RELIABILITY-BASED SCATTER FACTORS
Volume II: Computer Manual

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RIDGEWOOD, N. J. 07450

MARCH 1978

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AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433
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Project Engineer

RICHARD D. KROBUSEK
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Chief, Analysis and Optimization Branch

FOR THE COMMANDER

HOLLAND B. LOWNDES, JR.
Acting Chief, Structural Mechanics Division

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| 20. ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER) | A definition of scatter factor is introduced that is rational and at the same time can directly be related to the reality of aircraft design and certification as well as of the full-scale and also coupon fatigue test of structural elements or components. Specifically, the scatter factor is defined as the ratio of the MLE (maximum likelihood estimate) of the scale parameter of the two-parameter Weibull distribution assumedly describing the life distribution of structural elements or components, to the "time to first failure" among a
fleet of nominally identical elements or components subjected also to nominally identical operating conditions. Freudenthal has used the same definition of the scatter factor, however, under much simplified conditions: He assumes that the shape parameter of the Weibull distribution is known. This assumption is mathematically highly convenient since it permits the derivation of the distribution of the scatter factor in closed form and independent of the unknown scale parameter. Unfortunately, however, such an assumption is inconsistent with the reality where the Weibull shape parameter easily ranges from 2.0 to 10.0 reflecting the fact that structural elements or components suffer from a variety of sources of randomness in fatigue strength; not only from the probabilistic variation of the material property but also from the statistical variation in workmanship associated with, for example, drilling rivet holes in the process of airframe fabrications. The mathematical difficulty, however, multiplies when the Weibull shape and scale parameters are both assumed to be unknown. Procedures involving Monte Carlo techniques have been established to evaluate the scatter factor under these conditions, using the maximum likelihood estimates of the parameters. The fleet reliability can then be estimated on the basis of the scatter factor thus evaluated. The effect of the sample size to be used in the fatigue test, of the fleet size and of the reliability level on the accuracy of such estimation has also been discussed.
This users' manual report was prepared by Modern Analysis Inc., Ridgewood, N.J., under Air Force Contract No. F33615-77-C-3055. The work was monitored by the Air Force Flight Dynamics Laboratory, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, Project 2304N106, Reliability-Based Scatter Factors, with Dr. H. Leon Harter (AFFDL/FBRD) as Project Engineer.

The project was conducted under the general direction of Dr. M. Shinozuka, President, Modern Analysis Inc. with Mr. D. Li providing the programming and documentation efforts. The programs documented herein are written for operation on a CDC 6600 computer system.

The work reported herein was performed during the period March 1977 to March 1978. The final report in two volumes was submitted on March 7, 1978.
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SECTION I
INTRODUCTION

Statistical scatter factors to be used in the reliability assessment of aircraft structures were evaluated. Two programs were written for this purpose. The first program (A7701) calculates the expected value of the reliability $E[R']$ (Eq.22), the ratio $E[R']/R$ where $R$ is the assigned value of reliability, the coefficient of variation $V_{R'} = \sigma_{R'}/E[R']$ where the $\sigma_{R'} = \{E[(R')^2] - (E[R'])^2\}^{1/2}$ is given in Eq. 23, and the corresponding scatter factors from Eq. 6, $S$ based on $R$ and $S_l$ based on $E[R']$. The equation numbers correspond to equations in Volume I. The calculations in the program A7701 were based on the criteria that the shape parameter in the Weibull distribution is known and the scale parameter is found by maximum likelihood estimation (MLE). In the second program (A7720) the scatter factors are estimated for the case where both the scale and the shape parameters in the Weibull distribution are unknown. For this purpose the Monte Carlo simulation technique is introduced. The distributions of parameters $u^*, v_0^*$ and $Z$ defined in Eqs. 32, 33, 36 respectively, are empirically estimated. This program also calculates the two dimensional frequencies of $u$ and $v_0$ (Eqs. 28 and 29), the reliability ratio $E[R'']/R$, and the coefficient of variation $V_{R''} = \sigma_{R''}/E[R'']$ ($\sigma_{R''} = E[(R'')^2]$).
(E[R"])^{1/2} where E[R"] and E[(R")^2] are obtained from Eq. 41. The following sections of this document discuss these two programs, the input data, and the output.

SECTION II

SCATTER FACTOR WITH KNOWN SHAPE PARAMETER (Program A7701)

A simplified flow chart of Program A7701 is shown in Figure 1.

2.1 INPUT DATA

There are no input data cards used for this program. The necessary data parameters need to be specified at the beginning of the program through DATA statements. The input parameters are as follows: AM = grid size indicating the fleet size of aircraft; A = shape parameter in the Weibull distribution; R = fleet reliability; NPT = number of division points on the distribution of Z; DZ = increment size in the distribution.

2.2 PROGRAM DESCRIPTION

Program A7701 consists of a main program unit and two subroutines: GAMMA and SIMPN. The program listing is given in Appendix A.
Figure 1. Simplified Flow Chart of Program A7701
2.2.1 The Main Program Unit (A7701)

The main program unit calculates the expected value of $E[R']$ of $R'$ from Eq. 22, the ratio $E[R']/R$ where $R$ is given as the input parameter, the variance $VAR[R']$ from Eqs. 22 and 23 as $VAR[R'] = \left(\sigma_R\right)^2 = E[(R')^2] - (E[R'])^2$ and the scatter factors, $S$ based on $R$ and $S_I$ based on $E[R']$. The values for the Gamma function and the numerical integration necessary to complete Eqs. 22 and 23 are obtained from subroutines GAMMA and SIMPN.

