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TARADCOM R and D TECHNICAL REPORT

NO. 12325

U.S. ARMY TARADCOM

SEQUENTIAL TESTING:
BASICS AND BENEFITS

MARCH 1978

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by JOHN C. SCHMUHL
L.G. JONES
Systems and Cost Analysis Office
and
Product Assurance Directorate

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U.S. ARMY TANK-AUTOMOTIVE
RESEARCH AND DEVELOPMENT COMMAND
Warren, Michigan 48090

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Sequential Testing:
Basics and Benefits

Contents

Executive Summary

This report presents the sequential test method. The methods are described, a plan and computer program, to speed the use of the method, are included. This testing method could, if implemented on applicable testing problems, reduce test time and costs of testing. The added benefit of reduced energy needs are inherent in this testing method.

The text was originally released by the authors in 1972. The text has been thoroughly reviewed and is applicable to many Tank-Automotive system and component testing requirements. The principal efforts involved in preparing this report were performed by Mr. John Schmuhl, who is currently employed by TARADCOM.

I. Introduction and Summary

The purpose of this report is to present a method and a plan which could, if implemented, reduce the amount of time needed to life test items and at the same time reduce the costs of testing. The advantage of this method is that it would not be necessary to test an item through the complete length of time specified for a test. Necessary decision critiera as to the acceptability or unacceptability of a test can be determined much earlier thus saving time and money. The method employed to do this is known as sequential analysis and has been developed and used for approximately 25 years. A bibliography of articles and related topics to sequential analysis and testing is given in Appendix IV.

II. Sequential Analysis

One hundred percent conclusive and valid reliability demonstrations usually are extremely expensive in terms of time and money. To prove endurance capabilities, either long life tests must be run, or if time is a critical factor, large numbers of probably expensive items must be placed on test simultaneously.

Sequential analysis takes advantage of test information as it is accumulated and allows for previously agreed upon decisions to be made as the test develops. As the test progresses each failure is reported and plotted on a sequential analysis chart, such as that shown in Figure 1. This plot indicates one of three possible decisions which can be made each time a failure occurs: (1) Reject the item, either individually or the entire lot from which it was drawn; (2) accept the item; or (3) continue the test until more data becomes available.

To reach these decisions, four values are established as criteria against which test results are compared to establish compliance to the reliability requirements of the item. These are the lowest acceptable mean time between failure (MTBF) usually designated "θ_1", the desired, or upper limit of the MTBF usually designated "θ_0". Associated with the acceptance of either one of these MTBF values are risks, normally known as "consumer's risk" and "producer's risk." In this context consumers risk (statistically known as Type II error and designated "β") is the probability of accepting an item if its MTBF is equal to θ_1, or if the lot is actually bad. Producer's risk (statistically known as Type I error and designated "α") is the probability of rejecting an item if its MTBF is equal to θ_0, or if the lot is actually good. This assumes that θ_0 is greater than θ_1.

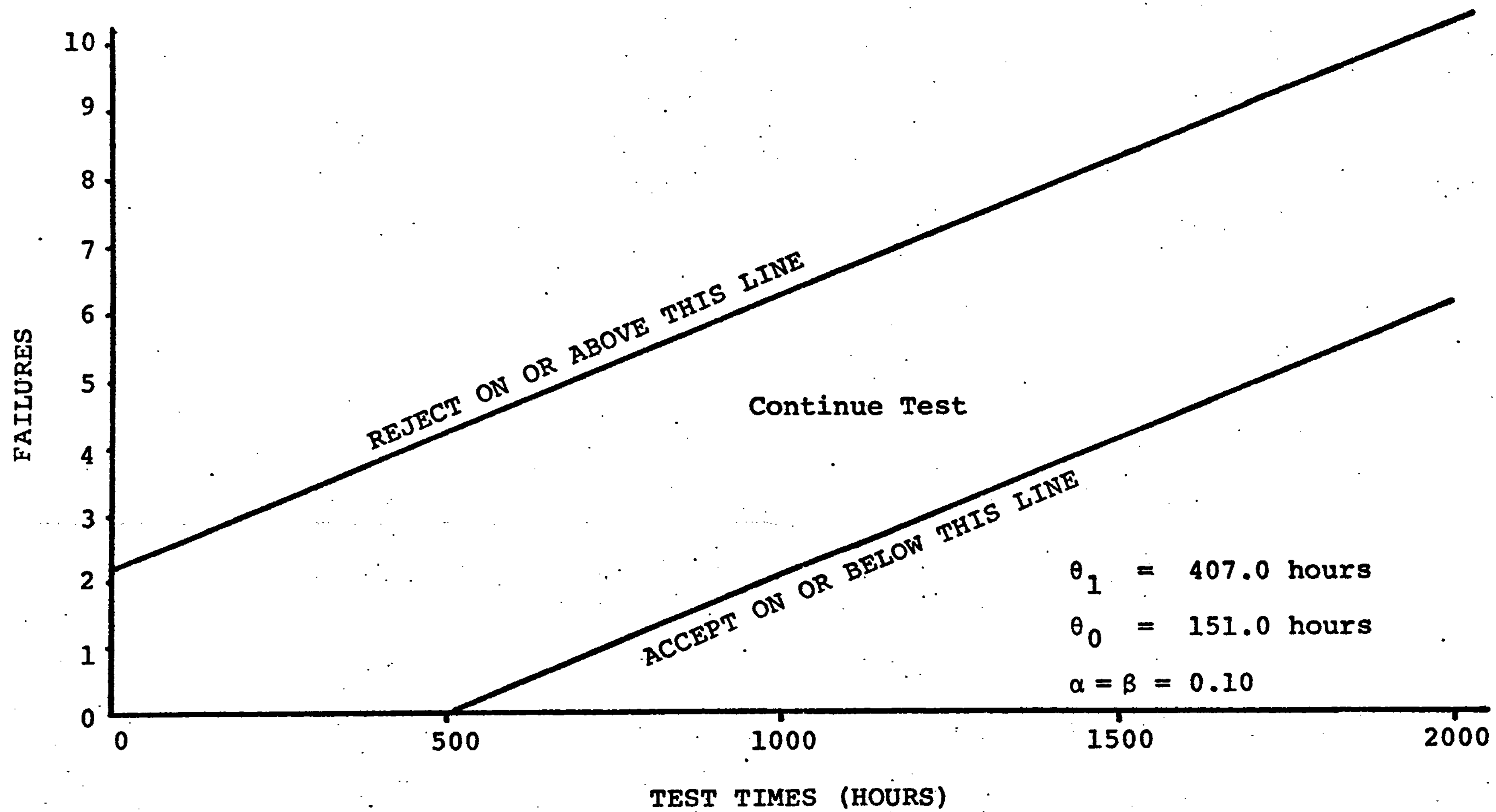


FIGURE 1 - SAMPLE SEQUENTIAL TEST CHART

Four values are required in light of the different interests of the "producer" and "consumer". Because the true MTBF of an item cannot be determined by any reasonable sample, two different MTBF values are determined, one high and the other low, between which the "true" value is expected to fall. These upper (reject) and lower (accept) limits can be expressed mathematically and factored into the sequential analysis chart. The most common distribution associated with sequential life testing is the exponential distribution, although it has been applied to the binomial, Weibull and Poisson distributions. The discussion which follows will deal with the exponential distribution as most work is centered on it and it is mathematically the most easy to deal with.

III. Mathematics of Sequential Testing

The analysis which follows assumes the following situation: n items are placed on life test and allowed to run until a failure occurs. That item which fails is either replaced or repaired and put back into service, thus at the end of any specified period of time there will be exactly n items on test. This case is known as the Replacement Case as opposed to the Non-Replacement Case, wherein the failed items are not replaced or repaired. Mathematically, the replacement case is easiest to use and in many applications the most reasonable. In addition, the underlying life distribution will be assumed to be exponential with probability density function:

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta} \quad ; \quad x > 0.$$

The variable "x" represents time and the unknown parameter $\theta (>0)$ can be thought of physically as the mean life. What is done in sequential analysis is to test the simple hypothesis that the true MTBF, θ, equals θ_0 (the upper limit of the MTBF) against the simple alternative hypothesis that θ equals θ_1 (the lower limit of the MTBF). The test is carried out by drawing n items at random from the population and placing them all on life test. The basic rationale for the formulation of sequential tests is derived from work done by A. Wald [44]. Wald's work on sequential analysis can be used virtually without modification in a situation where decisions are made continuously. A word on notation is in order at this point. When referring to the "hypothesis that the true MTBF equals θ_1", it is common to abbreviate this as:

$$H_1 \quad : \quad \theta = \theta_1$$

and the "hypothesis that the true MTBF equals θ_0" as:

$$H_0 \quad : \quad \theta = \theta_0$$

We wish to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ with producer's and consumer's risk α and β, respectively. Since information is available continuously, a continuous analogue of the sequential probability ratio test of Wald can be used. The decision to accept, reject, or continue the test depends on:

$$\beta < (\theta_0/\theta_1)^r \exp\left[-(1/\theta_1 - 1/\theta_0)V(t)\right] < A \tag{1}$$

where β and A are constants, depending on α and β such that $B < 1 < A$ and

$$B = \frac{\beta}{(1-\alpha)} \quad , \quad A = \frac{(1-\beta)}{\alpha} \tag{1a,1b}$$

The variable "r" represents the number of failures up to time "t". The decision to continue testing is made as long as the inequality (1) holds. At the time the experiment is stopped, if the first inequality in (1) is violated H_0 is accepted; if the second inequality is violated H_1 is accepted. $V(t)$ is a statistic which can be interpreted as the total life observed up to time t. In the replacement case:

$$V(t) = nt \tag{2}$$

Inequality (1) can be reformulated more conveniently as:

$$-h_1 + rs < V(t) < h_0 + rs \tag{3}$$

where h_1, h_0, and s are positive constants given by:

$$h_o = \frac{-\ln(B)}{1/\theta_1 - 1/\theta_0}$$

$$h_1 = \frac{\ln(A)}{1/\theta_1 - 1/\theta_0}$$

$$s = \frac{\ln(\theta_0/\theta_1)}{1/\theta_1 - 1/\theta_0}$$

r = failure number

In plotting the sequential test chart, the first part of inequality (3) forms a straight line representing the reject boundary while the second part of inequality (3) forms a straight line representing the accept boundary. Graphically, this is illustrated in Figure 2.

