta)$. Since $\epsilon > 1/100$ represents a crack length of less than 10^{-6}cm, and at such submicroscopic sizes other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes.

5. NUMERICAL CALCULATIONS AND DISCUSSION.

Calculations of the shear stress t_{yz}, given by (4.8) along the crack line, were carried out on a computer. The results are plotted for $\epsilon = 1/20$, $1/50$, $1/100$, $1/200$, in Figures 3 to 6. For a crack length of 20 atomic distances ($\epsilon \simeq 1/20$) the result is not very good in that the boundary condition at $|x|<\ell$, $y = 0$ is satisfied only very roughly. However, for a crack size of 100 atomic distances (Fig. 5) the shear stress boundary condition is fulfilled in a strong approximate sense. The relative error in this case is less than $1\frac{1}{2}$ °/o. Hence we conclude that the classical $A_o(\xi)$ given by (4.7) gives satisfactory results for crack lengths greater than 100 atomic distances.

The stress concentration occurs at the crack tip, and this is given by

$$(5.1) \qquad t_{yz}(\ell,0)/t_o = c_3/\sqrt{\varepsilon} \quad , \qquad \varepsilon \equiv a/2\beta\ell \quad ,$$

where c_3 converges to about

$$(5.2) \qquad c_3 \simeq 0.40 \ .$$

The following observations are very significant:

(i) The maximum shear stress occurs at the crack tip, and it is <u>finite</u> (eq. 5.1)

(ii) The shear stress at the crack tip becomes infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity.

(iii) <u>When</u> $t_{yz}(\ell,0) = t_c$ (= <u>cohesive shear stress</u>), <u>the plate will fail</u>. In this case

$$(5.3) \qquad \tau_o^2 \ell = C_G$$

where

$$(5.4) \qquad C_G = (a/2\beta c_3^2) t_c^{\,2}$$

Equation (5.3) is the expression of the Griffith fracture criterion for brittle fracture. We have arrived at this result via the maximum shear-stress hypothesis, rather than the surface energy concept used by Griffith and his followers. The significance of this result is that the fracture criteria are unified at both the macroscopic and the microscopic scales and that the natural concept of bond failure is employed.

(iv) The cohesive shear stress t_c may be estimated if one employs the Griffith's definition of the surface energy γ and writes

$$t_c^{2} a = K_c \gamma \quad , \tag{5.5}$$

where

$$K_c = 8\mu c_3^2 \beta / \pi(1-\nu) \tag{5.6}$$

Since some measurements exist on γ, by employing these values we can calculate the cohesive shear stress. For steel we have

$$\gamma = 1975 \text{ CGS} \quad , \quad \mu = 6.92 \times 10^{11} \text{ CGS}$$

$$\nu = 0.291 \quad , \quad a = 2.48 \text{ A}^\circ$$

$$t_c/\mu = 0.2568\ \beta^{1/2} \tag{5.7}$$

Let at a n atomic distance the nonlocal effects attenuate to 1% of its value at $x=0$. Using (2.4) we find that

(5.8) $\beta = 2.146/n$,

(5.9) $t_c/\mu = 0.3764\ n^{-1/2}$.

For n = 6 this gives

(5.10) $t_c/\mu = 0.14$

which is in the right range and well accepted by metallurgists. For example Kelly [10] gives $t_c/\mu = 0.11$. There is however a question of in-plane versus antiplane shear failure which need special examination. Compared to the results obtained in our previous work on the in-plane shear [4], the cohesive stress seems 30% higher. However, this is somewhat artificial since it is necessary to know the value of β or n in either case more precisely. This of course requires at least one experiment.

Acknowledgement: The author is indebted to Dr. Balta for carrying out computations.

REFERENCES

[1] A. C. Eringen and B. S. Kim, "Stress Concentration at the Tip of Crack," Mech. Res. Comm. 1, 233-237, 1974.

[2] A. C. Eringen, "State of Stress in the Neighborhood of a Sharp Crack Tip," Trans. of the twenty-second conference of Army mathematicians, 1-18, 1977.