2.2.2 Subroutine GAMMA (N, GA)

Subroutine GAMMA evaluates the Gamma function $\Gamma(n) = (n-1)!$ for specified positive integer values of $n$. The arguments for this subroutine are:

- $N = n$, argument of the Gamma function
- $GA = output value of the Gamma function$

2.2.3 Subroutine SIMPN (AF, NPOINT, DSTEP, AREA)

Subroutine SIMPN is a numerical integration routine which utilizes Simpson's rule to obtain area for a specified function. The arguments for this subroutine are:

- $AF = function to be integrated$
- $NPOINT = number of points or subdivisions on the function AF$
- $DSTEP = integration step$
- $AREA = area from the integral$
2.3 PROGRAM OUTPUT

Program A7701 prints out the output as shown in Table 1.

**TABLE 1.** Results from Program A7701

<table>
<thead>
<tr>
<th>SAMPLE SIZE N=</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEIBULL SHAPE A(1)</td>
<td>.50000</td>
</tr>
<tr>
<td>INCREMENT ON Z DZ=</td>
<td>.02500</td>
</tr>
</tbody>
</table>

| FLEET RELIABILITY LEVEL R= | .50000 |
| E(R") | .4246E+00 |
| VAR(R") | .66335E-01 |
| VR" | .60647E+00 |

<table>
<thead>
<tr>
<th>FLEET SIZE</th>
<th>S.F. BASED ON R</th>
<th>S.F. BASED ON E(R&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M= 1.</td>
<td>S= .10000E+01</td>
<td>S1= .54490E+00</td>
</tr>
<tr>
<td>M= 3.</td>
<td>S= .90000E+01</td>
<td>S1= .49041E+01</td>
</tr>
<tr>
<td>M= 5.</td>
<td>S= .25000E+02</td>
<td>S1= .13622E+02</td>
</tr>
<tr>
<td>M= 10.</td>
<td>S= .10000E+03</td>
<td>S1= .54490E+02</td>
</tr>
<tr>
<td>M= 25.</td>
<td>S= .62500E+03</td>
<td>S1= .34056E+03</td>
</tr>
<tr>
<td>M= 100.</td>
<td>S= .10000E+05</td>
<td>S1= .54490E+04</td>
</tr>
<tr>
<td>M= 250.</td>
<td>S= .62500E+05</td>
<td>S1= .34056E+05</td>
</tr>
<tr>
<td>M= 1000.</td>
<td>S= .10000E+07</td>
<td>S1= .54490E+06</td>
</tr>
</tbody>
</table>

N = sample size in fatigue test; n in the text
A(1) = shape parameter a in Weibull distribution
DZ = increment size dz for numerical integration
R = fleet reliability level
E(R") = expected value of R'; E[R'] in Volume I
E(R")/R = reliability ratio; E[R']/R in Volume I
VAR(R") = variance of R'
STD = standard deviation of R'; σ_r
VR" = coefficient of variation; V_r, in Volume I
M = aircraft fleet size
S = scatter factor, S, based on R; from Eq. 6
S1 = scatter factor, S, based on E[R']; from Eq. 6
The output such as that shown in Table 1 was generated for \( N = 1, 2, \ldots, 10; \ A = 0.5, 1, 2, 3, 4, 5, 10; \ R = 0.5, 0.6, 0.7, 0.8, 0.9, 0.99, 0.995, 0.999, 0.9999. \)

SECTION III
SCATTER FACTOR WITH UNKNOWN SHAPE AND SCALE PARAMETERS
(Program A7720)

A simplified flow chart of Program A7720 is shown in Figure 2.

3.1 INPUT DATA

There are no input data cards used for this program. The parameters are specified at the beginning of the program. The input parameters are as follows: \( R = \) fleet reliability, \( M = \) fleet size, \( NAM = \) classification ASCII code for results, \( N = \) sample size in fatigue test; \( n \) in the text, \( NSMPL = \) sample size in Monte Carlo simulation; \( N \) in the text, \( IO = \) input/output peripheral device number, \( IX = \) internal parameter in RANDU subroutine, \( NST = \) a lower bound in array \( N \), \( NND = \) an upper bound number in array \( N \), \( NU = \) number of divisions on \( u \), \( NV0 = \) number of divisions on \( v_0 \), \( UU = \) upper cut-off value on \( u \), \( UVO = \) upper cut-off value on \( v_0 \). The parameters \( u \) and \( v_0 \) are described in Volume I.

3.2 PROGRAM DESCRIPTION

Program A7720 consists of a main program and six
SUBROUTINES

START

INPUT DATA

VALUES FOR $v_0^*$, $u^*$, $v_0$, $u$

DISTRIBUTION OF $v_0^*$

DISTRIBUTION OF $u^*$

DISTRIBUTION OF $v_0$

DISTRIBUTION OF $u$

JOINT VALUES OF $u$ AND $v_0$

OUTSIDE SPECIFIED RANGE

JOINT FREQUENCY $F(u,v_0)$

VALUES FOR $w$, $Z$

DISTRIBUTION OF $Z$

$E[R^n]$, $\text{VAR } R^n$, $\sigma_R^n$, $E[R^n]/\lambda$

$V_R^n$, $Q^*$

END

Figure 2. Simplified Flow Chart of Program A7720
subroutines: SAMPL, ANS, RANDU, ASCEN, LIST and DGEN. The program listing is given in Appendix B.

3.2.1 The Main Program Unit A7720

The main program unit calculates distributions of $u$, $v_0$, $u^*$, $v_0^*$ and $Z$ where these parameters are defined by Eqs. 32, 33 and 36 respectively in Volume I. The two dimensional frequencies of $u$ and $v_0$ are obtained by utilizing the Monte Carlo simulation procedure. The reliability ratio $E[R'']/R$, the variance of $R''$, \( \text{VAR} = E[(R'')^2] - (E[R''])^2 \), the standard deviation of $R''$, \( \sigma_{R''} = \text{VAR}^{\frac{1}{2}} \), and the coefficient of variation \( V_{R''} = \sigma_{R''}/E[R''] \) are calculated from Eq. 41.

