FOR FURTHER TRAN

12

AD-E430 037

ADA055707

AD No.
DDC FILE COPY

MEMORANDUM REPORT ARBRL-MR-02831
(Supersedes IMR No. 550)

ON BALLOTING OF PROJECTILES

Kevin S. Fansler

April 1978

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (14) MEMORANDUM REPORT ARBRL-MR-02831	2. GOVT ACCESSION NO.
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) ON BALLOTING OF PROJECTILES,	5. TYPE OF REPORT & PERIOD COVERED (9) Final rept.,
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Kevin S. Fansler	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U. S. Army Ballistic Research Laboratory (ATTN: DRDAR-BLL) Aberdeen Proving Ground, MD 21005	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (16) RDT&E 1L161102AH43
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Armament Research and Development Command U. S. Army Ballistic Research Laboratory (ATTN: DRDAR-BL) Aberdeen Proving Ground, MD 21005	12. REPORT DATE (11) APR [redacted]78 13. NUMBER OF PAGES 55 (12) 48p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (18) SBIE (19) AD-E450 1737	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This report supersedes Interim Memorandum Report No. 550, April 1977.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Projectile-Gun Tube Interactions	Liquid Metals
Balloting	Lubrication
Friction, High Speed Sliding	Gun Barrels
Rotating Bands, Friction	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (ner)

The possibility for balloting-energy growth is examined. The basic equations are developed and the important parameters for balloting motion are obtained and discussed. It is concluded from the analysis that balloting-energy growth can not occur for the balloting model assumed in the present treatment.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

393411

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. INTRODUCTION

The energy contained in a shell's transverse motion directed toward a bore surface has supposedly become large enough on certain occasions to damage both shell and gun tube. As Gay[1] has shown, the theories of Reno[2] and Thomas [3] do not predict that such large balloting energies may develop. Reno does not consider sliding friction and assumes that the plane of yaw rotates with the shell. Thomas[3] does not use this constraint, but assumes that there exists sliding friction between bourrelet and bore. Chu and Soechting[4] extend Thomas'[3] theory to assume sliding friction between rotating band and bore, and also assume that the shell may have an eccentricity in its center of gravity; here also, the balloting energy should decrease with time.

More recently, Walker[5] has developed a theory that predicts that growth in balloting energy may occur. His theory extends Thomas' theory by further assuming friction between the bore and rotating band. According to Walker, the impact impulse generated by the bourrelet hitting a land is followed (due to the finite speed of the elastic wave) by a reaction force impulse that occurs on the opposite side of the shell at the rotating band. This causes an added frictional force on this part of the rotating band and results in an added torque impulse that will possibly increase the total transverse angular momentum possessed by the shell. The magnitude of this increment of added angular momentum is proportional to the effective coefficient of friction for the rotating band. Walker's theory is used in an attempt to explain the breakup of the 8-inch

---

1. *H. P. Gay, "Notes on the Yawing Motion of a Projectile in the Bore," BRL Report 2259, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, January 1973. AD 908456L.*

2. *F. V. Reno, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," BRL Report 320, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, February 1943. AD 491839.*

3. *L. H. Thomas, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," BRL Report 544, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, May 1945. AD PB22102.*

4. *S. H. Chu and F. K. Soechting, "Transverse Motion of an Accelerating Shell," Technical Report 4314, Picatinny Arsenal, Dover, NJ, June 1972. AD 894572L.*

5. *E. H. Walker, "Yawing and Balloting Motion of a Projectile in the Bore of a Gun with Application to Gun Tube Damage," BRL MR 2411, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1974. AD 923913L.*

M106 shell in the XM201 gun tube[5,6].

The motivation for the present investigation was a desire to apply Walker's[5] theory of balloting to Chu and Soechting's[4] computer description of in-bore motion. In studying Walker's theory, however, we decided that it needed revision. The values he assumed for the coefficients of friction are thought to be too large by an order of magnitude. When this and other errors are corrected, a completely different picture of balloting-energy growth and decay emerges. It is the purpose of this report to present this more accurate picture and to establish upper-bound limits on the energy growth rate or decay.

An apparent error in the Walker formulation concerns the magnitude of the reaction impulse at the rotating band. Walker claims that the reaction force at the rotating band is equal and opposite to the impact force on the bourrelet. Walker deduces this from his following statement: "The force component Y' is given by the requirement that the sum of the forces acting in the y direction is zero." Here Y' is the normal reaction force at the rotating band. If this were a statics problem, his approach might be valid. However, this is a dynamics problem; the projectile is required to rotate about the point C. With this requirement, it is found that the value of the reaction force depends upon the location of the center of gravity, the radius of gyration and the distance between the rotating band and bourrelet.

According to Walker's[5] theory, since the growth rate of balloting energy is a strong function of the value of the coefficient of friction at the rotating band, it is an important parameter. Walker used a rotating-band coefficient of friction of 0.55 for a gilding metal band, a value that might be expected for slow sliding velocities under small normal pressures. In order to arrive at reasonable values for the coefficient of friction, both the theory and experiment were investigated. According to a report[7] summarizing the Franklin

---

6. *C. M. Glass, "Fracture of an 8-inch M106 Projectile in an M110E2 Howitzer," BRL Report 1905, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, August 19 . AD B013343L.*

7. *R. S. Montgomery, "Friction and Wear at High Sliding Speeds," Wear, Vol. 36, 1976, pp. 275-298.*

Institute's experiments, the coefficient of friction decreases for increasing values of both sliding velocities and normal pressures. Herzfeld and Kosson[8] postulated that a hydrodynamic film exists between the rotating band and bore at the higher speeds; they found agreement with much of the experimental data available at the time. Their theory included plastic rotating bands; they found even lower values for the friction coefficients of plastic rotating bands. In contrast, Bowden[9] describes an experiment that shows no hydrodynamic film being generated at typical in-bore projectile velocities. Nevertheless, the normal pressures for this experiment were much less than found at the rotating band and bore interface. Thus, although Herzfeld and Kosson's[8] theory agrees with some experiments and appears plausible, the theory is not completely confirmed.

II. BALLOTING-ENERGY ANALYSIS

The fundamental dynamics of balloting are developed and examined for two simple models. Utilizing these dynamics, an upper-bound value for balloting energy growth can then be developed without appealing to a detailed numerical analysis.

A. Dynamics of Balloting

As Walker did, we use Thomas' assumptions and additionally assume that there is friction between the rotating band and bore. The description of the motion is given in terms of Eulerian angles. The rotations defining the Eulerian angles are given according to Goldstein[10] in Figure 1. For more clarity, Figure 2 gives some of the axes in terms of the gun-projectile system.

---

8. *C. M. Herzfeld and R. L. Kosson, "A Theory of Bore Friction," BRL Report 851, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, March 1953. AD 10639.*

9. *F. P. Bowden, "Recent Experimental Studies of Solid Friction," Friction and Wear, R. Davies (Editor), Elsevier Publishing Co., Princeton, NJ, 1959, pp. 84-109.*

10. *H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Co., Inc., Cambridge, MA, March 1956, pp. 93-175.*

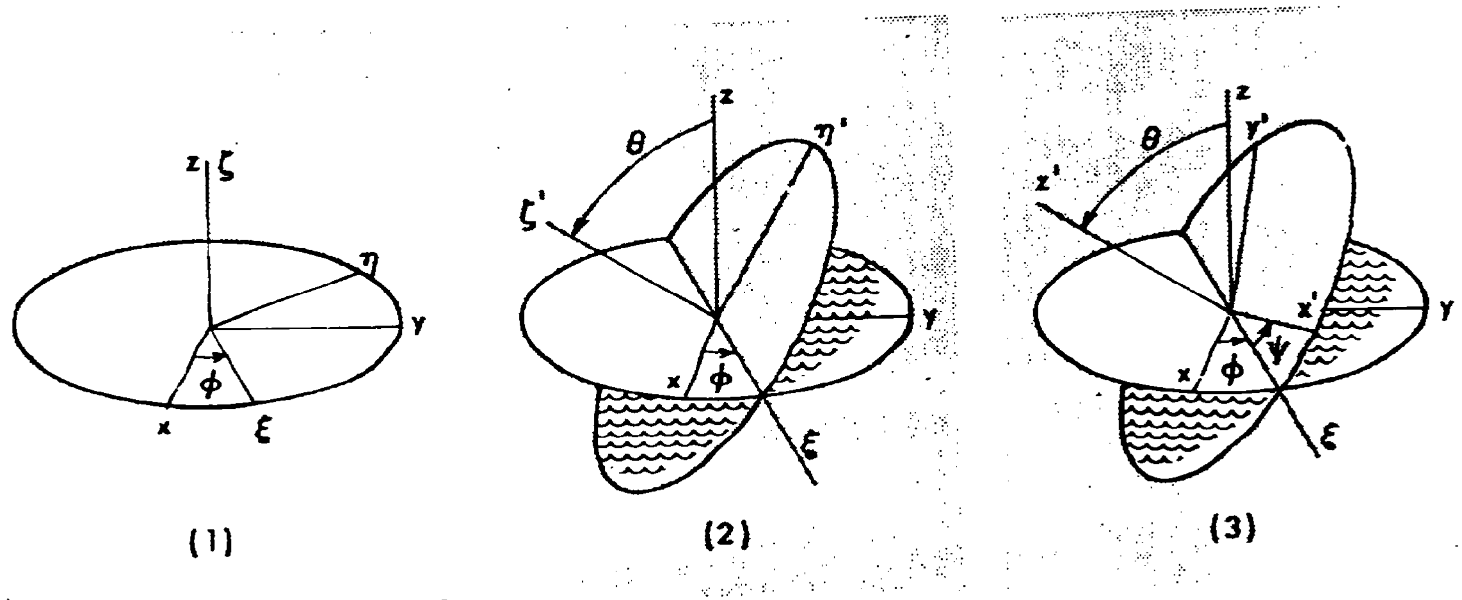

Figure 1. The coordinate axes and rotations defining the Euler angles. (The line of nodes is given by the ξ axis.)