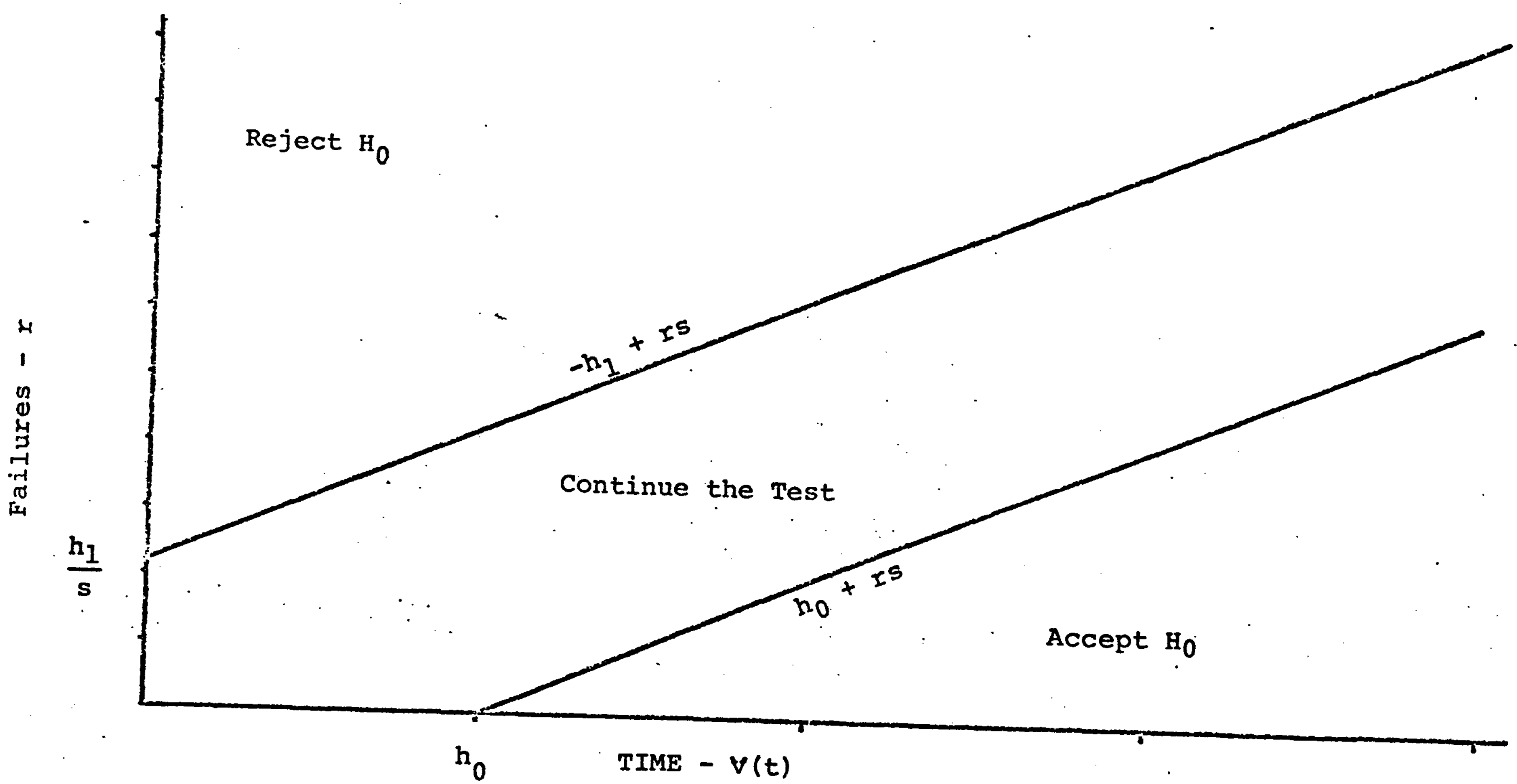


FIGURE 2 - SEQUENTIAL TEST CRITERIA

Further, it can be shown that the probability of accepting Ho, $L(\theta)$, when θ is the true parameter value, is given approximately by a pair of parametric equations:

$$L(\theta) = \frac{A^h - 1}{A^h - B^h} \qquad (7a, 7b)$$

$$\theta = \frac{(\theta_0/\theta_1)^h - 1}{h(1/\theta_1 - 1/\theta_0)}$$

by letting the parameter h run through all real values. The values of $L(\theta)$ at five points $\theta = 0$, θ_1, S, θ_0, and ∞ are enough to sketch the entire curve (known as the "Operating Characteristics" curve). These values are, respectively, 0, β, $\ln(A)/\ln(A)-\ln(B)$, $1-\alpha$, and 1.

Additionally, it is possible to determine the expected number of observations required to reach a decision when θ is the true parameter. This quantity is abbreviated $E_\theta(r)$ and is given approximately by:

$$E_\theta(r) \simeq \begin{cases} \dfrac{L(\theta)\ln\beta + [1-L(\theta)]\ln A}{\ln(\theta_0/\theta_1) - \theta(1/\theta_1 - 1/\theta_0)} = \dfrac{h_1 - L(\theta)(h_0+h_1)}{s-\theta} \ ; \theta \neq s & (9a) \\[2em] \dfrac{-\ln(A)\ln(B)}{[\ln(\theta_0/\theta_1)]} = \dfrac{h_0 h_1}{s^2} \ ; \theta = s & (9b) \end{cases}$$

Letting $k = \theta_0/\theta_1$, three important values of $E_\theta(r)$ become particularly simple when $\theta=\theta_1$, s, or θ_0. They are:

$$E_{\theta_1}(r) \simeq [\beta\ln(B) + (1-\beta)\ln(A)]/[\ln(k) - (k-1)/k] \qquad (10a)$$

$$E_s(r) \simeq -\ln(A)\ln(B)/[\ln(k)]^2 \qquad (10b)$$

$$E_{\theta_0}(r) \simeq [(1-\alpha)\ln(B) + \alpha\ln(A)/[\ln(k) - (k-1)] \qquad (10c)$$

In the replacement case where the number of items on test throughout is the same, namely n, it can be shown that the expected waiting time, $E_\theta(t)$, before a decision is reached if θ is the true MTBF, is given by:

$$E_\theta(t) = (\theta/n)E_\theta(r)$$

It is prudent, at this point, to stop and summarize the sequential decision criteria:

If $-h_1 + rs < V(t) < h_0 + rs$, continue the test, (12a)

If $V(t) \geq h_0 + rs$, stop the test and accept H_0, (12b)

If $V(t) \leq -h_1 + rs$, stop the test and reject H_0, (12c)

It can be seen that it is possible to satisfy (12a) for all t, hence making it impossible to ever make an accept or reject decision. In other words, the life test could run forever. What is done to prevent this situation from occurring is either to set a maximum time, T_0, at which to terminate the test or a maximum number of failures, r_0, at which to terminate the test or both. This guarantees that the test will run no longer than time T_0 or the time at which r_0 failures occur. Usually one knows T_0 and must find r_0. This is easily done, provided one has a table of chi-square values (Appendix II). Epstein [17] has shown that r_0 is the smallest integer for which:

$$\frac{\chi^2_{1-\alpha,2r}}{\chi^2_{\beta,2r}} > \frac{\theta_1}{\theta_0} \quad ; \quad r = r_0 \tag{13}$$

This can be easily related to T_0 from Epstein [17] by:

$$T_0 = \frac{\theta_0 \chi^2_{1-\alpha,2r}}{2n} \tag{14}$$

solving this for θ_0 yields:

$$\theta_0 = \frac{2n\ T_0}{\chi^2_{1-\alpha,2r}} \tag{15}$$

and substituting this into (13) gives:

$$\frac{\chi^2_{1-\alpha,2r}}{\chi^2_{\beta,2r}} \geq \frac{\theta_1 \chi^2_{1-\alpha,2r}}{2nT_0} \tag{16}$$

Rearrange (16) and simplify:

$$\frac{2nT_0}{\theta_1} \geq \chi^2_{\beta,2r} \tag{17}$$

$r_0 = r$ now the largest integer such that (17) is true. T_0 and r_0 are commonly referred to as the truncation time and truncation failures, respectively.

This essentially, summarizes the basics of exponential sequential analysis as applied to life testing. For more detailed presentations of this subject the bibliography in Appendix IV should be consulted.

IV. Examples of Sequential Analysis in Life Testing

EXAMPLE 1. Consider a sample of one item which is placed on replacement life test. The underlying distribution is exponential with the true mean time between failure located somewhere between 320.0 hours and 500.0 hours. A consumer's and producer's risk of 20% is decided upon and a truncated sequential test plan is desired.

SOLUTION:

$$\theta_0 = 500.0 \text{ hours} \qquad \alpha = 0.20$$
$$\theta_1 = 320.0 \text{ hours} \qquad \beta = 0.20$$
$$n = 1$$

To determine r_0, the truncation failures, (13) should be used along with Appendix (II). From Appendix II it is seen that when $\nu = 2r = 30$:

$$\frac{\chi^2_{1-\alpha,2r}}{\chi^2_{\beta,2r}} = \frac{\chi^2_{0,80,30}}{\chi^2_{0,20,30}} = \frac{23.364}{36.250} = 0.644 \geq \frac{\theta_1}{\theta_0} = \frac{320.0}{500.0} = 0.640$$

Thus:

$$r_0 = 2r/2$$
$$= 15$$

Knowing r_0 it is possible to determine the truncation time from (14):

$$To = \frac{\theta_0 \chi^2_{1-\alpha,2r}}{2n}$$
$$= \frac{500.0\chi^2_{0.80\ ,30}}{2(1)}$$
$$= 500.0\ (23.364)/2$$
$$= 5840.0 \text{ hours}$$

The sequential decision criteria are developed from (4), (5), and (6), namely:

$$B = \frac{\beta}{(1-\alpha)} = \frac{0.20}{0.80} = \frac{1}{4}\ ; \quad A = \frac{(1-\beta)}{\alpha} = \frac{0.80}{0.20} = 4$$

$$h_0 = \frac{-\ln B}{1/\theta_1 - 1/\theta_0} = \frac{-\ln\ (1/4)}{1/320 - 1/500}$$
$$= 1233.5$$

$$h_1 = \frac{\ln A}{1/\theta_1 - 1/\theta_0} = \frac{\ln (4)}{1/320 - 1/500}$$

$$= 1233.5$$

$$s = \frac{\ln (\theta_0/\theta_1)}{1/\theta_1 - 1/\theta_0} = \frac{\ln (500/320)}{1/320 - 1/500}$$

$$= 396.6$$

Thus:

If $-1233.5 + 396.6r < t < 1233.5 + 396.6r$, continue the test.