3.2.2 Subroutine SAMPL (U, V0, V0S, IX, IY, N, NSMPL, IO)

Subroutine SAMPL is used to generate values for $u$, $v_0$ and $v_0^*$ by using the Maximum Likelihood Estimation (MLE) procedure. The arguments for this subroutine are

- \( U = u \), output parameter
- \( V0 = v_0 \), output parameter
- \( V0S = v_0^* \), output parameter
- \( IX = \) any odd integer less than nine digits
- \( IY = \) internal parameter
- \( N = \) sample size in fatigue test; \( n \) in Volume I
- \( NSMPL = \) sample size in Monte Carlo simulation;
- \( N \) in Volume I
- \( IO = \) input/output peripheral device number
3.2.3 Subroutine ANS (A, NSMPL, NS, IO, NAM1, NAM2, M)

Subroutine ANS is used to arrange the input data in proper order and to print out the probability distribution. The arguments for this subroutine are

- A = input/output array
- NSMPL = sample size in Monte Carlo simulation
- NS = sample size
- IO = input/output peripheral number
- NAM1 and NAM2 = classification parameters for the results
- M = internal program parameter, or fleet size

3.2.4 Subroutine RANDU (IX, IY, RAN)

Subroutine RANDU generates independent random numbers uniformly distributed between 0 and 1. An odd integer, IX, needs to be specified in the main program. By changing IX, a new sequence of random numbers is generated. The arguments for this subroutine are

- IX = any odd integer less than 9 digits
- IY = internal parameter in the program
- RAN = output uniform random number between 0 and 1.

3.2.5 Subroutine ASCEN (S, NPT)

Subroutine ASCEN is used to rearrange given data in ascending order. The arguments of this subroutine are
S = an arbitrary input/output array

NPT = total number of values in the array

3.2.6 Subroutine LIST (A, NPT, IO, NAM1, NAM2)

Subroutine LIST is used for listing the input as a probability distribution. The arguments of this subroutine are

A = input data arranged in ascending order
NPT = total number of values in array A
IO = output peripheral device number
NAM1 and NAM2 = classification parameters for the list

3.2.7 Subroutine DGEN (Y, N, IX, RAN)

Subroutine DGEN generates exponential random numbers by utilizing a uniform distribution in subroutine RANDU. The arguments of this subroutine are

Y = the exponential random number
N = index indicating the array of random numbers
IX = defined in RANDU
RAN = defined in RANDU

3.3 PROGRAM OUTPUT

Program A7720 prints out the output as shown in Tables 2 - 9.
Table 2 Values for $v^*$

<table>
<thead>
<tr>
<th>SAMPLE NO.</th>
<th>PROB</th>
<th>$v^*$</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>.0500</td>
<td>.6230149E-01</td>
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<tr>
<td>200</td>
<td>.1500</td>
<td>.1618284E+00</td>
</tr>
<tr>
<td>300</td>
<td>.2759731E+00</td>
<td></td>
</tr>
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<td>400</td>
<td>.3760098E+00</td>
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<td>1998</td>
<td>.1077416E+08</td>
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<td>.7369510E+11</td>
<td></td>
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<tr>
<td></td>
<td>.2794072E+14</td>
<td></td>
</tr>
</tbody>
</table>

$V^0 = v^* \_0$

SAMPLE NO. = a positive integer such that the plotting position of a realization is given by

$(\text{SAMPLE NO.})/(\text{NSMPL} + 1)$

PROB = plotting position
Table 3  Values for $u^*$

<table>
<thead>
<tr>
<th>SAMPLE NO.</th>
<th>PROB</th>
<th>$u^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.0500</td>
<td>0.1752809E+00</td>
</tr>
<tr>
<td>200</td>
<td>0.1000</td>
<td>0.2507607E+00</td>
</tr>
<tr>
<td>300</td>
<td>0.1500</td>
<td>0.306521E+00</td>
</tr>
<tr>
<td>400</td>
<td>0.2000</td>
<td>0.3624199E+00</td>
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<td>500</td>
<td>0.2500</td>
<td>0.4018442E+00</td>
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<td>0.3000</td>
<td>0.4834875E+00</td>
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<td>0.5392864E+00</td>
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<td>800</td>
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<td>0.9040222E+00</td>
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<td>0.9601127E+00</td>
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<td>0.8000</td>
<td>1.055649E+01</td>
</tr>
<tr>
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<td>0.8500</td>
<td>1.151112E+01</td>
</tr>
<tr>
<td>1800</td>
<td>0.9000</td>
<td>1.286268E+01</td>
</tr>
<tr>
<td>1900</td>
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<td>1.554481E+01</td>
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<td>0.9800</td>
<td>1.786360E+01</td>
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<td>1.902801E+01</td>
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<tr>
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<td>0.9950</td>
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<td>1996</td>
<td>0.9980</td>
<td>2.475547E+01</td>
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<tr>
<td>1998</td>
<td>0.9990</td>
<td>2.891192E+01</td>
</tr>
<tr>
<td>1999</td>
<td>0.9995</td>
<td>2.940728E+01</td>
</tr>
</tbody>
</table>

$U^* = u^*$

SAMPLE NO. = a positive integer such that the plotting position of a realization is given by

$$\frac{\text{SAMPLE NO.}}{\text{NSMPL} + 1}$$

PROB = plotting position
Table 4  Values for $v_0$

<table>
<thead>
<tr>
<th>SAMPLE NO.</th>
<th>PROB</th>
<th>$v_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
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<td>200</td>
<td>.1000</td>
<td>.3662845E+00</td>
</tr>
<tr>
<td>300</td>
<td>.1500</td>
<td>.4507701E+00</td>
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<td>400</td>
<td>.2000</td>
<td>.5452315E+00</td>
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<td>500</td>
<td>.2500</td>
<td>.6394380E+00</td>
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<td>.7326998E+00</td>
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<tr>
<td>700</td>
<td>.3500</td>
<td>.8269664E+00</td>
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<td>.4000</td>
<td>.9212330E+00</td>
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<td>.1015898E+01</td>
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<tr>
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<td>.7000</td>
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<tr>
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<td>.7500</td>
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<td>1600</td>
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<td>.1678560E+01</td>
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<tr>
<td>1940</td>
<td>.9995</td>
<td>.2341222E+01</td>
</tr>
</tbody>
</table>