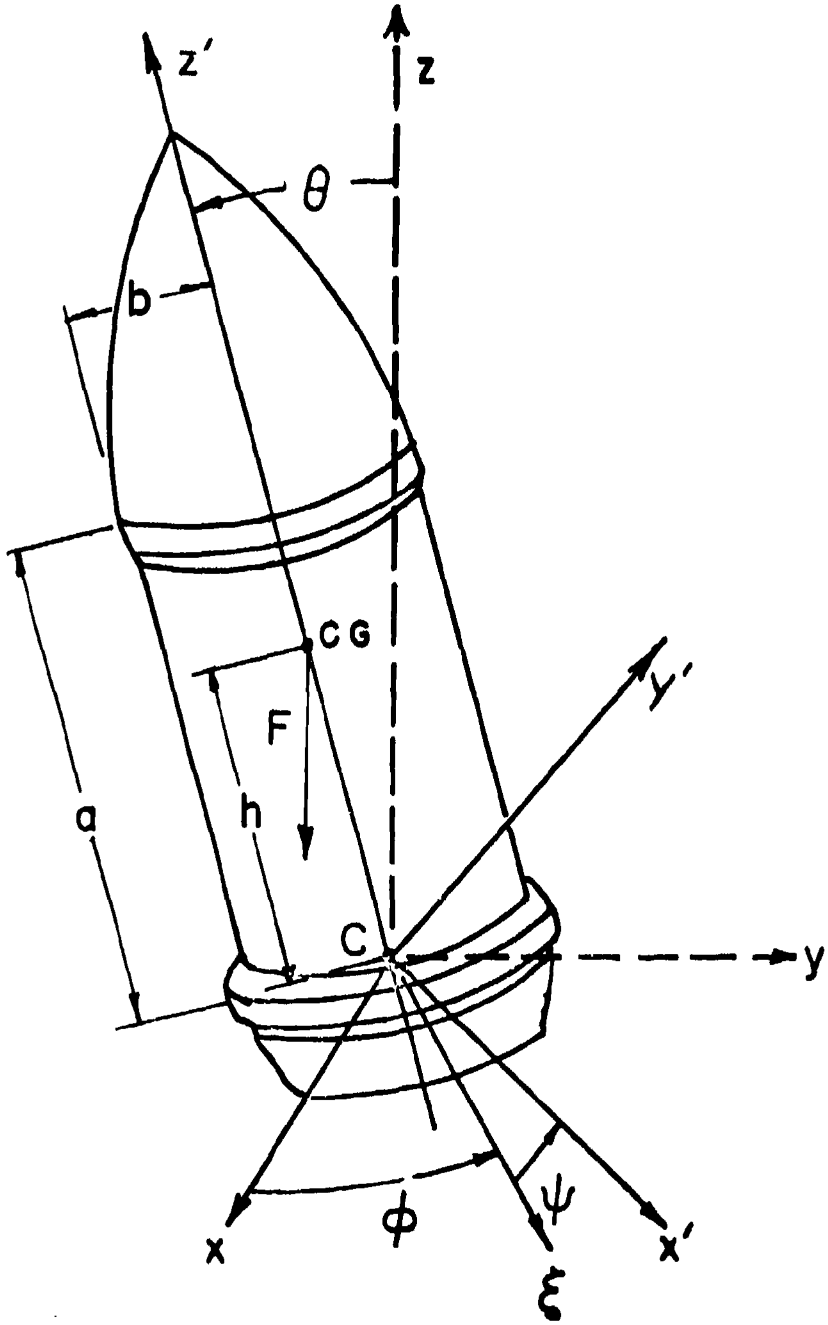

Figure 2. Description of projectile's orientation given in terms of Euler angles. The point C designates the center of the rotating band. The yaw about point C is defined by the angle θ and the yaw plane is defined by the angle ϕ between the x-axis and the line of nodes (ξ-axis). F is the inertial force along the bore line z; "a" is the distance between the rotating band and the bourrelet; "b" is the radius of the bourrelet, and h is the distance from C to the center of gravity.

The kinetic energy of an axially symmetric shell with respect to the point C is

$$T = \frac{1}{2} I(\omega_{y'}^2 + \omega_{x'}^2) + \frac{1}{2} A \omega_{z'}^2 \quad , \tag{1}$$

where ω_i is the angular velocity about the i-th axis and A is the moment of inertia about the line of axial symmetry. The transverse moment of inertia I about a line through C in the plane of the rotating band is

$$I = B + mh^2, \tag{2}$$

where B is the transverse moment of inertia about the c.g., m is the mass of the shell, and h is the distance from the point C to the c.g. of the shell. Transforming to the Eulerian angle description, one finds:

$$T = \frac{1}{2} I(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{1}{2} A (\dot{\psi} + \dot{\phi} \cos \theta)^2 \ . \tag{3}$$

For the projectile having the acceleration $\mathcal{G}$, the potential energy is

$$V = m \mathcal{G} h \cos \theta \ . \tag{4}$$

Following Walker now, except to vary the presentation for clarification purposes, we note that the Lagrangian is

$$L = T - V.$$

The Lagrange equations are

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad , \tag{5}$$

where

$$q_1 = \theta; \ q_2 = \phi; \ q_3 = \psi \tag{6}$$

and the Q_i are the generalized constraint forces imposed upon the projectile by the gun tube. The differential equations of motion are developed in detail in Appendix A. These expressions differ somewhat from Walker's formulation but not significantly for the purposes of this analysis.

As a first step in the investigation of balloting motion, we can attempt to construct a model that is consistent with Walker's claim that there exists a time lag between the impact impulse and the reaction impulse. A tentative hypothesis is made that the reaction impulse occurs immediately after the impact impulse occurs. In this model, the bourrelet impacts against the bore and the rotating band is assumed to be free of any constraining influences. Immediately after this impact process, the reaction impulse is applied to the rotating band and the bourrelet is assumed to be free of any constraining influences. In this rigid-body approximation, the value of the reaction impulse is determined by the constraint that the rotation must be about the point C in the plane of the rotating band (Figure 2).

It is convenient to express the kinetic energy in terms of the c.g. translational motion plus the rotational motion about the c.g. This expression is:

$$T = \frac{m}{2}\left(\dot{\eta}_{cg}^2 + \dot{z}_{cg}^2 + \dot{\xi}_{cg}^2\right) + \frac{m}{2} k^2 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{A}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 . \tag{7}$$

The subscript cg denotes the values for the center of mass for the projectile and k is the transverse radius of gyration for the projectile. Since the impact and reaction forces are of short duration, it is sufficient for the analysis to use the impulsive equations of motion and focus upon the motions immediately after an impulse occurs. For impulsive forces, it can be shown that

$$\left(\frac{\partial T}{\partial \dot{q}_i}\right)_1 - \left(\frac{\partial T}{\partial \dot{q}_i}\right)_o = P_{q_i} . \tag{8}$$

Here P_{q_i} is the generalized impulsive force for the q_i coordinate. The first term on the left is evaluated immediately after the impulsive force has acted and the term with the subscript o is evaluated immediately before the force has acted.

The details of the analysis are given in Appendix B. The effective coefficient of restitution, which includes the time lag effect, is given as

$$e_e = \frac{\left[e h' G \nu_e + h'(2a' - G\mu)(e+1) - 2h'^2 - 2k'^2\right]}{\left[h'(2h' + G\nu_e) + 2k'^2\right]} \tag{B15}$$

Here e is the usual coefficient of restitution, h' is the distance of the center of mass from the rotating band in calibers, a' is the distance between bourrelet and rotating band in calibers, k' is the radius of gyration in calibers, $G = 1/(1 + \pi^2/n^2)^{1/2}$ where n is the number of calibers traversed for rotation of the shell through one revolution, μ is the coefficient of friction at the bourrelet and ν_e is the effective coefficient of friction at the rotating band. Assuming the friction coefficients to be zero, a calculation of e_e may be easily made for the 8-inch projectile that was of interest to Walker. Here

$$a' = 1.135 \text{ calibers}, \; h' = 0.71 \text{ cal.},$$

$$k' = 1.066 \text{ cal.}, \; e = 0.7$$

and the twist is n = 20 cal/revolution. Substituting these values into Equation (B15), we obtain $e_e = -0.165$. Since e_e is negative, the resulting movement of the bourrelet is toward the bore and not away from the bore, as might be expected. With this simple model, the impacting process continues after the first reaction impulse. Thus, the hypothesis that the impacting process is completed before the reaction process is started is seen to be contradicted by the results from a model suggested by the hypothesis. Friction effects can not rescue the hypothesis from contradiction, since if $\mu > \nu_e$ (as will be noted later on), then e_e becomes negative.

These results do not rule out the possibility that part of the reaction process occurs after the impacting process is completed. Thus, there still exists the possibility that the presence of band friction might increase the effective coefficient of restitution.

If the impact forces and reaction forces overlap in time, another simple model is suggested. Let us consider that the impact and reaction forces occur simultaneously. The impulsive Lagrange equations for this case are

$$m(\dot{\eta}_{cg})_1 - m(\dot{\eta}_{cg})_o = P_\eta + (P_\eta)_b \tag{9}$$

$$mk^2\dot{\theta}_1 - mk^2\dot{\theta}_o = -(a-h)P_\eta + Gb\mu P_\eta + h(P_\eta)_b + Gb\nu_e(P_\eta)_b \tag{10}$$

Proceeding as before, we obtain

$$P_\eta = m\omega(1+e)\frac{[h(h+Gb\nu_e) + k^2]}{[a + Gb(\nu_e - \mu)]} \tag{11}$$

$$(P_\eta)_b = m\omega(1+e)\frac{[h(a-h-Gb\mu) - k^2]}{[a + Gb(\nu_e-\mu)]} \tag{12}$$

The ratio R, of $(p_\eta)_b$ to p_η, has the same values obtained for the delayed-impulse model. The value of R does not depend on the details of the impact and reaction processes; rather, it is determined only by the requirement that rotation must occur around the center of the rotating band, and hence should be invariant between the two models, as indeed it is. This value of R, given in terms of nondimensionalized quantities, is

$$R = \frac{h'(2a' - 2h' - G\mu) - 2k'^2}{h'(2h' + G\nu_e) + 2k'^2} \qquad (13)$$

Here it is seen that R essentially depends upon a number of parameters and is not equal to -1 as Walker has asserted.

Here, the effective coefficient of restitution e_e is simply e and no increment in balloting energy can be obtained from the effects of either band friction or bourrelet friction. Even though no possibility exists for balloting energy amplification, it would be desirable to minimize the magnitude of the impact impulse. Thus, according to Eq. (12), the distance "a" from rotating band to bourrelet should be as large as possible.

G. Soo Hoo[11] has recently modeled the bourrelet and rotating band as springs in his computer description of the projectile motion. It appears that the dynamics of his model could be approximated by utilizing a combination of the two models previously discussed. The previous models assume the forces are applied for an infinitesimal time. The finite application time of the forces in the springs model produces motions that differ somewhat from the impulsive model motions. Nevertheless, if the time of application of the spring forces on the bourrelet is short compared with the time between impacts, the impulsive models might be sufficient to describe the approximate dynamics.

If the spring at the rotating band is stiff compared to the bourrelet spring, the forces on the rotating band will closely follow the pattern of the impact forces. The resultant dynamics will then approximate that for the simultaneous model constructed earlier. If

---

11. *G. Soo Hoo, "A Theoretical Model for In-Bore Projectile Balloting/ Barrel Motion," Workshop on Dynamics of Precision Gun Weapons sponsored by U. S. Army Armament Command, General Thomas J. Rodman Laboratory, Rock Island Arsenal, January 1977.*

the rotating band spring is relatively soft, then the dynamics will tend toward that observed for the time-lag model. Now, if the rotating-band springs are somewhat softer than the bourrelet springs, the impacting process might be completed before the reaction process. That part of the reaction process taking place after the impacting process is completed could be approximated by the time-lag model. Part of the impact-impulse process would utilize the time lag model; the amount would be determined by the ratio R which has been previously discussed. The dynamics for the rest of the impaction and reaction processes would utilize the simultaneous-impulse model.