If $t \geq 1233.5 + 396.6r$, stop the test and accept H_0,

If $t \leq 1233.5 + 396.6r$, stop the test and reject H_0.

Graphically these decision rules appear in Figure 3. Assume the following failure times were noted during the test (in hours):

400, 1000, 1200, 2400, 3400

Note: These are measured from time equal to zero; they are not time between failure.

These points are plotted on Figure 3 and shown in Figure 4. It can be seen that the test can be terminated after 2800 hours, before the fifth failure occurs, with a decision to accept the hypothesis that the true mean time between failure is 500 hours (or greater). It is interesting to note that had no failure occurred before the first 1233.5 hours, the test could have been terminated with an accept decision.

Points for the Operating Characteristics curve, expected failure, and expected time curves are summarized below:

θ	$L(\theta)$	$E_\theta(r)$	$E_\theta(t)$
0.0	0.0	3.1	0.0
320.0	0.2	9.7	3092.2
396.6	0.5	9.7	3836.8
499.7	0.8	7.2	3586.8
∞	1.0	0.0	1233.5

These are plotted in Figures 5, 6, and 7 respectively.

In order to simplify the calculations required, a computer program has been written and implemented to develop sequential test plans. The program written in FORTRAN IV language has been developed for the GE-440 time-sharing system. The program is interactive and a listing of it is given in Appendix III. In order to illustrate its use the following example is given.