$V_0 = v_0$

SAMPLE NO. = a positive integer such that the plotting position of a realization is given by

\[(\text{SAMPLE NO.}) / (\text{NSMPL} + 1)\]

PROB = plotting position
Table 5 Values for \( u \)

<table>
<thead>
<tr>
<th>SAMPLE NO.</th>
<th>PROB</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
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<tr>
<td>200</td>
<td>0.1000</td>
<td>7.774431E+00</td>
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<tr>
<td>300</td>
<td>0.1500</td>
<td>8.587249E+00</td>
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<td>400</td>
<td>0.2000</td>
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<td>500</td>
<td>0.2500</td>
<td>1.020291E+01</td>
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<td>0.3000</td>
<td>1.106168E+01</td>
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<td>700</td>
<td>0.3500</td>
<td>1.189700E+01</td>
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<td>1996</td>
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<td>0.9990</td>
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<tr>
<td>1999</td>
<td>0.9995</td>
<td>2.698067E+01</td>
</tr>
</tbody>
</table>

\( U = u \)

SAMPLE NO. = a positive integer such that the plotting position of a realization is given by \((\text{SAMPLE NO.})/(\text{NSMPL} + 1)\)

PROB = plotting position
Table 6. Joint Values of $u$ and $v_0$ Outside the Specified Ranges $UU$ or $UVO$

$N = 3$ $NSMPL = 1999$ $IX = 25107$  
$NU = 50$ $NV0 = 20$ $UU = 25.0000$ $UVO = 5.0000$ $DU = .5000$ $DV0 = .2500$

OUTLIERS EITHER IN $U$ OR IN $V0$

<table>
<thead>
<tr>
<th>$U$</th>
<th>$V0$</th>
<th>$J$</th>
<th>$I$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>6</td>
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<td>.3221E+02</td>
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<td>7</td>
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<td>6</td>
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<tr>
<td>.7981E+02</td>
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<td>160</td>
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<td>3</td>
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</table>

$N =$ sample size in fatigue test; $n$ in Volume I  
$NSMPL =$ sample size in Monte Carlo simulation; $N$ in Volume I  
$IX =$ defined in RANDU  
$I, J =$ array of numbers  
$NU =$ number of divisions on $u$  
$NV0 =$ number of divisions on $v_0$  
$UU =$ upper cut-off for $u$  
$UVO =$ upper cut-off for $v_0$  
$DU =$ increment on $u$, $DU = UU/NU$  
$DV0 =$ increment on $v_0$, $DV0 = UVO/NV0$  
$U =$ $u$  
$V0 =$ $v_0$
Table 7. Joint Frequency of \( u \) and \( v_0 \), \( F(u, v_0) \)

\( F(u, v_0) \) for sample size \( n = 3 \)

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-16-
Table 8 Values for Z

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<td>2.000</td>
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<td>500</td>
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<td>-0.3405964F+00</td>
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<tr>
<td>600</td>
<td>3.000</td>
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<td>1100</td>
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<tr>
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<td>8.500</td>
<td>-0.349801E+01</td>
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<td>9.000</td>
<td>-0.2851978E+02</td>
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</table>

Z = Z

M = aircraft fleet size

SAMPLE NO. = a positive integer such that the plotting position of a realization is given by
(SAMPLE NO.)/(NSMPL + 1)

PROB = plotting position
Table 9 Expected Value, Variance, Standard Deviation, Reliability Ratio, and Coefficient of Variation of $R''$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$n_{SMPL}$</th>
<th>$n$</th>
<th>$N$</th>
</tr>
</thead>
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<td>1494</td>
<td>1</td>
<td>1</td>
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</table>

<table>
<thead>
<tr>
<th>$R$</th>
<th>$E(R''')$</th>
<th>$VAR$</th>
<th>$STD$</th>
<th>$E(R''')/R$</th>
<th>$VR''''$</th>
<th>$QS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000</td>
<td>0.478</td>
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<td>0.2107</td>
<td>0.4757</td>
<td>0.4318</td>
<td>1.3947E+01</td>
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<tr>
<td>0.6000</td>
<td>0.5747</td>
<td>0.0431</td>
<td>0.2076</td>
<td>0.4512</td>
<td>0.5637</td>
<td>2.1136E+01</td>
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<td>0.0362</td>
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<td>3.4182E+01</td>
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<tr>
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<td>0.7323</td>
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<td>0.4153</td>
<td>0.2349</td>
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<td>1.6770E+02</td>
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<td>0.0356</td>
<td>0.4785</td>
<td>0.0364</td>
<td>3.4388E+05</td>
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</tbody>
</table>

$N = \text{sample size in fatigue test; } n \text{ in Volume I}$

$NSMPL = \text{sample size in Monte Carlo simulation; } N \text{ in Volume I}$

$M = \text{aircraft fleet size}$

$R = \text{assigned reliability values}$

$E(R''') = \text{expected values of } R'' \text{ obtained from Eq. 41.}$

$VAR = E[(R'')^2] - E^2[R'']$, variance of $R'' \text{ in Volume I}$

$STD = \sigma_{R''}$, standard deviation of $R'' \text{ in Volume I}$

$E(R''')/R = \text{reliability ratio}$

$VR'''' = \text{coefficient of variation of } R'', STD/E[R''] \text{ in Volume I}$