B. Upper-Bound Value for Energy Growth

To obtain the maximum value possible for e_e and energy growth rate, three assumptions will be made. Firstly, it will be assumed that impact forces and reaction forces occur simultaneously. Secondly, it will be assumed that there is no axial torque. Thirdly, the corresponding torque impulse due to frictional forces occurring at the rotating band is assumed to occur immediately after the impaction and reaction processes. Since large balloting energies may be possible according to some evidence, these possibilities can be explored even though the dynamics of the motion may be complicated.

Comparing Eq. (11) and Eq. (B6), it is noticed that for conventionally designed projectiles, the impulses for simultaneous processes are larger than the impulses for the time-lag processes. To obtain a maximum value for the reaction impulse at the rotating band, the simultaneous model will be used.

In this model, it is assumed that there is no axial torque. Since axial torque is present, the effects of axial torque should be investigated. Goldstein[10] shows that when the projectile is not making contact with the bourrelet and there is no axial torque, $|\theta|$ depends only on θ. Thus, the balloting energy immediately after one impact would equal the balloting energy immediately before the next impact. With the presence of an axial torque, whether $|\theta|$ increases or decreases for subsequent equal values of θ may be examined by first integrating Eq. (A10)

$$A\omega_{z'} = A\omega_{z'_0} + N_a \int (\cos\theta)\, dt \tag{14}$$

where N_a is assumed constant. Here t is the elapsed time since $\theta = \theta_0$ where θ_0 is any specified possible value of θ. The quantity $\omega_{z'_0}$ is the value of $\omega_{z'}$ at the initial time. Likewise Eq. (A9) may be integrated in the same way and with some substitutions between the resulting equation and Eq. (A8), one can obtain

$$\dot{\phi} = \dot{\phi}_o \frac{\sin^2\theta_o}{\sin^2\theta} + \frac{N_a}{I \sin^2\theta}[t - \int(\cos\theta)dt] + \frac{A(\omega_z)_o}{I \sin^2\theta}(\cos\theta_o - \cos\theta) \quad (15)$$

Now consider when $\theta = \theta_o$ at a later time (but before the next impact). Substituting Eqs. (14) and (15) into Eq. (A8), one finds the difference in the angular acceleration is

$$I\,[\ddot{\theta}(t) - \ddot{\theta}(0)] = \quad (16)$$

$$\frac{N_a}{\sin\theta_o}[t - \int(\cos\theta)dt]\,[2\dot{\phi}_o \cos\theta_o - \frac{A(\omega_z)_o}{I} - \frac{N_a}{I}\int(\cos\theta)dt]$$

$$- \dot{\phi}_o N_a \sin\theta_o \int(\cos\theta)dt + \frac{N_a^2 \cos\theta_o}{I \sin^3\theta_o}[t - \int(\cos\theta)dt]^2$$

For small values of θ_o, the sum of the terms on the right hand side of Equation (16) is always negative. Hence, $\ddot{\theta}$ (t) is always less than or equal to $\ddot{\theta}$ (0). The torque, then, tends to align the axis of the projectile with the gun-bore axis, and hence results in softened subsequent impacts. Thus if we neglect the torque, we can obtain an upper bound value for balloting-energy growth rate. Moreover, the analysis becomes simpler.

By the third assumption, the friction force is applied at the rotating band immediately after the impact and reaction processes have occurred. Although the late application of the frictional force is not ruled out, such a possibility would be difficult to explain. However, the third assumption covers this possibility and at the same time leads to an upper bound on the energy growth rate. The value of e_e may then be greater than e. The expression for the maximum angular momentum due to this application of frictional force on the rotating band is, from Eqs. (3) and (8),

$$I\,\Delta\dot{\theta} = bG(P_\eta)_b\,\nu_e\,, \quad (17)$$

where it is assumed that $\nu_e = 0$ in the expression for $(P_\eta)_b$.

It is important that the torque due to friction be applied immediately after the occurrence of the simultaneous impulse; the analysis below shows this. The kinetic energy can be shown to vary according to

$$\frac{dT}{dt} = \vec{N}\cdot\vec{\omega} \quad . \tag{18}$$

Consider the change in transverse-motion energy caused by the torque $\vec{N}_t$ whose direction, according to Appendix C, lies along the line of nodes for the projectile's position at the time of the impact impulse. This torque $\vec{N}_t$ is caused by the presence of friction at the driving band. Then, integrating Eq. (18) and retaining only those terms giving the change in transverse-motion energy, we obtain:

$$\Delta T_t = \int \vec{N}_p\cdot\vec{\omega}_p \, dt. \tag{19}$$

Here, $\vec{N}_p$ and $\vec{\omega}_p$ are the vector projections of $\vec{N}_t$ and the angular velocity $\vec{\omega}$, respectively, onto the plane of the rotating band. The integrand is integrated over the duration of nonzero torque. According to a report of Gay[1] that gives the motion of the shell's C.G. relative to the bore axis, the angle between $\vec{\omega}_p$ and $\vec{N}_p$ will increase with time after impact so that the dot product of the two vectors will decrease. Furthermore, the magnitude of ω_p will be smaller with smaller values of θ. Thus, the largest increment of energy given to the total energy will occur if the reactive impulse occurs immediately after impact of the bourrelet.

From the third assumption, we find an expression for the angular velocity immediately after the frictional torque is applied:

$$\dot{\theta}_2 = \dot{\theta}_1 + \Delta\dot{\theta} \quad , \tag{20}$$

where $\dot{\theta}_1$ is the value of $\dot{\theta}$ immediately before the frictional torque is applied. According to the first assumption about coincidence of the impulses:

$$\dot{\theta}_1 = e\omega \quad , \tag{21}$$

where ω is the transverse angular speed before impact. The effective coefficient of restitution is simply

$$e_e \equiv - \dot{\theta}_2/\omega \quad . \tag{22}$$

Thus, using Eqs. (20), (21), (17) and (12) it is found that

$$e_e = e - (1+e) \left[\frac{GR}{2\,a' - G\mu}\right]\nu_e \quad , \tag{23}$$

where R is given by Eq. (13) and a' is the value of a in calibers.

According to the definition, Eq. (22), the balloting energy should grow if $e_e > 1$; if $e_e < 1$, the balloting energy should decay. From Eq. (23), whether e_e is greater or less than one depends upon the value of e, the expression in brackets and the value of ν_e. The value of e will probably not exceed 0.8 except for very soft balloting action. The expression in brackets will be near - 1/4 for many conventional projectiles. Thus, whether e_e is greater or less than one hinges upon ν_e. The effective coefficient of friction will be discussed in the next section.

An upper-bound value for energy growth rate can be obtained in terms of e_e. With the second assumption that the axial torque is ignored, the angular speed immediately before impact is e_e multiplied by the angular speed immediately before the last impact. After n impacts, $\dot{\Theta}$ would be

$$\dot{\Theta} = \omega e_e^{\,n} \tag{24}$$

where

$$\dot{\Theta} = |\dot{\theta}|$$

Now treating n as a continuous variable, n can be expressed as $n = \Theta/(2\Delta)$ where $\Delta = c/a$. Here, c is the clearance between the bore and bourrelet when the shell is centered in the bore. Substituting the expression for n into Eq. (24) and integrating, the following expression can be obtained:

$$t = \frac{2\Delta\ (1 - e_e^{-\Theta/(2\Delta)})}{\omega\ \ln e_e} \tag{25}$$

Since the balloting energy ε equals $\frac{1}{2} I\dot{\Theta}^2$, manipulation of Eqs. (24), (25) and the expression for n shows that

$$\varepsilon = \varepsilon_o/[1 - (\varepsilon_o/2I)^{1/2}\gamma t]^2 \tag{26}$$

where

$$\gamma = (\ln e_e)/\Delta$$

Equation (26) was also obtained by Walker, but with a completely different expression for γ.

III. FRICTION COEFFICIENTS

Since the band coefficient of friction is found to be an important parameter for describing balloting motion, it will be discussed in some detail. Friction experiments have been conducted by the Franklin Institute[7,12], by Bowden[9] and by Sauer[13], who obtained friction coefficients using a rocket sled. The pressures used in Sauer's experiments, though high, were much less than experienced by a rotating band in-bore. Nevertheless, he obtained evidence that a molten metal film existed, at least for the higher pressures. In addition, he developed a theory for wear and friction assuming molten metal at the interface[14]. In the present paper, it will be assumed that a liquid film exists at the rubbing interfaces for both plastics and metal rotating bands.

As mentioned earlier, experiments[7,12] show that the coefficient of friction decreases for increasing values of both sliding velocities and normal loadings. With these data and some data obtained for higher pressures by using a shell pusher, Pilcher and Wineholt[15], in a correlation study, have obtained the coefficient of friction as a function of velocity and pressure. These data were obtained for a steady-state process; i.e., the temperature profile through the thickness of the traveling block changes rapidly shortly after initiation of the friction process and thereafter approaches a steady-state profile. Since the shell's travel through the length of the gun-tube is a transient process, there exists the possibility

---

12. *W. W. Shugarts, Jr., "Frictional Resistance and Wear at High Sliding Speeds," The Franklin Institute Laboratories for Research and Development, IR No. I-2448-2, June 15, 1955.*

13. *F. M. Sauer, "Fundamental Mechanism of Wear and Friction of Unlubricated Metallic Surfaces at High Sliding Speeds," U.S. Naval Ordnance Test Station Report No. 1729, China Lake, CA, April 1957.*

14. *F. M. Sauer, "Analysis of Steady-State Metallic Friction and Wear Under Conditions of Molten Metal Film Lubrication," Stanford Research Institute, Technical Report No. 1, Project No. SU-1494, December 5, 1956.*

15. *J. O. Pilcher and E. M. Wineholt, "Analysis of the Friction Behavior at High Sliding Velocities and Pressures for Gilding Metal, Annealed Iron, Copper, and Projectile Steel," In-Bore Dynamics Symposium sponsored by the TTCP Technical Panel W-2, 1976.*

that steady-state conditions will not be approximated. Nevertheless, the application of an equation developed by H. G. Landau[16] shows that steady-state conditions will be approximated for most of the projectile's travel in-bore. The application of Herzfeld and Kosson's[8] theory confirms the results obtained by Landau's[16] equation.

The friction coefficient values obtained are for steady pressures of the band on the bore surfaces. The friction coefficient values we are interested in, however, correspond to the changes in friction forces caused by the reaction force. The cross product of the radius vector with the resultant incremental friction force, when integrated around the rotating band's periphery, yields an expression for the transverse torque. It is shown in Appendix C that the effective coefficient of friction to be used for the reaction process is

$$\nu_e = \nu + \frac{\partial \nu}{\partial p} \qquad \text{(C6)}$$

where p is the pressure on the rotating band. This effective friction coefficient may also be used in other applications where there is a net lateral force on the rotating band. To show how important the last term in this equation is, we first give the value of the friction coefficient ν in Figure 3. Using Pilcher and Wineholt's[15] formulation, ν was calculated for the 8-inch M106 projectile that broke up in-bore. Here it is seen that the coefficient of friction decreases with the distance that the shell has traveled in-bore. For most of the distance traveled, the friction coefficient is an order of magnitude lower than common handbook values.