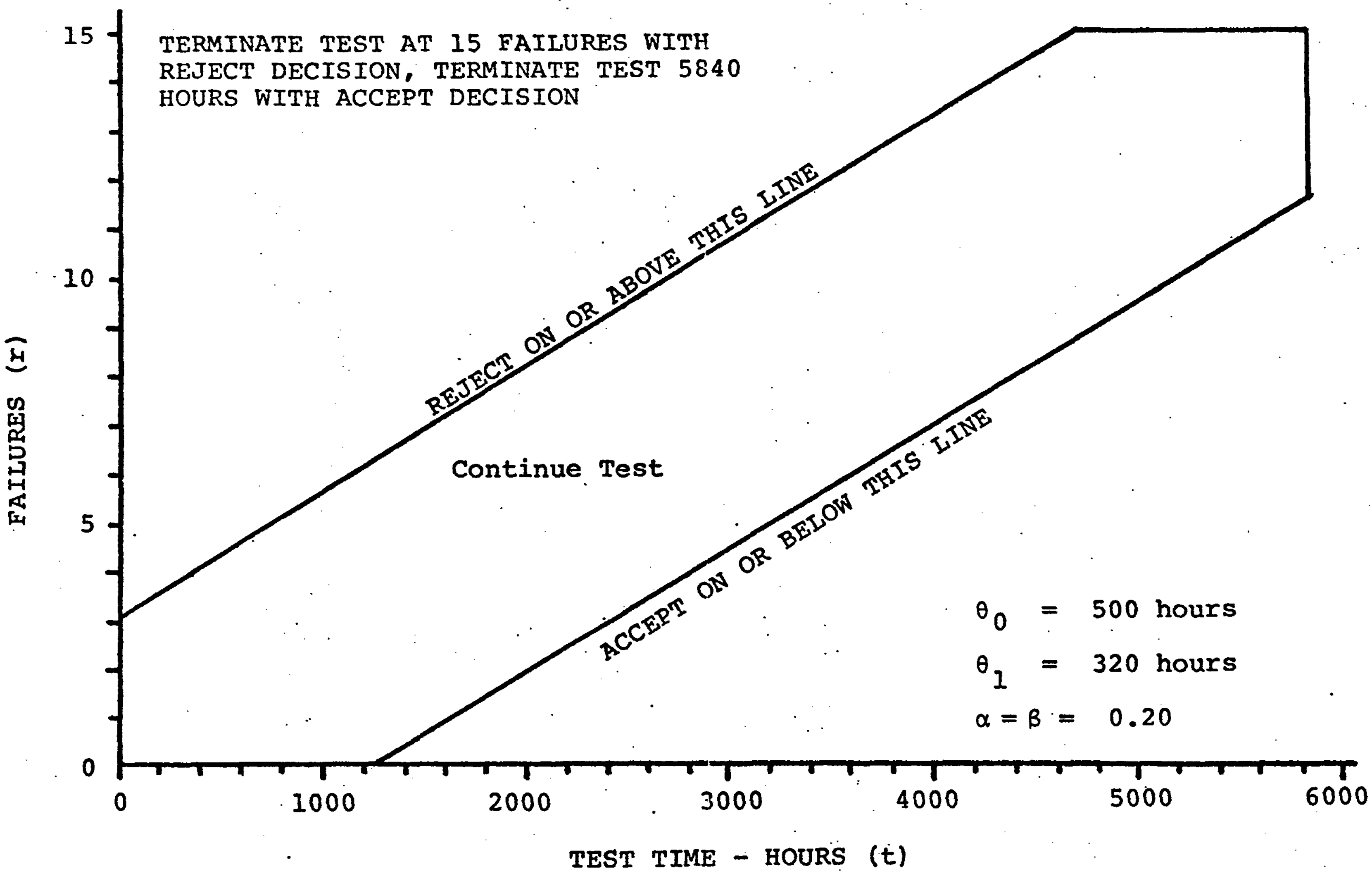


FIGURE 3 - SEQUENTIAL TEST CHART FOR EXAMPLE 1

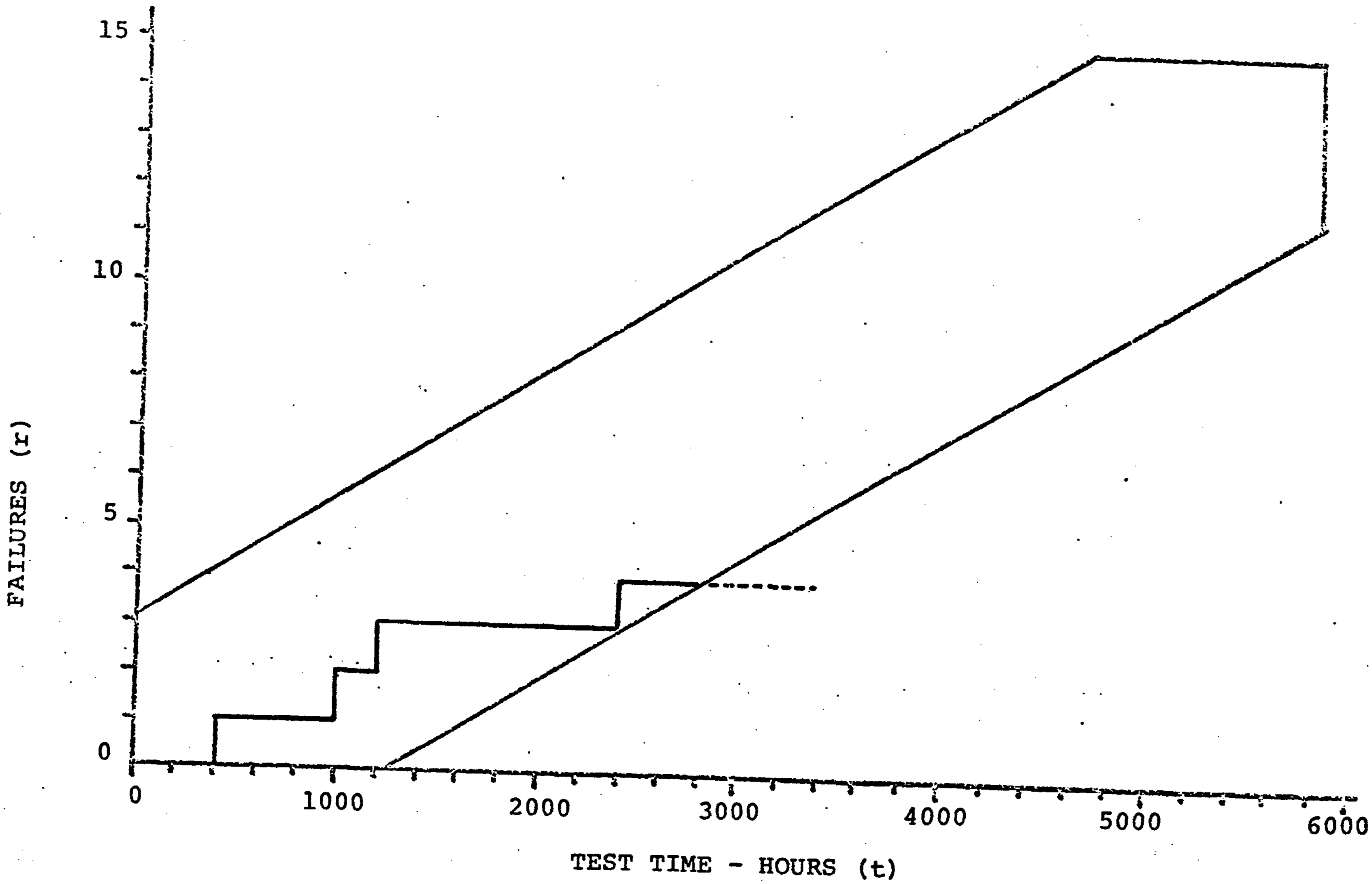


FIGURE 4 - SEQUENTIAL TEST CHART OF EXAMPLE 1
WITH FAILURES PLOTTED - ACCEPT BEFORE
4TH FAILURE OCCURS

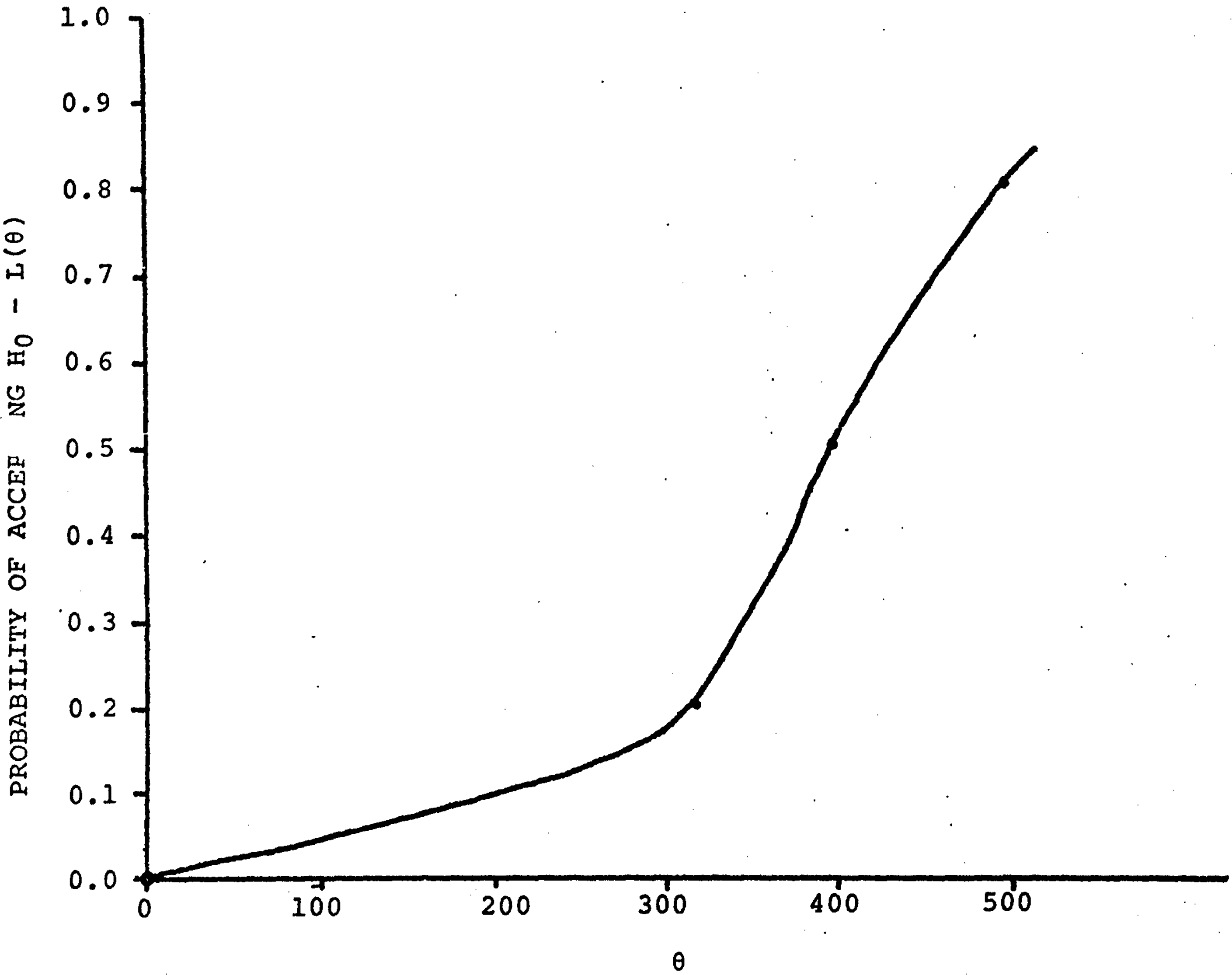


FIGURE 5 - PLOT OF $L(\theta)$ FOR EXAMPLE 1

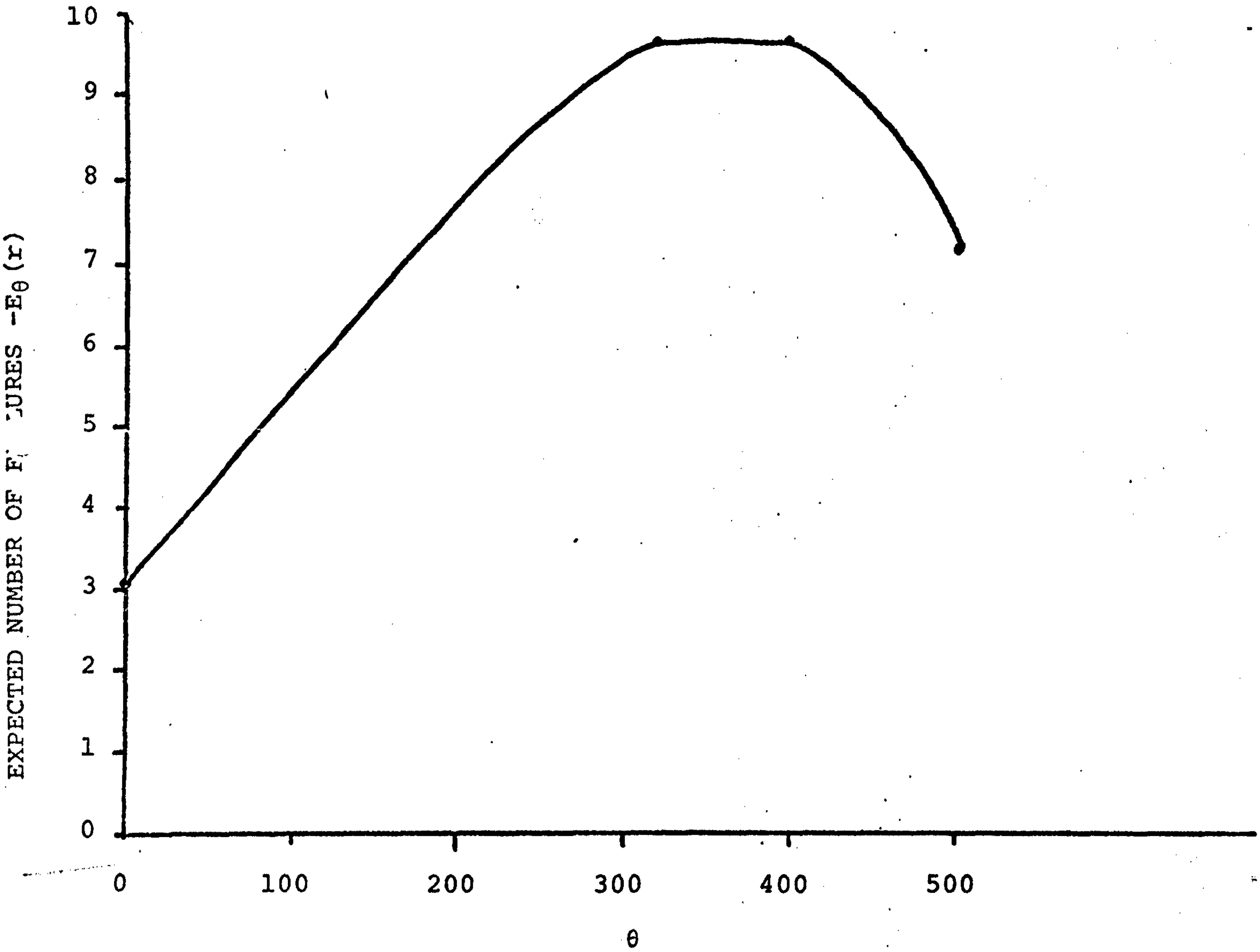


FIGURE 6 - PLOT OF $E_\theta(r)$ FOR EXAMPLE 1

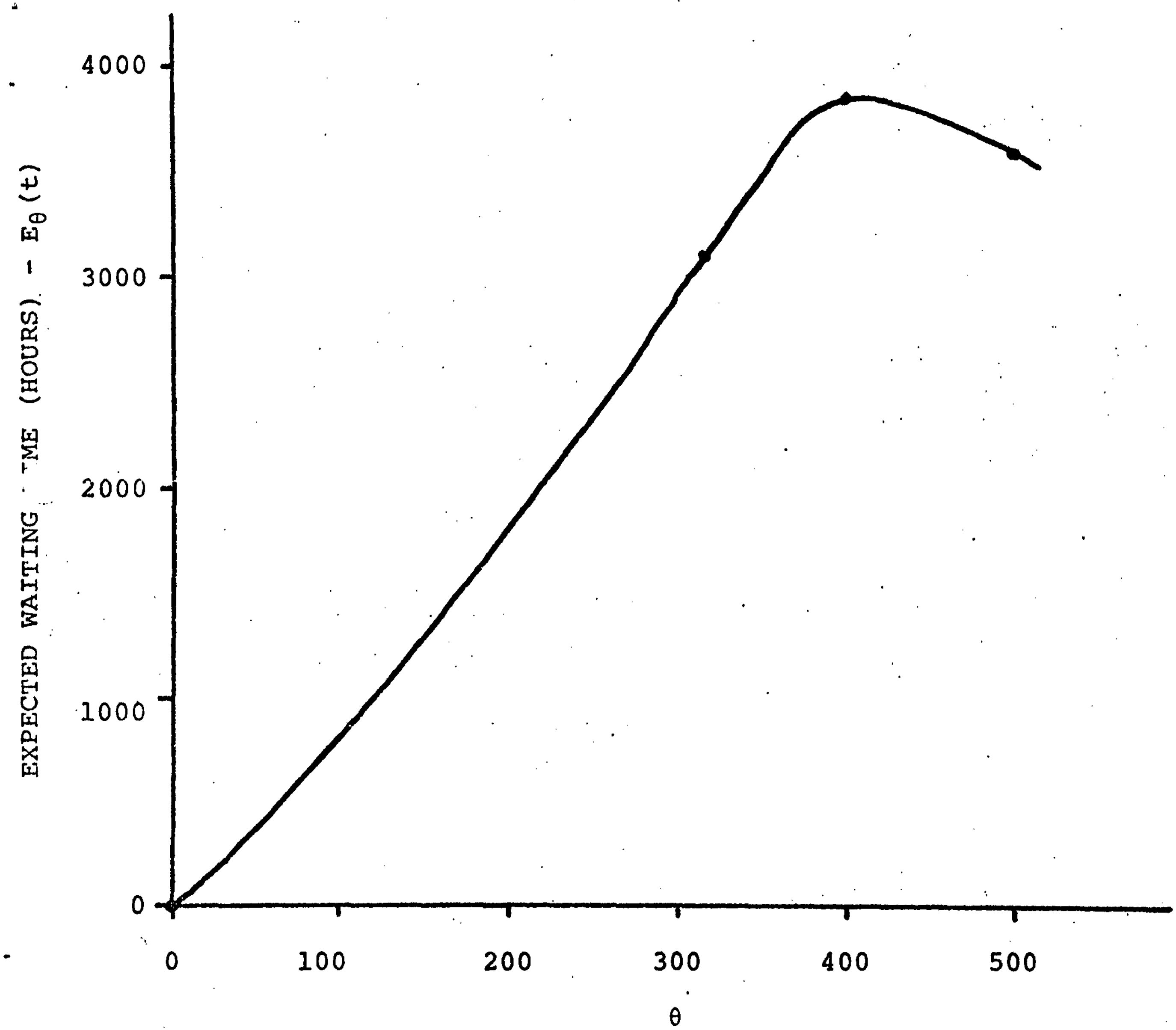


FIGURE 7 - PLOT OF $E_\theta(t)$ FOR EXAMPLE 1

EXAMPLE 2: Consider an engine which is to be tested for a maximum of 2000.0 hours. In order to accept the engine it must demonstrate a minimum mean time between failure of 151.0 hours. A producer's risk of 10 percent is desired with only a 1 percent consumer's risk. A sequential testing plan is required assuming the exponential distribution.

SOLUTION:

The following data is given:

$$T_O = 2000.0 \text{ hours} \qquad \alpha = 0.10$$

$$\theta_1 = 151.0 \text{ hours} \qquad \beta = 0.01$$

$$n = 1$$

The program, as written, will compute θ_O and r_O, however, they will be manually calculated in order to demonstrate the method. This can be done using (17):

$$\frac{2nT_O}{\theta_1} = \frac{2(1)(2000.0)}{151.0} = 26.5$$

$r_O = r$ is the largest integer such that:

$$26.5 \geq \chi^2_{\beta,2r} = \chi^2_{0.01,2r}$$

When $2r = 12$, $\chi^2_{0.01,\,12} = 26.217$, therefore,

$$r_0 = 6$$

Having determined r_0 it is possible to obtain θ_O from (15):

$$\theta_O = \frac{2nT_0}{\chi^2_{1-\alpha,2r}} = \frac{2(1)(2000.0)}{\chi^2_{0.90,12}} = \frac{4000}{6.304}$$

$$= 634.1 \text{ hours}$$

The sequential decision criteria can be found exactly as before. They are given below:

If $-454.4 + 294.4r < t < 891.8 + 284.4r$, continue the test,

If $t \geq 891.8 + 284.4r$, stop the test and accept H_0,

If $t \leq -454.4 + 284.4r$, stop the test and reject H_0.

Graphically, these decision criteria are shown in Figure 8. It is seen that the minimum time that the test could run is 891.8 hours for an accept decision or the time at which the sixth failure occurs for a reject decision. The complete computer dialogue and output for this example is shown in Figure 9. In addition, the program can automatically vary α and β between four values, namely, 0.01, 0.05, 0.10, 0.20 so that 16 possible

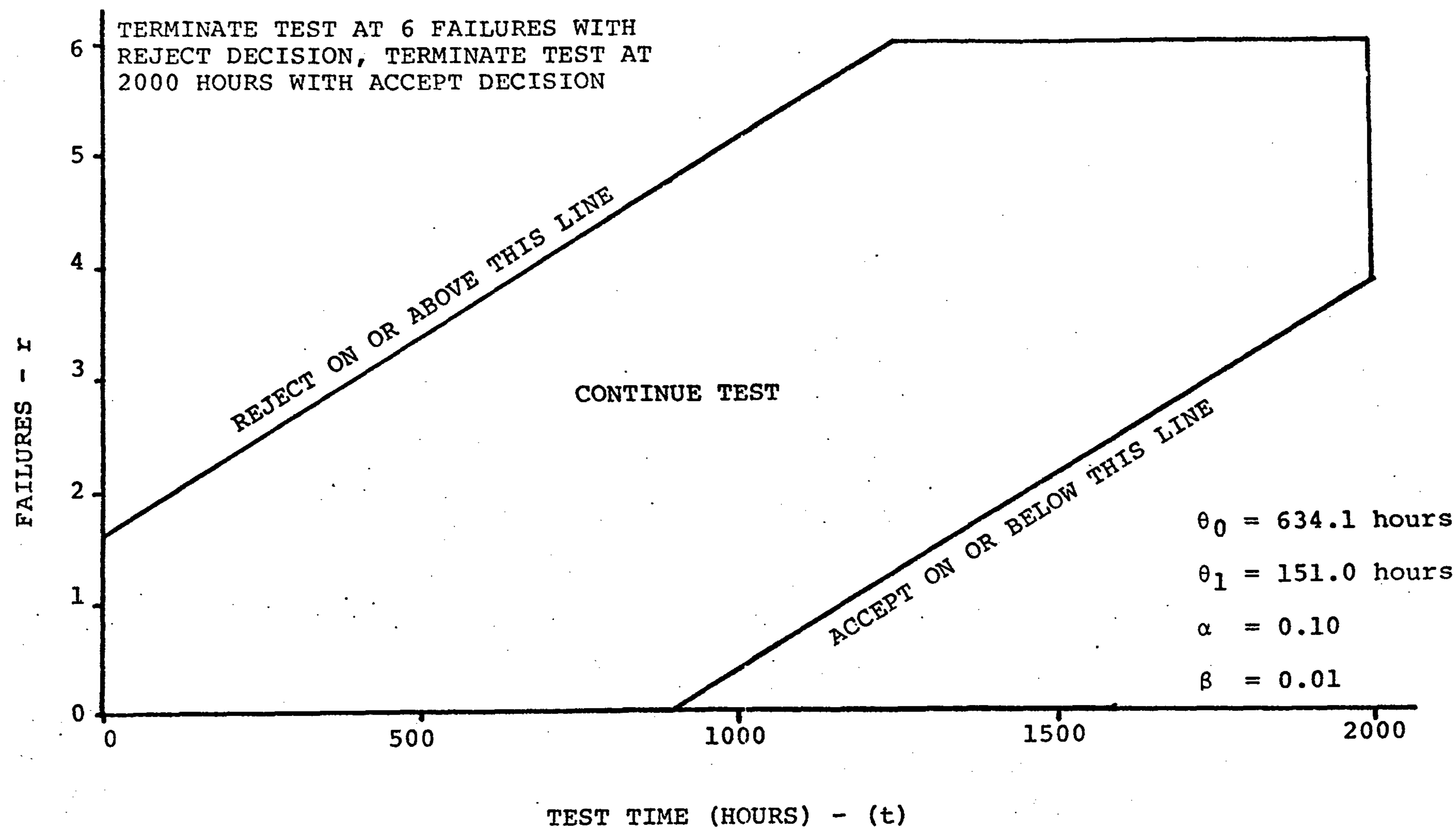