$QS = Q^* \text{ in Volume I}$
Appendix A

PROGRAM A7701( OUTPUT, TAPE6=OUTPUT)

C PURPOSE
C SCATTER FACTOR CALCULATION
C WITH KNOWN WEIBULL SHAPE PARAMETER.
C DESCRIPTION OF THE PROGRAM
C THIS PROGRAM CALCULATES $E(R')$, $E(R')/R$, $VAR(R')$, $VR'$ FOR GIVEV WEIBULL SHAPE PARAMETER A
C SAMPLE SIZE N, FLEET SIZE AM*, AND RELIABILITY LEVEL R.
C IT ALSO CALCULATES CORRESPONDING SCATTER FACTORS
C $S$ BASED ON $R$ AND $S1$ BASED ON $E(R')$.
C REMARKS
C INTEGRATION TO GET $E(R')$ AND $E((R')^2)$ IS CARRIED
C OUT BY SIMPSON'S METHOD.
C DATE 7/16/1977
C
C DIMENSION X(1000),A(50),CV(50),STD(50),Z(1000),NN(50)
C DIMENSION XX(1000),R(50),ERR(50),ARG3(1000),ARG2(1000)
C DIMENSION AM(50),ANZA(I000),VAR(50)

C ASSIGNMENT OF SPECIFIED ARRAY INPUT DATA.
C AM= FLEET SIZE
C A= WEIBULL SHAPE PARAMETER
C R= SPECIFIED RELIABILITY LEVEL
C NN= SAMPLE SIZE
DATA AM/1.,3.,5.,10.,25.,100.,250.,1000./
DATA A/0.5,1.,2.,3.,4.,5.,10./
DATA R/0.5,0.6,0.7,0.8,0.9,0.99,0.995,0.999,0.9999/
DATA NN/1,2,3,4,5,6,7,8,9,10,20/

C ASSIGNMENT OF ARRAY SIZE TO BE USED (UP TO 50).
C IN -- FOR SAMPLE SIZE NN
C IM -- FOR FLEET SIZE AM
C IA -- FOR WEIBULL SHAPE A
C IR -- FOR FLEET RELIABILITY
IN=11
IM=8
IA=7
IR=9

C TO SPECIFY OUTPUT PERIPERAL NUMBER 10.
10=6

C TO SPECIFY NUMBER OF INCREMENTS NPT FOR SIMPSON'S
C NUMERICAL INTEGRATION (UP TO 1000).
C USE NPT=400 IN THIS PROGRAM.
NPT=400
NPT1=NPT-1

C TO CALCULATE FOR EACH SAMPLE SIZE N.
280 DO 200 JN=1,IN
N=NN(JN)
AN=FLOAT(N)
AN1=1./AN
N2=2.*N
ANN=AN**N
CALL GAMMA (N,GA)
200 CONTINUE
TO CALCULATE FOR EACH WEIBULL SHAPE PARAMETER A:
DO 90 K=1,IA
DZ=0.025
IF (K .GE. 3) DZ=0.01
PRINT 25, N*K*A(K),DZ
25 FORMAT(1H1/!/!/!5X,"SAMPLE SIZE N=",I5//
15X,"WEIBULL SHAPE A(*,13,*),=",F10.5,
25X,"INCREMENT ON Z DZ=",F10.5/)
A1=1./A(K)
AKN1=A(K)*ANN/GA
ATN1=A(K)*AN-1.

TO CALCULATE FOR EACH RELIABILITY LEVEL R:
DO 50 L=1,IR
RR=ALOG(R(L))
PRINT 110,R(L)
110 FORMAT(15X,"FLEET RELIABILITY LEVEL R=",F10.5)/

TO CALCULATE ARGUMENTS IN EQS. (22) AND (23)
BY USING DENSITY FUNCTION OF Z:
DO 60 I=1,NPT
Z(I)=FLOAT(I)*DZ
ARG2(I)=-AN*Z(I)**A(K)
IF (ARG2(I) .LT. -100.) GO TO 40
ARG3(I)=AKN1*Z(I)**ATN1*EXP(ARG2(I))
ANZA(I)=AN*Z(I)**A(K)
40 IF (ARG2(I) .LT. -100.) GO TO 30
ARG=1./(1.-RR/ANZA(I))
80 XX(I)=X(I)**N
X(I)=ARG**N*ARG3(I)
GO TO 60
30 X(I)=0.
XX(I)=0.
60 CONTINUE

TO PERFORM INTEGRATION TO ESTIMATE E(R") AND E((R")**2).
CALL SIMPN(X,NPT1,DZ,AREA)
CALL SIMPN(XX,NPT1,DZ,AREX)

TO CALCULATE E(R")/R, VAR(R") , STD (=STANDARD DEVIATION),
AND VR" (=COEFFICIENT OF VARIATION) AND ALSO PRINT OUT.
ERR(L)=AREA/R(L)
VAR(L)=AREX-AREA**2
IF(VAR(L) .LT. 0.) VAR(L)=0.
STD(L)=SORT(VAR(L))
CV(L)=STD(L)/AREA
PRINT 10, AREA,ERR(L),VAR(L),STD(L),CV(L)
10 FORMAT(9X,"E(R")=",E12.5,5X,"E(R")/R=",E12.5/
19X,"VAR(R")=",E12.5,5X,"STD =",E12.5/
29X,"VR"=",E12.5/
318X,"FLEET SIZE=",5X,"BASED ON R",4X,
4S,F BASED ON E(R")/)

TO CALCULATE SCATTER FACTORS S BASED ON R
AND S1 BASED ON E(R") FOR EACH FLEET SIZE.
DO 70 J=1,IM
S1=(AM(J)/(AN*(1./AREA**AN1-1.)))**AI
SS=(AM(J)/(AN*(1./R(L)**AN1-1.)))**AI
PRINT 120, AM(J),SS,S1
120 FORMAT(17X,*M=*,F6.0,7X,**S= *,E12.5,4X,**S1= *,E12.5)
70 CONTINUE
50 CONTINUE
90 CONTINUE
200 CONTINUE
STOP
END