In Figure 4, the effective coefficient of friction is plotted as a function of the distance traveled from the breech. In contrast to the results of Figure 3, Pilcher and Wineholt's[15] formulation yields negative values of ν_e for the large pressures encountered at the rotating band. Although their formulation appears to be a creditable correlation of the data, the above results are contrary to what might be expected for liquid films.

Herzfeld and Kosson's[8] theory for liquid films may be used to get a value for the effective coefficient of friction. As shown in Appendix D, the Herzfeld and Kosson theory implies that ν_e is at most 10^{-3}. Herzfeld and Kosson's[8] theory is also valid for plastic rotating bands. The value of ν_e for plastic rotating bands might be

---

16. *H. G. Landau, "Heat Conduction in a Melting Solid," Quarterly Journal of Applied Mathematics* <u>8</u>, *No. 1, April 1950, pp. 81-94.*

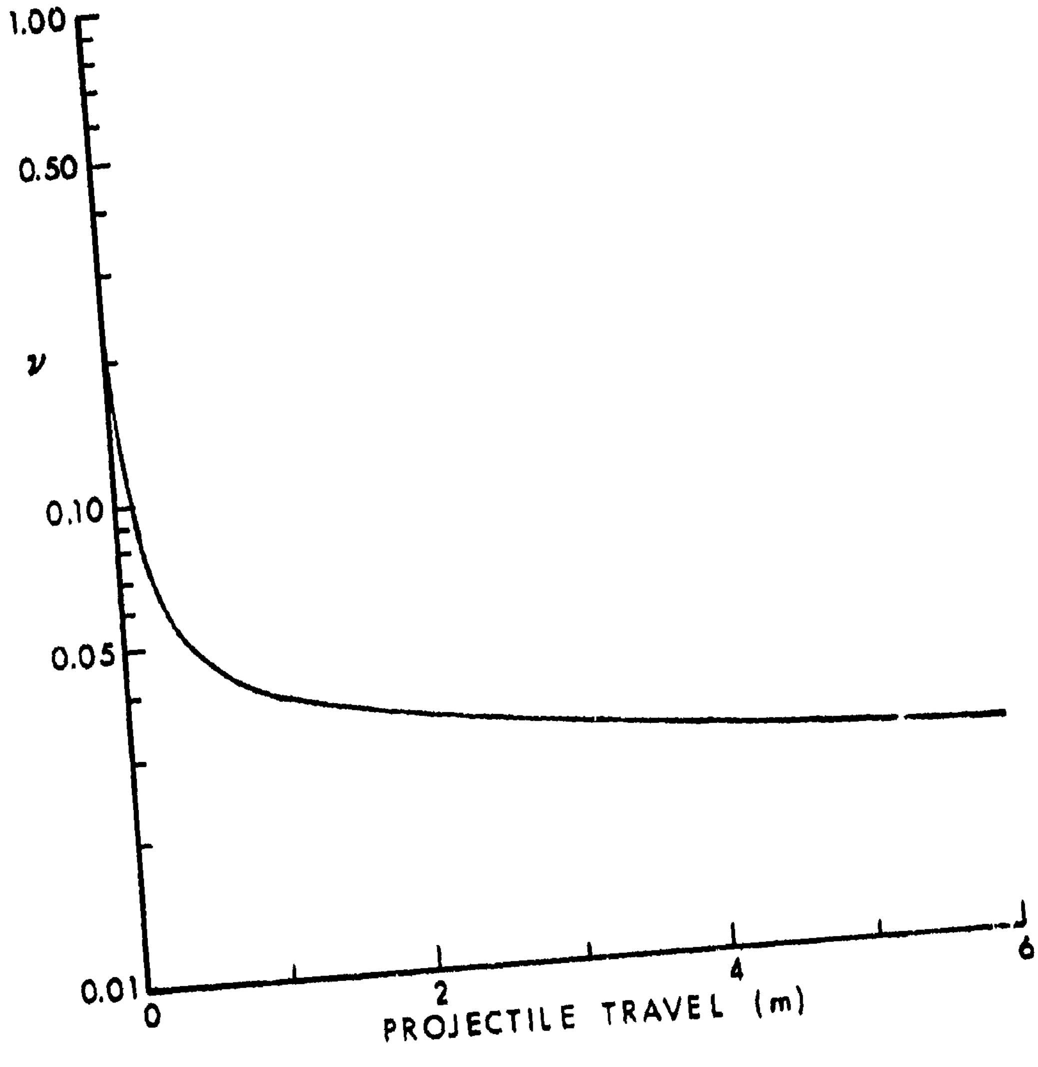

Figure 3. Rotating-band friction coefficient of 8-inch M106 projectile as a function of displacement from breech. The pressure at the rotating-band is estimated as 3.1×10^8 Pa.

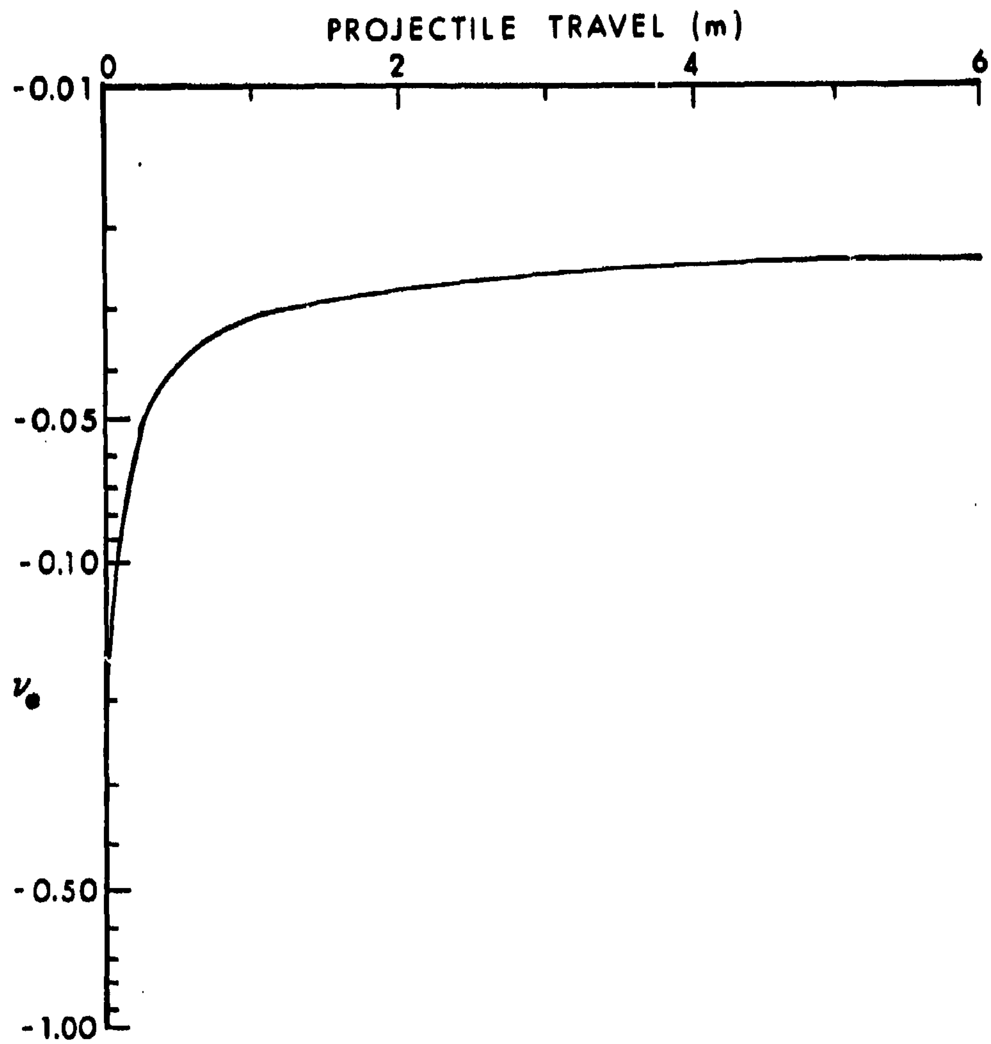

Figure 4. Effective friction coefficient of 8-inch M106 projectile as a function of displacement from breech.

of the same order of magnitude as found for gilding metal bands.

It might seem that during the reaction impulse, the liquid film could become much reduced in thickness and essentially solid-to-solid contact could be made. This form of contact would tend to raise the effective coefficient of friction. Appendix D discusses this possibility. In this Appendix, it is shown that the liquid layer is thinned only an insignificant amount and solid-to-solid contact is not made.

IV. DISCUSSION

It is not likely that large values for the effective friction coefficient would be encountered in practice. From liquid film theory, Appendix D shows that ν_e is at most 10^{-3}. For special circumstances, however, the effective coefficient of friction could possibly become much larger than the liquid theory would predict. Perhaps, for some projectiles, the rotating band could wear away to such an extent that the band pressure would be much reduced; a liquid film might no longer be present. Then, according to Bowden and also the Pilcher and Wineholt[15] formulation, the effective coefficient of friction might be near 0.2. Balloting energy growth still cannot occur for this situation. If, however, Walker's values for friction coefficients were used, the effective coefficient of restitution would be $e_e = 1.02$ ($e = 0.7$) and balloting energy growth could occur, although at a much slower maximum rate than predicted by Walker. Here, $R = -0.615$, while if Walker's value of $R = -1$ is used ("Equal to and opposite reaction force") the balloting energy growth would be almost as great as he obtained.

If the Pilcher and Wineholt[15] formulation is used, Figure 4 shows that the effective coefficient of friction is a negative quantity. According to Eq. (23) and since R is negative, $e_e < e$. In fact, since ν_e is negative, e would be the upper-bound value for the effective coefficient of restitution. These negative values for ν_e cause damping of the balloting motion and a more rapid balloting-energy decay rate.

As mentioned earlier, the negative values of ν_e are contrary to the results of liquid film theory. Although Pilcher and Wineholt's formulation appears to be generally creditable, no detailed experimental data were available for correlation studies at the higher pressures and velocities encountered in the gun-tube environment. They did, however, have data from shell-pusher experiments of the higher pressures but low velocities. They then extrapolated to obtain a formulation predicting the coefficients of friction at higher pressures and velocities. For ν, their formulation agrees roughly with the Herzfeld and Kosson theoretical results. Nevertheless, the corresponding effective coefficients of friction differ in a dramatic fashion, although both formulations predict small magnitudes for ν_c. Thus, it is asserted with some confidence that e_e will be less than one. Since gun-tube and projectile damage does occasionally occur, it would be interesting to hypothesize further as to how damage could have occurred. Suppose the impact and reaction impulses happened simultaneously. The resulting expression for the impact impulse is given by Eq. (11). It is assumed that the effective friction coefficient is positive, while the bourrelet friction coefficient is about 0.2. The value of R is normally negative. Now, suppose by some accident or design flaw, the effective value of "a" might be decreased considerably, even to the point where the magnitude of the first term would be comparable to the magnitude of the second term. Then, the impact impulse could possibly become large enough so that the gun-bore surface could be damaged even though the balloting energy might be small.