FIGURE 8 - SEQUENTIAL TEST CHART FOR EXAMPLE 2

```
RUN

SEQTST     15:25  ACTS  Ø6/27/72  TUE.


  ENTER N,THETA1,TØ
INPUT:ØØ146
 ? 1,151.,2ØØØ.


  ENTER Ø IF FULL DISPLAY IS DESIRED
  ENTER 1 IF NOT DESIRED
INPUT:ØØ154
 ? 1


  ENTER ALPHA,BETA DESIRED
INPUT:ØØ163
 ? Ø.1Ø,Ø.Ø1


                    SEQUENTIAL TESTING PLANS

                    EXPONENTIAL DISTRIBUTION

                      (WITH REPLACEMENT)

                    ************************

                        ALPHA   =     Ø.1Ø
                        BETA    =     Ø.Ø1
                        N       =        1
                        TØ      =   2ØØØ.Ø
                        THETA1  =    151.Ø

                        ****************

                        R       =        6
                        THETAØ  =    634.1

                        ****************

                        HØ      =    891.8
                        H1      =    454.4
                        S       =    284.4
```

FIGURE 9

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	891.8
1	0.0	1176.2
2	114.4	1460.6
3	398.8	1745.0
4	683.2	2000.0
5	967.6	2000.0
6	1252.0	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	1.6	0.0
151.0	0.010	3.3	499.1
284.4	0.338	5.0	1424.9
634.1	0.900	2.2	1373.0
INFINITY	1.000	0.0	891.8

****

STOP

RUNNING TIME: 4.4 CPUS ELAPSED TIME: 5.0 CPUS

FIGURE 9 (Continued)

sequential test plans can be generated. The output for these are given in Figures 10 through 25.

V. Conclusions

This report has dealt with the subject of sequential analysis as applied to life testing. Sequential testing has as its main advantage the capability of reducing the amount of test time required in order to make a decision regarding the acceptability or unacceptability of an item or its components. This can mean reduced costs and the possibility for a larger number of tests. From a statistical standpoint sequential testing is sound and in practice could be administered simply. The remainder of this report is devoted to Appendices and a bibliography which can be consulted if further investigation into the subject is necessary. It is this author's opinion that references [17] and [20], together, provide the most detailed and complete treatment of sequential testing and was used considerably in the preparation of this report.