[3] A. C. Eringen, C. G. Speziale and B. S. Kim, "Crack Tip Problem in Nonlocal Elasticity", J. Mech. and Phys. of Solids, 25, 339-355, 1977.

[4] A. C. Eringen, "Line Crack Subject to Shear", to appear in Int. J. Fracture.

[5] A. C. Eringen, "Linear Theory of Nonlocal Elasticity and Dispersion of Plane Waves", Int. J. Engng. Sci. 10, 233-248, 1972.

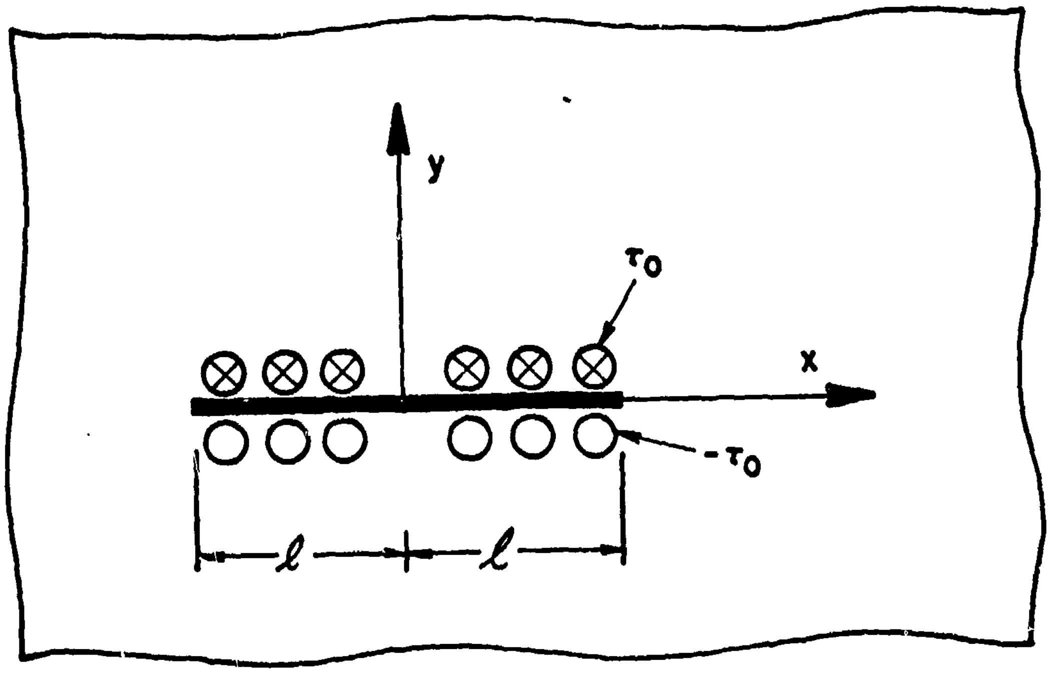
[6] A. C. Eringen, "Nonlocal Elasticity and Waves," Continuum Mechanics Aspects of Geodynamics and Rock Fracture Mechanics, (ed. P. Thoft-Christensen) Dordrecht, Holland, D. Reidel Publishing Co., pp. 81-105, 1974

[7] A. C. Eringen, "Continuum Mechanics at the Atomic Scale", Cryst. Lattice Defects, 7, pp. 109-130, 1977.

[8] I. S. Gradshteyn and I. W. Rhyzhik, "Tables of Integrals, Series and Products", New York; Acad. Press, 1965, pp. 480, 307, 338.