V. SUMMARY AND CONCLUSIONS

The dynamics of the projectile motion as proposed by Walker have been corrected. Then, instead of examining the details of the balloting motion to obtain energy growth rates, it was decided to simplify the analysis by seeking an upper bound for the energy growth rate. This was done by adopting the following model for projectile in-bore motion: the maximum value for the reaction impulse is used, axial torque on the projectile is ignored, and it is assumed that the reaction impulsive torque due to friction forces is applied immediately after the impact and reaction impulses occur. From this model, the effective coefficient of restitution is found to be a linear function of the effective friction coefficient for the driving band.

The range of these values is known approximately from experiment; since these values are small, it can be concluded that the balloting energy can only decrease from its initial value. This conclusion applies to both metallic and plastic rotating bands. Other mechanisms must be sought to explain balloting-energy growth, if it does occur.

ACKNOWLEDGMENTS

I have appreciated the help of Mr. G. D. Kahl and Mr. J. O. Pilcher.

REFERENCES

1. H. P. Gay, "Notes on the Yawing Motion of a Projectile in the Bore," BRL Report 2259, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, January 1973. AD 908456L.

2. F. V. Reno, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," BRL Report 320, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, February 1943. AD 491839.

3. L. H. Thomas, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," BRL Report 544, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, May 1945. AD PB22102.

4. S. H. Chu and F. K. Soechting, "Transverse Motion of an Accelerating Shell," Technical Report 4314, Picatinny Arsenal, Dover, NJ, June 1972. AD 894572L.

5. E. H. Walker, "Yawing and Balloting Motion of a Projectile in the Bore of a Gun with Application to Gun Tube Damage," BRL MR 2411, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1974. AD 923913L.

6. C. M. Glass, "Fracture of an 8-inch M106 Projectile in an M110E2 Howitzer," BRL Report 1905, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, August 1976. AD B013343L.

7. R. S. Montgomery, "Friction and Wear at High Sliding Speeds," Wear, Vol. 36, 1976, pp. 275-298.

8. C. M. Herzfeld and R. L. Kosson, "A Theory of Bore Friction," BRL Report 851, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, March 1953. AD 10639.

9. F. P. Bowden, "Recent Experimental Studies of Solid Friction," Friction and Wear, R. Davies (Editor), Elsevier Publishing Co., Princeton, NJ, 1959, pp. 84-109.

10. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Co., Inc., Cambridge, MA, March 1956, pp. 93-175.

11. G. Soo Hoo, "A Theoretical Model for In-Bore Projectile Balloting/Barrel Motion," Workshop on Dynamics of Precision Gun Weapons sponsored by U. S. Army Armament Command, General Thomas J. Rodman Laboratory, Rock Island Arsenal, January 1977.

12. W. W. Shugarts, Jr., "Frictional Resistance and Wear at High Sliding Speeds," The Franklin Institute Laboratories for Research and Development, IR No. I-2448-2, June 15, 1955.

13. F. M. Sauer, "Fundamental Mechanism of Wear and Friction of Unlubricated Metallic Surfaces at High Sliding Speeds," U.S. Naval Ordnance Test Station Report No. 1729, China Lake, CA, April 1957.

14. F. M. Sauer, "Analysis of Steady-State Metallic Friction and Wear Under Conditions of Molten Metal Film Lubrication," Stanford Research Institute, Technical Report No. 1, Project No. SU-1494, December 5, 1956.

15. J. O. Pilcher and E. M. Wineholt, "Analysis of the Friction Behavior at High Sliding Velocities and Pressures for Gilding Metal, Annealed Iron, Copper, and Projectile Steel," In-Bore Dynamics Symposium sponsored by the TTCP Technical Panel W-2. 1976.

16. H. G. Landau, "Heat Conduction in a Melting Solid," Quarterly Journal of Applied Mathematics 8, No. 1, April 1950, pp. 81-94.

17. H. Schlichting, Boundary Layer Theory, 6th ed., McGraw-Hill Book Company, Inc., New York, 1968, pp. 144-147, also pp. 108-114.

18. F. P. Bowden and D. Tabor, The Friction and Lubrication of Solids, Oxford University Press, Amen House, London, 1950, pp. 272-279.

APPENDIX A. DEVELOPMENT OF THE DIFFERENTIAL EQUATIONS OF MOTION

In this Appendix, the differential equations of motion are developed in detail. The kinetic energy, according to Eq. (3), is given as

$$T = \frac{1}{2}I(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{1}{2} A (\dot{\psi} + \dot{\phi} \cos\theta)^2 \tag{3}$$

The potential energy is given as

$$V = m \ddot{s}h \cos \theta \tag{4}$$

As before, the Lagrange equations are

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \tag{5}$$

where

$$q_1 = \theta; \; q_2 = \phi \; ; \; q_3 = \psi \tag{6}$$

The generalized force Q_1 in the θ direction is simply the torque about the line of nodes (the ζ axis in Figure 1). The expression for Q_1 is

$$Q_1 = - z \tilde{\eta} + \eta \tilde{z} - z_b \tilde{\eta}_b + \eta_b \tilde{z}_b \;, \tag{A1}$$

where $\tilde{\eta}$ and $\tilde{z}$ are the forces on the bourrelet along η and z respectively. The subscript b refers to the quantities at the rotating band. These forces and also the generalized forces are positive in the direction of their respective increasing coordinate values. From the geometry of Figure 2 we have that

$$z = a \cos \theta - b \sin \theta , \tag{A2-a}$$

$$\eta = -b \cos \theta - a \sin \theta , \tag{A2-b}$$

$$\eta_b = r \;, \tag{A2-c}$$

$$z_b = 0 \;. \tag{A2-d}$$

Where r is the radius of the rotating band. Thus,

$$Q_1 = - (a \cos \theta - b \sin\theta)\tilde{\eta} - (b \cos\theta + a \sin\theta)\tilde{z} + r \tilde{z}_b , \tag{A3}$$

Equations (A2-c) and (A2-d) follow from assuming that the rotating band deforms into a sphere of radius r.

For the ϕ coordinate, the torque will be in the z direction:

$$Q_2 = -\eta\,\tilde{\xi} - \eta_b\,\tilde{\xi}_b + N_a\,. \tag{A4}$$

Here N_a is the torque on the projectile transmitted through the rotating band.

From Eqs. (A2-b) and (A2-c), we have that

$$Q_2 = (b\cos\theta + a\sin\theta)\,\tilde{\xi} - r\,\tilde{\xi}_b + N_a\,, \tag{A5}$$

Again it is assumed that the rotating band deforms into a section of a sphere of radius r and thus the direction of N_a is along the gun-bore axis. Walker thought that N_a would lie along the projectile axis and so the last term in Eq. (A5) was multiplied by $\cos\theta$. For the ψ coordinate, the torque along z'-axis will be

$$Q_3 = -\eta'\tilde{\xi}' - \eta'_b\tilde{\xi}'_b + N_a\cos\theta \tag{A6}$$

Here we have that $\eta' = -b$, $\eta'_b = r$, $\tilde{\xi}' = \tilde{\xi}$, and $\tilde{\xi}'_b = \tilde{\xi}_b$. Thus we have

$$Q_3 = +b\,\tilde{\xi} - r\tilde{\xi}_b + N_a\cos\theta \tag{A7}$$

The Lagrange equations in Eulerian coordinates become, from Eqs. (3) through (6) and (A1) through (A7)

$$I\ddot{\theta} - I\dot{\phi}^2\sin\theta\cos\theta + A(\dot{\psi}+\dot{\phi}\cos\theta)\dot{\phi}\sin\theta - m\ddot{s}h\sin\theta =$$

$$-(a\cos\theta - b\sin\theta)\tilde{\eta} - (b\cos\theta + a\sin\theta)\tilde{z} + r\tilde{z}_b, \tag{A8}$$

$$\frac{d}{dt}[I\dot{\phi}\sin^2\theta + A(\dot{\psi} + \dot{\phi}\cos\theta)\cos\theta] =$$

$$(b\cos\theta + a\sin\theta)\,\tilde{\xi} - r\,\tilde{\xi}_b + N_a \tag{A9}$$

$$\frac{d}{dt}[A(\dot{\psi} + \dot{\phi}\cos\theta)] = +b\tilde{\xi} - r\tilde{\xi}_b + N_a\cos\theta. \tag{A10}$$

The final terms in Equations (A9) and (A10) differ from Walker's[5] formulation, as pointed out earlier; otherwise, Walker obtained essentially the same equations.

We can now examine the generalized forces in greater detail. The forces $\tilde{\eta}$, $\tilde{z}$ and $\tilde{\xi}$ act on the projectile during the impact of the bourrelet. The $\tilde{z}$ and $\tilde{\xi}$ forces are caused by the presence of friction during impact. Adopting Thomas' assumption,[3] one has

$$\tilde{\xi}^2 + \tilde{z}^2 = \mu^2 \tilde{\eta}^2 \ . \tag{A11}$$

The direction of the $\tilde{\xi}$ force will be opposite to the direction of the ξ component of velocity at the bourrelet's point of contact. Since the motion of the shell is in the positive z direction and $\tilde{z}$ is a frictional component of force, $\tilde{z}$ is in the negative z direction. Thus, according to Thomas:

$$\frac{\tilde{\xi}}{b(\dot{\psi}+\dot{\phi}\cos\theta) + a\,\dot{\phi}\sin\theta} = \frac{\tilde{z}}{\dot{z}} \ . \tag{A12}$$

The direction of $\tilde{\xi}$ may now be investigated. The quantity $\dot{\alpha} = \dot{\psi} + \dot{\phi}\cos\theta$ is simply the angular velocity about the body symmetry axis and is a known quantity from the relationship:

$$\dot{\alpha} = \pi\dot{z}/(rn) \ . \tag{A13}$$

Here n is the number of calibers traveled by the projectile while making one revolution. Since $\sin\theta$ is a small quantity and it is expected that $\dot{\phi}$ might be similar in magnitude to $\dot{\alpha}$, we can, using Eq. (A13), approximate Eq. (A12) by,

$$\tilde{\xi} = \pi\tilde{z}/n \ . \tag{A14}$$

Since the z-component of the impact force is negative, the ξ-component of the force is also negative from Eq. (A14). Substituting Eq. (A14) into Eq. (A11), we obtain

$$\tilde{z} = \mu\tilde{\eta}/(1+\pi^2/n^2)^{1/2} \ . \tag{A15}$$

It is a bit more involved to obtain the band frictional forces to be used in the expressions for torque. From Appendix C, it is obtained that

$$\tilde{\xi}_b = 0, \tag{A16}$$

$$\tilde{z}_b = \nu_e\tilde{\eta}_b/(1 + \pi^2/n^2)^{1/2} \ . \tag{A17}$$

Here ν_e is the effective friction defined in Appendix C. Walker's development differs only in Equations (A16) and (A17).

APPENDIX B. ANALYSIS OF TIME LAG MODEL

As discussed earlier, the time-lag model is constructed consistently with Walker's assertion that the reaction impulse can occur after the bourrelet impacts the lands of the bore. First consider the impact process.