SEQUENTIAL TESTING PLANS

EXPONENTIAL DISTRIBUTION

(WITH REPLACEMENT)

****

ALPHA	=	0.01
BETA	=	0.01
N	=	1
T0	=	2000.0
THETA1	=	151.0

****

R	=	6
THETA0	=	1134.3

****

H0	=	800.4
H1	=	800.4
S	=	351.2

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	800.4
1	0.0	1151.7
2	0.0	1502.9
3	253.3	1854.2
4	604.6	2000.0
5	955.8	2000.0
6	1307.1	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.3	0.0
151.0	0.010	3.9	591.5
351.2	0.500	5.2	1824.0
1134.3	0.990	1.0	1136.3
INFINITY	1.000	0.0	800.4

****

FIGURE 10

```
ALPHA   =    0.01
BETA    =    0.05
N       =       1
T0      =  2000.0
THETA1  =   151.0

****************

R       =       8
THETA0  =   692.9

****************

H0      =   576.5
H1      =   879.2
S       =   294.2
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	576.5
1	0.0	870.6
2	0.0	1164.8
3	3.3	1459.0
4	297.5	1753.2
5	591.6	2000.0
6	885.8	2000.0
7	1180.0	2000.0
8	1474.2	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	3.0	0.0
151.0	0.050	5.6	850.5
294.2	0.604	5.9	1722.9
692.9	0.990	1.4	976.4
INFINITY	1.000	0.0	576.5

****

FIGURE 11

ALPHA	=	0.01
BETA	=	0.10
N	=	1
T0	=	2000.0
THETA1	=	151.0

****

R	=	9
THETA0	=	573.3

****

H0	=	470.0
H1	=	922.4
S	=	273.5

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	470.0
1	0.0	743.4
2	0.0	1016.9
3	0.0	1290.4
4	171.5	1563.9
5	445.0	1837.4
6	718.5	2000.0
7	992.0	2000.0
8	1265.5	2000.0
9	1539.0	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	3.4	0.0
151.0	0.100	6.4	965.5
273.5	0.662	5.8	1585.1
573.3	0.990	1.5	872.0
INFINITY	1.000	0.0	470.0

****

FIGURE 12

ALPHA = 0.01
BETA = 0.20
N = 1
T0 = 2000.0
THETA1 = 151.0

****

R = 10
THETA0 = 486.4

****

H0 = 350.2
H1 = 959.6
S = 256.1

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	350.2
1	0.0	606.4
2	0.0	862.5
3	0.0	1118.7
4	65.0	1374.8
5	321.1	1631.0
6	577.3	1887.1
7	833.4	2000.0
8	1089.6	2000.0
9	1345.7	2000.0
10	1601.9	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	3.7	0.0
151.0	0.200	6.6	1001.9
256.1	0.733	5.1	1312.1
486.4	0.990	1.5	712.3
INFINITY	1.000	0.0	350.2

****

FIGURE 13

ALPHA	=	0.05
BETA	=	0.01
N	=	1
T0	=	2000.0
THETA1	=	151.0

****

R	=	6
THETA0	=	766.6

****

H0	=	856.3
H1	=	561.4
S	=	305.5

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	856.3
1	0.0	1161.8
2	49.6	1467.3
3	355.1	1772.8
4	660.6	2000.0
5	966.1	2000.0
6	1271.6	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	1.8	0.0
151.0	0.010	3.5	534.8
305.5	0.396	5.2	1573.6
766.6	0.950	1.7	1305.8
INFINITY	1.000	0.0	856.3

****

FIGURE 14

```
ALPHA   =    0.05
BETA    =    0.05
N       =       1
T0      =  2000.0
THETA1  =   151.0

****************

R       =       8
THETA0  =   502.8

****************

H0      =   635.4
H1      =   635.4
S       =   259.6
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	635.4
1	0.0	895.0
2	0.0	1154.7
3	143.4	1414.3
4	403.0	1673.9
5	662.6	1933.5
6	922.2	2000.0
7	1181.8	2000.0
8	1441.4	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.4	0.0
151.0	0.050	5.3	795.1
259.6	0.500	6.0	1555.4
502.8	0.950	2.4	1182.4
INFINITY	1.000	0.0	635.4

****

Figure 15

```
ALPHA  =    0.05
BETA   =    0.10
N      =       1
T0     =  2000.0
THETA1 =   151.0

****************

R      =       9
THETA0 =   426.2

****************

H0     =   526.5
H1     =   675.9
S      =   242.7
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	526.5
1	0.0	769.1
2	0.0	1011.8
3	52.1	1254.4
4	294.7	1497.1
5	537.4	1739.7
6	780.0	1982.4
7	1022.7	2000.0
8	1265.4	2000.0
9	1508.0	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.8	0.0
151.0	0.100	6.1	915.4
242.7	0.562	6.0	1466.4
426.2	0.950	2.5	1082.8
INFINITY	1.000	0.0	526.5

****

FIGURE 16

```
ALPHA   =    0.05
BETA    =    0.20
N       =       1
T0      =  2000.0
THETA1  =   151.0

****************

R       =      10
THETA0  =   368.8

****************

H0      =   398.4
H1      =   708.9
S       =   228.3
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	398.4
1	0.0	626.7
2	0.0	855.0
3	0.0	1083.4
4	204.4	1311.7
5	432.7	1540.0
6	661.1	1768.3
7	889.4	1996.7
8	1117.7	2000.0
9	1346.0	2000.0
10	1574.4	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	3.1	0.0
151.0	0.200	6.3	951.9
228.3	0.640	5.4	1236.9
368.8	0.950	2.4	900.5
INFINITY	1.000	0.0	398.4

****

FIGURE 17

ALPHA = 0.10
BETA = 0.01
N = 1
T0 = 2000.0
THETA1 = 151.0

****

R = 6
THETA0 = 634.1

****

H0 = 891.8
H1 = 454.4
S = 284.4

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	891.8
1	0.0	1176.2
2	114.4	1460.6
3	398.8	1745.0
4	683.2	2000.0
5	967.6	2000.0
6	1252.0	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	1.6	0.0
151.0	0.010	3.3	499.1
284.4	0.338	5.0	1424.9
634.1	0.900	2.2	1373.0
INFINITY	1.000	0.0	891.8

****

FIGURE 18

ALPHA	=	Ø.1Ø
BETA	=	Ø.Ø5
N	=	1
TØ	=	2ØØØ.Ø
THETA1	=	151.Ø

****

R	=	8
THETAØ	=	429.3

****

HØ	=	673.2
H1	=	524.4
S	=	243.4

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
Ø	Ø.Ø	673.2
1	Ø.Ø	916.6
2	Ø.Ø	116Ø.Ø
3	2Ø5.8	14Ø3.4
4	449.2	1646.8
5	692.6	189Ø.2
6	936.Ø	2ØØØ.Ø
7	1179.4	2ØØØ.Ø
8	1422.7	2ØØØ.Ø

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
Ø.Ø	Ø.ØØØ	2.2	Ø.Ø
151.Ø	Ø.Ø5Ø	5.Ø	759.2
243.4	Ø.438	6.Ø	145Ø.4
429.3	Ø.9ØØ	3.Ø	1277.9
INFINITY	1.ØØØ	Ø.Ø	673.2

****

FIGURE 19

```
ALPHA   =    0.10
BETA    =    0.10
N       =       1
T0      =  2000.0
THETA1  =   151.0

****************

R       =       9
THETA0  =   368.0

****************

H0      =   562.7
H1      =   562.7
S       =   228.1
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	562.7
1	0.0	790.8
2	0.0	1018.9
3	121.7	1247.0
4	349.8	1475.1
5	577.9	1703.2
6	806.0	1931.3
7	1034.1	2000.0
8	1262.2	2000.0
9	1490.3	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.5	0.0
151.0	0.100	5.8	881.4
228.1	0.500	6.1	1387.8
368.0	0.900	3.2	1184.1
INFINITY	1.000	0.0	562.7

****

FIGURE 20

ALPHA = 0.10
BETA = 0.20
N = 1
T0 = 2000.0
THETA1 = 151.0

****

R = 10
THETA0 = 321.4

****

H0 = 428.4
H1 = 592.3
S = 215.1

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	428.4
1	0.0	643.6
2	0.0	858.7
3	53.1	1073.8
4	268.2	1289.0
5	483.4	1504.1
6	698.5	1719.2
7	913.6	1934.4
8	1128.8	2000.0
9	1343.9	2000.0
10	1559.0	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.8	0.0
151.0	0.200	6.1	913.9
215.1	0.580	5.5	1179.6
321.4	0.900	3.1	987.3
INFINITY	1.000	0.0	428.4

****

FIGURE 21

ALPHA = 0.20
BETA = 0.01
N = 1
T0 = 2000.0
THETA1 = 151.0

****

R = 6
THETA0 = 511.5

****

H0 = 938.8
H1 = 342.7
S = 261.4

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	938.8
1	0.0	1200.2
2	180.1	1461.6
3	441.5	1723.0
4	702.9	1984.4
5	964.3	2000.0
6	1225.7	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	1.3	0.0
151.0	0.010	3.0	451.1
261.4	0.267	4.7	1230.7
511.5	0.800	2.7	1395.9
INFINITY	1.000	0.0	938.8

****

FIGURE 22

ALPHA = 0.20
BETA = 0.05
N = 1
T0 = 2000.0
THETA1 = 151.0

****

R = 8
THETA0 = 358.3

****

H0 = 723.6
H1 = 406.7
S = 225.5

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	723.6
1	0.0	949.1
2	44.4	1174.7
3	269.9	1400.2
4	495.4	1625.7
5	720.9	1851.2
6	946.5	2000.0
7	1172.0	2000.0
8	1397.5	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	1.8	0.0
151.0	0.050	4.7	709.5
225.5	0.360	5.8	1304.8
358.3	0.800	3.7	1342.6
INFINITY	1.000	0.0	723.6

****

FIGURE 23