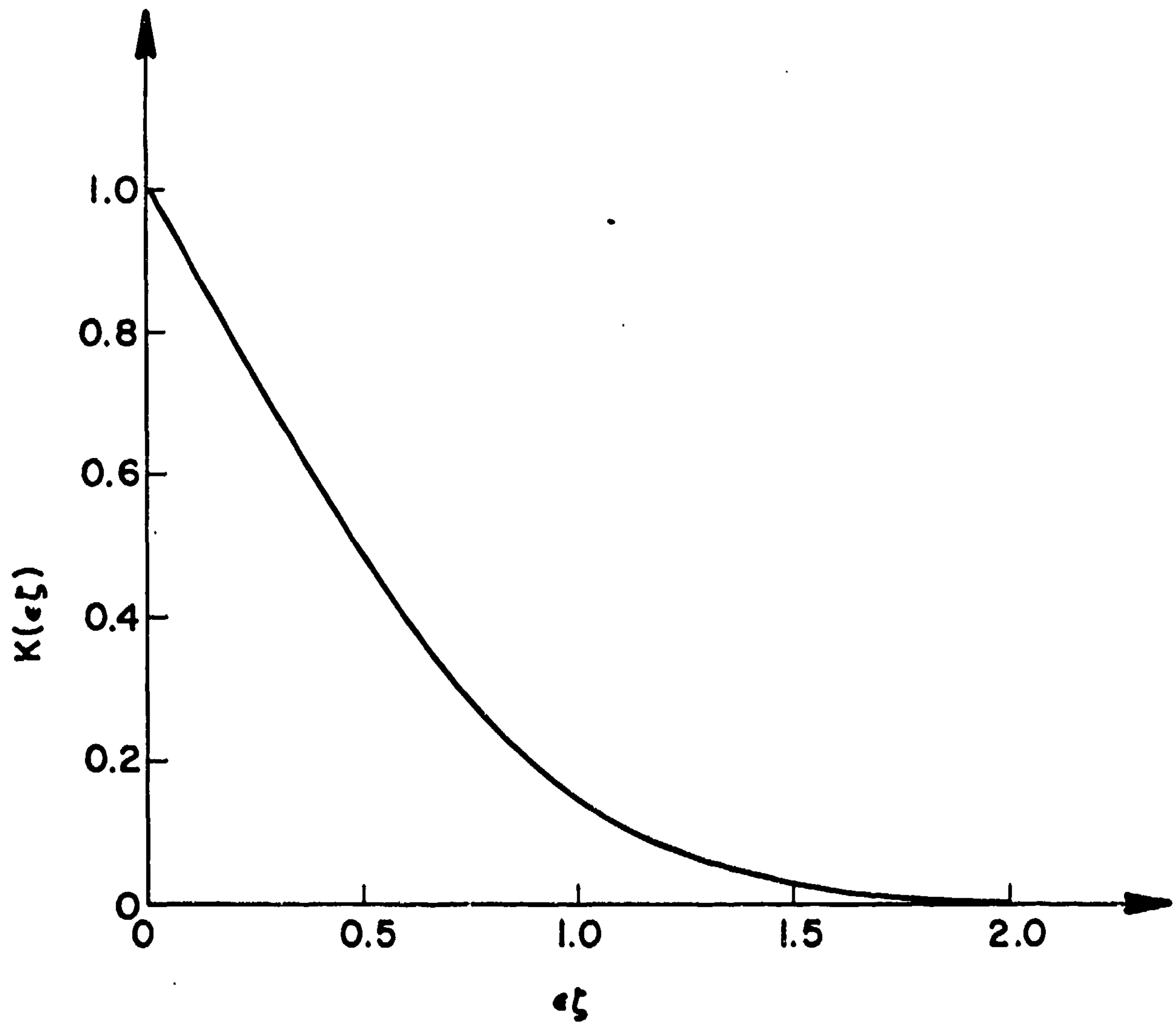
[9] I.N. Sneddon, "Mixed Boundary Value Problems in Potential Theory", North Holland Publishing Co., Amsterdam, 1966, pp. 106-108.

[10] A. Kelley, "Strong Solids", Oxford, 1966, p. 19.