From Eqs. (7) and (8), the transverse momentum given by the bourrelet impact is

$$m(\dot{\eta}_{cg})_1 - m\,(\dot{\eta}_{cg})_o = P_\eta \quad , \tag{B1}$$

where P_η is the impact impulse. Additionally, using Eq. (A15), the impulsive torque about the center of gravity may be obtained:

$$mk^2\dot{\theta}_1 - mk^2\dot{\theta}_o = -\,(a-h)\,P_\eta + \frac{b\mu P_\eta}{(1+\pi^2/n^2)^{\frac{1}{2}}} \quad . \tag{B2}$$

The other impulsive equations of motion obtained from Eq. (7) are not pertinent to the analysis. The initial conditions are given as

$$(\dot{\eta}_{cg})_o = -\,h\,\omega, \tag{B3}$$

$$\dot{\theta}_o = \omega\ . \tag{B4}$$

In addition, it is postulated that the coefficient of restitution is known or can be estimated so that

$$(\dot{\eta}_{cg})_1 = eh\omega \tag{B5}$$

where e is the coefficient of restitution. Immediately it is seen from Eqs. (B1), (B3) and (B5) that

$$P_\eta = mh\omega(e+1) \quad . \tag{B6}$$

Then substituting Eqs. (B6) and (B4) into Eq. (B2), we obtain

$$\dot{\theta}_1 = -\,\frac{\omega}{k^2}\,\{h(a-h-b\mu G)\,(e+1) - k^2\} \quad , \tag{B7}$$

where

$$G = 1/(1+\pi^2/n^2)^{\frac{1}{2}} \quad .$$

Now the reaction impulse process may be examined. From Eqs. (7) and (8), the transverse momentum given by the reaction impulse at the rotating band is

$$m(\dot{\eta}_{cg})_2 - m\,(\dot{\eta}_{cg})_1 = (P_\eta)_b \quad . \tag{B8}$$

The impulsive torque is given by

$$mk^2\dot{\theta}_2 - mk^2\dot{\theta}_1 = h\,(P_\eta)_b + bG\nu_e(P_\eta)_b \quad . \tag{B9}$$

In addition, since the projectile must rotate about the center of the rotating band,

$$(\dot{\eta}_{cg_2}) = -h\dot{\theta}_2 \quad . \tag{B10}$$

Eliminating $(P_\eta)_b$ between Eqs. (B8) and (B9), using Eq. (B7) to eliminate θ_1 from the result and finally using Eqs. (B5) and (B10) to eliminate $(\dot{\eta}_{cg})_1$ and $(\dot{\eta}_{cg})_2$, we obtain

$$\dot{\theta}_2 = -\frac{\omega[ehG\nu_e b-h^2+h\,(a-Gb\mu)\,(e+1)-k^2]}{[h(h+G\nu_e b)+k^2]} \quad . \tag{B11}$$

The value of $(P_\eta)_b$ may be found by substituting Eqs. (B5), (B10) and (B11) into Eq. (B8):

$$(P_\eta)_b = \frac{mh\omega(e+1)\,[h(a-h-Gb\mu)-k^2]}{h(h+G\nu_e b)+k^2} \quad . \tag{B12}$$

Utilizing Eqs. (B6) and (B12), the following ratio can be obtained:

$$R \equiv (P_\eta)_b/P_\eta = \frac{h(a-h-Gb\mu)-k^2}{h(h+G\nu_e b)+k^2} \quad . \tag{B13}$$

With the rebound angular velocity $\dot{\theta}_2$ obtained, the effective coefficient of restitution may be obtained. From Eq. (B11) it is seen that the effective coefficient of restitution is

$$e_e = \frac{[ehG\nu_e b-h^2+h\,(a-Gb\mu)\,(e+1)-k^2]}{h(h+G\nu_e b)+k^2} \, , \tag{B14}$$

where $e_e \equiv -\dot{\theta}_2/\omega$.

The quantities a, h, k and b may be nondimensionalized by the quantity 2b to obtain these quantities in calibers. One then obtains that

$$e_e = \frac{[eh' G\nu_e + h' (2a' - G\mu)(e+1) - 2h'^2 - 2k'^2]}{[h'(2h' + G\nu_e) + 2k'^2]}. \qquad (B15)$$

APPENDIX C. THE EFFECTIVE BORE COEFFICIENT OF FRICTION

The rotating band is subjected to a net transverse force of magnitude $|\tilde{n}|$. The frictional force per unit area "$\vec{f}$" on the rotating band will vary around the driving band since the pressure on the rotating-band's periphery vary. A net torque is then produced about C, shown in Figure 2, having a direction lying approximately in the plane of the rotating band. If the pressure distribution and coefficient of friction are known as functions of pressure, the total torque may be found according to

$$\vec{N} = \int (\vec{r} x f \hat{f}) \, d S + (\vec{N}_e)_a \tag{C1}$$

where $\hat{f}$ is a unit force vector that lies parallel to the land in the direction of the breech, f is the magnitude of $\vec{f}$, $\vec{r}$ is the position vector to the surface of the rotating band and S is the area of the rotating band. Here also, $(\vec{N}_e)_a$ is the axial torque developed by the rifling on the rotating band and lies along the gun-bore axis.

For F_t small enough, we can consider that f is perturbed from f_o, the value of f with no lateral forces, by the amount Δf. Equation (C1) then can be expressed as

$$\vec{N} = \int (\vec{r} \times f_o \hat{f}) \, dS + \int (\vec{r} \times \Delta f \hat{f}) \, dS + (\vec{N}_e)_a \tag{C2}$$

But since f_o is constant about the band's periphery, the first term is simply a vector along the gun-bore direction. The first and third term when added together will be called the net axial torque $\vec{N}_a$. Substracting $\vec{N}_a$ from $\vec{N}$, we obtain what is called the perturbation torque $\vec{N}_p$:

$$\vec{N}_p = \int (\vec{r} \times \hat{f} \Delta f) \, dS \ . \tag{C3}$$

But since

$$f = p \, \nu(p, V_p) \ , \tag{C4}$$

where ν is the bore coefficient of friction, then

$$\Delta f = (\nu + p \frac{\partial \nu}{\partial p}) \, \Delta p \ . \tag{C5}$$

If we define

$$\nu_e = \nu + p \frac{\partial \nu}{\partial p} , \tag{C6}$$

then substituting Eqs. (C5) and (C6) into Eq. (C3), we obtain

$$\vec{N}_p = \nu_e \int (\vec{r} \times \Delta p \, \hat{f}) \, dS \tag{C7}$$

since ν_e depends upon the unperturbed value of p.

If the pressure perturbation distribution is not known, a maximum value for the magnitude of $\vec{N}_p$ can be found by assuming that the pressure perturbation occurs only at the extrema of Δf. Furthermore, if we assume that the minimum value of Δf is just the negative of the maximum value of Δf, the following simple equation is obtained:

$$\vec{N}_p = - \nu_e \, b \, |\vec{\eta}| \, G\hat{\xi} \; , \tag{C8}$$

where b is the approximate radius of the rotating band and $G = - \hat{f} \cdot \hat{k}$. The unit vector $\hat{k}$ is in the direction of increasing z along the gun-bore axis. Here, we see that $\vec{N}_p$ is a vector lying along the line of nodes shown in Figure 1 and is simply a transverse torque. With these assumptions, the differential equations of motion are simplified somewhat from Walker's formulation[5].

The torque will also lie along the line of nodes for somewhat more general descriptions of the distribution of Δp. If Δp is an odd function of η and an even function of ξ, it can also be shown that a transverse torque is developed along the line of nodes. A more realistic value than obtained in Equation (C8) is then obtained for $\vec{N}_p$. We assume that Δp varies as the sine of the angular displacement from the line of nodes. The vector $\hat{N}_p$ is then found as

$$\vec{N}_p = - \nu_e G b \, F_t \hat{\xi}/2 \quad . \tag{C9}$$

APPENDIX D. EFFECTIVE FRICTION COEFFICIENT FOR A MOLTEN-FILM THEORY

We want to find the change in the band frictional forces on both the side where the reaction force is applied and the side opposite. These resultant forces will cause a transverse torque to be exerted on the projectile. A measure of the importance of this torque in determining the energy-growth rate is the effective coefficient of friction ν_e. An order of magnitude value for ν_e is obtained in this Appendix.

The liquid film theory of Herzfeld and Kosson will be used. They assume that the liquid layer has a linear velocity profile; therefore, we have that the friction force is

$$F_f = \mu_\ell V_p S/\delta. \tag{D1}$$

where μ_ℓ is the viscosity for the liquid, V_p is the velocity of the projectile and S is the surface area of the driving band. Here, δ is the initial thickness of the film and is determined from energy balance considerations. That is, the rate of doing work on the film equals the heat flow rate into the band plus the heat flow rate into the gun tube plus the rate at which heat is carried away with the material lost from the film.

The rotating band and bore configuration will be approximated by two plane surfaces with a block of mass m_e between them. This configuration is shown in Figure 5 where the width of the block is given as $2a_1$ and the length is, for the purposes of this analysis, much greater than the width. If the block is given a velocity $(V_b)_o$ downward, the space between one plane surface and the block will widen while the other space narrows. A torque will then be generated by the resultant unbalanced frictional forces. This unbalanced frictional impulsive force will be, from Equation (D1):

$$\int \Delta F_f dt = 2 \mu_\ell V_p a_1 z_o \int \left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) dt \tag{D2}$$

where z_o is the length of the block, δ_1 is the value of δ for the impacted side and δ_2 is the value for the other side. From Figure 5, it is apparent that $\delta_2 - \delta = \delta - \delta_1$. From some preliminary work, it was found that δ_1 is always near the initial value δ. Thus, Equation (D2) may be approximated as