```
ALPHA   =    Ø.2Ø
BETA    =    Ø.1Ø
N       =       1
TØ      =  2ØØØ.Ø
THETA1  =   151.Ø

***************

R       =       9
THETAØ  =   31Ø.8

***************

HØ      =   61Ø.6
H1      =   441.7
S       =   212.Ø
```

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
Ø	Ø.Ø	61Ø.6
1	Ø.Ø	822.6
2	Ø.Ø	1Ø34.6
3	194.4	1246.7
4	4Ø6.4	1458.7
5	618.4	167Ø.7
6	83Ø.4	1882.7
7	1Ø42.4	2ØØØ.Ø
8	1254.5	2ØØØ.Ø
9	1466.5	2ØØØ.Ø

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
Ø.Ø	Ø.ØØØ	2.1	Ø.Ø
151.Ø	Ø.1ØØ	5.5	832.6
212.Ø	Ø.42Ø	6.Ø	1272.Ø
31Ø.8	Ø.8ØØ	4.Ø	1258.6
INFINITY	1.ØØØ	Ø.Ø	61Ø.6

****

FIGURE 24

ALPHA	=	0.20
BETA	=	0.20
N	=	1
T0	=	2000.0
THETA1	=	151.0

****

R	=	10
THETA0	=	274.2

****

H0	=	466.0
H1	=	466.0
S	=	200.5

PLOTTING POINTS

FAIL. NO.	REJECT	ACCEPT
0	0.0	466.0
1	0.0	666.4
2	0.0	866.9
3	135.5	1067.4
4	336.0	1267.9
5	536.5	1468.4
6	737.0	1668.9
7	937.4	1869.3
8	1137.9	2000.0
9	1338.4	2000.0
10	1538.9	2000.0

****

THETA	L(THETA)	E(THETA,R)	E(THETA,T)
0.0	0.000	2.3	0.0
151.0	0.200	5.6	853.1
200.5	0.500	5.4	1082.9
274.2	0.800	3.8	1040.2
INFINITY	1.000	0.0	466.0

****

FIGURE 25

APPENDIX I

GLOSSARY OF SYMBOLS

Appendix I

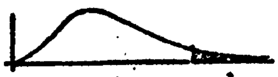
Glossary of Symbols

α	Producer's risk (Type I error)
A	Ratio of risks, equal to $(1-\beta)/\alpha$
β	Consumer's risk (Type II error)
B	Ratio of risks, equal to $\beta/(1-\alpha)$
$\chi^2_{\alpha,\nu}$	Chi-square variable with "ν" degrees of freedom with "α" area above the variable
$E_\theta(r)$	Expected number of failures if θ is the true MTBF
$E_\theta(t)$	Expected waiting time to reach a decision if θ is the true MTBF
$\exp[x]$	2.718... raised to the "x" power
h	Exponent used in calculating $L(\theta)$
h_0	Intercept of accept line in a sequential test
h_1	Intercept of reject line in a sequential test
H_i	Hypothesis concerning alternative i
k	Ratio of θ_0 and θ_1, equal to θ_0/θ_1
$L(\theta)$	Probability of accepting H_0 when θ is the true MTBF
MTBF	Mean Time Between Failure
n	Number of items on test
r	Number of failures at time t
r_0	Number of failures at which sequential test is truncated
T_0	Time at which sequential test is truncated
θ	True MTBF
θ_0	Upper limit of MTBF (desired MTBF)
θ_1	Lower Limit of MTBF (undesired MTBF)
s	Slope of accept and reject lines in a sequential test
V(t)	Accumulated test time (in replacement case)

APPENDIX II

CHI-SQUARE DISTRIBUTION PROBABILITY VALUES

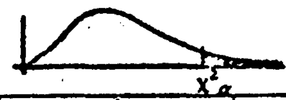
-- χ^2 DISTRIBUTION PROBABILITY VALUES



Values of $\chi^2_{\alpha,\nu}$

ν	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.80}$	$\chi^2_{.75}$	$\chi^2_{.70}$	ν
1	.0000393	.000157	.000982	.00393	.0158	.0642	.102	.148	1
2	.0100	.0201	.0506	.103	.211	.446	.575	.713	2
3	.0717	.115	.216	.352	.584	1.005	1.213	1.424	3
4	.207	.297	.484	.711	1.064	1.649	1.923	2.195	4
5	.412	.554	.831	1.145	1.610	2.343	2.675	3.000	5
6	.676	.872	1.237	1.635	2.204	3.070	3.455	3.828	6
7	.989	1.239	1.690	2.167	2.833	3.822	4.255	4.671	7
8	1.344	1.646	2.180	2.733	3.490	4.594	5.071	5.527	8
9	1.735	2.088	2.700	3.325	4.168	5.380	5.899	6.393	9
10	2.156	2.558	3.247	3.940	4.865	6.179	6.737	7.267	10
11	2.603	3.053	3.816	4.575	5.578	6.989	7.584	8.148	11
12	3.074	3.571	4.404	5.226	6.304	7.807	8.438	9.034	12
13	3.565	4.107	5.009	5.892	7.042	8.634	9.299	9.926	13
14	4.075	4.660	5.629	6.571	7.790	9.467	10.165	10.821	14
15	4.601	5.229	6.262	7.261	8.547	10.307	11.036	11.721	15
16	5.142	5.812	6.908	7.962	9.312	11.152	11.192	12.624	16
17	5.697	6.408	7.564	8.672	10.085	12.002	12.792	13.531	17
18	6.265	7.015	8.231	9.390	10.865	12.857	13.675	14.440	18
19	6.844	7.633	8.907	10.117	11.651	13.716	14.562	15.352	19
20	7.434	8.260	9.591	10.851	12.443	14.578	15.452	16.266	20
21	8.034	8.897	10.283	11.591	13.240	15.445	16.344	17.182	21
22	8.643	9.542	10.982	12.338	14.041	16.314	17.240	18.101	22
23	9.260	10.196	11.688	13.091	14.848	17.187	18.137	19.021	23
24	9.886	10.856	12.401	13.848	15.659	18.062	19.037	19.943	24
25	10.520	11.524	13.120	14.611	16.473	18.940	19.939	20.867	25
26	11.160	12.198	13.844	15.379	17.292	19.820	20.843	21.792	26
27	11.808	12.879	14.573	16.151	18.114	20.703	21.749	22.719	27
28	12.461	13.565	15.308	16.928	18.939	21.588	22.657	23.647	28
29	13.121	14.256	16.047	17.708	19.768	22.475	23.567	24.577	29
30	13.787	14.953	16.791	18.493	20.599	23.364	24.478	25.508	30
35	17.156	18.484	20.558	22.462	24.812	27.820	29.058	30.181	35
40	20.674	22.142	24.423	26.507	29.067	32.326	33.664	34.874	40
45	24.281	25.880	28.356	30.610	33.367	36.863	38.294	39.586	45
50	27.962	29.687	32.348	34.762	37.706	41.426	42.944	44.314	50
55	31.708	33.552	36.390	38.956	42.078	46.011	47.612	49.055	55
60	35.510	37.467	40.474	43.186	46.478	50.614	52.295	53.808	60
65	39.360	41.427	44.595	47.448	50.902	55.233	56.991	58.572	65
70	43.253	45.426	48.750	51.737	55.349	59.868	61.698	63.344	70
75	47.186	49.460	52.935	56.052	59.815	64.515	66.416	68.125	75
80	51.153	53.526	57.146	60.390	64.299	69.174	71.144	72.913	80
85	55.151	57.621	61.382	64.748	68.799	73.843	75.880	77.707	85
90	59.179	61.741	65.640	69.124	73.313	78.522	80.623	82.508	90
95	63.[illegible]	65.886	69.919	73.518	77.841	83.210	85.374	87.314	95
100	67.312	70.053	74.216	77.928	82.381	87.906	90.131	92.125	100
105	71.414	74.241	78.530	82.352	86.933	92.610	94.894	96.941	105
110	75.536	78.448	82.861	86.790	91.495	97.321	99.663	101.761	110
115	79.679	82.672	87.207	91.240	96.067	102.038	104.437	106.565	115
120	83.839	86.913	91.567	95.703	100.648	106.762	109.216	111.413	120

-- χ^2 DISTRIBUTION PROBABILITY VALUES
(continued)



Values of $\chi^2_{\alpha,\nu}$

ν	$\chi^2_{.50}$	$\chi^2_{.30}$	$\chi^2_{.25}$	$\chi^2_{.20}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	ν
1	.455	1.074	1.323	1.642	2.706	3.841	5.024	6.635	7.879	1
2	1.386	2.408	2.773	3.219	4.605	5.991	7.378	9.210	10.597	2
3	2.366	3.665	4.108	4.642	6.251	7.815	9.348	11.345	12.838	3
4	3.357	4.878	5.385	5.989	7.779	9.488	11.143	13.277	14.860	4
5	4.351	6.064	6.626	7.289	9.236	11.070	12.832	15.086	16.750	5
6	5.348	7.231	7.841	8.558	10.645	12.592	14.449	16.812	18.548	6
7	6.346	8.383	9.037	9.803	12.017	14.067	16.013	18.475	20.278	7
8	7.344	9.524	10.219	11.030	13.362	15.507	17.535	20.090	21.955	8
9	8.343	10.656	11.389	12.242	14.684	16.919	19.023	21.666	23.589	9
10	9.342	11.781	12.549	13.442	15.987	18.307	20.483	23.209	25.188	10
11	10.341	12.899	13.701	14.631	17.275	19.675	21.920	24.725	26.757	11
12	11.340	14.011	14.845	15.812	18.549	21.026	23.337	26.217	28.300	12
13	12.340	15.119	15.984	16.985	19.812	22.362	24.736	27.688	29.819	13
14	13.339	16.222	17.117	18.151	21.064	23.685	26.119	29.141	31.319	14
15	14.339	17.322	18.245	19.311	22.307	24.996	27.488	30.578	32.801	15
16	15.338	18.418	19.369	20.465	23.542	26.296	28.845	32.000	34.267	16
17	16.338	19.511	20.489	21.615	24.769	27.587	30.191	33.409	35.718	17
18	17.338	20.601	21.605	22.760	25.989	28.869	31.526	34.805	37.156	18
19	18.338	21.689	22.718	23.900	27.204	30.144	32.852	36.191	38.582	19
20	19.337	22.775	23.828	25.038	28.412	31.410	34.170	37.566	39.997	20
21	20.337	23.858	24.935	26.171	29.615	32.671	35.479	38.932	41.401	21
22	21.337	24.939	26.039	27.301	30.813	33.924	36.781	40.289	42.796	22
23	22.337	26.018	27.141	28.429	32.007	35.172	38.076	41.638	44.181	23
24	23.337	27.096	28.241	29.553	33.196	36.415	39.364	42.980	45.558	24
25	24.337	28.172	29.339	30.675	34.382	37.652	40.646	44.314	46.928	25
26	25.336	29.246	30.434	31.795	35.563	38.885	41.923	45.642	48.290	26
27	26.336	30.319	31.528	32.912	36.741	40.113	43.194	46.963	49.645	27
28	27.336	31.391	32.620	34.027	37.916	41.337	44.461	48.278	50.993	28
29	28.336	32.461	33.711	35.139	39.087	42.557	45.722	49.588	52.336	29
30	29.336	33.530	34.800	36.250	40.256	43.773	46.979	50.892	53.672	30
35	34.338	38.860	40.221	41.802	46.034	49.798	53.207	57.359	60.304	35
40	39.337	44.166	45.615	47.295	51.780	55.755	59.345	63.706	66.792	40
45	44.337	49.453	50.984	52.757	57.480	61.653	65.414	69.971	73.190	45
50	49.336	54.725	56.333	58.194	63.141	67.502	71.424	76.167	79.512	50
55	54.336	59.983	61.665	63.610	68.770	73.309	77.384	82.305	85.769	55
60	59.336	65.229	66.982	69.006	74.370	79.080	83.301	88.391	91.970	60
65	64.336	70.466	72.286	74.387	79.946	84.819	89.181	94.433	93.122	65
70	69.335	75.693	77.578	79.752	85.500	90.530	95.027	100.436	104.230	70
75	74.335	80.912	82.860	85.105	91.034	96.216	100.843	106.403	110.300	75
80	79.335	86.124	88.132	90.446	96.550	101.879	106.632	112.338	116.334	80
85	84.335	91.329	93.396	95.777	102.050	107.521	112.397	118.244	122.337	85
90	89.335	96.529	98.653	101.097	107.536	113.145	118.139	124.125	128.310	90
95	94.335	101.723	103.902	106.409	113.008	118.751	123.861	129.980	134.257	95
100	99.335	106.911	109.145	111.713	118.468	124.342	129.565	135.814	140.179	100
105	104.335	112.095	114.381	117.009	123.917	129.918	135.250	141.627	146.078	105
110	109.335	117.275	119.612	112.299	129.355	135.480	140.920	147.421	151.956	110
115	114.335	122.451	124.838	127.581	134.782	141.030	146.574	153.197	157.814	115
120	119.335	127.623	130.059	132.858	140.201	146.568	152.215	158.956	163.654	120

APPENDIX III

LISTING OF SEQUENTIAL TESTING PROGRAM