SUBROUTINE GAMMA(N,GA)
C TO CALCULATE GAMMA FUNCTION.
C USE FACTORIAL RELATIONSHIP SINCE N IS
C AN INTEGER IN EVERY CASE.
C N MUST BE LESS THAN 50.
C
GA=1.0
IF(N.EQ.1) RETURN
N1=N-1
DO 10 I=1,N1
10 GA=GA*FLOAT(I)
RETURN
END

SUBROUTINE SIMPN(AF,NPOINT,DSTEP,AREA)
DIMENSION AF(1)
C TO CARRY OUT INTEGRATION FOR GIVEN VALUES OF
C FUNCTION AF.
N=(NPOINT-1)/2
MN=N-1
ODD=0.
EVEN=0.
END=AF(1)+AF(NPOINT)
DO 10 I=1,NP
10 ODD=ODD+AF(2*I)
DO 20 I=1,MN
20 EVEN=EVEN+AF(2*I+1)
AREA=(4.*0*ODD+2.*0*EVEN+END)*DSTEP/3.0
RETURN
END
Appendix B

PROGRAM A7720 (OUTPUT, TAPE6=OUTPUT)
C PURPOSE
C SCATTER FACTOR CALCULATION
C WITH UNKNOWN WEIBULL SHAPE AND SCALE PARAMETERS.
C DESCRIPTION OF THE PROGRAM
C THIS PROGRAM FIRST GENERATES NSMPL NUMBER OF
C SAMPLE SETS OF SIZE N FOR U*, VO*, VO AND V0*,
C AND THEN CALCULATES THEIR PROBABILITY DISTRIBUTION
C AS WELL AS JOINT DENSITY F(U*V0*).
C SECOND, IT CALCULATES PROBABILITY DISTRIBUTION
C OF Z FOR SAMPLE SIZE N AND FLEET SIZE M BY GENERATING
C CORRESPONDING SAMPLES OF W.
C IT FINALLY CALCULATES E(R**), VAR(R**), E(R**)/R AND U*.
C REMARKS
C IN THE PROGRAM, "*" IS REPLACED BY "S", EXAMPLE: U*=US.
C DATE 10/25/77
DIMENSION NVOU(20,50), N(9), M(8), NAM(6)
EQUIVALENCE (VOS,Z)
C
C ASSIGNMENT OF SPECIFIED ARRAY INPUT DATA.
C N= SAMPLE SIZE.
C M= FLEET SIZE.
C R= SPECIFIED RELIABILITY LEVEL.
C NAM= DATA CLASSIFICATION ASCII CODE.
DATA N/2,3,4,5,6,7,8,9,10,20/
DATA M/1,3,5,10,25,100,250,1000/
DATA R/0.5,0.6,0.7,0.8,0.9,0.95,0.99,0.999,0.9999/
DATA NAM/2HU*,2H*,2HU*,2H*,2HZ/
C
C INPUT DATA ASSIGNMENT.
C NSMPL= NUMBER OF SAMPLE SETS OF SIZE N.
C IX= INITIAL VALUES FOR UNIFORM RANDOM NUMBER
G GENERATION SUBROUTINE RANDU(IX, IY, RAN).
C NST AND NND= CONTROL PARAMETER FOR DO-LOOP CALCULATION
FOR EACH SAMPLE SIZE N.
C NST= LOWER BOUND NUMBER IN ARRAY N.
C NND= UPPER BOUND NUMBER IN ARRAY N.
C NU= ARRAY SIZE FOR U.
C NV0= ARRAY SIZE FOR VO.
C UU= UPPER CUT-OFF VALUE FOR U.
C UVO= UPPER CUT-OFF VALUE FOR VO.
C LU AND 10= SELECT CODES OF DATA INPUT/OUTPUT PERIPHERAL.
C NSMPL=1999
IX=23451
NST=1
NND=10
NU=50
NV0=20
UU=25.
UVO=5.
C
C CALCULATION FOR EACH SAMPLE SIZE N(KN)
FNSMP1=FLOAT(NSMPL+1)
FNSMP2=1.0/FNSMP1
C
- 23 -
DO 100 KN=NS1, NND
NS=N(KN)
C
TO GENERATE NSMPL NUMBER OF SAMPLE SETS OF SIZE N
 FOR U, VO, AND VO*
 CALL SMPL(U, VO, VOS, IX, VS, NSMPL, IO)
C
TO CALCULATE PROBABILITY DISTRIBUTION OF V0*
 BY ARRANGING IN ASCENDING ORDER.
 CALL ANS(VOS, NSMPL, NS, IO, NAM(4), NAM(5), 0)
C
TO CALCULATE PROBABILITY DISTRIBUTION OF U*
 BECAUSE OF LIMITED MEMORY SIZE OF THE COMPUTER USED,
 AND BECAUSE OF VALUES U AND V0 TO BE RESERVED FOR
 CALCULATION OF Z, ARRAY VOS IS USED AS COMMON ARRAY.
 DO 110 I=1, NSMPL
 VOS(I)=1./U(I)
 CALL ANS(VOS, NSMPL, NS, IO, NAM(3), NAM(2), 0)
C
TO CALCULATE PROBABILITY DISTRIBUTION OF V0.*
 DO 120 I=1, NSMPL
 VOS(I)=VO(I)
 CALL ANS(VOS, NSMPL, NS, IO, NAM(4), NAM(2), 0)
C
TO CALCULATE PROBABILITY DISTRIBUTION OF U.*
 DO 130 I=1, NSMPL
 VOS(I)=U(I)
 CALL ANS(VOS, NSMPL, NS, IO, NAM(1), NAM(2), 0)
C
TO CALCULATE JOINT DENSITY F(U, V0), E(R**), VAR(R**),
 VR* E(PR*) AND Q*.
C
DU=UU/FLOAT(NU)
 DV0=VV0/FLOAT(NV0)
 DO 150 J=1, NU
 DO 150 I=1, NV0
 NV0(J, I)=0
150 CONTINUE
C
TO PRINT OUT NECESSARY INFORMATIONS FOR CHECK PURPOSE
 WRITE(6,600) NS, NSMPL, IX
 600 FORMAT(1H1////5X,* N=*, I5,* NSMPL=*, I5,* IX=*, I20)
 WRITE(6,610) NU, NV0, DU, DV0
 610 FORMAT(5X,* NU=*, I5,* NV0=*, I5,* UU=*, F8.4,* UV0=*,
           IF8.4,* DV0=*, F8.4,//)
 WRITE(6,611)
 611 FORMAT(5X,* OUTLIERS EITHER IN U OR IN V0*/)
 DO 160 K=1, NSMPL
 J=1
 I=1
 IF ( J.LE.NU .AND. I.LE.NV0 ) GO TO 10
 C
TO PRINT OUT NECESSARY INFORMATIONS FOR CHECK PURPOSE
 WRITE(6,630) U(K), V0(K), J, I
 630 FORMAT( 5X,* U=*, E12.4,3X,* V0=*, E12.4,3X,*, J=*, I6,3X,
           * I=*, I6)
IF(J>NU) J=NU
IF(I>NU) I=NU
10 NV0U(I,J)=NV0U(I,J)+1
160 CONTINUE
C TO PRINT OUT JOINT FREQUENCY F(U,V0).
C JOINT FREQUENCY F(U,V0) IS LISTED WITH U AS COLUMN,
C AND V0 AS ROW.
   WRITE(6,640) NS
   640 FORMAT(1H1,///5X,* F(U,V0) FOR SAMPLE SIZE N=*#15/
                 18X,* U AS COLUMN, AND V0 AS ROW.*7/
   WRITE(6,650) NV0U
   650 FORMAT(20(1X,#13))
C FOR BRIEF VERSION OF LISTING OF F(U,V0).
   WRITE(6,640) NS
   DO 170 J=1,40
      WRITE(6,660) ( NV0U(I,J),I=1,16)
   660 FORMAT(6X,16(1X,#13))
   170 CONTINUE
C TO CALCULATE DISTRIBUTION OF Z FOR EACH FLEET SIZE.
C SAME INITIAL VALUES FOR UNIFORM RANDOM NUMBER
C GENERATION SHALL BE USED FOR EACH CASE OF FLEET SIZE.
IXS=IX
   DO 200 KK=1,8
      MM=MM(KK)
      FM=FLOAT(MM)
      FNS=FLOAT(NSMPL)+1.
      IX=IXS
      DO 210 I=1,NSMPL
         CALL RANDU (IX,IY,RAN)
         IX=IY
         W=-ALOG (RAN)/FM
         VOW=V0(I)/W
         IF (VOW < 1.E-10) VOW=1.E-10
         Z(I)=U(I)*ALOG10(VOW)
   210 CONTINUE
C CALCULATE DISTRIBUTION OF Z FOR EACH FLEET SIZE
   CALL AMS(Z,NSMPL,NS,JO,NAM(5),NAM(2),MM)
C TO DEFINE THE CASE CALCULATED.
   WRITE(6,670) NS,NSMPL,MM
   670 FORMAT(1H1,///5X,* N=*/15,* NSMPL=*/16,* M=*/16)
   WRITE(6,680)
   680 FORMAT(/8X,* R=*/7X,* E(R")*##")*3X,* VAR*/6X,*
       1* STD*3X,* E(R")*##")/R*/1X,* VR"*##")*7X,* QS*/)
C TO ESTIMATE E(R")*, VAR(R")*, VR"*, E(R")/R*, AND Q*.
   DO 300 K=1,9
      FMALR=FM/ALOG(1./R(K))
      ER=0.0
      VAR=0.0
      DU2=DU/2.
      DV02=DV0/2.
      DO 310 J=1,NU
      DO 320 I=1,NU
   300 CONTINUE
C - 25 -
IF (NVOU(I,J) .EQ. 0) GO TO 320