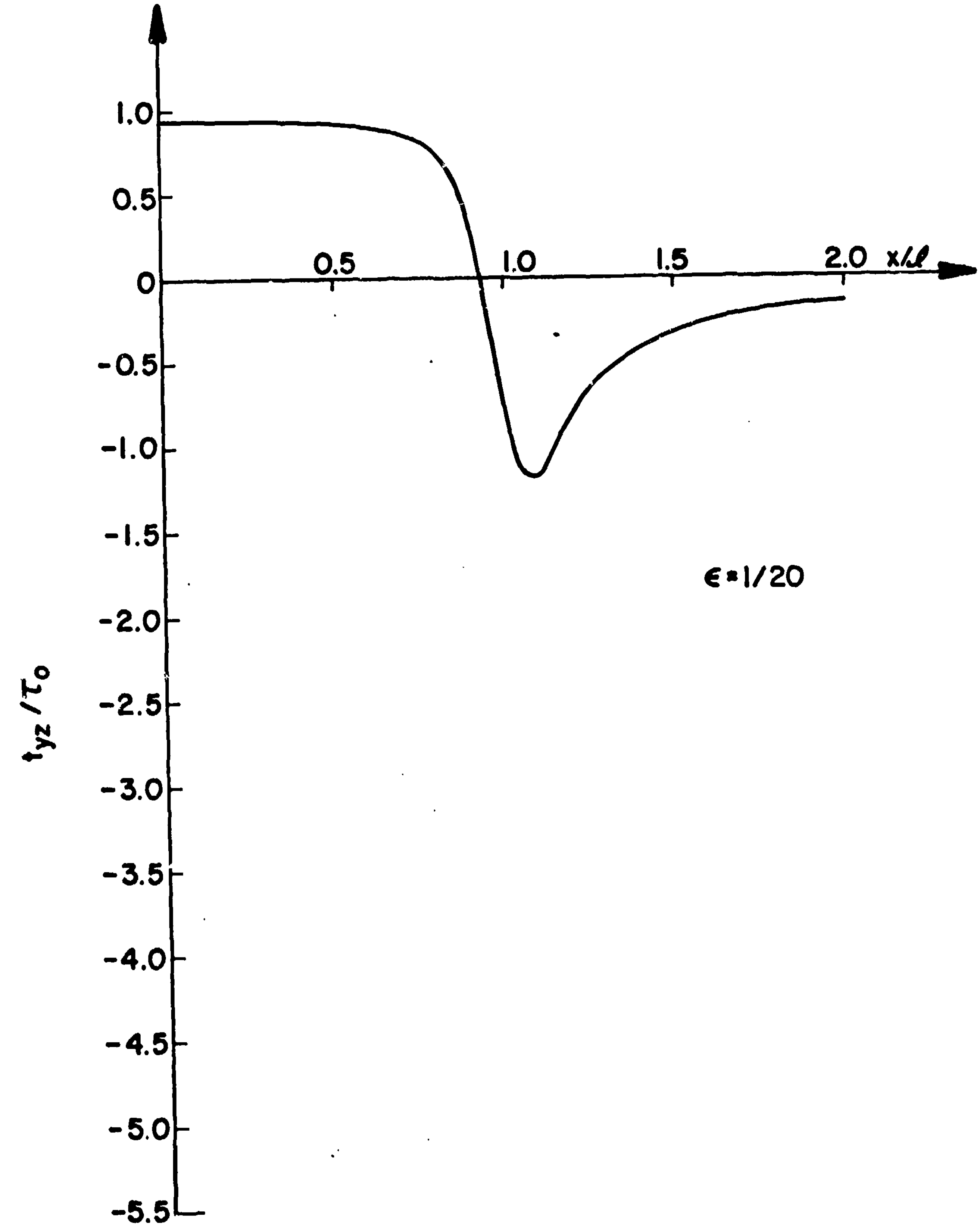
LINE CRACK SUBJECT TO ANTIPLANE SHEAR

