A RAPID CORRELATION ECHO RANGING SYSTEM. (U)

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UNCLASSIFIED  NEL-TM-141
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Foreword

This memorandum covers only a small phase of the problem of instrumentation for signal enhancement research and has been prepared primarily for internal distribution to aid others at NRL who may be interested in related problems. Only a limited distribution outside of the Laboratory is contemplated. Work to August 1955 is covered.
Introduction

In echo ranging systems using correlation techniques it is necessary to introduce an artificial delay in the comparison signal to match the travel time of the transmitted signal out and the echo back. For wide band systems the artificial delay time must usually be varied slowly to maintain an adequate averaging time when passing through the correlation function, and unless the required delay is known in advance to a fair degree of approximation, long periods of search (in range) may be required to locate the correlation function. To shorten the time spent in search some sort of multiple-processing or speed-up system is required. Thus it is possible to use a number of correlators in parallel time-wise, each operating simultaneously but on comparison signals with different time delays. Or by speeding up the signal it is possible to search more rapidly with a single correlator. It is the purpose of the present paper to describe an echo ranging system utilizing an Automatic Recycling Multiple Sampler (ARMS) as the device for speeding-up the signal for more rapid search in range.

Description of the Rapid Correlation Echo Ranger

In the simplified block diagram of the Rapid Correlation Echo Ranger shown in Figure I, a Pseudorandom Noise Generator...
2. R. G. Stephenson, Pseudorandom Noise Generators, NRL Internal Technical Memo. No. 109, 23 Nov 1953. (Confidential)


(PRNG #2) is used to produce the signals. PRNG #2 is driven by the recycling pulses from another PRNG (#1). PRNG #1 serves a dual purpose; it acts as a pulse frequency divider in the ARMS unit, and also provides a comparison signal for correlation with the speed-up signal output of the ARMS unit.

The block labeled Transmitter includes such filtering, modulation, and amplifying functions as may be required, including the transducer for placing the sound in the transmitting medium. Similarly, the block labeled Receiver includes the pick-up unit and any filtering, amplifying, and demodulation functions required after transmission and prior to speed-up. The block labeled Correlator is also quite general in function, containing any necessary additional filtering, limiting, amplifying, heterodyning, multiplying and rectifying functions including, if required, a bank of output filters for doppler determination. The blocks comprising the ARMS unit are enclosed by the rectangle indicated by dashed lines. By providing a separate, crystal-controlled pulse generator for driving PRNG #2, the system becomes one for one way transmission and reception instead of echo ranging.
Figure 1. Simplified Block Diagram of a Rapid Correlation Echo Ranging System
Operation of the Rapid Correlation Echo Ranger

The master pulse generator, which may itself be driven by a crystal controlled oscillator, provides \( \phi \) pulses per second for driving the \( R \)-stage binary shift register and PRNG #1. An \( n \)-stage PRNG recycles after \( N = 2^n - 1 \) pulses and can be connected to emit a pulse each time it recycles. PRNG #1 will thus provide \( \phi / (2^n - 1) \) pulses per second to drive the sampler, the selector, and PRNG #2. Since PRNG #2 is also an \( n \)-stage unit it will go through only one cycle while PRNG #1 is going through \( 2^n - 1 \) cycles. It follows that the output of PRNG #1 is simply a sped-up version of that of PRNG #2, the speed-up factor being \( 2^n - 1 \). Taking \( R = N - 1 = 2^n - 2 \), the output of the \( R \)-stage binary shift register in the ARMS unit is also a sped-up version of the received signal with a speed-up factor of \( 2^n - 1 \). Neglecting noise and doppler on the echo it is thus seen that the output of the binary shift register is nearly the same as that of PRNG #1.