UJX=FLOAT(J) *DV=DU2
VOX=FLOAT(I) *DV=DV0
PARM=VOX*FMALK
Z0=UJX*ALOG10(PARM)
PAR2=FLOAT(NVOU(I,J)) / FVS

DO 330 L=1,NSMPL
IF (Z0 .LE. Z(L)) GO TO 20

330 CONTINUE
20 FZ=FLOAT(L) /FNS
FZ2=FZ**2
ER=ER+FZ*PAR2
VAR=VAR+FZ2*PAR2

320 CONTINUE
310 CONTINUE
VAR=VAR-ER**2
STD=SQRT(VAR)
CV=STD/ER
RATIO=ER/R(K)
ZER=ER*FNSMPL
IZ=IFIX(ZER)
IZ1=IZ+1
FIZ=FLOAT(IZ) /FNSMPL
ZANS=Z(IZ)+(ER-FIZ)**2*(Z(IZ1)-Z(IZ))/FNSMPL2
QS=10.**ZANS
WRITE(6,690) R(K),ER,VAR,STD,RATIO,CV,QS

690 FORMAT(6x,9.4,1x, E12.5)

300 CONTINUE
200 CONTINUE
100 CONTINUE
STOP
END

SUBROUTINE ANS(NSMPL,NS,IO,NAM1,NAM2,M)
DIMENSION A(1)