```
      DIMENSION A(4),B(4),L(4),O(4),ER(4),EI(4)
      INTEGER R,R2
      REAL L,L5
      DATA A / Ø.Ø1,Ø.Ø5,Ø.1Ø,Ø.2Ø /
      DATA B / Ø.Ø1,Ø.Ø5,Ø.1Ø,Ø.2Ø /
      PRINT 7
    7 FORMAT("Ø ENTER N,THETA1,TØ")
      READ,N,O1,TØ
      L(1)=Ø.Ø
      O(1)=Ø.Ø
      PRINT 14
   14 FORMAT("Ø ENTER Ø IF FULL DISPLAY IS DESIRED",/,"  ENTER",
       " 1 IF NOT DESIRED")
      READ,ICHK
      IF(ICHK.NE.Ø) GO TO 16
      II1=1
      II2=4
      JJ1=1
      JJ2=4
      GO TO 17
   16 PRINT 18
   18 FORMAT("Ø ENTER ALPHA,BETA DESIRED")
      READ,AL,BE
      DO 23 I=1,4
      IF(AL.EQ.A(I)) II1=I
      IF(BE.EQ.B(I)) JJ1=I
   23 CONTINUE
      II2=II1
      JJ2=JJ1
   17 PRINT 1
    1 FORMAT(///,24X,"SEQUENTIAL TESTING PLANS",//,24X,
       "EXPONENTIAL DISTRIBUTION",//,27X,"(WITH REPLACEMENT)"
       ,//,24X,24("*"))
      DO 1Ø I=II1,II2
      DO 1Ø J=JJ1,JJ2
      A1=1.Ø-A(I)
      B1=1.Ø-B(J)
      AA=(1.Ø-B(J))/A(I)
      BB=B(J)/(1.Ø-A(I))
      PRINT 2,A(I),B(J),N,TØ,O1
    2 FORMAT(/,28X,"ALPHA  =",F8.2,/,28X,"BETA   =",F8.2,/,28X,
       "N      =",I8,/,28X,"TØ     =",F8.1,/,28X,"THETA1 =",
       F8.1,//,28X,16("*"))
      R1=2.Ø*FLOAT(N)*TØ/O1
      DO 11 K=2,2ØØ,2
      X2B2R=CHISQ(B(J),K)
      IF(X2B2R.LE.R1) GO TO 12
      GO TO 13
   12 IDX=K
      X21=X2B2R
   11 CONTINUE
   13 R=IDX/2
      OØ=2.Ø*FLOAT(N)*TØ/CHISQ(A1,IDX)
      PRINT 3,R,OØ
```

```
    3 FORMAT(/,28X,"R          =",I8,/,28X,"THETAØ =",F8.1,//,
     28X,16("*"))
      DENOM=1.Ø/O1-1.Ø/OØ
      HØ=-ALOG(BB)/DENOM
      H1=ALOG(AA)/DENOM
      S=ALOG(OØ/O1)/DENOM
      PRINT 4,HØ,H1,S
    4 FORMAT(/,28X,"HØ        =",F8.1,/,28X,"H1        =",F8.1,/,
     28X,"S         =",F8.1)
      PRINT 5
    5 FORMAT(/,28X,"PLOTTING  POINTS",//,14X,"FAIL. NO.",1ØX,
     "REJECT",1ØX,"ACCEPT",/)
      R2=R+1
      DO 15 K=1,R2
      K1=K-1
      VR=-H1+FLOAT(K1)*S
      VA=HØ+FLOAT(K1)*S
      IF(VR.LT.Ø.Ø) VR=Ø.Ø
      IF(VA.LT.Ø.Ø) VA=Ø.Ø
      IF(VR.GT.TØ) VR=TØ
      IF(VA.GT.TØ) VA=TØ
   15 PRINT 6,K1,VR,VA
    6 FORMAT(17X,I3,11X,F8.1,8X,F8.1)
      PRINT 8
    8 FORMAT(/,5X,62("*"),//,8X,"THETA",8X,"L(THETA)",8X,
     "E(THETA,R)",8X,"E(THETA,T)",/)
      O(2)=O1
      O(3)=S
      O(4)=OØ
      L(2)=B(J)
      L(3)=ALOG(AA)/(ALOG(AA)-ALOG(BB))
      L(4)=A1
      DO 19 K=1,4
      IF(K.EQ.3) GO TO 21
      ER(K)=(H1-L(K)*(HØ+H1))/(S-O(K))
      GO TO 19
   21 ER(K)=HØ*H1/(S*S)
   19 ET(K)=O(K)*ER(K)/FLOAT(N)
      PRINT 9,(O(K),L(K),ER(K),ET(K),K=1,4)
    9 FORMAT(6X,F8.1,9X,F5.3,2(1ØX,F8.1))
      L5=1.Ø
      ET1=HØ/FLOAT(N)
      PRINT 22,L5,L(1),ET1
   22 FORMAT(6X,"INFINITY",9X,F5.3,2(1ØX,F8.1),//,5X,62("*"),//)
   1Ø CONTINUE
      STOP
      END
      FUNCTION CHISQ(PROB,IV)
      P=1.Ø-PROB
      IF(P) 1,4,2
    1 PRINT 3
    3 FORMAT(//,"P IS NOT IN THE INTERVAL (Ø,1), ",
     "INCLUSIVE",///)
```

```
      GO TO 12
    2 IF(P-1.0) 7,6,1
    4 X=-0.999999E+74
    5 D=0.0
      GO TO 12
    6 X=0.999999E+74
      GO TO 5
    7 D=P
      IF(D-0.5) 9,9,8
    8 D=1.0-D
    9 T2=ALOG(1.0/(D*D))
      T=SQRT(T2)
      X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788
     *T+0.189269*T2+0.001308*T*T2)
      IF(P-0.5) 10,10,11
   10 X=-X
   11 X1=2.0/(9.0*FLOAT(IV))
      CHISQ=FLOAT(IV)*(1.0-X1+X*SQRT(X1))**3
   12 RETURN
      END
```

APPENDIX IV

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APPENDIX IV

BIBLIOGRAPHY

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER 12325	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Sequential Testing; Basics and Benefits		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John C. Schmuhl, L.G. Jones		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS USATARADCOM, Sys and Cost Anal Ofc, Warren, Michigan 48090		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS USATARADCOM, Product Assurance Directorate, Warren, Michigan 48090		12. REPORT DATE March 1978
		13. NUMBER OF PAGES 50
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability, Sequential Testing, Sequential Analysis, Life Testing, Computer Programs

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the sequential test method. An overview of sequential analysis and the mathematics of sequential testing is given. Examples are also given along with computer software for implementing a sequential test plan. This testing method can if implemented on applicable testing programs, reduce test time and costs of testing.