The essential difference is that the PRNG #1 recycles after \( N = 2^n - 1 \) counts while a sample in the shift register is recycled after \( R = 2^n - 2 \) counts from the master pulse generator. The output of the binary shift register is thus stepped along one stage with respect to the output of the PRNG #1 each time the latter recycles. After the first sample from PRNG #2 is introduced into the shift register it requires \( N - 2 \) \( = 2^n - 3 \) recycles of PRNG #1 to fill the shift register with samples from PRNG #2 and bring the output of the shift register in essential phase with the output of PRNG #1.
During the next recycling period the outputs are in step and peak correlation occurs. A slight theoretical improvement in performance of the system is achieved by having \( N \) stages in the shift register and using the output from the \( N \)th stage to recycle through the selector. The output from the \( N \)th stage will then be in step with the output of RNG #1 while the latter is going through its \( N \)th cycle. In this case, neglecting doppler, the peak correlation would extend over \( N \) instead of the previous \( N-1 \) samples. For large values of \( N \) the improvement is negligible.

Taking \( f_c \) as the center frequency of the signal prior to speed-up and \( \Delta f \) as the nominal frequency spread, the nominal number of cycles in the autocorrelation function may be taken as \( f_c/\Delta f \). For a rectangular spectrum the normalized autocorrelation function is \( \frac{\sin(\Delta f \tau)}{\pi \Delta f \tau} \) \( \cos(2\pi f_c \tau) \), from which it is seen that one cycle of the autocorrelation function corresponds to a differential delay of one cycle of the center frequency in the signal. Since the sampling rate is \( \varphi/N \), the number of samples per center frequency cycle is \( \varphi/f_c \). It follows that the number of stages occupied by one center frequency cycle in the shift register will be \( \varphi/f_c \). Since there is a differential delay of one stage each time RNG #1 recycles, it will recycle \( \varphi/f_c \) times to produce one cycle of the autocorrelation function. To produce \( f_c/\Delta f \) cycles it would need to recycle \( \varphi/\Delta f \) times. The recycling period is \( N/\varphi \), hence the time required to run through one cycle of the correlation function is \( 1/f_c \) and the nominal duration of the
correlation function is $1/\Delta f$.

A band pass filter with bandwidth $\Delta f$ and center frequency $f_c$ on the output of the correlator would therefore be adequate to achieve the maximum averaging time of $T_m = 1/\Delta f$. This averaging time could also be achieved by rectifying the output of the correlator and using a low pass filter with cutoff at $1/\Delta f$. Since all frequencies less than half the sampling rate are sped-up by a factor of $N$, the frequency spread of the signal after speed-up is $\Delta f_1 = N\Delta f$. The maximum processing gain is then given by

$$T_m \Delta f_1 = N = 2^p - 1$$

(1)

**Effect of Doppler in the Rapid Correlation Echo Rangefinder**

In the above development it was assumed that the doppler was zero on the signal being processed. In general this will not be true and the effect of doppler must be taken into account. For a velocity $v$ of the target relative to the echo-ranging vessel, the fractional doppler is $D = 2v/c_0$, $c_0$ being the velocity of sound in the transmitting medium. If $v_m$ is the maximum relative target speed to be expected, the maximum expected doppler will be $D_m = 2v_m/c_0$.

The effect of doppler is to produce a time-wise compression or expansion of the signal. A signal which with zero doppler occupied $N$ stages in the binary shift register will occupy $(1-D)N$ stages when subjected to a fractional doppler $D$. Suppose that at the beginning of a recycling period PRNG #1 is in step with the sped-up PRNG #2 signal on the output of the
binary shift register. At the end of the recycling period it would be DN stages out of step. That is, a differential delay of $DN^2 f_c / \phi$ center frequency cycles occurs during each recycling period. It follows that the correlator output is swept progressively through this number of cycles of the correlation function during each recycling period of PRNG #1. Since the duration of the recycling period is $N/\phi$ the output frequency during the period is $DN f_c$. An adjustment occurs at the end of the period in center frequency cycles so that the net differential delay over the complete recycling period is approximately $N f_c / \phi$, the same as for zero doppler. If $DN$ is small compared to unity the effect of the doppler will be to introduce a negligible perturbation on the correlator output.