CALL ASCEN (A,NSMPL)
IF (M .EQ. 0) GO TO 10
WRITE(6,100) NAM1,NAM2,M

100 FORMAT(1H1,//////5X,* DISTRIBUTION OF *,2A2,
1x FOR FLEET SIZE M=*,16//)
GO TO 20

10 WRITE(6,110) NAM1,NAM2,NS
110 FORMAT(1H1,//////5X,* DISTRIBUTION OF *,2A2,
1x FOR SAMPLE SIZE N=*,13//)

20 CALL LIST (A,NSMPL,IO,NAM1,NAM2)
RETURN
END
SUBROUTINE LIST(A,NPT,IO,NAM1,NAM2)
DIMENSION A(1),P(6)

C
TO LIST CORRESPONDING REALIZATION VALUES FOR
SPECIFIC PROBABILITY LEVELS.
C
DATA P/0.98,0.99,0.995,0.998,0.999,0.9995/
NPT1=NPT+1
ANPT1=FLOAT(NPT1)
ND=(NPT+1)/20
IF (ND .LT. 1) ND=1
DO 10 I=ND,NPT+ND
PROB=FLOAT(I)/ANPT1
WRITE(6,100) I,PROB,NAM1,NAM2,A(I)
10 CONTINUE
DO 20 I=1,6
J=IFIX(ANPT1*P(I))
WRITE(6,100) J*,A(I)
IF (J.EQ.NPT) GO TO 30
20 CONTINUE
100 FORMAT(5X,9*Sample No.*=*95,3X,*Prob=*,F7.4,3X)
30 RETURN
END

SUBROUTINE ASCEN(S,NPT)
DIMENSION S(1)

C
TO ARRANGE GIVEN ARRAY S IN ASCENDING ORDER
IF (NPT .LT. 2) RETURN
N2=NPT/2
DO 20 J=1,N2
MM=NPT-J+1
K=J
M=MM
DO 10 I=J,MM
IF (S(I) .LT. S(K)) K=I
IF (S(I) .GT. S(M)) M=I
10 CONTINUE
C TEST FOR J=K
IF (J .NE. K) GO TO 50
IF (MM .EQ. M) GO TO 20
TEMP=S(M)
S(M)=S(MM)
S(MM)=TEMP
GO TO 20
C TEST FOR MM=M
50 IF (MM .NE. M) GO TO 60
TEMP=S(J)
S(J)=S(K)
S(K)=TEMP
GO TO 20
60 IF (J .EQ. M) GO TO 30
C J/=M,MM/=K
TEMP=S(J)
S(J)=S(K)
S(K)=TEMP
TEMP1=S(MM)
S(MM)=S(M)
S(M)=TEMP
GO TO 20
30 IF (MM .EQ. K) GO TO 40
C
J=M
TEMP1=S(MM)
S(MM)=S(M)
S(M)=TEMP
TEMP=S(J)
S(J)=S(K)
S(K)=TEMP
GO TO 20
C
J=M, MM=K
40 TEMP=S(J)
S(J)=S(MM)
S(MM)=TEMP
<0 CONTINUE
RETURN
END

SUBROUTINE SAMPL(U, V0, V0S, IX, N, NSMPL, IO)
DIMENSION U(1), V0(1), V0S(1), Y(20), RAN(20), ALY(20)
C
TO GENERATE NSMPL NUMBER OF SAMPLE SETS OF SIZE N
FOR U, V0 AND V0S.
SAMPLE SIZE N MUST BE LESS THAN OR EQUAL TO 20 FOR
MINI-COMPUTER.
DU=0.2
NPT=400
DO 100 L=1, NSMPL
CALL OGEN(Y, N, IX, RAN)
KASE=0
PAR5=0.
DO 110 I=1, N
ALY(I)=ALUG(Y(I))
PAR5=PAR5+ALY(I)
110 CONTINUE
PAR5=PAR5/FLOAT(N)
UMIN=0.01
C
FIND THE SOLUTION OF PHI(U) IN MLE EQUATION.
DO 120 J=1, NPT
UJ=FLOAT(J-1)*DU+UMIN
10 PAR1=0.
PAR2=0.
DO 130 I=1, N
YIU=Y(I)*UJ
PAR1=PAR1+YIU*ALY(I)
PAR2=PAR2+YIU
130 CONTINUE
PHI2=PAR1/PAR2-1./UJ-PAR5
IF (ABS(PHI2) .LT. 0.001) GO TO 20
IF (J .EQ. 1) GO TO 30
IF (KASE .EQ. 1) GO TO 20
IF (PHI1 .GT. 0.) GO TO 40
IF (PHI2 .LT. 0.) GO TO 30
GO TO 50
40 IF (PHI2 .GT. 0.) GO TO 30
50 U1=U1-PHI2/(PHI2-PHI1)*U
KASE=1
GO TO 10
C
30 PHI1=PHI2
120 CONTINUE
20 U(L)=U1
US=1.*U1
VOS(L)=PAR2/FLOAT(N)
UV0=US*ALOG10(VOS(L))
IF (UV0 .GT. -30.) GO TO 60
VO(L)=O.
GO TO 100
60 IF (UV0 .LT. 30.) GO TO 70
VO(L)=1.E30
GO TO 100
70 VO(L)=VOS(L)**US
C
100 CONTINUE
RETURN
END

SUBROUTINE DGEN(YNIX, SAN)
DIMENSION Y(1), RAN(1)
C
C SUBROUTINE TO GENERATE EXPONENTIAL RANDOM NUMBER
C OF SIZE N
C BY Y=-LN(X) , WHERE X -- UNIFORM RANDOM NUMBER
C
DO 10 I=1,N
CALL RANDU(I, IY, RAN(I))
IY=IY
10 Y(I)=-ALOG(RAN(I))
RETURN
END

SUBROUTINE RANDU(I, IY, RAN)
M=2**32
K=2**7+1
IY=MOD(I*K+1, M)
RAN=FLOAT(IY)/FLOAT(M)
RETURN
END