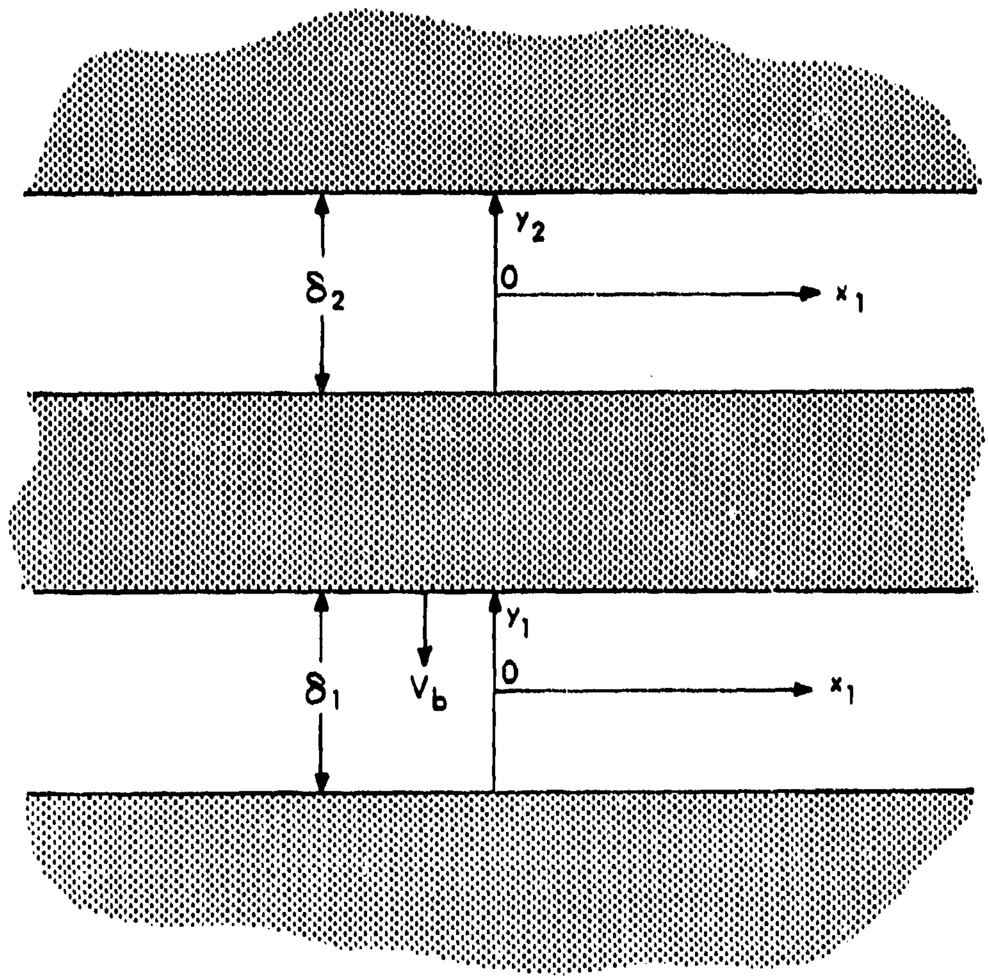

Figure 5. Movement of a block through liquid film that separates the block from two flat parallel surfaces.

$$\int \Delta F_f dt = \frac{4\mu_\ell V_p a_1 z_o}{\delta^2} \int (\delta - \delta_1)\, dt \tag{D3}$$

When the projectile has transited only a small fraction of the gun tube, Herzfeld and Kosson show that δ may be approximated as

$$\delta = \frac{\mu_\ell V_p^{3/2}}{2(\rho_T c_T)^{1/2} T_b} \left(\frac{2\pi a_1}{k_T}\right)^{1/2} \tag{D4}$$

Where ρ_T is the mass density of the gun tube, c_T is the heat capacity of the gun-tube material per unit mass, T_b is the melting temperature of the rotating band and k_T is the thermal conductivity of the gun tube material.

We want to find an expression for $\delta - \delta_1$ as a function of time so that the unbalanced impulsive frictional force may be determined. We return to Figure 5. If the block is given an initial velocity $(V_b)_o$, at some later time the velocity of the block will be V_b and the thicknesses of the upper and lower film will be δ_2 and δ_1 respectively. Coordinate systems may be constructed as shown with the origin of the coordinate systems placed halfway between the plane surfaces and the upper and lower surfaces of the block. The length of the block z_o is assumed large enough so that we can make the approximation that the fluid inflow and outflow travels essentially in a line parallel to the x_1 axis. For small thicknesses of film, order-of-magnitude considerations by Schlichting[17] show that the Navier-Stokes equations may be approximated by

$$\frac{\partial^2 U_1}{\partial y_1^2} = \frac{1}{\mu_\ell} \frac{dp_1}{dx_1}; \quad \frac{\partial^2 U_2}{\partial y_2^2} = \frac{1}{\mu_\ell} \frac{dp_2}{dx_1} \tag{D5}$$

with boundary conditions

$$y_{1,2} = \pm \delta_{1,2}/2, \; U_{1,2} = 0; \; x_1 = \pm a_1, \; p_{1,2} = 0 \tag{D6}$$

Here $U_{1,2}$ are the components of fluid velocity in the direction x_1 and

---

17. *H. Schlichting, Boundary Layer Theory, 6th ed., McGraw-Hill Book Company, Inc., New York, 1968, pp. 144-147, also pp. 108-114.*

$p_{1,2}$ are the pressures that vary only in the x_1 direction. The part of the solution of interest here is for p:

$$p_{1,2} = \pm \frac{6\mu_\ell V_b}{\delta^3_{1,2}} (a_1{}^2 - x_1{}^2) \quad , \tag{D7}$$

where the minus sign goes with the upper-part solution. Thus the resultant force per unit area at x_1 is

$$p_r = 6\ \mu_\ell V_b\ (a_1{}^2 - x_1{}^2)\ (\frac{1}{\delta_1^3} + \frac{1}{\delta_2^3}) \quad . \tag{D8}$$

Now the block loses kinetic energy according to

$$m_e\ V_b\ dV_b = -\ F_n\ d\delta_1 \quad , \tag{D9}$$

where F_n, the magnitude of the normal force, is given as

$$F_n = 2z_o \int_o^{a_1} p_r\ d\ x_1 \ . \tag{D10}$$

Integrating Equation (D10), substituting the resulting expression for F_n into Equation (D9), and again integrating, we obtain, for $\delta_{1,2}$ near δ:

$$V_b = V_o + \frac{32z_o\ \mu_\ell\ a_1{}^3}{m_e} (\frac{1}{\delta^2} - \frac{1}{\delta_1{}^2}) \tag{D11}$$

Bowden and Tabor[18] obtained a somewhat similar result for a cylinder impinging upon a plane surface. Since $V_b = -\ d\delta_1/dt$ and δ is comparable in value to δ_1, we obtain

$$\frac{d(\delta-\delta_1)}{dt} = -\ \frac{64z_o\mu_\ell a_1{}^3}{m_e\ \delta^3} (\delta-\delta_1) + V_o \quad . \tag{D12}$$

---

18. *F. P. Bowden and D. Tabor, The Friction and Lubrication of Solids, Oxford University Press, Amen House, London, 1950, pp. 272-279.*

Defining a time constant τ as

$$\tau \equiv \frac{m_e \, \delta^3}{64 z_o \mu_\ell a_1^3} , \tag{D13}$$

Equation (D12) may be integrated to obtain

$$\delta - \delta_1 = V_o \tau \, [1 - \exp(-t/\tau)] \ . \tag{D14}$$

Here, we see that τ is the time required for $\delta - \delta_1$ to be $(1 - e^{-1})$ times the maximum value for $\delta - \delta_o$ and $V_o \tau$ is that maximum value.

We find the frictional impulsive force by substituting Equation (D14) into Equation (D3) and then integrating to obtain

$$\int_o^{t_v} \Delta F_f \, dt = \frac{m_e \, V_p \, V_o \, \delta}{16 \, a_1^2} \, [t_v - \tau + \tau \exp(-t_v/\tau)] \ . \tag{D15}$$

The effective coefficient of friction would then be the frictional impulsive force divided by the lateral impulsive force $m_e V_o$. Thus,

$$\nu_e = \frac{\delta \, V_p}{16 \, a_1^2} \, [t_v - \tau + \tau \exp(-t_v/\tau)] \ . \tag{D16}$$

Here, t_v is the maximum time for which these equations approximate the actual conditions. This maximum time is unknown but might be expected to be governed by the actual fluid-flow details and melt processes. The value of δ may be determined for the 8-inch M106 projectile. From Herzfeld and Kosson, we have

$$\mu_\ell = 0.035 \text{ poise}$$

$$k_T = 0.105 \text{ cal/cm/s/K}$$

$$\rho_T \, c_T = 1.053 \text{ cal/cm}^3\text{/K}$$

$$T_b \doteq 10^3 \text{K}$$

$$V_p = 8 \cdot 10^4 \text{ cm/s}$$

The value of a_1 for the rotating band is

$$a_1 = 2.5 \text{ cm.}$$

Substituting these values in Equation (D4), we obtain $\delta \doteq 7 \cdot 10^{-4}$cm.

The factors multiplying the bracket would be such that

$$\nu_e = 0.56 \, [t_v - \tau + \tau \exp(-t_v/\tau)]$$

If it is assumed that the equivalent mass m_e is approximately the mass of the shell ($m = 4.43 \cdot 10^5$gm) and $z_o = 15$ cm, τ from Equation (D13) has the value $\tau = 3 \cdot 10^{-7}$s. As mentioned earlier, it is not known how long these differences in film depth will be maintained. Certainly, if no other processes limited t_v, the values for t_v would be limited by the time between impacts. To obtain a time between impacts, the clearance between bourrelet and bore, distance between rotating band and bourrelet, moment of inertia and balloting energy all need to be known. From Walker, the clearance c is $3.6 \cdot 10^{-4}$m, $a = 0.231$m and $I = 6.17$ kg m^2. A possible value for ε with no balloting growth is 5 joules. With these values, it is found that the time between impacts would be .0024. Thus, <u>we see that the effective coefficient of friction is at most approximately 10^{-3}, an exceedingly small value.</u>

It has previously been asserted that the final value of δ_1 is near δ. This is an important result since, if appreciable thinning were to occur, solid-to-solid contact might occur, and ν_e might become large. The extent of this thinning may be investigated with Equation (D11). When $V_b = 0$, then if δ_1 is near δ,

$$\frac{\delta - \delta_1}{\delta} \approx \frac{m_e V_o \delta^2}{64 \, z_o \, \mu_\ell \, a_1^3} \qquad \text{(D17)}$$

Now equating $m_e V_o$ with the expression for the reaction impulse given the rotating band at or after impact, rearranging the expression so it may be given in terms of the balloting energy ε and substituting into Equation (D17), one finds:

$$\frac{\delta - \delta_1}{\delta} = \frac{(2I \, \varepsilon)^{1/2} \, (1+e) \, |R| \, \delta^2}{64 \, z_o \, \mu_\ell \, a \, a_1^3} \qquad \text{(D18)}$$

Substituting the values previously used for the 8-inch M106 projectile, we obtain $(\delta - \delta_1 / \delta \approx 0.006$.