For $DN$ large compared to unity the doppler must be taken into account. Suppose a bank of overlapping bandpass filters are placed on the output of the correlator to match the doppler. The bandwidth of individual filters should be at least $2 \cdot \phi/N$ to accept side bands introduced by the recycling frequency, and the center frequencies would range from $f_c$ to $D_m N f_c$ in steps of $2 \phi / N$. If $f_c$ is taken as $\phi/3N$ (one third the sampling rate), the center frequencies of the filters would vary in steps of $6 f_c$ and the number of filters required would be about $1 + D_m N/6$. Since the correlator output is swept through $DN^2 f_c / \phi$ cycles of the correlation function during each recycling period of PRNG #1, there should be at least this many cycles in the nominal duration of the correlation function to avoid serious attenuation in the output.

This gives

$$f_c / \phi \geq DN^2 f_c / \phi$$  \hspace{1cm} (2)

or

$$1/(\Delta f)_m = D_m N^2 / \phi$$  \hspace{1cm} (3)
whence
\[(\Delta f)_{m} = \frac{\varphi}{D_{m}N^{2}}\]  
\[= 3f_{c}/D_{m}N, \quad D_{m}N \leq 3\]
\[= f_{c}, \quad D_{m}N < 3\]

when \(f_{c}\) is taken as \(\varphi/3N\).

The maximum doppler to be expected may thus place serious limitations on the bandwidth of the signal which may be employed. If this limitation is adhered to, however, the nominal duration of the output signal will remain \(T_{m} = 1/\Delta f\) and the maximum processing gain will be \(2^{P-1}\) as before. This maximum processing gain can be achieved by rectifying and low-pass filtering the output of the band pass filters.

**Discussion**

The maximum value which can be used for the master pulse frequency \(\varphi\) depends to some extent on the type of binary shift register which is employed. Transistor-type shift registers can be operated at rates as high as 3 or 4 megacycles for example, but there seems to be some question as to their reliability. Miniature magnetic shift registers can apparently be reliably operated at 100 kc. For the moment, therefore we will take the maximum driving frequency for the shift register as

\[\varphi_{m} = 100 \text{ kc}\]  

(5)

Taking the velocity of underwater sound as approximately 5000 ft/sec and assuming a maximum relative target range rate of 30 knots gives \(D_{m} \approx 0.02\)  

(6)

as the maximum fractional doppler to be expected in echo ranging studies. For ship-to-shore transmission studies slower speeds may be expected. In a particular application being considered.
6 knots may be taken as the maximum ship-to-shore speed to be expected. This would give
\[ D_m = 0.002 \] (7)

Taking these two values of \( D_m \) as extremes, it seems worth while to tabulate the corresponding values of \((\Delta f)_m\) for various values of \( R \) in equation (4). Since the sampling rate is \( \Phi/N \), the maximum center frequency may be represented by the equation
\[ (f_c)_m = \frac{\Phi}{3N} \] (8)

Equation (8) is based on the assumption
\[ \Delta f \leq (f_c)_m \] (9)
since the maximum frequency in the signal should not exceed one half of the sampling rate. In the following table \((\Delta f)_m\) has been given the value of \((f_c)_m\) when equation (4) would have given a larger value.

Table I

<table>
<thead>
<tr>
<th>( n )</th>
<th>( N )</th>
<th>( E )</th>
<th>( (f_c)_m )</th>
<th>( (\Delta f)_m ) ( D_m = 0.02 )</th>
<th>( (\Delta f)_m ) ( D_m = 0.002 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>63</td>
<td>62</td>
<td>529 cps</td>
<td>529 cps*</td>
<td>529 cps*</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>126</td>
<td>262</td>
<td>262*</td>
<td>262*</td>
</tr>
<tr>
<td>8</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>510</td>
<td>65</td>
<td>19</td>
<td>65*</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>1022</td>
<td>33</td>
<td>4.8</td>
<td>33*</td>
</tr>
<tr>
<td>11</td>
<td>2047</td>
<td>2046</td>
<td>16</td>
<td>1.2</td>
<td>12</td>
</tr>
</tbody>
</table>

* It should be pointed out that while these values represent broadband operation in the sense that \((\Delta f)_m = (f_c)_m\), they may not be broadband in the ordinary meaning of the term. \( f_c \) may be the center frequency after demodulation and hence may be very much less than the actual carrier frequency. For \( \Phi = 3 \), all values in the last three columns of Table I would be increased by a factor of 30.