FIGURE 1



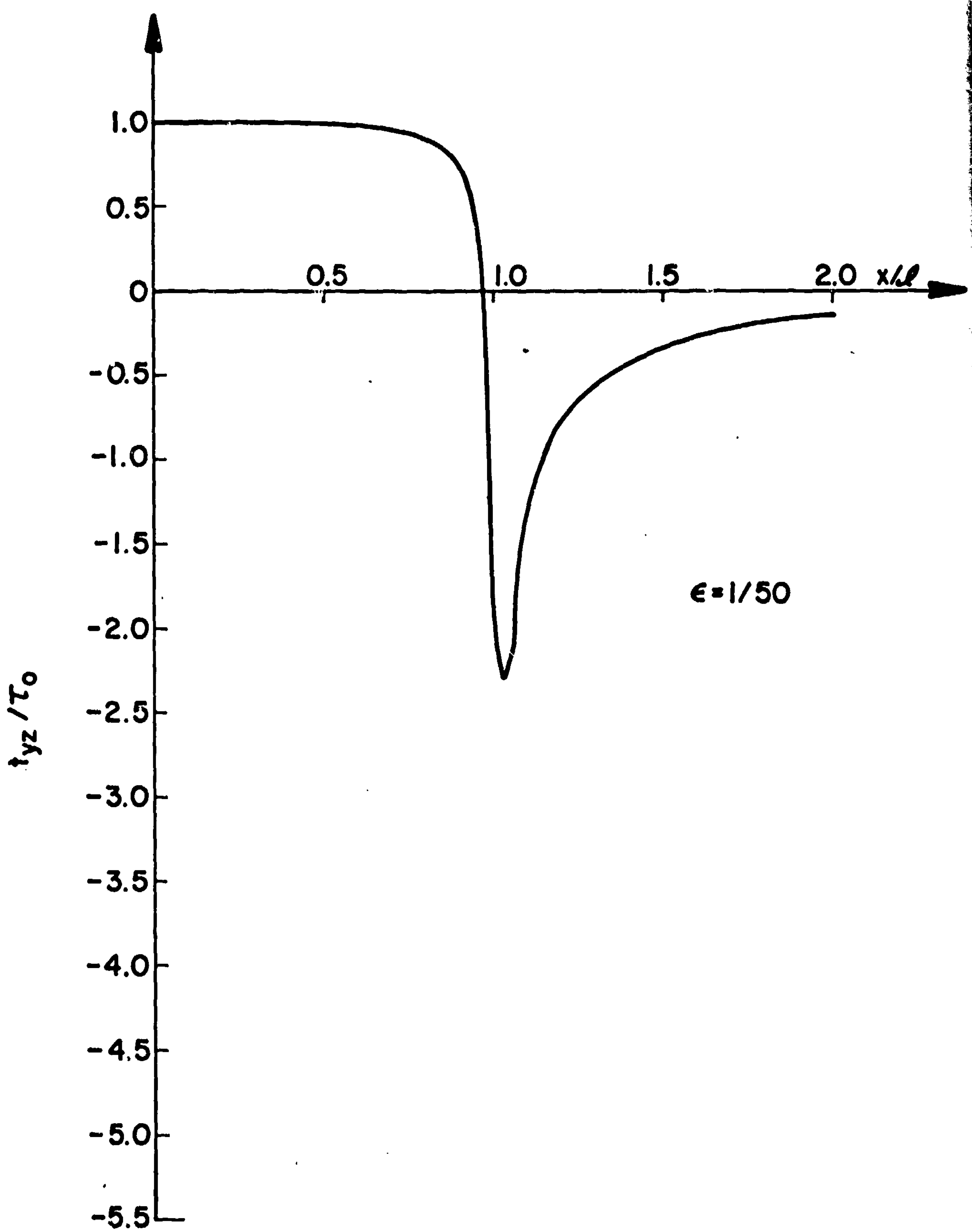
KERNEL FUNCTION K(εζ)