LIST OF SYMBOLS

a	distance from bourrelet to rotating band
a'	nondimensioned value of a in calibers
a_1	half width of the rotating band
A	axial moment of inertia
b	radius of the shell at the bourrelet
B	transverse moment of inertia about the CM of shell
c	clearance between bourrelet and bore of gun for zero yaw
c_T	specific heat capacity for gun tube
C	center of the rotating band
e	coefficient of restitution
e_c	effective coefficient of restitution with the late transverse torque taken into account
f	friction force per unit area at the rotating band
F	inertia force on shell
F_f	total frictional force at band
F_t	net reaction transverse force
G	$= 1/(1+\pi^2/n^2)^{\frac{1}{2}}$
h	distance from shell's center of mass to the plane of the rotating band
I	transverse moment of inertia about an axis in the plane of the rotating band
k	transverse radius of gyration about shell's center of mass
$\hat{k}$	unit vector in direction of increasing z
k_T	heat conductivity of the gun tube
L	system Lagrangian

PRECEDING PAGE BLANK-NOT FILMED

m	mass of shell
n	twist of the rifling in calibers per revolution
p	pressure at the rotating band and bore interface
p_{q_i}	impulsive momentum imparted along the i'th coordinate
q_i	generalized coordinates
Q_i	generalized forces of constraint
r	radius of gun-tube bore
s	distance projectile has traveled in-bore
S	surface area of rotating band
t	time
T	kinetic energy of system
$U_{1,2}$	fluid velocity as a function of x_1 and $y_{1,2}$
V_p	velocity of projectile
$(V_b)_o$	initial velocity imported to the block in Figure 5
V_b	velocity of block in Figure 5
x	x-component of nonrotating rectilinear coordinates fixed with reference to the gun-bore axis, the origin of the coordinates is the point C, the center of the rotating band
X'	x' component of rectilinear coordinates fixed with reference to the shell; illustrated in Figure 1
y	the coordinate whose direction is perpendicular to the x coordinate and the z coordinate
y'	coordinate fixed with reference to shell; illustrated in Figure 1
z	coordinate along gun tube axis whose origin is the center of the rotating band
z'	coordinate along shell's axis whose origin is the center of the rotating band

$\hat{z}$	unit vector in direction of increasing z
z_o	length of block which represents rotating band in liquid film theory
$\dot{\alpha}$	shell spin rate as given by Thomas, $\dot{\alpha} = \pi\dot{z}/(rn)$
δ	initial thickness of liquid film layer at rotating band
δ_1	thickness of liquid film layer at position where reaction force is applied
δ_2	thickness of liquid film layer at position opposite to where reaction force is applied
ζ, ζ'	coordinates involved in description of Euler angles; illustrated in Figure 1
η, η'	coordinates involved in description of Euler angles which are perpendicular to the line of nodes, illustrated in Figure 1
θ	yaw angle, inclination of the shell axis from the gun-tube bore axis
Θ	total angle swept out by the shell's axis in the θ direction, $\Theta = \int d\|\theta\|$
μ	coefficient of friction between bourrelet and bore
μ_ℓ	viscosity for the liquid film
ν	coefficient of friction between rotating band and bore
ν_e	effective coefficient of friction between rotating band and bore
ξ, ξ'	coordinate along the line of nodes in the Euler angle scheme; see Figure 1
$\hat{\xi}$	unit vector lying along the line of nodes
ρ_Γ	mass density for gun-tube material
τ	a measure of the duration of the rotating band's motion after application of a reaction impulse

LIST OF SYMBOLS (Continued)

ϕ	angular position of the line of nodes; see Figure 1
ψ	spin angle about the projectile's axis; see Figure 1
$\vec{\omega}$	angular velocity of shell

Subscripts

a	refers to gun bore axis
b	rotating-band conditions
cg	center of mass conditions
e	effective values of quantities
f	refers to the frictional part of quantity
ℓ	. ..rs to liquid phase conditions
p	properties of projectile; perturbation properties
r	resultant conditions for a quantity
T	refers to the gun-tube material
v	refers to the maximum value of the time for equations to be valid

Superscripts

$\rightarrow$	denotes vector quantities
$\hat{}$	denotes unit vector quantities
$\sim$	denotes force components for the corresponding coordinates

DISTRIBUTION LIST

No. of Copies	Organization
12	Commander Defense Documentation Center ATTN: DDC-TCA Cameron Station Alexandria, VA 22314
1	Director Defense Nuclear Agency Washington, DC 20305
2	Commander US Army Materiel Development and Readiness Command ATTN: DRCDMA-ST DRCDL 5001 Eisenhower Avenue Alexandria, VA 22333
3	Commander US Army Aviation Research and Development Command ATTN: DRSAV-E DRCPM-AAH, M. Corgiatt Product Manager, AH-1, D. Brewer 12th and Spruce Streets St. Louis, MO 63166
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
1	Commander US Army Electronics Command ATTN: DRSEL-RD Fort Monmouth, NJ 07703
1	Commander US Army Jefferson Proving Ground ATTN: STEJP-TD-D Madison, IN 47250
3	Commander US Army Missile Research and Development Command ATTN: DRDMI-R DRDMI-RBL DRDMI-RDK Redstone Arsenal, AL 35809
1	Commander US Army Tank Automotive Research & Development Cmd ATTN: DRDTA-RWL Warren, MI 48090
1	Commander US Army Mobility Equipment Research & Development Cmd ATTN: DRDME-WC Fort Belvoir, VA 22060
5	Commander US Army Armament Materiel Readiness Command ATTN: Technical Lib DRSAR-RDG, J. Blick Rock Island, IL 61299
5	Commander US Army Armament Research and Development Command ATTN: DRDAR-TSS (2 cys) DRDAR-TDS, Mr. Lindner DRDAR-LC-F, Mr. A.Loeb Mr. E. Friedman Dover, NJ 07801
4	Commander US Army Armament Research and Development Command ATTN: DRDAR-LCV, Mr. Barrieres Mr. R. Reisman DRDAR-SCN, Mr. Kahn PM, XM788/789, LTC Delany Dover, NJ 07801

DISTRIBUTION LIST

No. of Copies	Organization
4	Commander US Army Watervliet Arsenal ATTN: Tech Lib Mr. W. Dock Dr. G. Carofano Mr. P. Alto Watervliet, NY 12189
1	Commander US Army Harry Diamond Labs ATTN: DRXDO-TI 2800 Powder Mill Road Adelphi, MD 20783
1	Commander US Army Materiels and Mechanics Research Center ATTN: DRXMR-ATL Watertown, MA 02172
1	Commander US Army Natick Research and Development Command ATTN: DRXRE, Dr. D. Sieling Natick, MA 01762
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL, Tech Lib White Sands Missile Range NM 88002
1	HQDA (DAMA-WSA, MAJ Fite) Washington, DC 20310
1	Commander US Army Research Center ATTN: CRD-AA-EH P. O. Box 12211 Research Triangle Park NC 27709
1	Director US Army BMD Advanced Technology Center P.O. Box 1500, West Station Huntsville, AL 35807
1	Commander US Army Ballistic Missile Defense Systems Command Huntsville, AL 35804
3	Commander Naval Air Systems Command ATTN: AIR-604 Washington, DC 20360
3	Commander Naval Ordnance Systems Command ATTN: ORD-9132 Washington, DC 20360
2	Commander and Director David W. Taylor Naval Ship Research & Development Ctr ATTN: Tech Lib Aerodynamic Lab Bethesda, MD 20084
3	Commander Naval Surface Weapons Center ATTN: Code 6X Mr. F. H. Maille Dr. J. Yagla Dr. G. Moore Dahlgren, VA 22448
1	Commander Naval Surface Weapons Center ATTN: Code 730, Tech Lib Silver Spring, MD 20910

DISTRIBUTION LIST

No. of Copies	Organization
1	Commander Naval Weapons Center ATTN: Code 553, Tech Lib China Lake, CA 93555
1	Commander Naval Research Laboratory ATTN: Tech Info Div Washington, DC 20375
1	Commander Naval Ordnance Station ATTN: Code FS13A, P.Sewell Indian Head, MD 20640
1	AFRPL/LKCB, Dr. Horning Edwards AFB, CA 93523
2	AFATL (DLDL, D.C. Daniel; Tech Lib) Eglin AFB, FL 32542
1	AFWL/DEV Kirtland AFB, NM 87117
1	ASD/XRA (Stinfo) Wright-Patterson AFB, OH 45433
1	Director National Aeronautics and Space Administration George C. Marshall Space Flight Center ATTN: MS-I, Lib Huntsville, AL 35812
1	Director Jet Propulsion Laboratory ATTN: Tech Lib 2800 Oak Grove Drive Pasadena, CA 91103
1	Director National Aeronautics and Space Administration Langley Research Center ATTN: MS 185, Tech Lib Langley Station Hampton, VA 23365
1	Director NASA Scientific & Technical Information Facility ATTN: SAK/DL P. O. Box 8757 Baltimore/Washington International Airport, MD 21240
1	AAI Corporation ATTN: Dr. T. Stastny Cockeysville, MD 21030
1	Advanced Technology Labs ATTN: Dr. J. Erdos Merrick & Steward Avenues Westbury, NY 11590
1	Aerospace Corporation ATTN: Dr. T. Taylor P. O. Box 92957 Los Angeles, CA 90009
1	ARO, Inc. ATTN: Tech Lib Arnold AFS, TN 37389
1	ARO, Inc. Von Karman Gasdynamics Facility ATTN: Dr. J. Adams Arnold AFS, TN 37389
1	ARTEC Associates, Inc. ATTN: Dr. S. Gill 26046 Eden Landing Road Hayward, CA 94545

DISTRIBUTION LIST

No. of Copies	Organization
1	AVCO Systems Division ATTN: Dr. W. Reinecke 201 Lowell Street Wilmington, MA 01887
1	Battelle Columbus Laboratories ATTN: J. E. Backofen, Jr. 505 King Avenue Columbus, OH 43201
1	Technical Director Colt Firearms Corporation 150 Huyshore Avenue Hartford, CT 14061
2	General Electric Corporation Armaments Division ATTN: Mr. R. Whyte Mr. J. MacNeil Lakeside Avenue Burlington, VT 05401
1	Honeywell, Inc. ATTN: Mail Station MN 112190 (G. Stilley) 600 Second Street, North Hopkins, MN 55343
3	Hughes Helicopter Company Bldg. 2, MST22B ATTN: Mr. T. Edwards Mr. J. Johnson Mr. R. Forker Centinella & Teel Streets Culver City, CA 90230
1	Martin Marietta Aerospace ATTN: Mr. A. J. Culotta P. O. Box 5387 Orlando, FL 32805
1	Winchester-Western Division Olin Corporation New Haven, CT 06504

No. of Copies	Organization
1	Rockwell Int'l Science Center ATTN: Dr. Norman Malmuth P. O. Box 1085 1000 Oaks, CA 91360
1	Sandia Laboratories ATTN: Aerodynamics Dept Org 5620, R. Maydew Albuquerque, NM 87115
1	S&D Dynamics, Inc. ATTN: Dr. M. Soifer 755 New York Avenue Huntington, NY 11743
1	Guggenheim Aeronautical Lab California Institute of Tech ATTN: Tech Lib Pasadena, CA 91104
1	Franklin Institute ATTN: Tech Lib Race & 20th Streets Philadelphia, PA 19103
1	Director Applied Physics Laboratory The Johns Hopkins University Johns Hopkins Road Laurel, MD 20810
1	Massachusetts Institute of Technology Dept of Aeronautics and Astronautics ATTN: Tech Lib 77 Massachusetts Avenue Cambridge, MA 02139
1	Ohio State University Dept of Aeronautics and Astronautical Engineering ATTN: Tech Lib Columbus, OH 43210

DISTRIBUTION LIST

No. of Copies	Organization
2	Polytechnic Institute of New York Graduate Center ATTN: Tech Lib Dr. G. Moretti Route 110 Farmingdale, NY 11735
1	Director Forrestal Research Center Princeton University Princeton, NJ 08540
1	Forrestal Campus Library Princeton University ATTN: Dr. M. Summerfield P. O. Box 710 Princeton, NJ 08540
1	Southwest Research Institute ATTN: Mr. Peter S. Westine P. O. Drawer 28510 8500 Culebra Road San Antonio, TX 78228

Aberdeen Proving Ground

Marine Corps Ln Ofc
Dir, USAMSAA
Cdr, USA CSL/EA (Bldg E3516)
ATTN: A. Flatau, SAREA-DE-W