FIGURE 2



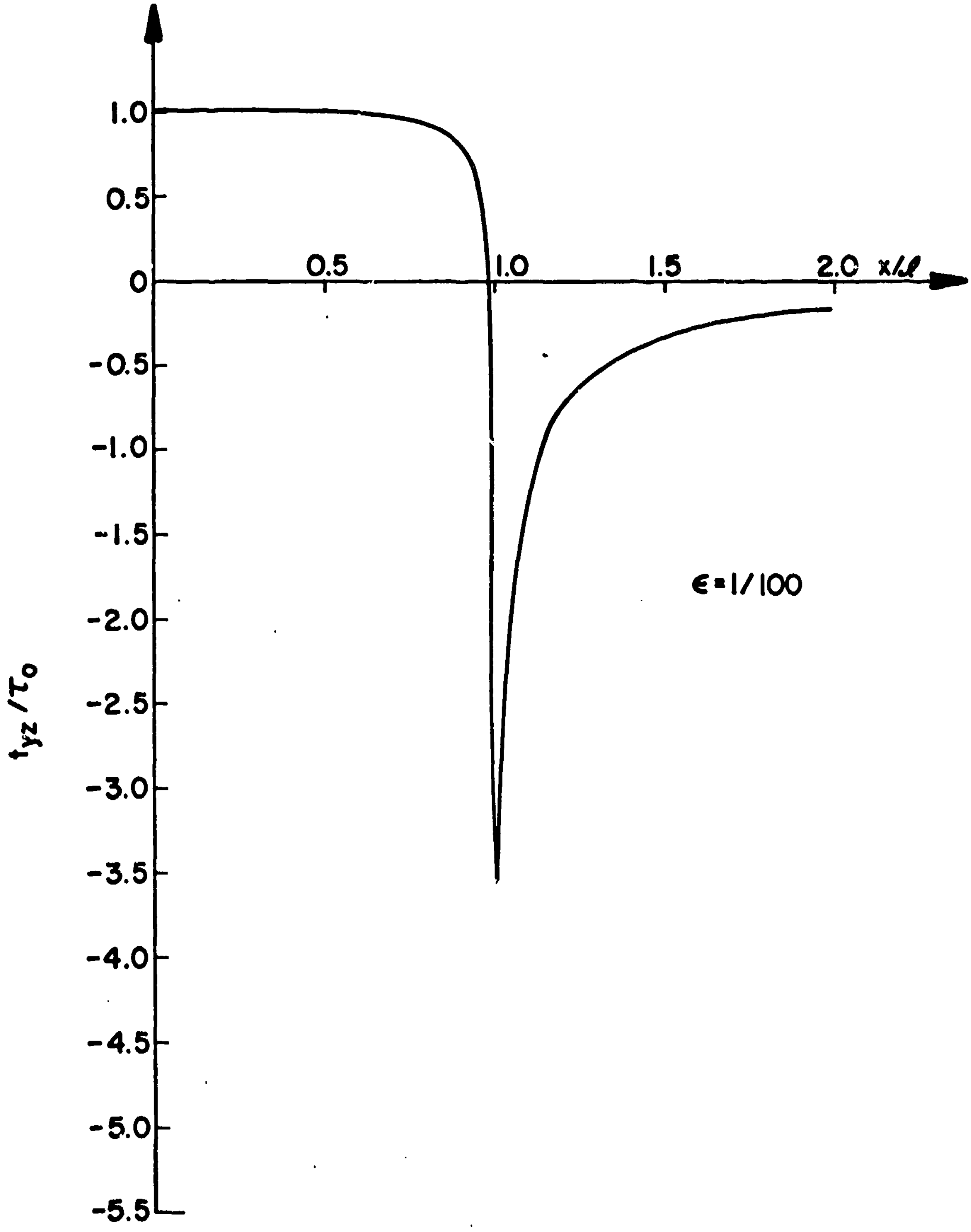
ANTIPLANE SHEAR STRESS

FIGURE 3



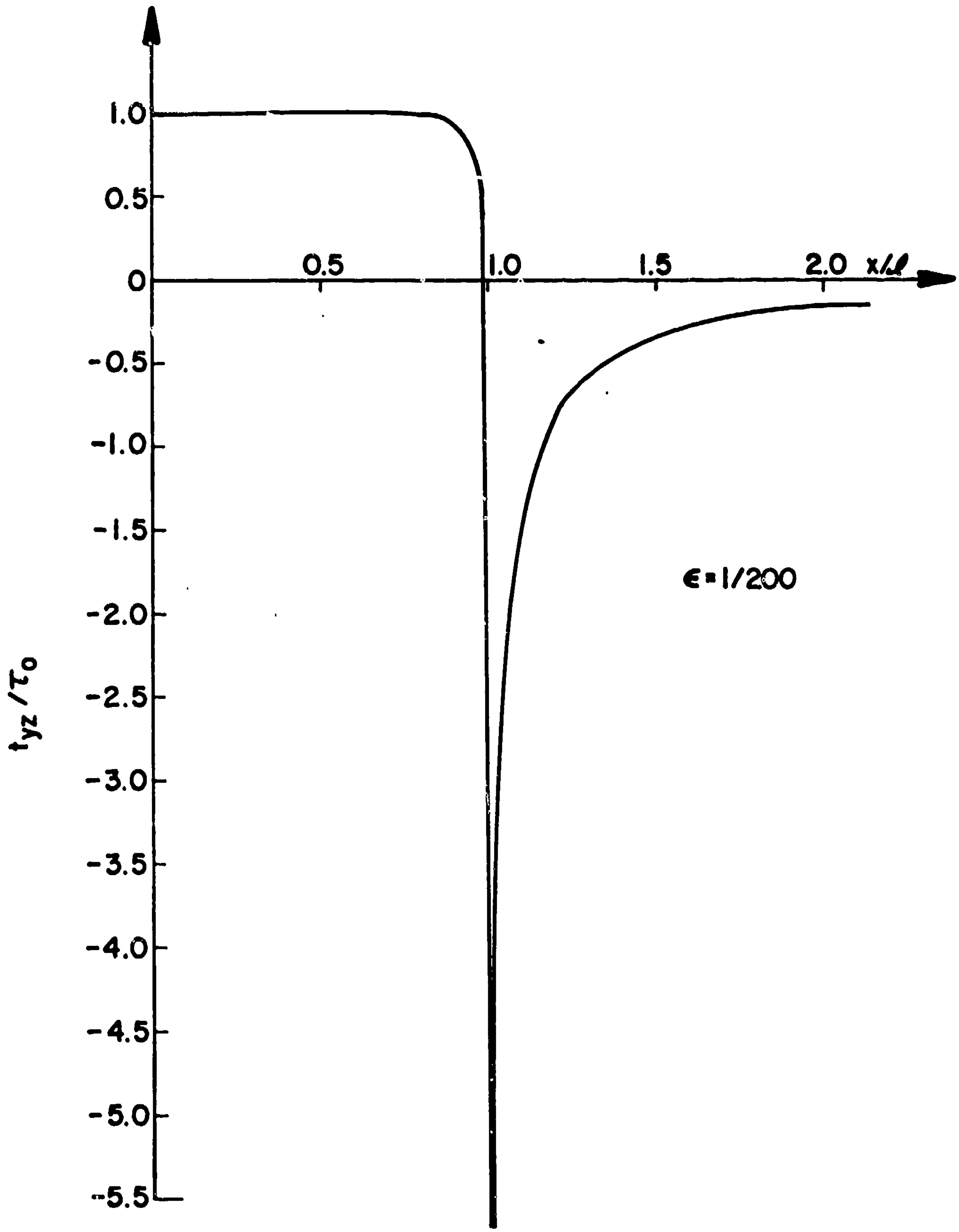
ANTIPLANE SHEAR STRESS

FIGURE 4



ANTIPLANE SHEAR STRESS

FIGURE 5



ANTIPLANE SHEAR STRESS

FIGURE 6

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4. TITLE (and Subtitle)
Line Crack Subject to Antiplane Shear

5. TYPE OF REPORT & PERIOD COVERED
Technical Report

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
A. Cemal Eringen

8. CONTRACT OR GRANT NUMBER(s)
N00014-76-C-0240

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Princeton University
Princeton, NJ 08540

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
P00002

11. CONTROLLING OFFICE NAME AND ADDRESS
Office of Naval Research (Code 471)
Arlington, VA 22217

12. REPORT DATE
July, 1978

13. NUMBER OF PAGES
14 Pages

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
fracture mechanics, line crack, antiplane shear, nonlocal theory, crack tip problems, cohesive stress

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
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